Involving Adults in the Learning of Mathematics

by

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Abstract

Preparing adults for the mathematics skills necessary for the twenty first century is a challenge. It is especially challenging if those adults have had past traumatic experiences learning mathematics. Involving students in the learning process involves more than the presentation of mathematical concepts by the teacher and the simple regurgitation of rote memorization by the students. It requires a concerted effort by both the teacher and the student. The teacher must present mathematical concepts that are inclusionary, challenging, in an environment created for learning, in class settings that are technologically advanced, and geared to the student's level. The student must assimilate the information through cooperative learning exercises and writing assignments, which incorporate the learning objectives of the current class and the ideas learned in previous classes.

Involving Adults in the Learning of Mathematics

I begin each mathematics class with the same question: "Who in this class would consider themselves 'math-phobic'?" Invariably about 90% of the class raise their hands. (I have always suspected that the other 10% would like to, but are afraid of something!) After I survey the response, I usually ask one of the students how much one apple would cost if two apples cost $1.00. "Fifty cents," exclaims the student, "but *that* was easy!" My response is always the same: "That was *mathematics!*"

After some classroom discussion, we generally come to the realization that it isn't the mathematics that frightens the students, rather it was some teacher (disguised as an educator, parent or other tutor) who showed some form of disgust (anger, frustration, etc) when an immediate response wasn't forthcoming from the student. These students aren't math-phobic, they are *teacher*-phobic!

Teachers of mathematics are, perhaps, the most likely to spread "teacher-phobia." The question, then, begs to be answered: "What can I do to help my students overcome teacher-phobia?" "How can I ensure that I am not the cause of *teacher-phobia?"* The answers to these questions may seem like common sense, but to paraphrase Rousseau, common sense isn't all that common.

The National Council of Teachers of Mathematics (NCTM) must have had this growing gulf of math anxiety and rampant spread of teacher phobia in mind when it developed its goals for the teaching of mathematics. These goals are designed to replace the rote memorization of mathematical rules and concepts with innovative, interactive activities which help not only to develop the concepts, but also to anchor them in the minds of the students. This process of cooperative learning creates a fresh environment in which students begin to realize that learning mathematical concepts is not an impossible task, and can even be exciting.

The NCTM encourages teachers to emphasize the learning of the value of mathematics, build confidence in the students' ability to do mathematics, perform problem solving techniques, reason mathematically, and to communicate their knowledge of mathematics both orally and verbally (NCTM, 1989).

The accomplishment of these goals were not left to chance. In 1991, the NCTM published the "Professional Standards for Teaching Mathematics" which, according to Showalter,

> recommends that teachers select interesting and intellectually stimulating mathematical tasks, present opportunities for students to deepen their understanding of mathematics and its applications, promote the investigation of mathematical ideas, use technology to pursue these investigations, find connections to previous and developing knowledge, and employ cooperative-learning experiences (p. 5).

While the NCTM's focus is primarily on the development of the mathematics skills of K - 12 grade students, the lessons and guidelines serve as intelligent guideposts for anyone teaching these necessary, yet often frustratingly confusing, skills. These goals represent the collective thoughts of the literally thousands of years of NCTM's members' teaching experience, and focus on the student's ability to achieve rather than memorize.

The NCTM is not the only mathematics organization to show concern for the epidemic lack of mathematics enthusiasm. In 1995 the American mathematical Association of Two Year colleges released its report emphasizing the need to establish a "new paradigm," in which the main focus was placed on the teaching of the older student. The guidelines stressed the need to involve these adults in their education through the use of technology, hands on investigations, and cooperative learning strategies.

This establishment of standards for educational institutions echoes the concern put forward by the National Council of Supervisors of Mathematics in a 1977 point paper. This position paper described "the essential mathematical competencies" that will be required by all students for "responsible adulthood" (NCSM, 1994, p. 388). These competencies had twelve components: Problem solving, communicating mathematical ideas, mathematical reasoning, applying mathematics to everyday situations, alertness to the reasonableness of results, estimation, appropriate computational skills, algebraic thinking, measurement, geometry, statistics, and probability (p. 389).

The first eight of these components deal with the less concrete of the concepts, and are the focus of this paper. The last four may be satisfied by curriculum changes which may incorporate these valuable concepts. However, we must not be lulled into the belief that merely *presenting* the material is synonymous with *learning* the material. Offering the last four components must be done with the vigorous institution of the first eight, for it is this first group that is the foundation of the last four.

As mathematics teachers we should keep the goals of these, and similar organizations, in mind as we present our material. The "how's" of each goal vary and can be as personal as what we wear to class. However, there are some common elements in teaching that emphasize these important guidelines.

<u>Use of Modern Technology</u>

There is no question that the use of modern computing tools prepares the student for the mathematics of the work place. The important technological developments of the past couple of decades have enabled engineers and accountants, alike, to apply the mathematics of their respective fields effectively and efficiently. It follows that if our institutions are the breeding grounds for such professions then our students must be not only aware of the applications of this technology within their chosen fields, but also feel comfortable with the applications.

Computers seem ubiquitous. At every turn some form of modern computing machine can be found. Yet computer usage among students, beyond word processing and games, is not as common as some teachers and administrators believe, or would hope. Although this statement is strictly based on empirical data, it merits a close look by all science-based educators.

Administrators and educators understand the importance of using modern technology in the classroom, and often discuss new technological methods to improve learning. They understand that it is incumbent upon each educator in a mathematical setting to "model the use of appropriate technology" (AMATYC, 13). In the classroom, teachers use computer-generated overlays and large screen monitors to demonstrate mathematical concepts. Although this technological advance in the process of learning has merit, an important element is generally omitted. Since not all students have a computer at their disposal, there is little opportunity to apply the "computer approach" to mathematics to solidify the learning of the day. Even those students who have a computer, usually do not have the software needed to practice the concepts.

Small regional centers and community colleges have the most difficulty in maintaining a bank of computers for students. Even though many have libraries that provide computer resources, these resources are routinely restricted to word processing. In some cases, the Center Director has a separate computer for student use. But even then, the resource can be used only by one student at a time, and only during office hours.

There are four approaches to these problems. First, mathematics classes should be taught in a computer laboratories, when available. This will give each student the opportunity to "play with mathematics" and explore the more complex concepts more freely. This does not mean that small regional colleges must expend large portions of their budget to get reach a goal of "one computer, one student." In fact this single computer approach may "limit students' decision making process," and robs them of experiencing the problem solving processes associated with group work (Heid, 1990). If computer laboratories are not available, then hand held graphing calculators should be available for the students.

Second, as educators we must feel comfortable with the use of computers within our field. This should include our comfort with *when* to use technology and *when not* to use it. The AMATYC encourages students to use the technology as an enhancement to the study of mathematics, not as a replacement classroom instruction. In this vein, the use of technology allows for the exploration of curves, the impact of changing variables, and a myriad other exciting explorations that used to take precious classroom time to demonstrate. The fear by many educators is that too many class hours will be devoted to teaching the software and not the applications. To this end the AMATYC encourages curriculum designers to assess the amount of time spent "learning how to use computers and calculators effectively" to ensure that it is "compatible with the expected gain in learning mathematics" (AMATYC, 1995). Heid recommends devoting "a reasonable amount of time to preparing students for the use of computer software." (Heid, 197). This can be done effectively by using software that engages the student (either through role playing or another interactive game) while at the same time teaching them to perform mathematical manipulations. The earlier this is done in the educational process the less time must be spent later when using a computer is essential.

Addtionally, we should take care to not replace the underlying mathematics involved in problem solving with straight application on computers. An important man once said, "Those who know 'how' will always work for those who know 'why'." Students should be able to recognize when data has been entered incorrectly, or software has the dreaded "glitch." Teaching the "why's" of mathematical concepts goes a long way in determining if the answer received is the answer expected.

Third, the students should be encouraged strongly to purchase a computer or a hand held graphing calculator when they first sign up for courses in mathematics. As stated earlier, involving students in the problem solving and decision making with the use of computers and calculators can enhance their ability to perform more complex problem solving in later courses.

Fourth, software and hardware should be made available at discounted rates to help defray the costs of such a large investment. While this may seem an unreasonable concept on the surface, it has merit. College bookstores, like many large stores, can buy in such large quantities that their costs are dramatically lower. While it is understandable that these stores are not "altruistic," they could offer essential items like these are reduced profit margins.

If the old adage, "Practice makes perfect," is true, we owe it to our mathematics students to give them the opportunity to practice as much as possible. Technology can help only if it is available.

Mathematical Proofs

In mathematics, there is little that is more elegant, and little that is more exciting than to perform a proof. Teachers of mathematics know this and enjoy the ability to "show off their skills" in front of the class. For those curricula that require advanced mathematics, the proofs are an integral part of the lesson. However, many in our profession forget that it is not necessarily in the best interest of students in non-technical curricula to overwhelm them with complex numerical manipulations.

Am I suggesting the teachers never do proofs? Not at all! However, the teacher must evaluate the level of the class, the necessity of the proof, and whether the proof will contribute more to the understanding of the lesson than to the confusion of the students.

We should be able to divide our students into two distinct groups. The first are those who wish to go on to "bigger and better things" in mathematics. This group usually has a firm foundation in mathematics, is convinced that mathematics is "learnable"," and can be enjoyable. Their background is such that proofs encourage them "to explore mathematical topics; develop and refine their own ideas, strategies, and methods; and reflect on and discuss mathematical concepts and procedures (Garofalo, 1989, p. 504)."

The second group, however, are those who have been overwhelmed by the concepts of mathematics. They are the classic "teacher phobes" discussed earlier. Both the NCSM and NCTM standards encourage the type of reasoning described for the first group. This group, however, will take some time, patience, and understanding before they are able to assimilate the mathematical reasoning associated with proofs.

For instance, when I teach an algebra class to students in group two, I always present the Arabic proof of the quadratic equation. As we work through the proof, I pause at each logical juncture and emphasize that the previous step was a procedure that we had covered in a previous lesson. By the end of the proof, the students are often amazed that the proof was so easy to perform, and made up of manipulations that they knew. Their mathematics skills, reasoning, and confidence have been reenforced.

Additionally, with the emergence of new classroom computer technology, there is some argument of classical proofs over exhaustive computer proofs. Although that argument will probably persist for years to come, there is some merit for computer proofs, especially in non-technical mathematics classes. Should a teacher feel that a proof is absolutely necessary, she should evaluate using computers to perform an exhaustive proof rather than the classical theoretical proof that may confuse more than help.

Make Mathematics Relevant and Fun

The classic presentation of theoretical mathematics with no immediate application to activities relevant to the student is boring at the best, demoralizing at the worse. The mistake that some mathematics teachers at colleges and universities make is in presenting a relatively complex concept with only a cursory mention of how it is used in nature. "The Fibonnaci series that we have discussed can be seen on roses. Now let's move on."

This approach does little to involve the student in the use of mathematics, and does nothing to reenforce the concepts presented during the class. Teachers should be encouraged to derive scenarios that involve the lesson and the students.

My students seem to enjoy being spies. The class is broken up into teams, consisting of a two to four students with varying ability. (It is important to know your students well enough to be able to assign teams that are not entirely made up of strong students.) During the class the teams assemble outside of the classroom and watch as aircraft circle the field on approach. Their task is to use the angular velocity and trigonometric principles to calculate the approach velocity and height of the aircraft. In other scenarios, they use calculus to estimate the acceleration of rockets, or rate of infection caused by biological contaminants. In more basic classes, we use logs to analyze the current population growth and the decline in food supplies.

Each team is required to present and justify their findings using the mathematics they have learned during the semester. In all cases discussions about the findings are encouraged, and students are challenged by "what if" type questions. It is through this dialog that students begin to use the concepts of the class in developing logical argument. Thinking and analyzing in mathematical terms has begun to take shape.

Once a student in England came to me and said that he could no longer look at all the "things around without seeing the math that is involved." This student was not an "A" mathematics student and had earlier professed to be a "math phobe," yet he had begun to witness the beauty that is mathematics.

Use Terminology That is Relevant to the Class

Teach in terms they understand, with examples that are relevant to "them." Once while I was teaching at a federal penal institution in California, I was having difficulty getting my students to remember to convert to "like units of measurement" when solving problems. The example that I used was an area problem with measurements in feet and yards. Routinely, 60% of the students would forget to convert either to feet or yards before performing the calculations. One day in sheer exasperation, I surrendered. At that point one of the brighter students stood up and said, "Look, if you bought two kilos to sell for $20 a gram, how much would you make?" Every student had the correct answer before I did.

I had not been able to communicate the concepts to them because I was using examples that had no impact on them. Once the "light went on" for them, I was able to use that tie to their past lives to facilitate concepts in their new ones. Although I would never advocate the use of drug examples in a traditional setting, the point remains that it is incumbent on us, as teachers, to find the common ground.

<u>Ethno-mathematics</u>

What do Bannecker, Al-Khwarismi, Seki Kawa, and Ada Byron have in common? They are NOT European, white, male mathematicians. Yet each had a profound impact on the world of mathematics. *Ethnomathematics*, coined by Ubratan D'Ambrosio, "is the study of the concepts, practices and artifacts through which we discover mathematical elements among peoples living outside or on the margins of Western culture" (Struik, 1995, pg 1).

The traditional emphasis on Eurocentric mathematics has had a deleterious effect on mathematics learning in non-European countries, and may have impacted the diverse culture of our schools as well (Struik, 1995). Given the potential of such exclusionary teaching, it stands to reason that including the mathematics of various cultures, side by side with the concepts of European cultures, would empower *everyone*, not just the few.

To this end, I require at least one term paper in each class. Students are challenged to write a paper on the mathematical contributions of a non-traditional mathematician or culture. As an integral part of the paper, the student must use the concepts learned during the current class to explain those contributions and their meaning to society as a whole.

Developmental mathematics classes have presented papers on the *quipos* of the Inca civilizations. Others have compared the calculus work of Seki Kawa with that of Newton and Leibnitz. In each case, the students' learning and processing of mathematical ideas increased. More importantly they began to realize that each culture has contributed to the body of mathematical knowledge.

At the end of one Business Mathematics class, a student approached me about her paper. She expressed her surprise and delight at having learned the degree that African Americans have been involved in mathematical discoveries. This simple paper had empowered her to search out other scientific contributions by minorities and had inspired her to continue studying mathematics.

For some, this opportunity to write is the empowerment of their cultural heritage, for others it is a challenge to stereotypical misperceptions. In an algebra class, one of my students was surprised that women had been involved in mathematics. He had "always heard" that women "weren't good at math." Although it would be naive to believe that this single experience changed years of prejudice, it, at least, had increased his exposure and broadened his horizons.

<u>Mathemantics</u>

Mathemantics is a term I used to describe the process of writing about mathematical concepts in everyday language, using terms that are clear and easily understood.

As stated earlier, I require at least one paper per semester from each student. This serves two purposes: it helps the student to assimilate the mathematical concepts more completely, and it gives the students feedback on their ability to write technical data clearly and effectively.

I am, if nothing else, a realist. Six months after the end of a semester, I am not sure how much a non-technical student in a required mathematics class will remember. However, there is one thing of which I am certain: If that student has written a paper on a mathematics concept, that concept will be deep seated in her, or his, memory.

During the research the students must review the principles of mathematics that deal with their topic. It is during this review, that I often get office visits asking for clarification of confusing ideas, and in depth questions concerning the applicability of specific concepts. Students generally confront concepts that were more advanced than those taught during the class, and, thus, learn even more than was originally intended. As the student begins to write, the mathematical concepts and their use are reframed into the language best understood by the student. As teachers, we often lose our ability to explain mathematics in terms that all of our students can easily grasp. The reframing of concepts allows the writer to describe the processes in a way that we may not have thought of using. Additionally, it allows us to evaluate how the student has assimilated the course material presented in the paper.

Having read many Graduate Research Projects, it has come to light that our current system is producing students in technical curricula that can perform quality mathematical manipulations, but cannot put the words together well enough to convey their thoughts. Teaching students to write technical papers at an early stage in the educational experience prepares them for the rigors of the work place. It is there that our students will be put to the true test of their ability, and it is there that the true worth of their degree will be seen.

Remember What It Is Like

A professor that I know begins each semester the same way. She introduces herself and explains that she has been doing "this" level mathematics since she was eleven years old. This well meaning and brilliant professor, through this story, begins each class under a shroud of intimidation.

From the students' point of view, it will be difficult to approach her with questions without feeling "stupid" or inadequate. This professor cannot establish an identity power base with her students, because, again from their point of view, she clearly won't know what it is like to be completely lost in a mathematics class.

In some cases, that last view of the student is correct. As teachers, it is important to attempt to "walk a mile in the student's shoes." If you were slow to understand mathematics, try to remember the frustration of being "clueless" during the class, and then try to convey the sheer joy at the enlightenment of mathematics discovery. If you are one of those fortunate mathematicians who have been a "math whiz" since second grade, at least try to understand the students' misery by thinking of something that you had difficulty learning. It is never appropriate to tell untruths to a class, but one must filter one's discussions to exclude possible stumbling blocks.

Oddly enough, I truly enjoy telling my students that the only class I ever failed in high school was a mathematics class! They are usually astounded that "he's been there," and know that I can appreciate their struggle. Even more importantly, they can see that mathematics can be conquered, even by those of us who were slow learners.

If At First You Don't Succeed ...

If one approach doesn't work, try another. One of the most frustrating classes I have ever taken was a graduate seminar in stochastic modeling conducted by a doctoral teaching assistant (TA). Most of the students in the class were struggling with the concepts, and on one particular day the TA had presented the same problem at least a half a dozen times, in the same way. Finally one brave soul exclaimed, "I'm sorry, I don't understand how that was done. Could you explain it a different way?" The now frustrated TA yelled, "I only know one way. You'll just have to live with it!" From that point on most of us got little from the seminar.

Unlike most of our students, we go in to each session prepared with examples and solutions. In our preparation time it is in our best interest, and certainly the best interest of the student, to prepare a separate approach to the more complex areas. This serves a two-fold purpose. First, it actually gives the student a different perspective of the problem. Second, it helps to maintain our credibility in the class.

Be careful, however! It is not always in the best interest of the class to present different ways if all but one student understood the first way. In fact, it may do more to confuse than to help. Usually a quick "I'll be glad to show you a different way later," helps.

Be Careful of What You Say

Many of our adult learners, especially those in the developmental mathematics classes, may have suffered greatly from a teacher who told them that they were just not smart enough to do the mathematics. For instance, I know a young 4th grade student who is already convinced that she cannot do mathematics. This first step toward mathematics anxiety is based simply on a series of quick timed tests. The teacher told her that she certainly wasn't going to be an engineer, but social work would be okay!

The Pygmallion Effect, or Rosenthal Syndrome, is a psychological concept that emphasizes that the way we think about people impacts their behaviors. If we, as teachers, think of particular students as "mathematically challenged," then we will begin treating them as though they are slower than the others. These exhibited behaviors on our behalf triggers self-doubt on the behalf of the student. This apprehension to perform mathematical manipulations simply re-enforces our original belief, resulting in the proverbial "See, I told you that student couldn't do math!"

We **must** counter such actions as often as we can. Perhaps the best way is through positive re-enforcement, when appropriate. Give the struggling student an easy problem to answer. When they get it right, praise them appropriately, and issue a problem with more of a challenge. Eventually, the student will begin to realize that they can do mathematics, and will start a positive spiral in the Pygmallion cycle.

Create the Environment For Learning

Intimidation and fear must be replaced with cooperation and understanding. If each time a student ventures a guess, he is belittled, made to feel inadequate, or embarrassed, he will stop volunteering information. Eventually, he may stop listening and learning. Students need to explore the wonder of mathematics in a non-threatening environment. One in which a wrong answer is not held up to ridicule.

This does not mean that we are responsible for "coddling" our students. However, we are responsible for ensuring that their psychological safety is not threatened each time their hand is raised to answer.

Be Enthusiastic

Most of my students think I'm crazy! And that's wonderful! I make every attempt to infect them with the excitement of mathematics. There is no other subject, in my eyes, that can be as beautiful, yet as functional as mathematics.

Rarely do I lecture at a podium from the front of the room. Usually I am wandering about the room discussing a problem that I placed on the board. This "out and about" approach to teaching allows me to see the problem from the students' perspective and tends to get the students involved in discussion rather than simply yelling out answers. Of course the added benefit is that it serves as a reminder that if you fall asleep in my class, I will probably start talking rather loudly right next to your desk!

How wonderful it is to hear students say they were in class because they didn't want to miss what I was doing that day. Sometimes it is the math discussions about the tuition rate hikes, election year statistics, or maybe it is just my bizarre collection of outlandish "Friday" neckties. Whatever the cause of this excitement, it is happening in a mathematics class!

Summary

Remember that many adult learners of mathematics are teacher phobes, who have a significant hurdle to surmount. It is our job to help them do it. The student is not here for us, rather we are here for the student. We must do our level best to present material that conveys the concepts of mathematics in a language that is easy for them to grasp, represents all cultures, and continually engages them. Empower your students with the knowledge that mathematics is not above them, rather it is there for their taking.

One day towards the end of a developmental mathematics class, an adult learner approached me to thank me. She said her daughter was having trouble with mathematics at her school. For the first time in her daughter's life, my student did not have to say "Go see your Dad." They sat down together and worked through the problems. They were both proud of her new ability. I was proud, too.

Empower your students with mathematics that is inclusionary. Teach them; don't just throw facts and formulae at them. If you do, one day all our adult students will be proud to say that "math is easy."

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