Escape Velocity

**Background Story:**
A MASSIVE meteorite is coming, you are in great DANGER and you need to leave planet “**” as soon as you can. Your scientists spotted a safer place “not too far”, planet “%%” should be a great place for your colony. To reach planet “%%”, you first need to escape your current planet. For that, you need to figure out the escape velocity!

**Scientific Background:**
The Universal Law of Gravitation gives the force of attraction (in N) between two bodies of masses \( m_1 \) and \( m_2 \) (in kg) which are at a distance \( r \) (in m) apart via: \( F = \frac{Gm_1 m_2}{r^2} \), where \( G \) is the gravitational constant.

To escape from the planet “**”, we first need to calculate work done by the shuttle “WiS” against gravity. This is work is directly proportional to the gravitational potential energy and calculated by using the attractive force via: \( \text{Work} = \int F \, dr = \frac{GMm}{R} \) \( m \) being the total mass of the rocket \( M \) The mass of the planet (in kg) and \( R \) the radius of the planet (in m). The initial speed of the shuttle “WiS” is called the “escape velocity”. Once the shuttle “WiS” is escaped from the planet “**”, its speed and work against the gravity will be negligible and hence can be assumed as 0.

**Data, Equations, and Laws:**
- For an ideal shuttle getting into orbit, the payload should make up 6% of the total mass. The shuttle engines, fuel tanks and so on should be 3%. The propellants should be 91%.

- Gravitational constant of the planet “**” is \( 6.674 \times 10^{-11} \) N m\(^2\) kg\(^{-2}\)
- Gravitational acceleration of the planet “**” is 10 ms\(^{-2}\)
- The radius of “**” is 3001 miles
- The shuttle has 2 millions liters of fuel, the total density of the fuel is 2.5 g /ml
- The average density of planet “**” is 4.41 g/cm\(^3\)

- Equations of Motion : \( v = u + a.t \) ; \( S = u.t + \frac{1}{2} a t^2 \)

where \( u \) and \( v \) are initial and terminal velocities of the rocket, \( a \) is the rocket's acceleration, \( S \) is the displacement, and \( t \) is the time interval.
In physics, the law of conservation of energy states that the total energy of an isolated system remains constant. Here this means that to find the escape velocity you need to equal the work require to escape and the kinetic energy.

Kinetic energy = \(0.5 \times m \times v^2\) where \(m\) is the mass of the body and \(v\) its velocity.

**Procedure:**
1) Convert the radius of planet “**” in m
2) Find the volume of planet “**”.
3) Find the mass of planet “**”.
4) Find the total mass of the fuel.
5) Find the total mass of the shuttle “WiS” with fuel.
6) Apply the conservation of energy principle for the shuttle “WiS”.
7) Find the escape velocity “\(v_e\)”.
8) Find how long it takes for the shuttle “WiS” to escape the planet “**”.
9) Use the equations of motion to derive: \(S = \left(\frac{u+v}{2}\right) \times t\)
10) Find how far the shuttle “WiS” has gone when it escape from the planet “**”

Congratulations you are securely escaped from the planet “**”!