McNair Scholars Research Journal

Volume 1

Article 7

2014

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Christian M. Lewis
Embry-Riddle Aeronautical University

Recommended Citation

Lewis, Christian M. (2014) "Trajectory Shaping Study for Various Ballistic Missile Scenarios," *McNair Scholars Research Journal*: Vol. 1 , Article 7.
Available at: https://commons.erau.edu/mcnair/vol1/iss1/7

TRAJECTORY SHAPING STUDY FOR VARIOUS BALLISTIC MISSILE SCENARIOS

By

Christian M. Lewis

A Summer Researcher Report Submitted to the
Embry-Riddle Aeronautical University McNair Scholars Program
in Partial Fulfillment of the Requirements of
A Summer Research Experience

Embry-Riddle Aeronautical University

Daytona Beach, Florida

August 2008

ACKNOWLEDGEMENTS

The author would like to express heartfelt thanks to the undergraduate research mentor, Dr. Bogdan Udrea, without whom, such an undergraduate opportunity would not have been possible. Thank you for taking the time to show me a glimpse of what it will take to continue on the path of education.

Appreciation and gratitude also goes out to the McNair Scholars Program, currently directed by Dr. Maranda McBride, for allowing me the opportunity to participate in such an esteemed and invaluable program in the pursuit of higher education.

ABSTRACT

Author: Christian M. Lewis

Title: Trajectory Shaping Study for Various Ballistic Missile Scenarios

Institution: Embry-Riddle Aeronautical University

Degree: Bachelor of Science in Aerospace Engineering

Year: 2008

As the world continues to change and threatening countries begin to develop weapons capable of reaching all corners of the Earth, it becomes necessary for U.S. to find new ways of protecting the sovereignty it its people and borders of its nation.

It is the purpose of this research to determine a more effective way of neutralizing ballistic missile warheads before they reenter Earth's atmosphere and simulate a scenario showing this possibility in action.

The ballistic missile code is written in MATLAB and initialized in the Entry Analysis Tool for Exploration Missions (EATEM). EATEM is a tool originally designed for the analysis of a Lander entering Martian atmosphere. It was designed by Shaun Deacon in the pursuit of his M.S. in Aerospace Engineering at Embry-Riddle Aeronautical University.

The ballistic missile scenario is designed to show how a ballistic missile may be used to intercept an opposing ballistic missile. The intercept missile will launch a few minutes after the hostile launch has been detected and will initiate an intercept trajectory as it ascends through the atmosphere and into orbit.

NOMENCLATURE

Δt = timestep

N = effective navigation ratio

t = time (s)

t = time vector

g = gravity (ft/s)

$x_{t,0}$ = initial target position, x-direction (ft)

$y_{t,0}$ = initial target position, y-direction (ft)

V_t = initial target velocity (ft/s)

n_t = target acceleration command (ft/s^2)

β = target flight path angle (rads)

$\dot{\beta}$ = rate of change of target flight path angle (rads/s)

$V_{t,x}$ = target velocity, x-direction (ft/s)

$V_{t,y}$ = target velocity, y-direction (ft/s)

x_m = missile position, x-direction (ft)

y_m = missile position, x-direction (ft)

V_m = missile velocity (ft/s)

$V_{m,x}$ = missile velocity, x-direction (ft/s)

$V_{m,y}$ = missile velocity, y-direction (ft/s)

$a_{m,x}$ = missile acceleration, x-direction (ft/s^2)

$a_{m,y}$ = missile acceleration, y-direction (ft/s^2)

x_t = target position, x-direction (ft)

y_t = target position, y-direction (ft)

R_0 = initial range between target and missile (ft)

R_x = range between target and missile in the x-direction (ft)

R_y = range between target and missile in the y-direction (ft)

R = range between target and missile (ft)

λ = line of sight angle (rads)

$\dot{\lambda}$ = rate of change of line of sight angle (rads/s)

L = lead angle (rads)

HE = heading error (rads)

$V_{t\text{-}m,x}$ = relative target to missile velocity, x-direction (ft/s)

$V_{t\text{-}m,y}$ = relative target to missile velocity, y-direction (ft/s)

$V_{t\text{-}m}$ = relative target to missile velocity (ft/s)

V_c = closing velocity (ft/s)

n_c = acceleration command (ft/s^2)

INTRODUCTION

The world has many nations and hostile regimes that would take any and every opportunity to bring terror and fear upon the United States of America. While more developed countries already have the capability of long range ICBM strikes, smaller developing countries are currently in the process of developing such technologies. While we have advanced, if not superior, technologies when compared to other nations, it is still almost impossible to predict the exact time, method, or target of future threats. For this reason it is necessary to prepare ourselves against every possible attack imaginable. While not the easiest to disguise, ballistic missiles represent a major threat. Treaties are in place preventing the use of multiple warhead attacks, but it would be naïve of us to assume no such can occur. This is why it is necessary to prepare for the possibility of multiple warheads and decoys launched against us from hostile threats.

It is the purpose of this research to determine a more effective way of neutralizing ballistic missile warheads before they reenter Earth's atmosphere and simulate a scenario showing this possibility in action. This simulation is designed to look at multiple scenarios associated with ballistic missile launches and orbital intercepts performing different maneuvers.

This code is written and utilized in MATLAB under Entry Analysis Tool for Exploration Mission (EATEM) with the purpose of having a model that simulates real time launches that can further examine through different avenues of approach. Currently this model is being designed for launch, atmospheric ascent (including multi-stage rocket design), orbital trajectory shaping and kinetic kill intercepts.

After the explanation and validation of the simulation a section will examine its potential application with regards to future Lidar projects and other atmospheric based scenarios. Further application possibilities are examined but no analysis details are presented.

Overview

Introduction

This section is included in an effort to provide the reader with a background and to present the general approach without the rigorous detail which is present in the main body of the thesis.

Methodology

The approach taken begins with recreation of the code presented in chapter 1 of *Tactical and Strategic Missile Guidance.* This code has a missile, using proportional guidance law, track and intercept a target performing a linear maneuver. Chapter 1 presents the basic coding using the proportional guidance law. This is the law used to issue acceleration commands based on the current position, velocity, and Line of Sight (LOS) of the missile to the target.

After developing a comparable code, the target was given simple maneuvers to carry out with the goal of testing the design of the missile guidance code. The target first traveled with a constant velocity in a linear fashion toward the target at an arbitrary altitude. It was then made to accelerate in the same fashion a square root function would. Finally, it was made to follow a circular path.

The atmospheric model was developed to be integrated into the EATEM code for the purpose of simulating Earth's atmosphere. The data needed for an accurate representation was taken from *Fundamentals of Astrodynamics and Applications* and applied as a call function. Two methods were tried, but it was eventually determined that the use of conditional statements would be the fastest and most effective use of code. The atmosphere is assumed to be broken into multiple discrete segments, but the functions that model these segments actually force them to become a continuous piece-wise function.

Modeling Simplifications

For the coding aspect of the missile–to-target program it was assumed that all encounters were to happen in a two dimensional environment. This caused no difference in the simulation other than to expedite the replication of the codes seen in the 4th edition of *Tactical and Strategic Guidance*. Inertia and specific missile/target dynamics were also not taken into account. Essentially, both the missile and target were treated as point masses in this scenario.

The atmospheric model was completed by taking the atmosphere to be multiple segments modeled by a continuous piecewise function. It is assumed that each segment is completely uniform throughout the globe and that the Earth is a perfect sphere with no differences at either pole or the equator.

Results Comparison

The results ascertained from the model created are almost identical to those which are provided. If the models are brought to a significantly high zoom of magnification it can be seen that there are differences, but these occur on an order of magnitude of 1e2; whereas, the entire simulation takes place on a 1e3 order of magnitude.

Example of Simulation Application

Once completed, the simulation will be able to model the intercept of hostile ballistic missile warheads already in the orbital phase of their launch. Simple simulations will begin with one warhead and will expand to more as the model shows its full potential.

Conclusion and Recommendations

The paper concludes with a recap of the success of the code development. Further applications are examined and future work is recommended.

Basic Intercept Code Development

Introduction

Acquisition and intercept of a target by a missile is governed by a closed loop system designed to evaluate current conditions and make the appropriate actions. Current conditions cover a range of concepts, but for this portion of the research, it will be confined to the following topics: Guidance Theory, Simulation Theory, Simulation Process and Engagement Simulations.

Guidance Theory

Proportional Navigation was developed to allow an object, moving at high rates of speed, to adjust its heading in order to cause the target and missile to attain the same position at the same time [3]. Simple models of proportional navigation simply cause the missile and target to reside in the same place at the same time, whereas trajectory shaped models have the missile coinciding with the target in a particular manner (i.e., perpendicular to rather than head on.). This is done to maximize effectiveness and/or minimize fuel consumption. The equation defining proportional navigation is shown below.

$$n_c = N' * V_c * \lambda' \qquad (1)$$

The acceleration command, n_c , is the variable that holds the final acceleration command delivered to associated fins or ailerons. The Effective Navigation Ratio, N', is a number between 3 and 5 used to determine the amount of motion each acceleration command will deliver. This is a user chosen number based off the amount of rapid acceleration a particular system can handle. The higher the number the more rapid a response will be. V_c represents the magnitude of the closing velocity between the missile and target. In this simulation this magnitude is based on the actual missile and target velocities; whereas, real world scenarios would have to guesstimate the information regarding the target. The last variable is the rate of change of the LOS. This variable is based off of the rapidly changing positions and velocities of both the missile and target. The acceleration command issues commands perpendicular to the thrust of the missile in order to cause the body to change its LOS angle. [4]

Simulation Theory

The work I have produced in MATLAB is comparable to the FORTRAN work referenced from *Tactical and Strategic Missile Guidance*, 4th edition. The target and missile code are written and executed in the same editor, restricting the simulation to one overarching timestep instead of allowing the missile to vary its timestep according to range. Another advantage of having them written in the same code is the missile has a constant stream of data precise to each timestep, whereas real world models can only make estimations of range and velocities. The information is first initialized within a

“For” loop, to set the total number of iterations per program execution, followed by an “if”, to change the simulation timestep as necessary. Within a 500 ft distance the timestep changes from 20 iterations per second to 1000 iterations per second, in order to allow the missile enough time, at such high speeds, to adjust its LOS via acceleration commands.

Simulation Process

The simulation begins with the declaration of global variables, which include initial time, effective navigation ratio, timestep, and various conversion factors. The following simulation is similar, both in variables used and numerical values attained, to ones performed in the 4th edition of Paul Zarchan’s *Tactical and Strategic Missile Guidance* book. Initial information for the target is inputted, and associated vectors are determined:

$$x_{t,0} = 40000\,ft;\ y_{t,0} = 10000ft;\ V_t = 1000\,ft/s$$

$$n_t = 3g\,ft/s^2;\quad \beta = 0\ rads$$

$$V_{t,x} = -V_t \cos\beta;\quad V_{t,x} = -V_t \sin\beta$$

$$x_t = V_{t,x} * t + x_{t,0};\quad y_t = V_{t,y} * t + y_{t,0};$$

Following the target, the missile information is entered beginning with initial information, angular information, vector information, and acceleration information:

$$x_m = 0\,ft;\ y_m = 10000\,ft;\ V_t = 3000\,ft/s$$

$$R_x = x_t - x_m;\ R_y = y_t - y_m$$

$$\lambda = \tan^{-1} {}^{R_y}\!/_{R_x}$$

$$L = \sin^{-1}\left({}^{(V_t * \sin\lambda)}\!/_{V_m}\right)$$

$$HE = 0$$

$$V_{m,x} = V_m \cos L + \lambda + HE * \frac{\pi}{180};\quad V_{m,y} = V_m \sin L + \lambda + HE * \frac{\pi}{180}$$

$$a_{m,x} = 0;\quad a_{m,y} = 0$$

When being entered into the “if” loop there are a few more variables that must be defined:

$$x_t = x_{t,0};\quad y_t = y_{t,0}$$

$$R_0 = \left(R_x^2 + R_y^2\right)^{\frac{1}{2}}; \quad R = R_0$$

The actual “if” loop is used to calculate each set of equations 4000 times or until the missile gets within one foot of the target. Each iteration of the loop represents a timestep. These timesteps are plotted to create a graph depicting different forms of information. Following are the equations that are calculated for each iteration:

$$t = \Delta t * t$$

$$x_t = V_{t,x} * \Delta t + x_t; \quad y_t = V_{t,y} * \Delta t + y_t$$

$$x_m = V_{m,x} * \Delta t + x_m; \quad y_m = V_{m,y} * \Delta t + y_m$$

$$R_x = x_t - x_m; \quad R_y = y_t - y_m$$

$$R_0 = \left(R_x^2 + R_y^2\right)^{\frac{1}{2}}$$

$$\dot{\beta} = \frac{n_t}{V_t}$$

After calculating the rate of change for the target flight path, the new flight path angle must be adjusted:

$$\beta = \beta + \Delta t * \dot{\beta}$$

Target velocity is continuously updated:

$$V_{t,x} = -V_t \cos\beta; \quad V_{t,x} = -V_t \sin\beta$$

The missile velocity is adjusted (keep in mind the first iteration does not take into account missile acceleration because it has yet to be calculated):

$$V_{m,x} = a_{m,x} * \Delta t + V_{m,x}; \quad V_{m,y} = a_{m,y} * \Delta t + V_{m,y}$$

The relative velocity is the difference between the velocities of the missile and target at any given time:

$$V_{t-m,x} = V_{m,x} - V_{t,x}; \quad V_{t-m,y} = V_{m,y} - V_{t,y}$$

$$V_{t-m} = \left(V_{t-m,x}^2 + V_{t-m,y}^2\right)^{\frac{1}{2}}$$

The closing velocity is similar to the relative velocity, except it takes into account the range and is negative. The negative sign indicates the objects are still moving toward one another. A change to positive indicates either they have passed each other or the target is moving away faster than the missile can keep up:

$$V_c = -\frac{R_x * V_{t-m,x} + R_y * V_{t-m,y}}{R}$$

The LOS is updated according to the x and y ranges of the missile and target:

$$\lambda = \tan^{-1} R_y / R_x$$

$$\dot{\lambda} = \frac{R_x * V_y - R_y * V_x}{R^2}$$

The change in the rate of line of sight, closing velocity, and effective navigation ratio are then computed into the acceleration command:

$$n_c = N * V_c * \dot{\lambda}$$

The acceleration command is then put into actual accelerations in the x and y directions, which are transmitted to the fins and ailerons:

$$a_{m,x} = -n_c \sin \lambda ; \quad a_{m,y} = -n_c \cos \lambda$$

Engagement Simulation

The engagement simulation geometry is presented below:

FUNDAMENTALS OF TACTICAL MISSILE GUIDANCE

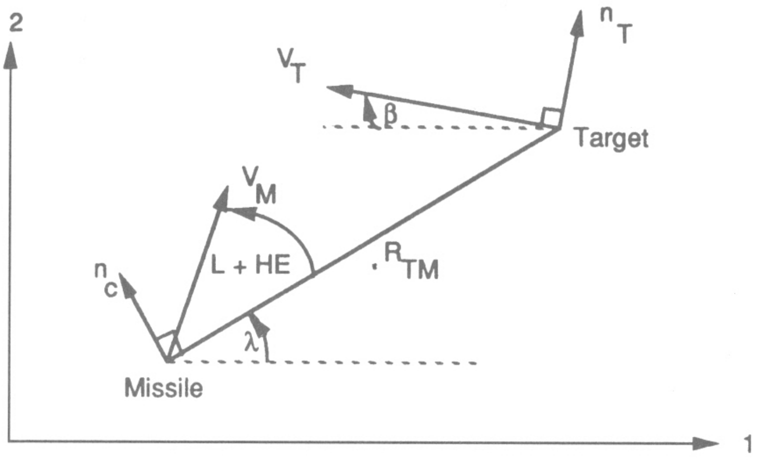

Fig. 2.1 Two-dimensional missile-target engagement geometry.

Figure 1: 2-D target to missile simulation geometry [4]

The graphs pertaining to the simulation described above are displayed below.

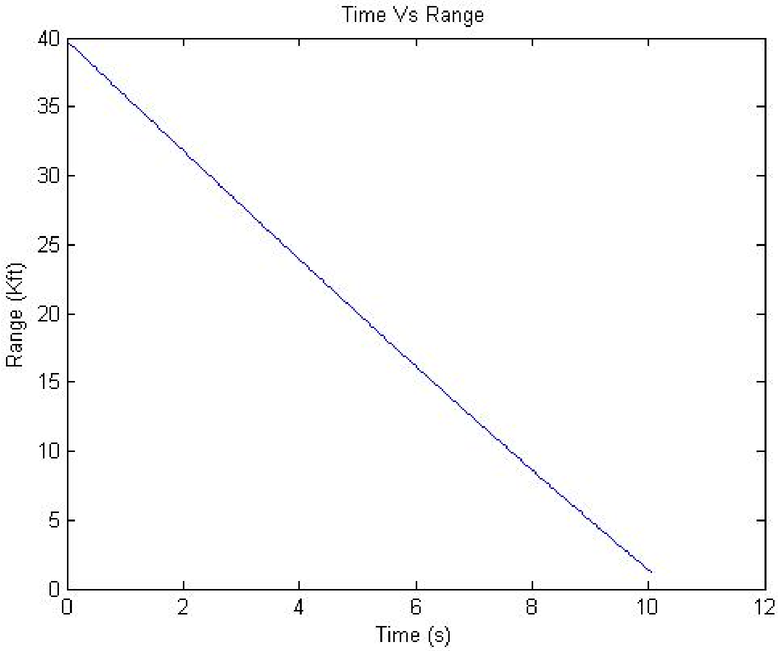

Figure 2: Plot of time vs. range

The figure above depicts the decrease of range between the missile and target as the simulation continues.

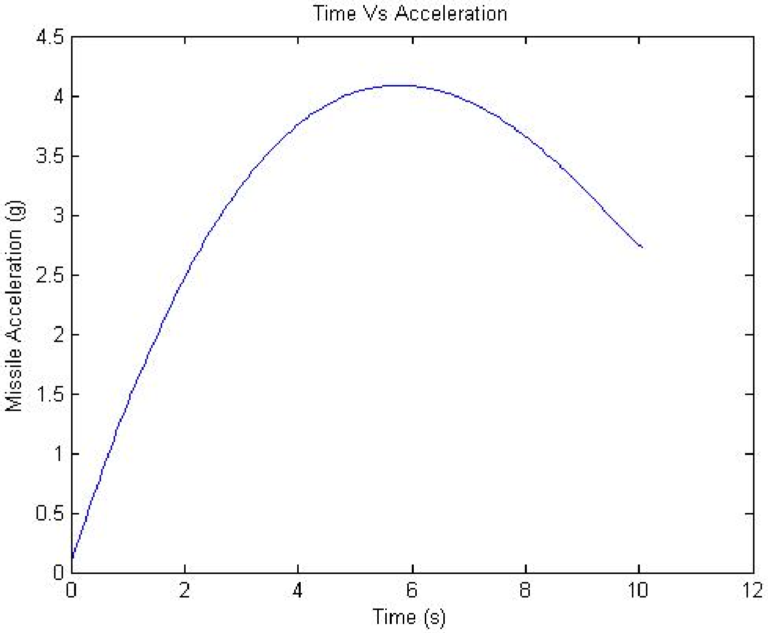

Figure 3: Plot of time vs. acceleration

This figure depicts the amount of acceleration, in g's, the missile exerts in order to adjust its heading towards that of the target.

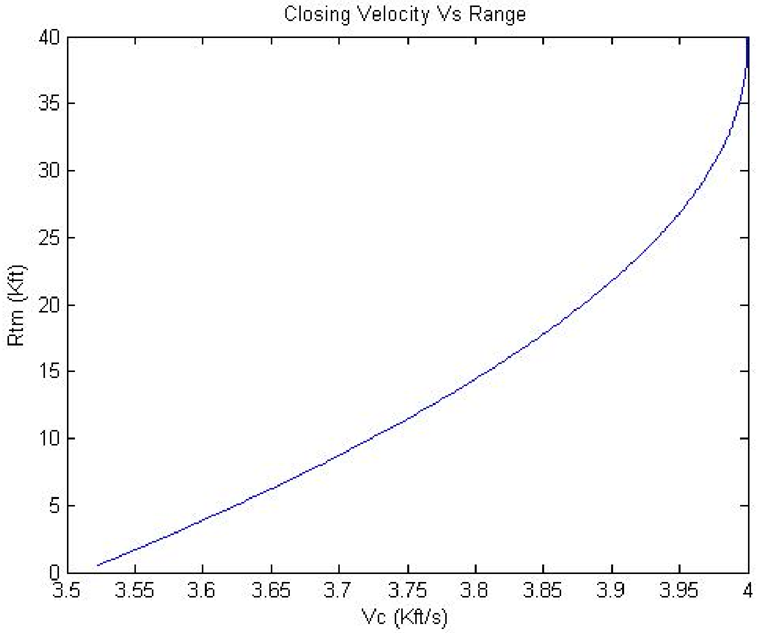

Figure 5: Plot of closing velocity vs. range

This figure depicts the change in closing velocity as the missile closes the range. The graph begins in the upper right hand corner.

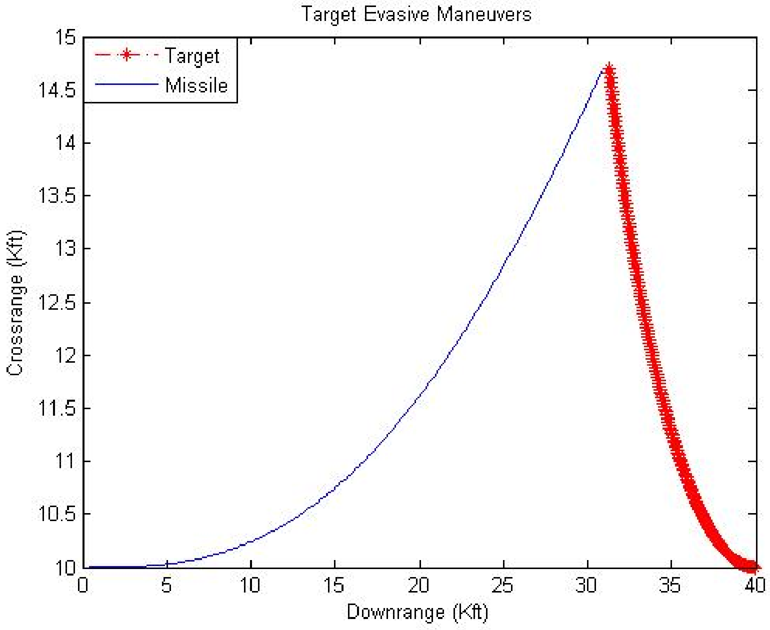

Figure 6: Plot of target evasive maneuvers

The figure above depicts the missile adjusting its LOS to intercept a missile performing a 3g maneuver.

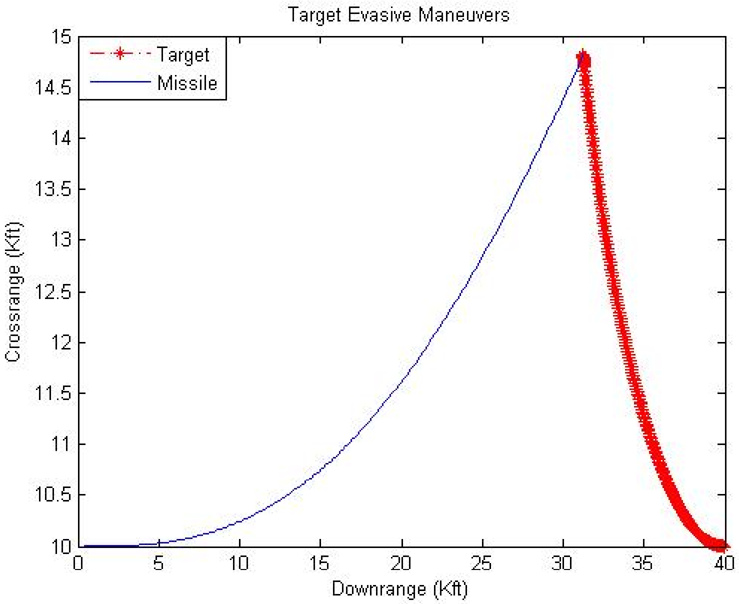

Figure 7: Plot of Timestep target evasive maneuvers

This is a continuation of the previous graph, but with the simulation continuing until the missile is within 500ft of the target. You can see the change in timestep as the almost constant line of asterisks toward the end of the simulation. The missile is within 2ft of the target.

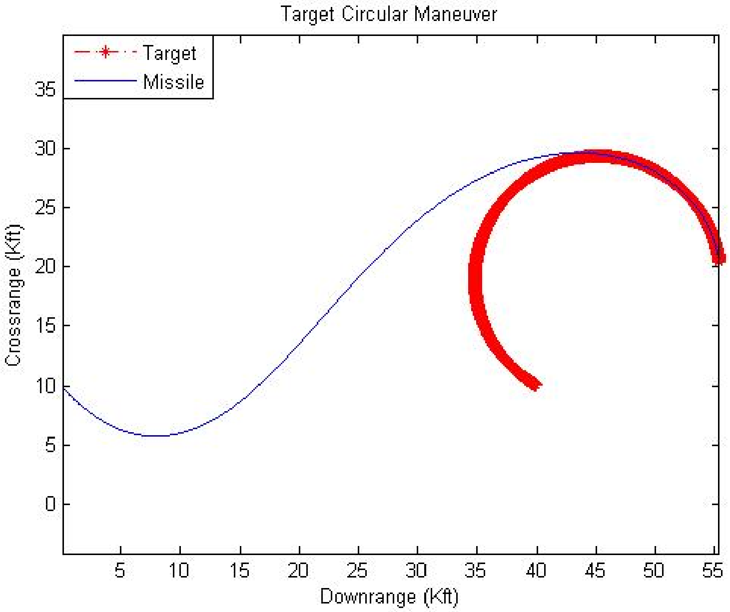

Figure 8: Plot of Timestep target circular maneuvers

This is a more complex model where the target was required to perform a circular maneuver. This was accomplished by having it adjust one degree for every time the loop completed an iteration.

Atmospheric Model

Introduction

The purpose of the atmospheric model is to accurately simulate the effects of Earth's atmosphere on the body of a ballistic missile [2]. The previous model used in the EATEM program was adjusted for Earth and tested for errors. There were two methods tested in when choosing how to call the function. The first is a series of conditional IF, ELSEIF, and ELSE statements to describe the segmented atmosphere. The proper statement for the segment of the atmosphere was determined based on altitude in kilometers. The second method tried was a sort of spline, where the values were interpolated through a series of arrays. A comparison of the two methods proved the conditional statements to process faster than the spine. Thus, this method was chosen.

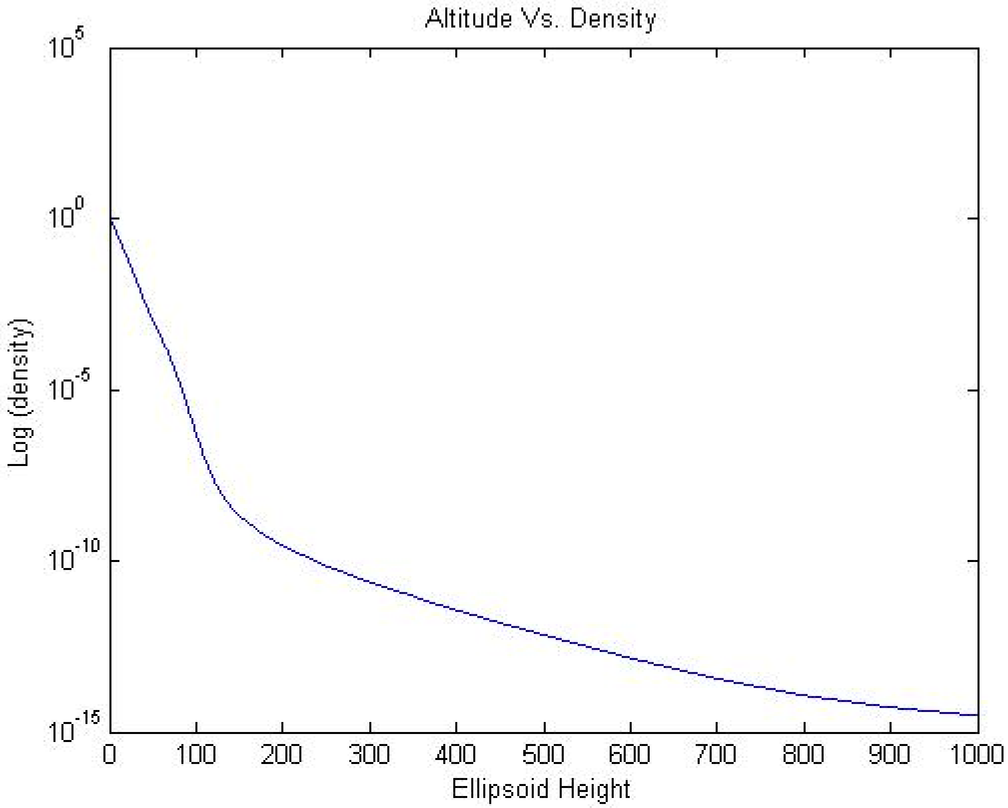

Figure 9: Plot of Atmospheric model

CONCLUSION AND RECOMMENDATIONS

Most of the coding for this simulation was reproduced through Paul Zarchan's *Tactical and Strategic Missile Guidance* and thus is highly comparable to the work that has already been produced. This is because his work was the guiding subject matter for study.

This simulation is set up to allow follow up testing with a wide variety of ideas for missile defense and/or other operations involving exoatmospheric rendezvous. This would be easy to alter by changing the object being launched and adjusting the amount of distance before a successful intercept.

One idea for a next step intercept would be the implementation of a Lidar imaging scenario. This type of scenario involves an intercept missile that filters out its targets by utilizing lasers to discern between an actual warhead and a decoy.

While this simulation represents a fairly large step in the right direction towards missile defense, there are many aspects that can be expanded upon to make the simulation more realistic.

This simulation could be expanded to be used in SIMULINK in order to allow for the target and missile to utilize different timesteps. This would allow the missile to issue acceleration commands more periodically than the target has time to adjust.

Utilizing all of the concepts out of *Tactical and Strategic Missile Guidance* was beyond the scope of this work. Thus, there is much room for expansion into Noise Analysis, Zero-miss closed loop functions, Weaving targets, and much more.

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