

Publications

3-1-1994

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Walterscheid, R. L., G. Schubert, and M. P. Hickey (1994), Comparison of theories for gravity wave induced fluctuations in airglow emissions, *J. Geophys. Res.*, 99(A3), 3935–3944, doi: <https://doi.org/10.1029/93JA03312>.

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Comparison of theories for gravity wave induced fluctuations in airglow emissions

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Abstract. A comparison is undertaken of theories for the gravity wave induced fluctuations in the intensity of airglow emissions and the associated temperature of the source region. The comparison is made in terms of Krassovsky's ratio η_E for a vertically extended emission region (η_E is the ratio of the vertically integrated normalized intensity perturbation to the vertically integrated normalized intensity-weighted temperature perturbation). It is shown that the formulas for η_E in the works by Tarasick and Hines (1990) and Schubert et al. (1991) are in agreement for the case of an inviscid atmosphere. The calculation of η_E using the theory of Tarasick and Hines (1990) requires determination of their function χ ; we show that χ is simply related to the "single-level" Krassovsky's ratio η of Schubert et al. (1991). The general relationship between χ and η is applied to a simple chemical-dynamical model of the O₂ atmospheric airglow and the altitude dependence of these quantities is evaluated for nonsteady state chemistry. Though the Tarasick and Hines (1990) formula for η_E does not explicitly depend on the scale heights of the minor constituents involved in airglow chemistry, η_E implicitly depends upon these scale heights through its dependences on chemical production and loss contained in χ . We demonstrate this dependence of η_E for the OH nightglow on atomic oxygen scale height by direct numerical evaluation of η_E ; in this case the dependence originates in the chemical production of perturbed ozone.

Introduction

In a series of recent papers, Walterscheid et al. [1987], Schubert and Walterscheid [1988], Hickey [1988a, b], and Schubert et al. [1991] have developed an Eulerian theory for the fluctuations in OH nightglow driven by the passage of gravity waves through the OH nightglow emission region. The theory relates the fluctuations in OH nightglow intensity and in the temperature of the emission region to the characteristics of the gravity waves and, accordingly, it serves as a basis for the quantitative interpretation of spectrophotometric observations of OH nightglow fluctuations [e.g., Hecht et al., 1987; Viereck and Deehr, 1989; Swenson et al., 1990]. The theory has gone through several stages of improvement, and in its most recent form [Schubert et al., 1991] accounts for emission from an extended region and for effects of dissipation due to eddy viscosity and eddy thermal diffusivity.

Hines and Tarasick [1987], Tarasick and Hines [1990], and Tarasick and Shepherd [1992a, b] have developed an independent theory of wave-driven fluctuations in airglow emissions. Tarasick and Shepherd [1992a, b] incorporate a relatively sophisticated chemical scheme for reacting airglow

species, but they do not include eddy viscosity and eddy thermal diffusivity shown to be important by Hickey [1988a, b] and Schubert et al. [1991] for waves with short vertical wavelengths.

The main purpose of the present paper is to compare and elucidate these theories by applying them to the same airglow model. The quantity most readily comparable is Krassovsky's ratio η_E [Krassovsky, 1972] for a vertically extended emission region given by

$$\eta_E = \frac{\langle I' \rangle / \langle \bar{I} \rangle}{\langle T_I' \rangle / \langle \bar{T}_I \rangle} \quad (1)$$

where I is intensity, T_I is intensity weighted temperature, primes refer to Eulerian fluctuations, overbars denote the time-independent basic state, and angular brackets denote integration over the thickness of the emission region [Hines and Tarasick, 1987; Schubert and Walterscheid, 1988]. Variables denoting Eulerian fluctuations are the height-dependent complex amplitudes of quantities varying sinusoidally in time and the horizontal plane.

We first show that the formulas for η_E derived by Tarasick and Hines [1990] and Schubert et al. [1991] for any airglow emission are in agreement in the case of an inviscid atmosphere. As a by-product of this we identify how the function χ of Tarasick and Hines [1990] is related to the "single-level" Krassovsky's ratio η of Schubert et al. [1991]. We apply the general theory to a model of the O₂ atmospheric airglow and evaluate how η and χ vary with altitude when nonsteady state chemistry is important. Krassovsky's ratio

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η_E for the emission from a vertically extended region is explicitly calculated for the O₂ atmospheric airglow. Finally, we discuss how the vertical scale heights of airglow species influence η_E even though the expression for η_E given by *Tarasick and Hines* [1990] does not explicitly depend on minor constituent scale heights. We analyze the scale height dependence, which may originate in chemical production and loss. For the OH nightglow this occurs through the dependence of η_E on the chemical production of perturbed ozone from perturbed atomic oxygen. The dependence is demonstrated for a model of the OH nightglow.

Comparison of Theories

Krassovsky's Ratio

Krassovsky's ratio η_E can be directly calculated from (1) using, for example, the Eulerian theory of *Schubert et al.* [1991]. With the help of (10) and (12) of *Schubert and Walterscheid* [1988], (1) can be rewritten in terms of the wave-induced temperature and intensity fluctuations

$$\eta_E = \frac{\langle \bar{T}_I \rangle \langle I' \rangle}{\langle \bar{T}' \rangle + \langle \bar{T}_I' \rangle - \langle \bar{T}_I \rangle \langle I' \rangle} \quad (2)$$

Hines and Tarasick [1987] and *Tarasick and Hines* [1990] have derived an equation for η_E using a modified Lagrangian approach [*Tarasick and Hines*, 1990, Equation (68)]. We can obtain a formula identical to theirs by proceeding from (2) as follows. Consider the term $\langle I' \rangle$ in the numerator of (2). In terms of the single-level Krassovsky's ratio η [*Schubert and Walterscheid*, 1988] we can relate I' to the temperature fluctuation T' by

$$\frac{I'}{\bar{I}} = \eta \frac{T'}{\bar{T}} \quad (3)$$

The temperature perturbation can in turn be related to the divergence of the wave velocity field $\nabla \cdot \mathbf{v}$ by the polarization factor f_1 [*Schubert et al.*, 1991]

$$\frac{T'}{\bar{T}} = \frac{1}{f_1} \nabla \cdot \mathbf{v} \quad (4)$$

Substitution of (3) and (4) into the expression for $\langle I' \rangle$ gives

$$\langle I' \rangle = - \left\langle \frac{\bar{I}(\nabla \cdot \mathbf{v})\eta}{f_1} \right\rangle \quad (5)$$

We define the function χ by

$$\chi \equiv \frac{-i\omega\eta}{f_1} - \frac{f_2}{f_1} \frac{d \ln \bar{I}}{dz} \quad (6)$$

where f_2 is the wave polarization factor relating wave vertical velocity w to the temperature perturbation

$$w \equiv f_2 \frac{T'}{\bar{T}} \quad (7)$$

and ω is the angular frequency of the wave. We will show later that χ as defined by (6) is equivalent to the function χ used by *Hines and Tarasick* [1987] and *Tarasick and Hines* [1990]. Substitution of (6) and (7) into (5) together with an integration by parts gives

$$\langle I' \rangle = \frac{1}{i\omega} \left\{ - \langle \bar{I}(\nabla \cdot \mathbf{v})\chi \rangle + \left\langle \bar{I} \frac{dw}{dz} \right\rangle \right\} \quad (8)$$

provided $\bar{I}w$ vanishes at the limits of integration.

The last step in rewriting the expression for $\langle I' \rangle$ involves introduction of another polarization factor relating dw/dz to $\nabla \cdot \mathbf{v}$ [*Hines and Tarasick*, 1987; *Tarasick and Hines*, 1990]

$$\frac{dw}{dz} \equiv (\mu + i\nu)\nabla \cdot \mathbf{v} \quad (9)$$

Use of (9) to rewrite (8) results in

$$\langle I' \rangle = \frac{1}{i\omega} \langle \bar{I}(\nabla \cdot \mathbf{v})(\mu + i\nu - \chi) \rangle \quad (10)$$

The numerator of (2) can be put in a form identical to the numerator of the *Tarasick and Hines* [1990] equation (68) for η_E by substitution of (10) for $\langle I' \rangle$ and by evaluation of the integrand of the right side of (10) at the displaced position $z - w/i\omega$, where $w/i\omega$ is the Eulerian estimate of parcel vertical displacement when the parcel velocity is w . Expressions like (10), with the integrands evaluated at z and $z - w/i\omega$, are equal to first order, and for notational simplicity we ignore the difference except as noted.

The denominator of (2) can be rewritten in a form identical to that of the denominator of the *Tarasick and Hines* [1990] equation (68) for η_E by proceeding along the lines of the above development of the expression for $\langle I' \rangle$. Use must be made of the polarization relations introduced above and integration by parts. The final result of these algebraic manipulations is the following expression for η_E :

$$\eta_E = \frac{\langle \bar{I}(\nabla \cdot \mathbf{v})(\mu + i\nu - \chi) \rangle}{\left\langle \bar{I}(\nabla \cdot \mathbf{v}) \left(\frac{\bar{T}}{\langle \bar{T}_I \rangle} - 1 \right) (\mu + i\nu - \chi) \right\rangle - \left\langle \bar{I}(\nabla \cdot \mathbf{v})(\gamma - 1) \frac{\bar{T}}{\langle \bar{T}_I \rangle} \right\rangle} \quad (11)$$

Equation (11) is valid only for an inviscid atmosphere since its derivation employs the inviscid formula for the polarization factor f_1 (see the appendix for the forms of the polarization factors).

In the case of an isothermal atmosphere (11) simplifies considerably since $\bar{T} = \langle \bar{T}_I \rangle = \text{con}$ and μ and ν are also constants. The simplified form of (11) in this case is

$$\eta_E = \frac{1}{(\gamma - 1)} \frac{\langle \bar{I}(\nabla \cdot \mathbf{v})\chi \rangle}{\langle \bar{I}(\nabla \cdot \mathbf{v}) \rangle} - \frac{(\mu + i\nu)}{(\gamma - 1)} \quad (12)$$

The algebraic formula for η_E given by *Hines and Tarasick* [1987]

$$\eta_E = \frac{\chi - \mu - i\nu}{(\gamma - 1)} \quad (13)$$

follows from (12) if χ can be considered constant and the atmosphere is both isothermal and inviscid. We will evaluate this simplification in the following section when we calculate $\chi(z)$ for the O₂ atmospheric airglow.

Finally, it is simple to show directly from (1), (3), (4), and

(A4) that for an isothermal, inviscid reference state atmosphere (12) is equivalent to

$$\eta_E = \frac{\langle \bar{I}(\nabla \cdot \mathbf{v}) \eta \rangle}{\langle \bar{I}(\nabla \cdot \mathbf{v}) \rangle} \quad (14)$$

Accordingly, in this case η_E is just a normalized vertical integration of the single level η with the contributions of η weighted by $\bar{I}(\nabla \cdot \mathbf{v})$ in the integration over altitude. The expression for η_E in (12) involves a similar vertical integration of χ with the same weighting function.

Relation Between χ and η

The above derivation of (11) provides a connection (6) between the single-level Krassovsky ratio η [Schubert and Walterscheid, 1988] and the function χ introduced by Hines and Tarasick [1987] to relate Lagrangian-based perturbations in airglow intensity from an atmospheric parcel to the compression or dilatation of the parcel $\nabla \cdot \mathbf{v}$

$$\chi \equiv \left(\frac{\delta I}{\bar{I}} \right) \left(\frac{i}{\omega} \nabla \cdot \mathbf{v} \right)^{-1} \quad (15)$$

The symbol δ denotes a perturbation defined by $\delta\psi = \psi(x, y, z, t) - \bar{\psi}(z_0)$, where ψ is any quantity, and $z_0 = z - w/i\omega$. Thus δ denotes the difference between the undisturbed, basic-state value of ψ at z_0 and the total value of ψ at z . The quantity $w/i\omega$ is the Eulerian estimate of the displacement of a parcel carried from z_0 to z by steady state motion.

It can be readily demonstrated that χ defined by (15) is equivalent to χ given by (6) as follows. Eulerian- and Lagrangian-based perturbations in intensity can be related by

$$\delta\psi = \psi' + \frac{w}{i\omega} \frac{d\bar{\psi}}{dz} \quad (16)$$

Combination of (16) with (3), (4), and (7) gives

$$\frac{\delta I}{\bar{I}} = \left(\frac{\eta}{f_1} + \frac{f_2}{i\omega f_1} \frac{d \ln \bar{I}}{dz} \right) (\nabla \cdot \mathbf{v}) \quad (17)$$

which, when compared with (15), results in

$$\chi = \frac{-i\omega\eta}{f_1} - \frac{f_2}{f_1} \frac{d \ln \bar{I}}{dz} \quad (18)$$

Equation (18) for χ is identical to (6). Thus the single-level Krassovsky ratio η and the function χ are simply related.

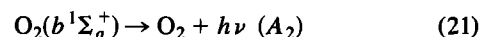
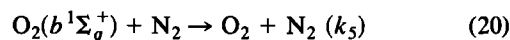
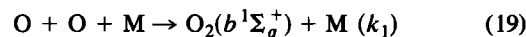
O₂ Atmospheric Airglow

Determination of χ and η

To further elucidate the relationship between χ and η , we apply the general theory of the preceding section to the wave-driven fluctuations in the O₂ atmospheric airglow [Tarasick and Shepherd, 1992a; Hickey et al., 1993]. We calculate χ by simplifying the chemistry in the model to include direct production of the O₂(*b*¹Σ_g⁺) state through the Chapman mechanism alone, while loss of the *b*¹ state is due to radiation or quenching only by N₂. This is the same chemistry considered by Tarasick and Shepherd [1992a], who also considered additional chemical schemes. To simplify evaluation of the theoretical results, we impose isother-

mality on the background atmosphere; the scale height of the major gas and the gravitational acceleration are similarly set to constant values. However, the atomic oxygen scale height varies with altitude. To facilitate comparison with Tarasick and Shepherd [1992a], we neglect the effects of eddy diffusion of heat and momentum in the gravity wave dynamics. The gravity wave vertical wavenumber is given in terms of the wave period and horizontal wavelength by (A3).

The chemical equations may be summarized as follows:



The reaction rates k_1 and k_5 and the radiative transition probability A_2 are defined by Hickey et al. [1993] and are identical to those used by Tarasick and Shepherd [1992a] (our k_5 is equivalent to their k_4). From (19)–(21) we can write expressions for the chemical and radiative production P and loss L rates of the *b*¹ state and the loss rate of atomic oxygen $L(\text{O})$

$$P(b^1) = k_1 n^2(\text{O})n(\text{M}) \quad (22)$$

$$L(b^1) = n(b^1)\{k_5 n(\text{N}_2) + A_2\} \quad (23)$$

$$L(\text{O}) = k_1 n^2(\text{O})n(\text{M}) \quad (24)$$

where n is a number density. In the background reference state, (22) and (23) give

$$\bar{P}(b^1) = \bar{k}_1 \bar{n}^2(\text{O})\bar{n}(\text{M}) \quad (25)$$

$$\bar{L}(b^1) = \bar{n}(b^1)\{k_5 \bar{n}(\text{N}_2) + A_2\} \quad (26)$$

Thus, if the production and loss rates of the *b*¹ state are in equilibrium in the undisturbed reference state, that is, if $\bar{P}(b^1) = \bar{L}(b^1)$, then (25) and (26) give

$$\bar{n}(b^1) = \frac{\bar{k}_1 \bar{n}^2(\text{O})\bar{n}(\text{M})}{(k_5 \bar{n}(\text{N}_2) + A_2)} \quad (27)$$

Equations (22) and (23) also provide linearized expressions for the Eulerian perturbation production and loss rates of the *b*¹ state

$$\frac{P'(b^1)}{\bar{P}(b^1)} = \frac{k'_1}{\bar{k}_1} + \frac{2n'(\text{O})}{\bar{n}(\text{O})} + \frac{n'(\text{M})}{\bar{n}(\text{M})} \quad (28)$$

$$L'(b^1) = n'(b^1)\{k_5 \bar{n}(\text{N}_2) + A_2\} + k_5 \bar{n}(b^1)n'(\text{N}_2) \quad (29)$$

The perturbation loss rate for atomic oxygen $L'(\text{O})$ is equal to $P'(b^1)$ from (22) and (24).

The number density perturbations appearing in (28) and (29) can be determined by the linearized Eulerian continuity equation

$$i\omega n' = P' - L' - w \frac{d\bar{n}}{dz} - \bar{n} \nabla \cdot \mathbf{v} \quad (30)$$

Application of (30) to O, M, N₂, and the *b*¹ state gives

$$i\omega n'(\text{M}) = -w \frac{d\bar{n}(\text{M})}{dz} - \bar{n}(\text{M}) \nabla \cdot \mathbf{v} \quad (31)$$

$$i\omega n'(N_2) = -w \frac{d\bar{n}(N_2)}{dz} - \bar{n}(N_2)\nabla \cdot \mathbf{v} \quad (32)$$

$$i\omega n'(O) = -L'(O) - w \frac{d\bar{n}(O)}{dz} - \bar{n}(O)\nabla \cdot \mathbf{v} \quad (33)$$

$$i\omega n'(b^1) = P'(b^1) - L'(b^1) - w \frac{d\bar{n}(b^1)}{dz} - \bar{n}(b^1)\nabla \cdot \mathbf{v} \quad (34)$$

Equations (22)–(29) and (31)–(34) together with relations from linear acoustic gravity wave theory (see appendix) essentially suffice to determine η and χ , but we also need to use

$$\frac{I'}{\bar{I}} = \frac{n'(b^1)}{\bar{n}(b^1)} \quad (35)$$

which follows from (21). The single-level Krassovsky ratio is thus

$$\eta = \frac{n'(b^1)/\bar{n}(b^1)}{I'/\bar{I}} \quad (36)$$

and χ is given by (6). After some algebra we obtain

$$\eta = \{i\omega + k_5\bar{n}(N_2) + A_2\}^{-1} \left\{ \frac{\bar{n}(O)\bar{P}(b^1)}{\bar{n}(b^1)[i\omega\bar{n}(O) + 2\bar{P}(b^1)]} \cdot \left[i\omega(f_3 - 2) + \frac{2f_2}{\bar{H}(O)} - 2f_1 \right] + \frac{f_2}{\bar{H}(b^1)} - f_1 - k_5\bar{n}(N_2)f_3 \right\} \quad (37)$$

$$\chi = (\gamma - 1)\eta + \frac{f_2}{f_1} \frac{1}{\bar{H}(b^1)} \quad (38)$$

In writing (37) and (38) we have introduced the background scale height

$$\bar{H} = \left(-\frac{1}{\bar{n}} \frac{d\bar{n}}{dz} \right)^{-1} \quad (39)$$

and the additional gravity wave polarization relation

$$\frac{n'(M)}{\bar{n}(M)} \equiv f_3 \frac{T'}{\bar{T}} \quad (40)$$

(In the isothermal case, $i\omega f_3 = -f_1 + f_2[\bar{H}(M)]^{-1}$). The vertical scale height of the b^1 state in the background atmosphere is the same as the intensity scale height of the reference state atmosphere since $\bar{I} \propto \bar{n}(b^1)$. From (27), and for an isothermal atmosphere it follows that

$$\frac{1}{\bar{H}(b^1)} = \frac{2}{\bar{H}(O)} + \frac{A_2}{\{A_2 + k_5\bar{n}(N_2)\}} \frac{1}{\bar{H}(M)} \quad (41)$$

where we have assumed $\bar{H}(N_2) = \bar{H}(M)$.

Discussion of χ and η

Our discussion of χ and η is facilitated by rewriting the combination of (37) and (38) as

$$\chi(1 + \alpha)(1 + \beta) = \chi_{ss} + \alpha(1 + \beta) + \beta \left\{ \frac{[1 - f_3(\gamma - 1)]\bar{H}(M)}{\bar{H}(b^1)} - \frac{(\gamma - 1)f_3k_5\bar{n}(N_2)}{k_5\bar{n}(N_2) + A_2} \right\} \quad (42)$$

$$\eta(\gamma - 1)(1 + \alpha)(1 + \beta) = \chi_{ss} + \alpha(1 + \beta) - \frac{\beta(\gamma - 1)f_3k_5\bar{n}(N_2)}{k_5\bar{n}(N_2) + A_2} - \frac{\bar{H}(M)}{\bar{H}(b^1)} \cdot \{1 - f_3(\gamma - 1)\} \{1 + \alpha(1 + \beta)\} \quad (43)$$

where

$$\alpha = \frac{i\omega}{k_5\bar{n}(N_2) + A_2} \quad (44)$$

$$\beta = \frac{2\bar{P}(b^1)}{i\omega\bar{n}(O)} \quad (45)$$

$$\chi_{ss} = 5 - 2\gamma - \frac{k_5\bar{n}(N_2)}{k_5\bar{n}(N_2) + A_2} \quad (46)$$

In the above, χ_{ss} is the expression for χ given by *Tarasick and Shepherd* [1992a] (their equation (26)), who limited their investigation to the case of steady state chemistry. The function χ accounts for nonsteady state chemistry; it is a complex function that varies with height while χ_{ss} is purely real.

There are two conditions that must be met for χ given by (42) to reduce to χ_{ss} , that is, for nonsteady state chemistry to be unimportant. Both α and β must satisfy

$$|\alpha| \ll 1 \quad |\beta| \ll 1 \quad (47)$$

The second condition is equivalent to the neglect of the loss term (24) in the atomic oxygen continuity equation (33). Numerical evaluation of $|\beta|$ shows that $|\beta| \ll 1$ even for periods of 10 h and longer. Numerical estimates of $|\alpha|$ show, however, that the first inequality in (46) can be violated for periods shorter than a few minutes corresponding to acoustic or evanescent waves. When both conditions (47) are satisfied, χ_{ss} is a good approximation to χ and η is given by

$$\eta \approx \frac{\chi_{ss}}{(\gamma - 1)} - \frac{\bar{H}(M)}{\bar{H}(b^1)} \left\{ \frac{1}{(\gamma - 1)} - f_3 \right\} \quad (48)$$

When only the second of conditions (47) is satisfied,

$$\chi \approx \frac{\chi_{ss}[k_5\bar{n}(N_2) + A_2] + i\omega}{i\omega + k_5\bar{n}(N_2) + A_2} \quad (49)$$

and η is given by (48) with χ_{ss} replaced by (49).

In the extremely short period acoustic limit $\omega \rightarrow \infty$,

$$\chi \rightarrow 1 + \frac{1}{i\omega} \{(4 - 2\gamma)(k_5\bar{n}(N_2) + A_2) - k_5\bar{n}(N_2)\} \quad (50)$$

$$\eta \rightarrow \frac{1}{(\gamma - 1)} - \frac{\bar{H}(M)}{\bar{H}(b^1)} \left\{ \frac{1}{(\gamma - 1)} - f_3 \right\} \quad (51)$$

and in the extremely long period gravity wave limit $\omega \rightarrow 0$,

Table 1. Parameter Values Employed in the Numerical Calculations of χ , η , and η_E for the O₂ Atmospheric Airglow

Parameter	Numerical Value	Reference
\bar{T}	204.3 K	March, 18°N at $z = 75$ km <i>Garcia and Solomon</i> [1985] same as for \bar{T}
$\bar{H}(M)$	6290 m	
g	9.579 m s^{-2}	
A_2	0.083 s^{-1}	<i>Deans et al.</i> [1976]
k_1	$4.7 \times 10^{-45} (300/\bar{T})^2 \text{ m}^6 \text{ s}^{-1}$ $= 1.0 \times 10^{-44} \text{ m}^6 \text{ s}^{-1}$	<i>Campbell and Gray</i> [1973]
$\bar{n}(\text{O}_2)$	$0.21 \bar{n}(M)$	
$\bar{n}(\text{N}_2)$	$0.79 \bar{n}(M)$	
k_5	$2.2 \times 10^{-21} \text{ m}^3 \text{ s}^{-1}$	<i>Martin et al.</i> [1976]
k_6	$4 \times 10^{-23} \text{ m}^3 \text{ s}^{-1}$	<i>Martin et al.</i> [1976]
k_5^*	$k_5 + \frac{\bar{n}(\text{O}_2)}{\bar{n}(\text{N}_2)} k_6$ $\cong 2.21 \times 10^{-21} \text{ m}^3 \text{ s}^{-1}$	

$$\chi \rightarrow \frac{\gamma \bar{H}(M)}{\bar{H}(b^1)} + (\gamma - 1) \frac{k_5 \bar{n}(\text{N}_2)}{[k_5 \bar{n}(\text{N}_2) + A_2]} \quad (52)$$

$$\eta \rightarrow \frac{k_5 \bar{n}(\text{N}_2)}{[k_5 \bar{n}(\text{N}_2) + A_2]} + \frac{\bar{H}(M)}{\bar{H}(b^1)} (1 + f_3) \quad (53)$$

The approximation of the function χ by χ_{ss} is invalid only when the period of the waves is near either the very short acoustic periods or the very long gravity wave periods.

Numerical Evaluation and Discussion of χ , η , and η_E

We now illustrate the theoretical developments of the previous sections by numerically evaluating χ , η , and η_E for the simplified O₂ atmospheric airglow model already described. The undisturbed model atmosphere is taken to be that of *Garcia and Solomon* [1985] corresponding to conditions at 18°N in March. The temperature, major gas scale height, and acceleration of gravity in the reference state are set to their values at a height of 75 km in the *Garcia and Solomon* [1985] model atmosphere. Table 1 summarizes the values of parameters used in the numerical evaluations of

this section. The quantity k_5^* given in this table is a modified quenching coefficient, which was defined to take into account the quenching of O₂(b^1) by N₂ and O₂. It is related to the quenching coefficient for N₂ alone (k_5) and for O₂ alone (k_6) by $k_5^* = k_5 + \bar{n}(\text{O}_2)k_6/\bar{n}(\text{N}_2)$. We found that k_5^* differs from k_5 by less than one percent. Figure 1a gives the altitude-dependent basic state inverse scale heights for M = O₂ + N₂, atomic oxygen, and the excited b^1 state of O₂ (also the reference state intensity inverse scale height). The b^1 state inverse scale height or intensity inverse scale height changes sign just above 90 km while $(\bar{H}(\text{O}))^{-1}$ changes sign several kilometers higher. The sign changes in $(\bar{H}(\text{O}))^{-1}$ and $(\bar{H}(b^1))^{-1}$ are of course associated with the maxima in the altitude profiles of $\bar{n}(\text{O})$ and $\bar{n}(b^1)$ (Figure 1b). The basic state scale heights of O and b^1 are negative and have large magnitudes below the maxima in the altitude profiles of $\bar{n}(\text{O})$ and $\bar{n}(b^1)$, corresponding to the rapid increase of $\bar{n}(\text{O})$ and $\bar{n}(b^1)$ with height in this region of the atmosphere. Accordingly, $(\bar{H}(\text{O}))^{-1}$ and $(\bar{H}(b^1))^{-1}$ have small negative values below the maxima in the number density altitude profiles.

Figure 2 compares altitude profiles of χ (from (42)) and χ_{ss} (from (46)) for periods of 1, 10, and 100 min and a horizontal

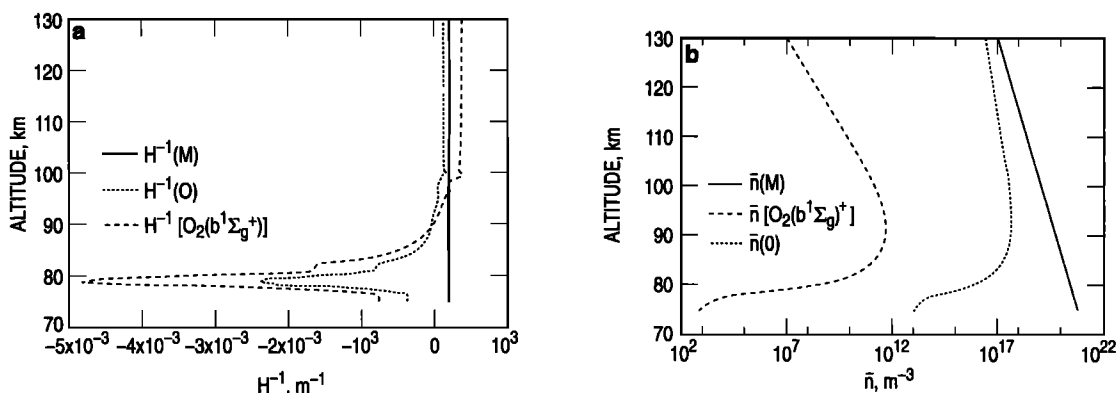


Figure 1. (a) Altitude profiles of inverse scale height for M = O₂ + N₂ (solid), O (short dash), and the b^1 state of O₂ (long dash) in the background or reference state atmospheric model of *Garcia and Solomon* [1985] for March and 18°N. (b) Height profiles of \bar{n} for M, O, and b^1 in the same model atmosphere as in Figure 1a.

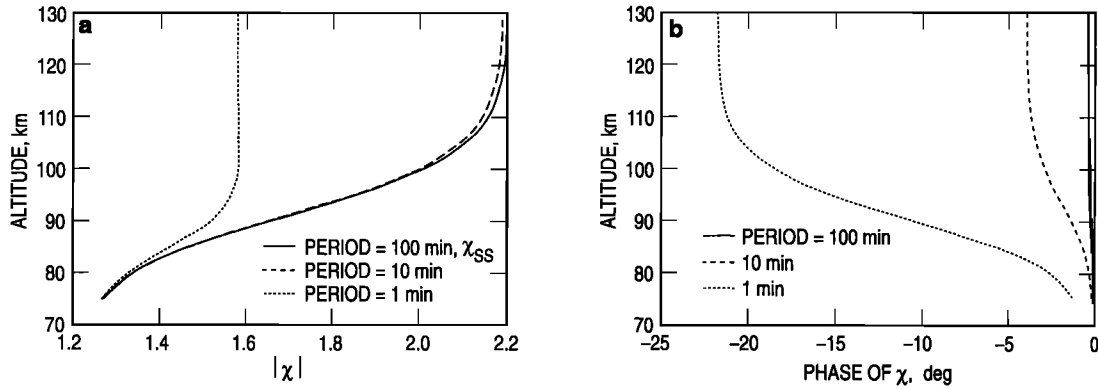


Figure 2. (a) Amplitude of χ and χ_{ss} and (b) phase of χ versus height for the O_2 atmospheric airglow for a horizontal wavelength of 100 km and periods of 1 (dotted), 10 (dashed), and 100 (solid) min. The value χ_{ss} is a real number and is independent of period. At a period of 100 min, $|\chi|$ is essentially identical to χ_{ss} .

wavelength of 100 km. At a period of 100 min, $|\chi|$ is essentially identical to χ_{ss} (Figure 2a) and the phase of χ is nearly 0° (Figure 2b). At the shorter period of 10 min there is a clear difference between $|\chi|$ and χ_{ss} at altitudes above about 95 km; in addition, the phase of χ is nonzero, decreasing from about 0° at a height of about 75 km to about -4° at altitudes above about 110 km. At a period of 1 min, $|\chi|$ is very different from χ_{ss} above about 80 km; at the highest altitudes in Figure 2a, $|\chi|$ is only about 70% of χ_{ss} . The phase of χ is also very different from 0° at a period of 1 min; at altitudes above about 110 km the phase of χ is approximately -22° .

In general, χ varies significantly with altitude (Figure 2). The magnitude of χ increases monotonically with height and approaches a value independent of height at large altitudes. The phase of χ decreases monotonically with height and also approaches a constant negative value high in the model atmosphere. When the integrals in (11) are not too sensitive to the local values of χ , the constant χ formula (13) can be used with a representative value of χ for the emission layer to provide a good approximation to (11) [Tarasick and Shepherd, 1992a, b]. The integrals in (11) become sensitive to the local values of χ when $\nabla \cdot \mathbf{v}$ varies rapidly through the emission layer and results in strong destructive interference between emission fluctuations coming from different heights with different χ weightings.

Figure 3 shows altitude profiles of the amplitude (Figure

3a) and phase (Figure 3b) of η as given by (43) for periods of 1, 10, and 100 min and for a horizontal wavelength of 100 km. Unlike χ , η varies strongly in both amplitude and phase as a function of height. The large variations in η versus height reflect directly the changes in altitude of the inverse scale height of the b^1 state (see (43) for the dependence of η on $(\bar{H}(b^1))^{-1}$). The peak in $|\eta|$ just below about 80 km altitude (Figure 3a) reflects the sharp minimum in $(\bar{H}(b^1))^{-1}$ at this height (Figure 1a). The dramatic change in the phase of η just below about 100 km altitude (Figure 3b) reflects the change in sign of $(\bar{H}(b^1))^{-1}$ at this height (Figure 1a).

Figure 4 shows the vertically integrated Krassovsky ratio η_E as a function of period, obtained by carrying out the integration of χ over height according to (11) (or alternatively, by carrying out the integration of η over height using (14)). Figure 4 also shows η_E obtained by vertically integrating χ_{ss} . The horizontal wavelength in Figure 4 is 100 km. The differences between η_E calculated from χ or η , and η_E calculated from χ_{ss} , are significant at acoustic periods. In general, $|\eta_E|$ increases with period, though peaks and troughs occur in $|\eta_E|$ associated with evanescent waves and with interference effects (Figure 4a). The phase of η_E varies strongly with period, especially at evanescent periods and in association with interference effects (Figure 4b); η_E is nearly real only in a small range of periods at the shortest gravity wave periods.

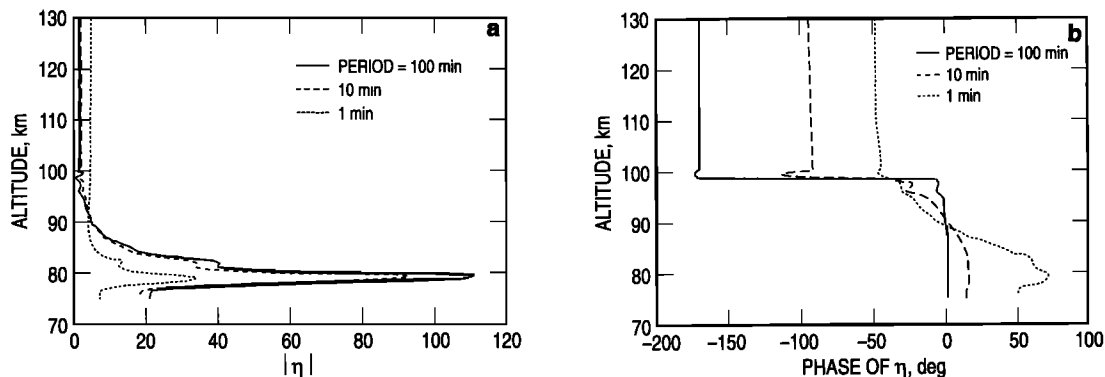


Figure 3. Similar to Figure 2 for η .

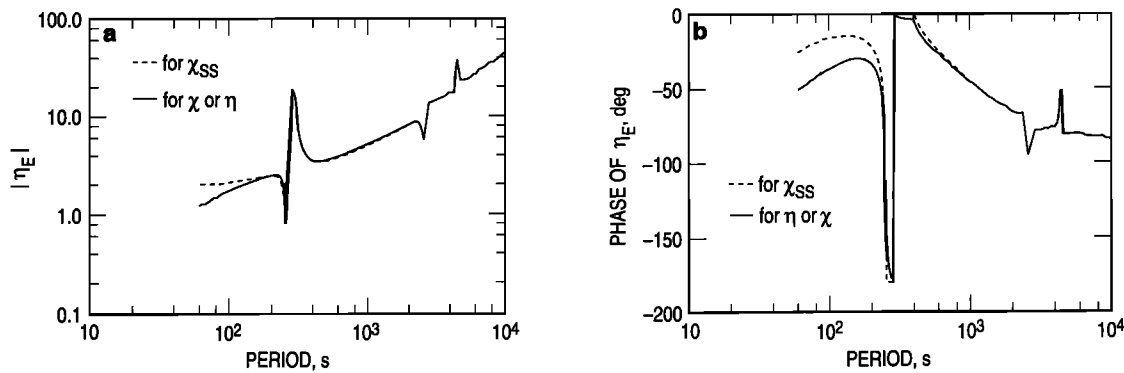


Figure 4. (a) Amplitude and (b) phase of the vertically integrated Krassovsky ratio η_E for the O_2 atmospheric airglow as a function of period using χ or η (solid) and χ_{SS} (dashed). The horizontal wavelength is 100 km.

Minor Constituent Scale Heights and OH Nightglow

We apply the theory of gravity wave induced fluctuations in airglow to the emissions of excited OH^* in order to explore the effects of minor constituent scale heights on Krassovsky's ratio. *Tarasick and Hines* [1990, p. 1110] have noted that "the concentration gradient for the reactive species [does not] appear anywhere in their development," ... "a significant difference from the work of authors who have used a thin-layer, rather than a vertically integrated model (Walterscheid et al., 1987; Frederick, 1979)." Vertical displacement can effect emissions by the dilation and contraction of airglow layers [*Hines and Tarasick*, 1987] and by perturbing chemical production and loss. The nonappearance of concentration gradients in the formulation of *Hines and Tarasick* [1987] refers to the fact that integrated wave-driven fluctuations that are caused by dilation and contraction alone assume the form $(\bar{I} dw/dz)$ (see (8)). The dependence explored here and in the work by *Walterscheid et al.* [1987] originates in wave-disturbed chemical production.

Dependence of η_E on O Scale Height

Walterscheid et al. [1987] considered the effects of wave-driven fluctuations in the OH nightglow emitted from an infinitesimal layer of the emission region. The nightglow arises from the radiative decay of excited OH^* produced by the reaction $O_3 + H \rightarrow OH^* + O_2$. They found that the vertical advection of atomic oxygen exerts a strong control on the single-level η , an effect manifest in terms of a sensitivity of η to the atomic oxygen scale height $\bar{H}(O)$. The sensitivity of η to the O scale height is a result of O_3 production related to O transport via the reaction $O + O_2 + M \rightarrow O_3 + M$. The smaller the O scale height the greater the O perturbation for a given vertical velocity. We will show here that η_E also depends on $\bar{H}(O)$ though that dependence is different from the dependence of the single-level η on $\bar{H}(O)$. The influence of $\bar{H}(O)$ on η_E arises because the effects of positive and negative perturbations of O on O_3 do not cancel along the line of sight through an extended emission region. Ozone production depends on the product of O, O_2 , and M, and a given perturbation of O has a different effect on O_3 production depending on the specific altitude of the perturbation.

To examine the dependence of η_E on $\bar{H}(O)$, we parameterize the altitude profile of atomic oxygen concentration so that the bottomside scale height can be controlled through a single parameter. Varying $\bar{H}(O)$ does not affect the basic-state emission profile in these calculations. The parameterization is adjusted to give a good fit overall to the nightside profile of O given by *Winick* [1983]. The bottomside scale height for the *Winick* [1983] model is 0.75 km. We varied the scale height about this value by a factor of 3 to give scale heights of 0.25 and 2.25 km, in addition to the nominal value of 0.75 km. Other basic state quantities are the same as those used by *Schubert et al.* [1991].

Figure 5a shows the results for the amplitude of η_E versus period for the three different choices of O scale height and for a horizontal wavelength of 1000 km. To simplify the application of the theoretical formulas, the calculations are performed for an isothermal atmosphere without dissipation. The dependence on $\bar{H}(O)$ is quite pronounced. Peak values of $|\eta_E|$ range over 2 orders of magnitude as $\bar{H}(O)$ varies between 0.25 and 2.25 km. The greatest differences occur at longer wave periods where strong interference effects are evident. At these periods there is a general tendency for $|\eta_E|$ to increase as the magnitude of $\bar{H}(O)$ is decreased. At shorter periods the curves for $\bar{H}(O) = 0.75$ and 2.25 km cross so that $|\eta_E|$ first increases and then decreases with increasing scale height. Figure 5b shows the corresponding phases. The differences in the phase of η_E due to different atomic oxygen scale heights are not dramatic until interference effects become dominant at long periods. At gravity wave periods near evanescence (where the curves terminate) there is a tendency for the phase to increase with decreasing magnitude of $\bar{H}(O)$.

The results agree with the single-level results in showing a dependence of η_E on atomic oxygen scale height. However, the results of the present calculations for η_E are substantially different from those for η . The single-level results of *Walterscheid et al.* [1987] do not exhibit interference effects. The single-level results show a systematic increase in the amplitude of Krassovsky's ratio with decreasing magnitude of $\bar{H}(O)$ and a systematic increase of phase with increasing magnitude of $\bar{H}(O)$. Where interference effects are not dominant in the present calculations these systematic dependencies are not seen. In fact, the dependence of the phase of η_E (Figure 5b) on atomic oxygen scale height is reversed.

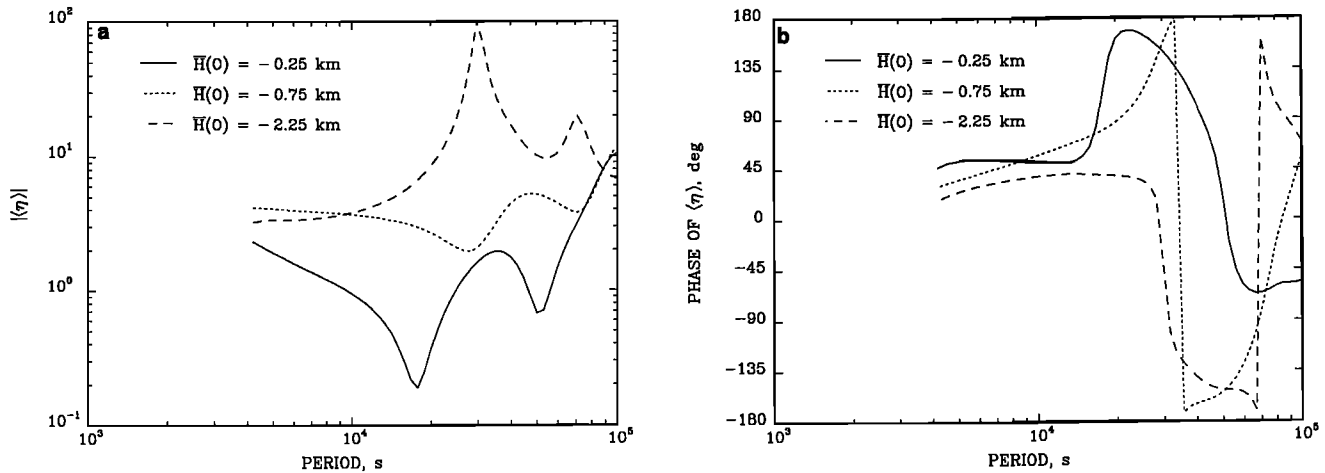


Figure 5. (a) Amplitude and (b) phase of η_E versus period for OH airglow fluctuations for three different values of the basic-state atomic oxygen scale height and for a horizontal wavelength of 1000 km. The calculations are for an isothermal atmosphere without dissipation.

The differences in the present extended layer results and the single level results are due to interference effects, especially at longer periods, and also to the fact that the perturbing effects of O advection compete with other perturbing effects of dynamics including the vertical advection of other species, notably, O_3 . This competition dictates in a complicated way how I' and T'_l vary over the emission layer.

We have not changed the basic-state profiles of other constituents, especially O_3 , in response to the changes in the basic-state O profile associated with the scale heights that depart from the *Winick* [1983] model. While self-consistency argues that this be done, we believe that such modifications to the basic state will not alter the conclusion that η_E depends on O scale height. The vertical advection of minor constituents can affect perturbed production minus loss when the vertical advection of basic-state production minus loss $\bar{P} - \bar{L}$ is nonzero. In steady state, nonzero $\bar{P} - \bar{L}$ is maintained by vertical diffusion. Calculations based on *Winick* [1983] not reported here (see also *Schubert and Walterscheid* [1988] and *Allen et al.* [1984]) show that the vertical gradient of $\bar{P} - \bar{L}$ can be significant below ~ 85 km. Variations in $\bar{H}(O)$ should alter the vertical gradient of $\bar{P} - \bar{L}$. When η_E is sensitive to $\bar{H}(O)$, practical application of the theory would obviously benefit from measurements of the altitude profile of atomic oxygen. However, if the O scale height is unknown, the theory can be used to estimate it by varying the scale height until model predictions for η_E agree with measured values, assuming that the other parameters required for interpretation of model results are also known.

Discussion

The basic result of this paper is that the formulas for η_E for ground-based airglow observations developed by *Tarasick and Hines* [1990] and *Schubert et al.* [1991] are equivalent for the case of an inviscid atmosphere. The demonstration of this fact is considered nontrivial since the numerical integrations required to evaluate (11) have not been extensive with one exception when quantities such as χ vary with height [*Tarasick and Shepherd*, 1992a, b]. *Hines and Tarasick* [1987] have asserted that their results could be obtained

from a linearized Eulerian approach through integration by parts, but details of this demonstration were not provided.

Though agreement between the two theories has been demonstrated for an inviscid atmosphere, it is nonetheless essential to include viscous and thermal diffusion for a realistic evaluation of η_E for airglow emissions originating at mesospheric heights and above [*Hickey*, 1988a, b; *Schubert et al.*, 1991]. The theory of *Tarasick and Hines* [1990], represented by (11) in this paper, is presently limited by its neglect of these diffusive properties for waves with short vertical wavelengths.

Conclusions

We have compared theories for gravity wave induced fluctuations in the intensity and associated temperature variations of airglow emissions. The theories considered are the Eulerian formalism of *Schubert et al.* [1991] and the modified Lagrangian formalism of *Hines and Tarasick* [1987] and *Tarasick and Hines* [1990]. We applied both theories to the analysis of O_2 atmospheric airglow fluctuations [*Tarasick and Shepherd*, 1992a]. Finally, we clarified the effects of minor constituent scale heights on vertically integrated emissions when vertical displacement perturbs chemical production. Our major findings are

1. The formulas for η_E by *Tarasick and Hines* [1990] and *Schubert et al.* [1991] are in agreement for ground-based observation and an inviscid atmosphere.
2. The calculation of η_E using the theory of *Tarasick and Hines* [1990] requires determination of their function χ ; we show that χ is simply related to the single-level Krassovsky's ratio η of *Schubert et al.* [1991].
3. The general relationship between χ and η is applied to a simple chemical-dynamical model of the O_2 atmospheric airglow and the altitude dependence of χ and η is evaluated for different periods. The assumption of steady state chemistry is found to be very good at gravity wave periods but not at acoustic periods ≤ 1 min.
4. The *Tarasick and Hines* [1990] formula for η_E does not explicitly depend on the scale heights of the minor constituents involved in airglow chemistry. However, the

formula for η_E implicitly depends upon these scale heights through χ when vertical displacement perturbs chemical production minus loss. Model calculations for the OH nightglow show that η_E depends on atomic oxygen scale height.

Appendix: Polarization Factors

For an isothermal inviscid atmosphere μ and ν are given by

$$\mu = \frac{1 - \frac{\gamma}{2} + \left(1 - \frac{\omega^2}{k_x^2 \bar{c}^2}\right) \frac{\omega^2 \bar{c}^2}{g^2}}{1 - \frac{\omega^4}{k_x^2 g^2}} \quad (\text{A1})$$

$$\nu = \frac{-k_z \bar{c}^2 / g}{1 - \frac{\omega^4}{k_x^2 g^2}} \quad (\text{A2})$$

In (A1) and (A2), $\bar{c}^2 = \gamma R \bar{T}$, γ is the ratio of specific heats, R is the gas constant for dry air, k_x is the horizontal wave number of the perturbation, k_z is the vertical wave number, ω is the wave angular frequency, and g is the acceleration of gravity. The vertical wave number is obtained from the dispersion relation for gravity waves in an isothermal inviscid atmosphere

$$k_z^2 = k_x^2 \left\{ \frac{(\gamma - 1)g^2 / \bar{c}^2}{\omega^2} - 1 \right\} + \frac{\omega^2 - \frac{\bar{c}^2}{4\bar{H}^2(M)}}{\bar{c}^2} \quad (\text{A3})$$

where \bar{H} is the vertical scale height of the major gas $M = \text{O}_2 + \text{N}_2$ in the undisturbed background state. For an isothermal inviscid atmosphere the polarization factor f_1 is given by

$$f_1 = \frac{-i\omega}{(\gamma - 1)} \quad (\text{A4})$$

$$f_2 = \left(\frac{\omega}{k_x}\right) \frac{(\alpha + i\delta)(1 - \beta) - i\delta}{i\delta(\delta - i\alpha) - (1 - \beta)[-i\beta + i\delta(\delta - i\alpha)]} \quad (\text{A5})$$

where

$$\alpha = \frac{1}{k_x \bar{H}} \quad (\text{A6})$$

$$\beta = \frac{\omega^2}{g \bar{H} k_x^2} \quad (\text{A7})$$

$$\delta = \frac{k_z}{k_x} + \frac{i\alpha}{2} \quad (\text{A8})$$

The polarization factors f_1 , f_2 , μ , and ν given above neglect the mean wind. The mean wind profile does affect airglow fluctuations mainly by causing the wave frequency to be Doppler shifted to higher or lower frequencies in the airglow layer. The practical application of the theory requires a determination of the winds in the emission region, especially for waves with slow horizontal phase speeds, or sufficient information about the waves to deduce the wind

speed [Hecht and Walterscheid, 1991]. The implications of mean winds for model results have been dealt with by Schubert and Walterscheid [1988]. Since the focus of the present paper is a comparison of theories and not interpretation of specific data sets, this aspect of the general problem is not addressed in the present paper.

Acknowledgments. This work was supported by the Aerospace Sponsored Research Program (R.L.W. and G.S.), by NSF under grant ATM90-00216 (R.L.W.), and by NASA under grants NAS8-38333 (M.P.H.) and NAGW-2887 (R.L.W.). We thank Michael McNab for carrying out the calculations of OH nightglow fluctuations.

The Editor thanks C. O. Hines, D. W. Tarasick, R. A. Viereck, and another referee for their assistance in evaluating this paper.

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(Received February 4, 1993; revised November 12, 1993; accepted November 17, 1993.)