# Mean Field Games with state constraints

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Mean Field Games (MFG) with state constraints are differential games with infinitely many agents, each agent facing a constraint on his state. In this case, the existence and uniqueness of Nash equilibria cannot be deduced as for unrestricted state space because, for a large set of initial conditions, the uniqueness of solutions to the minimization problem which is solved by each agent is no longer guranteed. Therefore, we attack the problem by interpreting equilibria as measures in a space of arcs and we introduce the definition of mild solution for MFG with state constraints. More precisely, we define a mild solution as a pair $(u, m) \in C([0, T] \times \overline{\Omega}) \times C([0, T]; \mathcal{P}(\overline{\Omega}))$, where $m$ is given by $m(t) = e_t \sharp \eta$ for some constrained MFG equilibrium $\eta$ and

$$u(t, x) = \inf_{\substack{\gamma \in \Gamma \\ \gamma(t) = x}} \left\{ \int_t^T \Big[ L(\gamma(s), \dot{\gamma}(s)) + F(\gamma(s), m(s)) \Big] \, ds + G(\gamma(T), m(T)) \right\}.$$

For more details see [1]. The aim of this talk is to provide a meaning of the PDE system associated with these games, the so-called Mean Field Game system with state constraints. For this, we will analyze the regularity of mild solution and we will show that it satisfies the MFG system in suitable point-wise sense. These results have been obtained in collaboration with Piermarco Cannarsa (Rome Tor Vergata) and Pierre Cardaliaguet (Paris-Dauphine).

## References

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[3] Cannarsa, P., Capuani, R., and Cardaliaguet, P., *Mean Field Games with state constraints: from mild to pointwise solutions of the PDE system*, submitted, *https://arxiv.org/abs/1812.11374*, 2018.