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Honors Directed Study Abstract - PS 303 Modern Physics Honors Projects

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Honors Directed Study - PS 303 Modern Physics

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For this directed study, I was asked to first explore special relativity by comparing fixed target particle accelerators with colliders using their center-of-mass energies. Then, I was asked to explore quantum mechanics by finding various solutions to the quantum harmonic oscillator and observing how the correspondence principle applied to the harmonic oscillator. For the comparison of particle accelerators, fixed targets had significantly less energy than colliders, though the fixed target colliders were the earliest, dating back to 1931 with the earliest cyclotron. Colliders had the largest values, as exhibited by the Large Hadron Collider with a center-of-mass energy of $\sqrt{s} = 8$ TeV. For the quantum harmonic oscillator, the lower quantum numbers exhibited a very non-intuitive probability distribution, while as the quantum number increased, the probability distribution became more even and equal, mimicking that of a classical system.

I. THEORY

A. Particle Accelerators (Relativity)

Particle accelerators operate by smashing a beam of particles of given momentum into a target and observing the resultant reaction, including release of energy and new particles. Generally, these accelerators are designed to be fixed target, where a beam collides with a target, or to be colliders, where a beam collides with another beam travelling in the opposing direction. For a fixed target, the momentum 4-vectors are [1]:

$$p^{\mu}_{beam} = (E_{beam}, p_{beam}c) \tag{1}$$

$$p^{\mu}_{target} = (m_{target}, 0) \tag{2}$$

So, if s is defined to be the square of the center-of-mass energy, s is equal to:

$$s = (p^{\mu}_{beam} + p^{\mu}_{target})^2 = (E_{beam} + m_{target})^2 - p^2_{beam} \tag{3}$$

Assuming that the relativistic equations for energy and momentum hold true and the mass of the beam is equal to the mass of the target, then the equation becomes:

$$s = 2mc^2 K_{beam} + 4m^2c^4 \tag{4}$$

And thus the center-of-mass energy $\sqrt{s}$ becomes:

$$\sqrt{s} = 2mc^2\sqrt{1 + \frac{K_{beam}}{2mc^2}} \tag{5}$$

However, for colliders, the beams both have momentum and energy. Assuming one beam has energy E_1 and the other E_2, it is safe to assume that [2]:

$$\sqrt{s} = E_1 + E_2 \tag{6}$$

B. Quantum Harmonic Oscillators (Quantum Mechanics)

Simple harmonic motion is another system that can be modeled both classically and quantumly. Classically, simple harmonic motion is modeled by sinusoidal waves whose probability densities should follow $\frac{1}{2}kx^2$. However, in quantum mechanics, the wave function ψ determines the characteristics of the wave. Using the Schrodinger equation [3]:

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$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}+\frac{1}{2}m\omega^2x^2\psi=E\psi \tag{7}$$

From this, the equation for ψ can be found to be [4]:

$$\psi_n(x)=(\frac{m\omega}{\pi\hbar})^{1/4}(\frac{1}{\sqrt{2^n n!}})H_{\beta n}(x)e^{-\frac{\beta^2x^2}{2}} \tag{8}$$

And here, $\beta=\sqrt{\frac{m\omega}{\hbar}}$.

Depending on the quantum number n, different characteristics can be achieved. As n grows larger, it is predicted that the system should become more and more like the classical model until it is almost exactly the same; this is known as the Correspondence Principle.

II. PROJECT 1 RESULTS

First, I identified the fixed target particle accelerators, the years of their operation, their beams, and when available their targets and discoveries.

The first series of fixed target accelerators were the Berkeley cyclotrons, which involved 5 cyclotrons in operation between 1931 and 1942. The first cyclotron that provided proof of the concept in 1931 used an H_2^+ beam, the second in 1932 a proton beam, and the others a deuteron beam. Besides being the first particle accelerators, these cyclotrons also investigated deuteron-nucleus interactions and discovered isotopes. Further, there was a 95 inch cyclotron at Harvard University, in experimental use from 1949 to 1961 and from 1961 to 2002 used in medical research, that used a proton beam and was actually 10 times as powerful as the earlier Berkeley cyclotrons.

Other accelerators include the Cambridge Electron Accelerator, in operation from 1962 to 1974 and using electron beams; the SLAC linear collider, in operation from 1966 to the present and using electron/positron beams; the Bates Linear Accelerator, in operation from 1967 to 2005 and using polarized electrons; the Fermilab synchotrons in operation from 1970 to the present and using protons/antiprotons; the High Current Proton Accelerator and Los Alamos, in operation from 1972 to the present and using protons and used in high energy neutron research; the PSI High Intensity Proton Accelerator, in operation from 1974 to the present and using a proton beam and graphite target that releases neutrons and used to produce mesons and neutrons [5]; the TRIUMF Cyclotron, also in operation from 1974 to the present and using proton beams; the Mainz Microtron, in operation from 1975 to the present and using polarized electrons; the Tevatron, in operation from 1983 to 2011 and using protons and made the discovery of the top quark (also a collider); the ISIS neutron source, in operation from 1984 to the present and using protons to produce neutrons; ELSA, in operation from 1987 to the present and using electrons for structure and material analysis; the antiproton decelerator at CERN, used from 2000 to the present and using proton/antiproton beams and an iridium target [6]; the Spallation Neutron Source, in operation from 2006 to the present and involving the use of proton beams with a liquid mercury target; the Japan Proton Accelerator Research Complex, in operation from 2007 to the present and using proton beams to produce hadrons; and lastly ALBA, which has been in operation from 2010 to the present and uses electron beams.

Then, the colliders were identified. Besides the Tevatron, which was both a fixed target and collider accelerator and made the discovery of the top quark in 1995, there was the Anello Di Accumulazione, in operation from 1961 to 1964 and used electron/positron beams to discover the first e^-e^- interactions; the Princeton-Stanford collider, in operation from 1962 to 1967 and using electron/positron beams; the VEPP series of colliders, which were in operation from 1964 to present day and involved a series of 5 colliders, all using electron/positron beams, and were responsible for the discovery of $\gamma\gamma$ production, ϕ production, and the decays of ρ, ω, and ϕ mesons; the Stanford Positron Electron Asymmetric Rings, which were in operation from 1972 to 2003 and are being upgraded, used electron/positron beams to discover the J/ψ meson, charmonium states, and the τ particle; the DESY colliders, which have been in operation roughly since 1960 to today, though most of the original colliders have been shut down, and used electron/positron beams to help discover quarks and B mesons and gluons; the Cornell Electron Storage Ring, which was in operation from 1979 to 2002 and used electron/positron beams to observe B mesons; the Stanford Linear Collider, which has been in operation from 1988 to today and uses electron/positron beams to help in discoveries such as quark coupling; the Beijing Electron-Positron Colliders, in operation from 1989 to 2004 and from 2008 to today; the KEKB, which was in operation from 1999 to 2009 and used electron/positron beams; the Double Annular ϕ Factory for Nice Experiments, in operation from 1999 to today and using electron/positron beams; the Large Electron-Positron Collider at CERN, in operation from 1989 to 2000; the Intersecting Storage Rings at CERN, in operation from 1971 to 1984 and using proton beams; the Super Proton Synchotron at CERN, in operation from 1981 to 1984 and using proton/antiproton beams to discover the Z and W bosons; the Relativistic Heavy Ion Collider, in use from 2000 to today and using either proton beams or ion beams; and lastly the Large Hadron Collider at CERN, in operation from 2008 to today and, using proton beams or lead ion beam, is most noted for the discovery of the Higgs boson [7].

Plotting the center-of-mass energy against the first year of operation, it can be seen how the energy of accelerators has increased over time. First, for the fixed targets, the plot of the data is shown in Figure 1. Due to the varied nature of the center-of-mass energies, the data were plotted logarithmically.

Further, the colliders were plotted separately, as their center-of-mass energies were typically higher. The spread is shown in Figure 2.

Finally, to compare all the center-of-mass energies, the two sets were plotted together in Figure 3 and 4.

In all, the colliders had higher center-of-mass energies than the fixed target accelerators. However, there were more and earlier fixed target accelerators due to the simple and relatively small and cheap design.

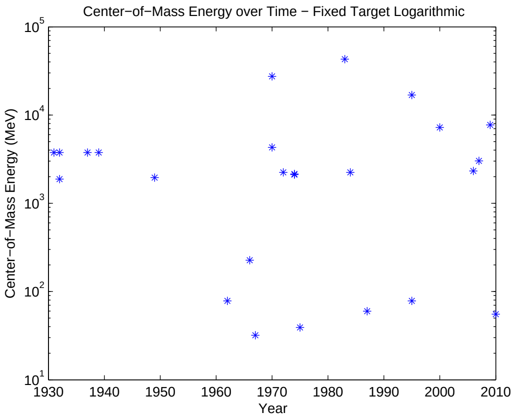

FIG. 1. Fixed target particle accelerator years and center-of-mass energies, shown logarithmically to show spread. As can be seen, due to the large discrepacy in energy values, there are many energy values that are a fraction of an MeV. However, it exhibits an upward trend for the most part, though many accelerators remain at low energies due to the nature of the experiments being ran. The highest energy value belonged to the Tevatron, which also ran as a collider, which is why its center-of-mass energy is higher than other fixed target accelerators.

III. PROJECT 2 RESULTS

After finding the wave function for a hydrogen-like atom, the probability density functions could be graphed. Plotting together for n=2, n=10, and n=50, the probability densities were obtained in Figure 5.

To obtain the probability densities separately, the probability $\psi(x) * \psi(x)$ was plotted against the distance x for each quantum number. As the quantum number increases, the Correspondence Principle predicts that the distribution will become parabolic, much like the classical probability.

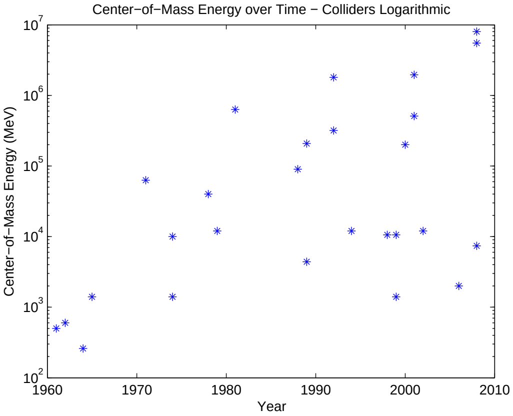

FIG. 2. Collider particle accelerator years and center-of-mass energies, shown logarithmically to show spread. As can be seen, due to the large discrepacy in energy values, there are many energy values that are a fraction of an MeV or several TeV. However, it exhibits an upward trend, though many accelerators remain at low energies due to the nature of the experiments being ran. The highest energy value is that of the Large Hadron Collider.

Since the Correspondence Principle implies that, for high value quantum numbers, the probability densities should mirror that of the classical models, it becomes apparent that the models here do just that. For the probability densities, classically it is a concave up parabola, whereas the quantum probability is concave down for small quantum numbers, and further the classical is a smooth distribution. With quantum number 2, it is neither smooth nor indicative of a parabolic curve, as the density is still highest nearer the center and there is no probability at the edges, where the classical system has the highest probability. At quantum number 10, while the graph is concave up, there are uneven peaks in the distribution. This however is decreased in quantum number 50, which gives an almost flat, even distribution with little gaps. At high n values, the particle has the highest probability at the edges, at $\pm x_0$. This is consistent with the classical model, since the

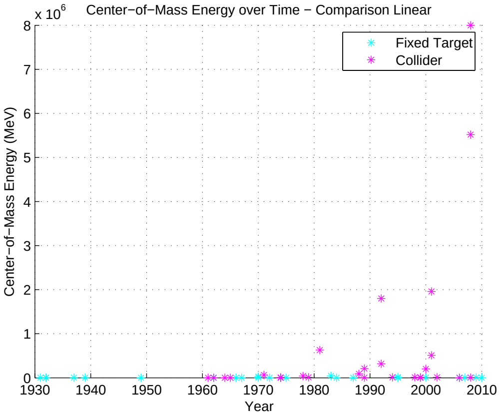

FIG. 3. Combination of all center-of-mass energies, for both fixed targets (cyan) and colliders (magenta), versus year of operation.

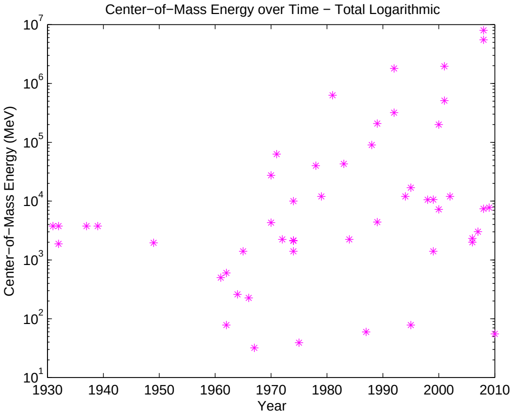

FIG. 4. Combination of all center-of-mass energies on a logarithmic scale to show spread, for both fixed targets and colliders, versus year of operation. Here, type of accelerator not distinguished.

particle is going fastest at x_0 and thus has the least chance of being found there, while it is slowest as it changes direction at $\pm x_0$, or the peaks and troughs, and thus has the greatest chance of being found at these points.

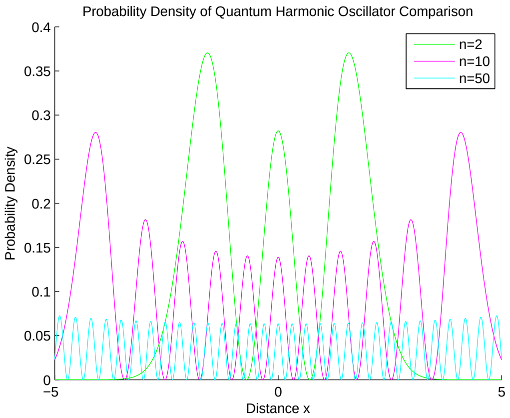

FIG. 5. Combination of n=2, n=10, and n=50 probability densities for a quantum oscillator. The green is n=2, magenta n=10, and cyan is n=50. As the quantum number increases, the probabilities become more equal and even, and at n=50 the probability is almost an even spread, thus verifying the Correspondence Principle.

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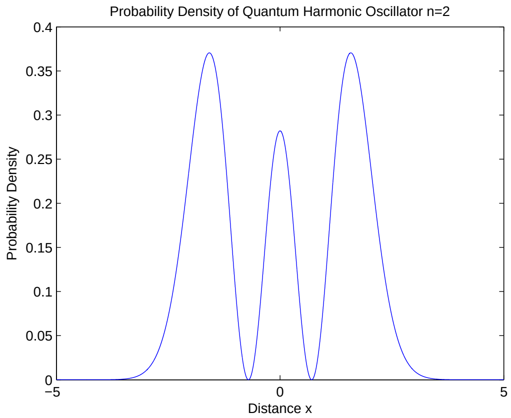

FIG. 6. Probability density of the n=2 state of a quantum harmonic oscillator, taken from the $\psi * \psi$ function and distance x. This is not consistent with a classical simple harmonic oscillator, as the probability is still higher towards the center.

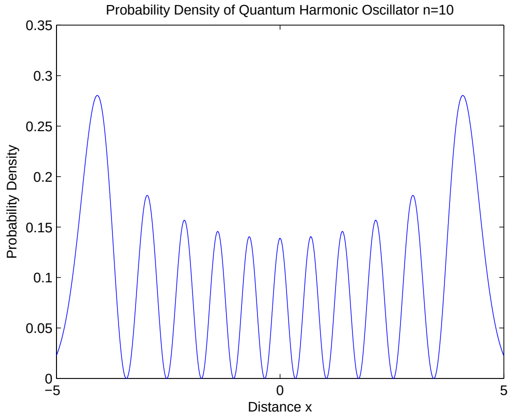

FIG. 7. Probability density of the n=10 state of a quantum harmonic oscillator, taken from the $\psi * \psi$ function and distance x. This is closer to a classical simple harmonic oscillator, as it is more parabolic but still has gaps.

[6] Baird, S. et al. (1996). “Design Study of the Antiproton Decelerator: AD”. CERN/PS 96-43.

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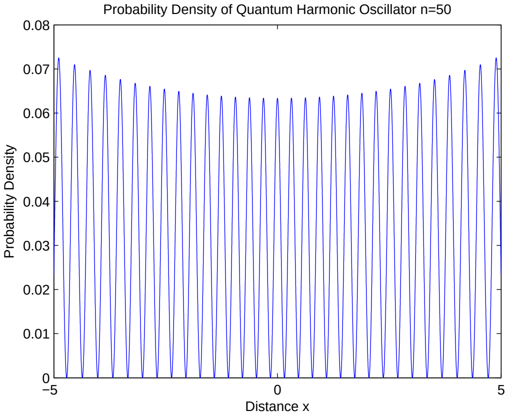

FIG. 8. Probability density of the n=50 state of a quantum harmonic oscillator, taken from the $\psi * \psi$ function and distance x. This is almost completely consistent with a classical simple harmonic oscillator, as it follows the parabola $\frac{1}{2}kx^2$.

cds.cern.ch/record/261062/files/p1_2.pdf.