A Constructive Solution to The Ornstein-Uhlenbech Equation on a Separable Banach Space of Infinite Dimension

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Gill and Myers [GM] proved that every separable infinite-dimensional Banach space, denoted $\mathcal{B}$, has an isomorphic, isometric embedding in $\mathbb{R}^\infty = \mathbb{R} \times \mathbb{R} \times \cdots$. They used this result and a method due to Yamasaki [YA] to construct a sigma-finite Lebesgue measure $\lambda_\mathcal{B}$ for $\mathcal{B}$ and defined the associated integral $\int_\mathcal{B} \cdot \, d\lambda_\mathcal{B}$ in a way that equals a limit of finite-dimensional Lebesgue integrals.

The objective of this talk is to apply this theory to developing a constructive solution to the Ornstein-Uhlenbech equation :

$$\frac{\partial u(x,t)}{\partial t} = \triangle u(x,t) + x \cdot \bigtriangledown u(x,t) \ , \quad u(x,0) = \phi(x) \tag{1}$$

where $x \in \mathcal{B}$, $\phi \in \mathcal{C}_0^2[\mathcal{B}]$, and $t \in [0, \infty)$. Our approach is constructive in the sense that the solution $u(x,t)$ of equation (1) is expressible as an integral $\int_\mathcal{B} \cdot \, d\lambda_\mathcal{B}$ which, by the aforementioned definition, equals a limit of Lebesgue integrals on Euclidean space as the dimension $n \to \infty$. Thus with this theory we may evaluate infinite-dimensional quantities, such as the solution $u(x,t)$, by means of finite-dimensional approximation.

Keywords : Ornstein-Uhlenbech, constructive solution, separable Banach space

References

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