

Escape Velocity

Background Story:

A MASSIVE meteorite is coming, you are in great DANGER and you need to leave planet “***” as soon as you can. Your scientists spotted a safer place “not too far”, planet “%%” should be a great place for your colony. To reach planet “%%”, you first need to escape your current planet. For that, you need to figure out the escape velocity!

Scientific Background :

The Universal Law of Gravitation gives the force of attraction (in N) between two bodies of masses m_1 and m_2 (in kg) which are at a distance r (in m) apart via: $F = \frac{Gm_1m_2}{r^2}$, where G is the gravitational constant.

To escape from the planet “***”, we first need to calculate work done by the shuttle “WiS” against gravity. This work is directly proportional to the gravitational potential energy and calculated by using the attractive force via: $Work = \int F dr = \frac{GmM}{R}$ m being the total mass of the rocket M The mass of the planet (in kg) and R the radius of the planet (in m). The initial speed of the shuttle “WiS” is called the “escape velocity”. Once the shuttle “WiS” is escaped from the planet “***”, its speed and work against the gravity will be negligible and hence can be assumed as 0.



Data, Equations, and Laws:

- For an ideal shuttle getting into orbit, the payload should make up 6% of the total mass. The shuttle engines, fuel tanks and so on should be 3%. The propellants should be 91%.
- Gravitational constant of the planet “***” is $6.674 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$
- Gravitational acceleration of the planet “***” is 10 ms^{-2}
- The radius of “***” is 3001 miles
- The shuttle has 2 millions liters of fuel, the total density of the fuel is 2.5 g/ml
- The average density of planet “***” is 4.41 g/cm^3
- Equations of Motion : $v = u + a.t$; $S = u.t + \frac{1}{2}at^2$

where u and v are initial and terminal velocities of the rocket, a is the rocket's acceleration, S is the displacement, and t is the time interval.

- In physics, the law of *conservation of energy* states that the total **energy** of an **isolated system** remains constant. Here this mean that **to find the escape velocity you need to equal the work require to escape and the kinetic energy.**
- *Kinetic energy* = $0.5 m v^2$ where *m* is the mass of the body and *v* its velocity.

Procedure:

- 1) Convert the radius of planet “**” in m
- 2) Find the volume of planet “**”.
- 3) Find the mass of planet “**”.
- 4) Find the total mass of the fuel.
- 5) Find the total mass of the shuttle “WiS” with fuel.
- 6) Apply the conservation of energy principle for the shuttle “WiS”.
- 7) Find the escape velocity “ v_e ”.
- 8) Find how long it takes for the shuttle “WiS” to escape the planet “**”.
- 9) Use the equations of motion to derive : $S = \left(\frac{u+v}{2} \right) . t$
- 10) Find how far the shuttle “WiS” has gone when it escape from the planet “**”

Congratulations you are securely escaped from the planet “”!**