#### INCORPORATING GRAPHING CALCULATORS INTO THE ART AND SCIENCE OF TEACHING STATISTICS AND DECISION MATHEMATICS

by

#### Dr. Mary Ellen K. Landers, Assistant Professor Embry-Riddle Aeronautical University Fort Worth Center

#### ABSTRACT

The main paper purpose was to present curriculum appropriateness for requiring student use of either the TI-82 or TI-83 (TI-82/83) in Elementary Statistics and Decision Mathematics. An Embry-Riddle student in either course formulates problems, carries out ensuing calculations, and interprets results. More high-level studies could be pursued and accomplished if intervening calculations were not as inordinately time consuming as they are for students who use only minimal-type calculators.

A further purpose was to present possibilities for use of graphing calculators without resorting to user-created programs. The TI-82/83 has features which are applicable in areas of descriptive statistics and inferential statistics taught in elementary statistics courses. In creating frequency tables, sorting data, calculating one-variable statistics, drawing histograms, and making box plots the TI-82/83 is appropriate. Procedures for making inferences about a population are taught in inferential statistics. TI-82/83 has features which are beneficial for students to use in computations necessary in carrying out hypothesis testing, generating confidence intervals, and calculating linear regression coefficients.

Applicable technology, which will improve the quality of Elementary Statistics and Decision Mathematics courses is available. Students would be well served to have the TI-82/83 graphing calculator as an integral part of both courses.

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#### Introduction

During the slide-rule age, calculations in applied mathematics and engineering courses were carried out using a device called a slide rule. Students in mathematics and engineering walked proudly carrying their slide rules conspicuously attached to them. Two-hundred years of the tool's development and use came to an end with the advent of personal calculators.

Today, personal calculators have ever expanding capability to do calculations for mathematics, science, and business problems. Although evermore user friendly, their realm remains in the tool world. Like their increasingly distant predecessor the slide rule, calculators do not formulate problems nor do they interpret results.

#### Statement of the Problem

Students in Embry-Riddle courses Elementary Statistics and Decision Mathematics have as tasks: formulating problems, carrying out ensuing calculations, and interpreting results. More high-level work could be accomplished if intervening calculations were not as inordinately time consuming as they are for students using only minimal-type calculators. Consequently, students would be well served to have either the TI-82 or TI-83 graphing calculator (TI-82/83) as an integral part of both courses.

# Purpose of the Paper

The following discussion will point out ways the TI-82/83 graphing calculator can be used in Elementary Statistics and Decision Mathematics. Although TI-82/83 graphing calculators are programmable computers, use of user-written programs will not be necessary in order to incorporate suggested techniques into course curriculum.

#### **Elementary Statistics**

Most elementary statistics courses consist of: descriptive statistics, probability and probability distributions, and inferential statistics. Descriptive statistics encompasses the earliest applications of statistical science: problems of data and graphs describing the data (Triola, 1995). Probability distributions are basic to understanding inferential statistics--conclusions drawn from sample statistics concerning population parameters.

# **Descriptive Statistics**

Descriptive statistics is defined as the collecting and classifying of data. Some of the important techniques for classifying data which are taught in elementary statistics include: creating frequency tables, sorting data, calculating one-variable statistics, drawing histograms, and making box plots. The TI-82/83 has features which are applicable in these areas.

#### **Creating Frequency Tables**

To create frequency tables using a TI-82/83 graphing calculator, the student would key STAT followed by the first EDIT menu selection (key STAT 1). The list screen will appear. It is possible to record experiment results in list one (L1) followed by corresponding outcome frequencies in list two (L2). An example of list one's and two's content for an experiment in which a die is rolled 60 times is shown in Table 1.

Table 1

# Frequency Table for Rolling a Die 60 Times

L1	L2
1	11
2	8
3	12
4	7
5	6
6	6

# Sorting Data

The EDIT menu also allows students to choose to sort a list, or list pairs, in ascending or descending order. The lists need to be specified and separated with commas. From the same menu, students may also erase designated lists by similarly using choice four--ClrList.

# Calculating One-variable Statistics

When a frequency table has been created in list form, it is possible to use the first CALC menu selection to carry out one-variable statistics on the table. The student would exit from the list screen via the 2nd QUIT keys. Then by using STAT CALC followed by "1 2nd L1 comma

L2" students can carry out one-variable statistics on the frequency table stored in L1 and L2 (see Table 1).

The initial screen of one-variable statistics consists of: (a) mean; (b) sum of L1; (c) sum of the squares of entries in L1; (d) standard deviation calculated using division by n minus one, where n is the sample size--sixty in Table 1; (d) standard deviation calculated using division by n; and (e) n--sum of L2 entries. Using the down arrow to reveal the remainder of the calculations yields the five-number data summary: (a) minimum, (b) first quartile, (c) median, (d) third quartile, and (e) maximum.

# **Drawing Histograms**

The graphics capability of the TI-82/83s is employed to pictorially represent aspects of onevariable statistics. Frequency distributions are represented in histogram form by selecting the plot menu via 2nd STAT PLOT keys. Once Plot1, or another plot has been selected, the student would use ENTER to mark the choices "On" and histogram, by symbol, and specify which two lists hold the frequency table. Then by keying WINDOW, the student is able to select horizontal axis range (Xmin to Xmax) and histogram bar width (Xscl) as well as vertical axis range (Ymin to Ymax) and histogram bar height markings (Yscl). The student may key GRAPH to display the histogram and key TRACE along with arrow keys to show specifics of the distribution pictured.

# Making Box Plots

Box plots which display the five-number summary of data are also able to be produced by selecting the plot menu via 2nd STAT PLOT keys and proceeding, as in the case of histograms, with box plot chosen. The student may turn off the vertical axis markings by setting Yscl equal zero.

# **Probability Distributions**

In the sections on probability distributions, the material moves the student from using sample data for descriptive purposes of relative frequency tables and histograms to probability distributions--theoretical models (See L4 in Table 2). In a probability distribution the possible outcomes, along with their expected frequencies, are presented for circumstances which are understood concerning a population (Triola, 1995).

# **Relative Frequency Distributions**

Probability distributions based on empirical relative frequency can be created from frequency tables. When L1 and L2 holds a frequency table for which L2 is the frequency of outcomes in

L1, the student would locate the cursor on "L3" and enter 2nd L2 divided by the following: 2nd LIST MATH menu selection 5 followed by 2nd L2. Processing Table 1, which is stored in L1 and L2, would result in L3 entries in Table 2. L4 table entries display expected relative frequency for a fair die-1/6.

Table 2

Relative	Freau	encv T	able	for	Rolling	a	Die	60	Times
						_			_

L1	L2	L3	L4
1	11	.22	.166
2	8	.16	.166
3	12	.24	.166
4	7	.14	.166
5	6	.12	.166
6	6	.12	.166

Relative frequency is always a number between zero and one. When elementary statistics students work with univariate probability distributions, they cannot take advantage of one-variable-statistics feature, which requires frequency counts. Mean and standard deviation calculations involving use of lists would require entries in a column for L1\*L3 (L4) as well as one for L1^2\*L3 (L5). The distribution's expected value (mean) results from summing column L4. To obtain the standard deviation, the student would sum L5, subtract the calculated-mean squared, and take the square root of the result on the home screen.

# **Binomial Distributions**

List features may be used in binomial-probability table calculations. Students would enter each value of the random variable in L1 (see Table 3).

# Table 3

|--|

L1	L2	L3	
0	1		
1	4		
2	6		
3	4		
4	1		

After moving the cursor to the first (then the next following) item in L2, students may enter the largest value that the random variable assumes (i.e. the number of independent trials--n), key MATH, select 3 from the PRB menu, and L1's entry to obtain the first binomial coefficient (how many times each specific Bernoulli result occurs). In Table 3, the second entry in L2 is  ${}_{4}C_{1}$ , for example.

To complete the table, the student would place the cursor on L3 and key 2nd L2 times p raised to the L1 power times (1-p) raised to the "(" n minus L1 ")" power. L3 will display the binomial probabilities for the n-independent trials with probability p of "success"--desired binomial outcome. L3 exhibits the probabilities  $P(x) = {}_{n}C_{x} p^{x} (1-p)^{n-x}$ .

# Table 4

Binomial Distribution with Four Independent Trials and Success Probability of .27 (n=4 and p=.27)

L1	L2	$L3 = L2^{*}.27^{L1^{*}.73^{(4-L)}}$
0	1	.28398241
1	4	.42013836
2	6	.23309046
3	4	.05747436
4	1	.00531441

# **Inferential Statistics**

In inferential statistics the student uses sample data to make deductions concerning a population. Some of the important procedures for making inferences about a population which are taught in this portion of elementary statistics include: hypothesis testing, confidence intervals, and linear regression. TI-82/83 has features which are appropriate for students to use in intervening calculations.

#### Hypothesis Testing

Hypothesis testing is a decision-making process studied in elementary statistics courses. The decision-making process starts by identifying a premise of concern, progresses by formulating two opposing hypotheses, and concludes using statistical information to arrive at a decision about the supposition which was posed (Johnson, 1996).

#### Inferences involving one population

Sometimes an L1 listing is useful when testing hypotheses. Whenever sample data is provided for which mean and standard deviation must be calculated, one-variable statistics may be used on data listing entered in L1 to obtain statistics. The keys 2nd QUIT exit the EDIT feature. Test-statistic calculation would then be carried out on the home screen.

# Inferences involving two populations

List features make dependent samples of paired data especially easy to handle for students. It is possible for students to enter the first observation in L1 and the second in L2 for each pairing. Then by placing the cursor on L3 and keying 2nd L1 - 2nd L2, the difference (d) listing is obtained in L3. L3 may be treated as a one population sample which is outlined in the preceding explanation.

# Inferences involving independence in multinomial experiments

Similar techniques are useful for students in calculating the Chi-squared statistic. Lists L1, L2, and L3 would be used as follows: L1 for observed frequencies (O), L2 for expected frequencies (E), and L3 for calculation of Chi-squared addends. Each addend,  $(O-E)^2/E$ , is calculated by keying  $(L1-L2)^2/L2$  in L3's heading location. The Chi-squared statistic is the sum of L3 entries. To obtain Chi-squared for given sample data, students would move their cursors past L3's last entry and key 2nd LIST MATH 5 2nd L3 to sum the addends. The total would appear in the last entry of L3.

# Confidence Intervals

Equally useful is an L1 listing for students determining confidence intervals for the true population parameter. As in hypothesis testing, whenever sample data is provided for which mean and standard deviation statistics must be calculated, one-variable statistics may be used to obtain mean and standard deviation statistics. Another beneficial feature--2nd ENTRY--allows minimal keystrokes to be necessary when students carry out the final calculations on the home screen for the required parameter confidence-interval. 2nd ENTRY is easy to use and makes it possible to compute interval upper- and lower-endpoints by only entering the mean minus the critical value multiplied by the appropriate standard error. Keying 2nd ENTRY, after obtaining one endpoint, allows the student to change the minus sign in the keyed expression to a plus sign (see Table 5). The ENTER key yields second end-point computation.

#### Table 5

# Second-Entry Home Screen Display

Home Screen	Key Stroke
45-1.65(12.6/8)	ENTER
41.913	2nd ENTRY
45-1.65(12.6/8)	(Position cursor, key $+$ )
45 + 1.65(12.6/8)	ENTER
48.087	

# Linear Regression

Bivariate data in list L1 and L2 may be processed from the STAT CALC menu. If students are required to calculate the linear regression equation themselves, list features make it easy to calculate L1\*L2 and L1- or L2-squared in lists L3 and L4. List summation may be done as a final entry in any list. The student places the cursor at the end of the list and enters 2nd LIST MATH 5 2nd followed by the list number to obtain the list's sum of its entries. Final calculation of  $b_0$  and  $b_1$  would take place on the home screen after 2nd QUIT had been keyed.

# **Decision Mathematics**

Decision mathematics courses present quantitative methods of management science/operations research (MS/OR) to assist in decision making (Anderson, Sweeney, & Williams, 1995). Consequently, the Embry Riddle Aeronautical University course makes extensive use of quantitative methods. Elementary Statistics is prerequisite to Decision

Mathematics, and many calculator techniques would already have been mastered by the continuing student. The TI-82/83 is an advantageous tool in both courses because of its features.

#### Lists and Tables

In many MS/OR decision-making procedures, use of lists and tables would facilitate calculation. An example is in using Bayes Rule to compute posterior probabilities. Lists would be useful in computations of expected value and expected utility as well. Some forecasting computations, such as exponential smoothing, could be carried out nicely in list form.

# Linear Regression

When students use regression analysis in forecasting, features such as STAT CALC linear regression expedite the process. With less time spent in computations, more time would be available for interpretation of results. If actual calculation of linear regression coefficients is required, lists and tables aid the student with speedy computations.

# Windows and Graphing

Another area in which the decision mathematics student would benefit from TI-82/83 features is that of graphical solution of linear-programming problems. The ability to graph linear constraints in two variables to scale would aid students in understanding feasible region of linear programming problems. In graphing with the TI-82/83 for hands-on knowledge of a linear program's feasible region and optimal points students would use y=, WINDOW, and GRAPH keys. Time would be used efficiently and learning would be facilitated by the ease of graphing accurate linear models.

# Conclusion

The TI-82/83 graphing calculator has many capabilities not mentioned here. Only a few very basic essentials have been addressed in confirming the great benefit to the student use of a TI-82/83 graphing calculator would bring to bear upon improving the Elementary Statistics and Decision Mathematics courses by using applicable technology.

#### References Cited

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