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Thrust Mount and Launch Stand Structural Design Criteria for Large Missiles and Space Vehicles

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THRUST MOUNT AND LAUNCH STAND STRUCTURAL DESIGN CRITERIA FOR LARGE MISSILES AND SPACE VEHICLES

By

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Martin Company, Denver, Colorado

SUMMARY

Often it is necessary to begin design studies of a missile launch stand and thrust mount before the final missile configuration has been determined. The thrust mount structure should provide: (1) dynamic stability for the missile-stand combination during captive and hold-down firings, (2) sufficient stiffness to prevent excessive displacements during engine start and shutdown, (3) adequate stiffness for hot and cold engine gimbaling checkouts, and (4) sufficient stiffness to provide safe dynamic loads that may result from engine firing malfunctions. This paper provides generalized basic design criteria for the stiffness or spring constant requirements that should exist at the vehicle attachment points for booster thrusts ranging between 300,000 and 30,000,000 lb. Failure to meet these criteria could result in damage to a costly space vehicle or missile. These criteria also include overall vertical, lateral, rotational, and torsional stiffness requirements that are presented graphically as thrust versus stiffness. From a plot of launch stand stiffness versus vehicle-stand frequency, it is shown that the fundamental frequency increases with increasing stiffness up to a definite value of stand stiffness. Thereafter, almost no frequency increase is gained by strengthening the stand beyond this stiffness value.

The method of determining launch stand stiffness requirements is based on a dimensionless matrix formulation involving free flexural vibrations of missiles mounted on a flexible stand. The effects of variable boundary conditions at the base support, variable bending stiffness along the vehicle, and variable mass distribution along the vehicle are considered. A thrust-to-weight ratio of 1.3 is used since it was found that a ±15 percent variance of this ratio did not appreciably change the stiffness requirements of the launch stand. Bending stiffness at the base of the vehicle is established as a function of booster thrust. Also, the average length of the vehicle as a function of thrust is established based on existing missiles and space vehicles.
Any number of lumped masses along the vehicle can be used. For a parameter study of this nature, ten masses are chosen to ensure sufficient accuracy. The problem is matrically formulated in standard eigenvalue form for rapid solution on an electronic digital computer.

The criteria presented here are approximate and cannot apply to all launch stands exactly; however, the criteria requirements are reasonably accurate and should permit design of a particular launch stand to begin early in the program. For example, these criteria were used in establishing the Saturn V launch stand stiffness requirements at the request of NASA early in 1963. Similar criteria could now be established for a Post-Saturn launch stand or any other large missile or space vehicle launch facility.
NOTATION

\( a_i \)  Coefficient relating flexural stiffness at \( i^{th} \) station to flexural stiffness at the first station

\( b_i \)  Coefficient relating mass at \( i^{th} \) station to mass at the first station

\( E \)  Modulus of elasticity in lb/in.\(^2\)

\( f \)  Natural frequency in cps

\( h \)  Spacing between stations along vehicle length in inches

\( i \)  Subscript which refers to parameters at a particular station

\( I_i \)  Average area moment of inertia of vehicle cross section at \( i^{th} \) station in inch\(^4\)

\( k_i \)  Longitudinal spring constant between \( i^{th} \) and \( i^{th-1} \) stations along vehicle length in lb/in.

\( k_h \)  Lateral spring constant at base of vehicle in lb/in.

\( k_v \)  Vertical spring constant at base of vehicle in lb/in.

\( L \)  Total length of idealized vehicle in inches

\( m_i \)  Mass at \( i^{th} \) station in lb-sec\(^2\)/in.

\( q \)  Last or highest numbered station

\( r_i \)  Radius of idealized vehicle at \( i^{th} \) station in inches

\( R \)  Rotational spring constant at base of vehicle in in.-lb/rad

\( R_T \)  Torsional spring constant at base of vehicle in in.-lb/rad

\( T \)  Booster thrust of idealized vehicle in pounds

\( u_1 \)  Boundary condition parameter

\( u_2 \)  Boundary condition parameter

\( W \)  Weight of idealized vehicle in pounds
\( y_i \)  Lateral deflection at \( i^{th} \) station
\( \epsilon \)  Dimensionless boundary condition parameter
\( \lambda \)  Dimensionless natural frequency parameter
\( \omega \)  Circular frequency
1. TECHNICAL APPROACH

The method of determining launch stand stiffness requirements is based primarily on a dimensionless matrix formulation given in Reference (1). Total stiffness requirements will be developed for the lateral, rotational, vertical and torsional directions. Figure 1 shows the idealized vehicle attached to the launch stand springs. From Reference (1), the basic equation in matrix form for the solution of eigenvalues and eigenvectors is given as:

\[ [AB + \varepsilon S + \zeta N - \lambda (HM - \alpha C - \beta D)]Y = 0 \ldots \ldots \ [1] \]

where:

- \( k_h \) = Boundary spring parameter = \( \frac{k_nh^3}{EI} \)
- \( \lambda \) = Eigenvalue = \( \frac{m_1\omega^2h^3}{EI} \)
- \( \varepsilon \) = Axial load parameter,
- \( \zeta \) = Shear parameter,
- \( \alpha \) = Rotary inertial parameter.

Matrices will be defined after modifying Equation [1].

For preliminary design purposes, one can neglect the effects of axial load, shear distortion and rotary inertia in the calculation of launch stand spring constants. Therefore, Equation [1] can be reduced to:

\[ [AB + \varepsilon S - \lambda HM]Y = 0 \ldots \ldots \ldots \ldots \ldots \ldots \ [2] \]
FIGURE 1  IDEALIZED MODEL FOR LATERAL AND ROTATIONAL SPRING STUDIES

Let \( A'B' = AB + \epsilon S \).

Now Equation [2] becomes:

\[
[A'B' - \lambda HM]Y = 0
\]  \[3\]
\[ A^*B^* = \begin{bmatrix} a_2 & -a_2 & -a_2 & \cdots & \cdots & \cdots & \cdots & a_q(u_1-1/a) & a_q(u_2-2) \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \end{bmatrix} \]

\( u_1 \) and \( u_2 \) are boundary condition parameters defined in Reference (1) as:

\[
\begin{align*}
    u_1 &= \frac{(R/2) - (EIq/h)}{(R/2) + (EIq/h)} \\
    u_2 &= \frac{(2EIq/h)}{(R/2) + (EIq/h)} 
\end{align*}
\]

It can be shown that: \( u_1 + u_2 = 1 \).

All matrices of Equation [3] are in dimensionless form except for the eigenvector, \( Y \). This feature enables one to make an extensive parameter study of a broad range of missiles and space vehicles. Average mass and bending stiffness ratio distributions must be determined along a typical multistage space vehicle. Parameters to be varied are the stand springs. The practical upper and lower limits of these springs for any given thrust can then be determined by inspecting the variance of the frequency parameter with the spring stiffness parameters. This inspection reveals that, up to a point, increasing spring stiffness increases the frequency parameter; however, beyond a certain point or range, a continued increase in spring stiffness does not appreciably increase the frequency parameter, \( \lambda \).
II. DETERMINATION OF LATERAL AND ROTATIONAL STAND SPRINGS

Studies were made to determine the bending stiffness at the base of several U.S. missiles and space vehicles. Results are shown in Figure 2 which shows how EI at the vehicle base varies with thrust. The relationship between missile length and thrust is also needed, which is shown in Figure 3. Although some variation is certain to exist, these curves depict the average length and bending stiffness as a function of thrust of known missiles and space vehicles. The existing trend is then projected for thrusts up to 30,000,000 pounds.

The required values or stiffnesses of the lateral stand spring, \( k_n \), and the rotational stand spring, \( R \), may be determined as a function of thrust. For a solution of \( k_n \), the rotational spring is made infinite. Similarly, in solving for \( R \), the lateral spring is made infinite. A lumped mass system of ten masses is considered to ensure sufficient accuracy in the final results. The following mass and bending stiffness ratio distribution (Table 1) which agrees reasonably well with existing vehicles, is assumed for all booster thrusts:

<table>
<thead>
<tr>
<th>Station</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent of Total Mass</td>
<td>0.03</td>
<td>0.04</td>
<td>0.05</td>
<td>0.08</td>
<td>0.09</td>
<td>0.11</td>
<td>0.14</td>
<td>0.15</td>
<td>0.15</td>
<td>0.16</td>
</tr>
<tr>
<td>Percent of ( EI_{10} )</td>
<td>0.125</td>
<td>0.1667</td>
<td>0.250</td>
<td>0.4167</td>
<td>0.500</td>
<td>0.6667</td>
<td>0.8333</td>
<td>0.9167</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

In the solution for lateral stiffness, \( k_n \), a missile or space vehicle with any given thrust in the range under consideration may be used for establishing the stiffness criteria of the entire thrust range, since dimensionless parameters are used. With reference to Equation [3] and Table 1, it follows that:

\[
A = \begin{bmatrix}
1.33 \\
2.00 \\
3.33 \\
4.00 \\
5.33 \\
6.67 \\
7.33 \\
8.00 \\
8.00 \\
1.00 \\
\end{bmatrix}
\]

since \( a_2 = \frac{EI_2}{EI_1} \), \( a_3 = \frac{EI_3}{EI_1} \), etc.
FIGURE 2: VEHICLE FLEXURAL STIFFNESS VS THRUST

THRUST (LB x 10^6)

VEHICLE FLEXURAL STIFFNESS
Figure 3: Vehicle Length vs Thrust

LENGTH, L (FT)

THRUST (LB x 10^6)

0 5 10 15 20 25 30

0 100 200 300 400 500

Post-Saturn Configuration

Saturn V

SIB/SII RIFT

SIB RIFT

Titan II

75
Notice that actual values need not be known since matrix $A$ contains the ratios of average bending stiffnesses along the vehicle which of course apply to all sizes of vehicles under consideration.

In writing matrix $B^\prime$, the rotational spring, $R$, is made infinitely stiff. Thus, with reference to Equation [4], $u_1 = 1$ and $u_2 = 0$.

$$
B^\prime = \begin{bmatrix}
1 & 2 & 1 \\
1 & 2 & 1 \\
1 & 2 & 1 \\
1 & 2 & 1 \\
1 & 2 & 1 \\
1 & 2 & 1 \\
1 & 2 & 1 \\
1 & 2 & 1 \\
1 & 2 & 1 \\
\end{bmatrix}
$$

Matrix $B^\prime$ contains first order difference patterns as well as the only variable, $\varepsilon$. This variable will be given several values after defining the remaining matrices of Equation [3].

$$
H = \begin{bmatrix}
1 & & & & & & & & \\
2 & 1 & & & & & & & \\
3 & 2 & 1 & & & & & & \\
4 & 3 & 2 & 1 & & & & & \\
5 & 4 & 3 & 2 & 1 & & & & \\
6 & 5 & 4 & 3 & 2 & 1 & & & \\
7 & 6 & 5 & 4 & 3 & 2 & 1 & & \\
8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & \\
9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
\end{bmatrix}
$$

where $H$ is a summing matrix.
M is a matrix of mass ratios. From Equation [2] and Table 1, it is seen that:

\[
M = \begin{bmatrix}
1 & 1.33 & 1.67 & 2.67 & 3.00 & 3.67 & 4.67 & 5.00 & 5.33 \\
1.33 & 1.67 & 2.67 & 3.00 & 3.67 & 4.67 & 5.00 & 5.33 \\
1.67 & 2.67 & 3.00 & 3.67 & 4.67 & 5.00 & 5.33 \\
2.67 & 3.00 & 3.67 & 4.67 & 5.00 & 5.33 \\
3.00 & 3.67 & 4.67 & 5.00 & 5.33 \\
3.67 & 4.67 & 5.00 & 5.33 \\
4.67 & 5.00 & 5.33 \\
5.00 & 5.33 \\
5.33
\end{bmatrix}
\]

The variable, ε, is a boundary spring parameter defined in Equation [1]. Five different values are assigned to this variable to plot a dimensionless curve from which the upper and lower limits of lateral stiffness can be chosen. Experimental solutions indicate that for a practical range of the eigenvalue, \( \lambda_1 \), the highest value of \( \varepsilon \) should be 3.08, while the lowest value should be 0.0308 with factors of ten in between. Hence, five cases are matrically formulated from Equation [3] for solution on a digital computer. Ten eigenvalues and modes are found for each case with the primary interest directed toward only the first eigenvalue, \( \lambda_1 \). The square roots of these values are plotted against the boundary spring parameters, \( \varepsilon \), in Figure 4. The square root of the eigenvalue is plotted because the frequency varies with the square root of \( \lambda \). From this curve one may select an upper and lower limit of \( \varepsilon \) as shown in Figure 4. As a check, the fundamental frequency, \( f_1 \), is plotted against the stand lateral spring, \( k_h \), which is shown in Figure 5 for a particular example where \( T = 3,000,000 \) pounds. This curve is applicable only for a vehicle having a booster thrust of 3,000,000 pounds, whereas the curve of Figure 4 is dimensionless and applicable for any thrust. The values of 4 and 9 for \( \varepsilon \) in Figure 4 are used in determining the lateral stand-stiffness requirements. From the definition of \( \varepsilon \) under Equation [1], one can compute the value of \( k_h \) for various thrusts since the length (or \( h \)) and base bending stiffness, \( EI_q \), have been established as a function of thrust in Figures 2 and 3 respectively. The results of these computations are shown in columns 13 and 14 of Table 2 and also in Figure 6.
RANGE FOR DETERMINATION OF $k_h$ ($\epsilon = 4$ AND 9)

NOTE:
$R = \infty$.
$\lambda_1 =$ FUNDAMENTAL EIGENVALUE.

FIGURE 4 SELECTION OF LIMITS FOR $k_h$
Figure 5 Selection of Limits Verification

Range for verification from \( \epsilon \) vs \( \sqrt{\lambda} \) curve

\( k_h = 0.380 \times 10^6 \) and \( 0.854 \times 10^6 \)

Note:

\( R = \infty \)

\( T = 3 \times 10^6 \) LB.
<table>
<thead>
<tr>
<th>T</th>
<th>V</th>
<th>$n_0$</th>
<th>$E_0$</th>
<th>$E_1$</th>
<th>$n_1$</th>
<th>$h$</th>
<th>$E_0^3$</th>
<th>$E_1^3$</th>
<th>$E_0$/h</th>
<th>$E_1/h$</th>
<th>$n_0$</th>
<th>$n_1$</th>
<th>$\omega$</th>
<th>$f_0$</th>
<th>$f_1$</th>
<th>$E_0$ Upper Limit</th>
<th>$E_1$ Lower Limit</th>
<th>$n_0$ Upper Limit</th>
<th>$n_1$ Lower Limit</th>
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<td>17.95</td>
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<td>0.3093</td>
<td>9.73</td>
<td>38.2</td>
<td>15.90</td>
<td>2.88</td>
<td>1.288</td>
<td>1.513.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30.0</td>
<td>23.08</td>
<td>1794.</td>
<td>800.</td>
<td>109.0</td>
<td>640.</td>
<td>2662.9</td>
<td>262.</td>
<td>0.3822</td>
<td>12.50</td>
<td>46.6</td>
<td>20.5</td>
<td>3.44</td>
<td>1.532</td>
<td>2.11.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

TABLE 2 COMPREHENSIVE DERIVATION OF ALL SPRING CONSTANTS AND FREQUENCIES

<table>
<thead>
<tr>
<th>$\omega$</th>
<th>$f_0$</th>
<th>$f_1$</th>
<th>$E_0$ Upper Limit</th>
<th>$E_1$ Lower Limit</th>
<th>$n_0$ Upper Limit</th>
<th>$n_1$ Lower Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.25</td>
<td>0.0210</td>
<td>0.0281</td>
<td>0.105</td>
<td>0.966</td>
<td>0.279</td>
<td>0.126</td>
</tr>
<tr>
<td>5.52</td>
<td>0.0846</td>
<td>0.135</td>
<td>0.129</td>
<td>0.469</td>
<td>0.280</td>
<td>0.180</td>
</tr>
<tr>
<td>8.55</td>
<td>0.1460</td>
<td>0.613</td>
<td>0.270</td>
<td>0.526</td>
<td>0.264</td>
<td>0.264</td>
</tr>
<tr>
<td>10.16</td>
<td>0.256</td>
<td>0.963</td>
<td>0.426</td>
<td>0.679</td>
<td>0.280</td>
<td>0.264</td>
</tr>
<tr>
<td>12.12</td>
<td>0.330</td>
<td>1.263</td>
<td>1.62</td>
<td>0.902</td>
<td>0.400</td>
<td>1.418.</td>
</tr>
<tr>
<td>14.47</td>
<td>0.469</td>
<td>1.83</td>
<td>0.980</td>
<td>0.430</td>
<td>1.418.</td>
<td></td>
</tr>
<tr>
<td>19.98</td>
<td>0.679</td>
<td>2.43</td>
<td>1.025</td>
<td>0.453</td>
<td>1.418.</td>
<td></td>
</tr>
<tr>
<td>25.80</td>
<td>0.902</td>
<td>3.00</td>
<td>1.125</td>
<td>0.526</td>
<td>1.418.</td>
<td></td>
</tr>
<tr>
<td>38.20</td>
<td>0.980</td>
<td>3.00</td>
<td>1.125</td>
<td>0.526</td>
<td>1.418.</td>
<td></td>
</tr>
<tr>
<td>46.60</td>
<td>1.025</td>
<td>3.00</td>
<td>1.125</td>
<td>0.526</td>
<td>1.418.</td>
<td></td>
</tr>
</tbody>
</table>
FIGURE 6  STAND LATERAL SPRING CONSTANT VS THRUST
In the solution for rotational stiffness, $R$, the same idealized vehicle is used. For this solution, the lateral spring, $k_h$, is made infinitely stiff. This condition restricts the mass at $q$ (Fig. 1) from translation. Hence, this restriction results in a 9 degree of freedom system.

\[
A = \begin{bmatrix}
1.33 \\
2.00 \\
2.33 \\
4.00 \\
5.33 \\
6.67 \\
7.33 \\
8.00 \\
8.00
\end{bmatrix}
\]

Matrix $B'$ is somewhat different since the variable, $e$, is replaced by a new variable, $u_1$, defined in Equation [4].

\[
B' = \begin{bmatrix}
1 - 2 & 1 \\
1 - 2 & 1 \\
1 - 2 & 1 \\
1 - 2 & 1 \\
1 - 2 & 1 \\
1 - 2 & 1 \\
1 - 2 & 1 \\
1 - 2 & 1 \\
(1 + u_1)
\end{bmatrix}
\]

\[
H = \begin{bmatrix}
1 \\
2 \\
3 \\
4 \\
5 \\
6 \\
7 \\
8 \\
9
\end{bmatrix}
\]
\[
M = \begin{bmatrix}
1 \\
1.33 \\
1.67 \\
2.67 \\
3.0 \\
3.67 \\
4.67 \\
5.0 \\
5.0
\end{bmatrix}
\]

$u_1$ is a function of the rotational spring constant, $R$, the bending stiffness at the base of the vehicle, $EI_q$, and the spacing between mass stations, $h$. $EI_q$ and $h$ are fixed quantities for a given thrust. The rotational spring constant is assigned five different values so that a smooth curve may again be obtained for selecting the upper and lower bounds of this spring constant.
It is obvious from Equation [4] that $u_1$ may vary from -1.0 to +1.0. Five cases were matrically formulated for solution on a digital computer. As in the previous parameter study, the formulation and solution of five different cases is very rapid since only one element of the $B'$ matrix requires different values. Again, ten eigenvalues and modes are found for each of the five cases. A dimensionless plot of $u_1$ vs $\sqrt{\lambda_1}$ (where $\lambda_1$ is the fundamental eigenvalue) is shown in Figure 7. Due to the gradual change in slope of the right side of this curve, it is very difficult to select definite limits for the parameter, $u_1$. Hence, a plot of fundamental frequency vs the rotational spring, $R$, is given in Figure 8 for $T = 3,000,000$ pounds. A more definite slope change is seen here from which the limits of $R$ are selected and also established for the dimensionless plot. The values of $1.24 \times 10^{11}$ and $2.28 \times 10^{11}$ correspond with values for $u_1$ of -0.10 and 0.30, respectively. These latter parameter values may be used to determine the rotational spring stiffness requirements of launch stands for the entire range of vehicle thrusts considered. From Equation [4], for the upper and lower values of $u_1$, one may calculate $R$ for different thrust values since $EI_q$ and $L$ (and hence $h$) are given in Figures 2 and 3 as a function of thrust. These $R$ values vs thrust are computed in columns 11 and 12 of Table 2 and are plotted in Figure 9.

III. DETERMINATION OF VERTICAL SPRING CONSTANTS

In the solution for vertical stiffness, $k_v$, a slightly different approach is used in the development of the required vertical stiffness for launch stands. The idealized model is shown in Figure 10 which considers longitudinal motion only. The vertical spring in the launch stand is represented by $k_v$, while longitudinal vehicle springs exist between each mass. The matric formulation for this problem is based on a method given in Reference (2). Essentially, a matric formulation is developed for the free vibrations of a linearly coupled vibrating system of many springs and masses. For application to this problem, the equations of motion may be expressed as:

$$(C - \lambda D) X = 0$$

[5]
FIGURE 7 DIMENSIONLESS PLOT OF $\sqrt{\lambda}$ VS $u_1$ ($k_h = \infty$)
SELECTED RANGE ($R = 124 \times 10^9$ AND $282 \times 10^9$)

NOTE:
$k = \infty$
$T = 3 \times 10^6$ LB.

FIGURE 8 SELECTION OF LIMITS FOR $R$
FIGURE 9 STAND ROTATIONAL SPRING CONSTANT VS THRUST
FIGURE 10 IDEALIZED MODEL FOR VERTICAL SPRING DETERMINATION
where $C$ is the spring matrix, $D$ is the mass matrix, and $X$ is the eigenvector. Since a parameter study is desired, both the $C$ matrix and the $D$ matrix can be expressed in nondimensional form. If each element in matrix $C$ is divided by $k_2$ with the resulting matrix designated $C'$ and if each element in matrix $D$ is divided by $m_1$ with the resulting matrix designated $D'$, then Equation [5] may be expressed as:

$$(C' - \lambda D')X = 0 \quad \ldots \ldots \ldots \ldots \ldots \quad [6]$$

The nondimensional eigenvalue, $\lambda$, is now expressed as:

$$\lambda = \frac{m_1}{k_2} \omega^2 \quad \ldots \ldots \ldots \ldots \ldots \quad [7]$$

The spring constant, $k_2$, must be found as well as the relative values of the remaining springs. Longitudinal springs in a missile are a complex function of tank dome flexibility as well as axial deformation in the outer ring and stringers. As a general approximation, the spring constant is based on the deformation of a thin ring that has the same area as the skin stringer combination but is reduced by a factor to compensate for tank bottom deformation. For a cylinder of length, $h$, and having a relatively small thickness, the longitudinal spring constant may be expressed as:
\[ k_i = \frac{2EI_i}{h r_i^2} \quad i = 2 \ldots 10, \quad [8] \]

where, \( E I_i \) is the average stiffness of the section and \( r_i \) is the radius of the particular section. To account approximately for tank dome flexibilities, Equation [8] may be reduced by a factor of 2. Hence,

\[ k_i = \frac{EI_i}{h r_i^2} \quad i = 2 \ldots 10. \quad [9] \]

Based on actual base diameters of existing missiles and space vehicles, the radius at the base of vehicles is shown in Figure 11 as a function of the thrust range under consideration. Current design practice has shown that the ratios of vehicle longitudinal springs should be approximately like that shown in matrix \( C' \). The value of \( \lambda \) in Equation [6] can be determined with \( k_v/k_2 \) as the only variable. The matrices are:

\[
C' = \begin{pmatrix}
\left(\frac{k_v}{k_2} + 1\right) & -1 \\
-1 & 2 & -1 \\
-1 & +2 & -1 \\
-1 & +2 & -1 \\
-1 & 1.75 & -0.75 \\
-0.75 & 1.50 & -0.75 \\
-0.75 & 1.50 & 1.25 & -0.50 \\
-0.50 & 1.00 & -0.50 & -0.50 \\
-0.50 & 0.50 & -0.50 & 0.50
\end{pmatrix}
\]
FIGURE 11 VEHICLE BASE RADIUS VS THRUST
\[
D' = \begin{bmatrix}
1.0 \\
0.9375 \\
0.0375 \\
0.875 \\
0.6875 \\
0.5625 \\
0.50 \\
0.3125 \\
0.25 \\
0.1875 \\
\end{bmatrix}
\]

where \(D'\) is based on the same mass ratios used in Table 1.

Values of the ratios of \(k_v/k_2\) have been assumed from 0.12 through 100, and the corresponding values of \(\lambda\) have been determined from Equation [6] for a selected number of ratios of \(k_v/k_2\). The results of the solutions and the range for determining \(k_v\) are shown in Figure 12. Values of \(k_2\) can be determined from Equation [9] by reference to Figures 2, 3, and 11 for various thrust values. From Figure 12, the selected lower and upper limits of \(k_v/k_2\) are 2 and 5. Therefore, since \(k_2\) is known as a function of thrust, \(k_v\) may be established as a function of thrust for the lower and upper limits of 2 and 5. Calculations for \(k_v\) are shown in columns 22 and 23 of Table 2 and finally in Figure 13.

**IV. DETERMINATION OF TORSIONAL SPRING CONSTANTS**

It can be shown that the total torsional spring constant at the vehicle attachment points is:

\[
R_T = 2k_h r^2
\]

[10]

Using the above relationship and the results of Figures 11 and 6, one can develop Figure 14 which shows the torsional spring constant requirements vs thrust.
FIGURE 12 SELECTION OF LIMITS FOR $k_v$
FIGURE 13  STAND VERTICAL SPRING CONSTANT VS THRUST
FIGURE 14 STAND TORSIONAL SPRING CONSTANT VS THRUST
V. FUNDAMENTAL FREQUENCY CONSIDERATIONS

Although the primary objective of this paper is to establish the overall total stiffness requirements that should exist at the vehicle attachment points for a large range of booster thrusts, the prediction of fundamental structural frequencies may also be of interest to the vehicle designer. From the expression for \( \lambda \) under Equation [1], and using 0.0065 as an approximate value for \( \lambda_1 \), obtained from Figure 4, one can easily find the lateral fundamental frequency of the vehicle-stand combination for any thrust. The calculations are made in Columns 15 thru 18 in Table 2, and the results are shown in Figure 15. Similarly from Equation [7] and Figure 12, one can solve for the vertical fundamental frequency of the system shown in Figure 10 for any thrust. These calculations are made in columns 24 through 27 of Table 2 while the results are shown in Figure 16.

VI. CONCLUSIONS

Once the approximate size of a proposed launch vehicle has been established, the launch or test stand design engineer may establish the thrust-mount stiffness requirements at the vehicle attachment points simply by referring to Figures 6, 9, 13, and 14. Therefore, design of the launch stand can proceed long before the final flight vehicle mass and stiffness data are available. Experience on the Titan and Saturn programs has shown that the stiffness criteria requirements presented in this paper provide the flight vehicle with economically and technically sound boundary conditions. For example, these stiffness criteria were used early in 1963 for establishing the Saturn V launcher stiffness requirements prior to design of the launcher-umbilical tower as given in Reference (3). Detailed analysis has shown that the dynamic loads imposed on the flight vehicle from wind, engine start malfunctions, and engine gimbaling will not be unreasonably excessive when the launch stand stiffness requirements recommended herein are met.
FIGURE 15  FUNDAMENTAL LATERAL FREQUENCY VS THRUST
FIGURE 16  FUNDAMENTAL VERTICAL FREQUENCY VS THRUST
REFERENCES

