Title: Invariant Manifolds in the Hamiltonian–Hopf Bifurcation

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Abstract We study the evolution of the stable and unstable manifolds of an equilibrium point of a Hamiltonian system of two degrees of freedom which depends on a parameter \( \nu \) . The eigenvalues of the linearized system are pure imaginary for \( \nu < 0 \) and complex with nonzero real part for \( \nu > 0 \). (These are the same basic assumptions as found in the Hamiltonian-Hopf bifurcation theorem of the authors.)

For \( \nu > 0 \) the equilibrium has a two-dimensional stable manifold and a two-dimensional unstable manifold, but for \( \nu < 0 \) there are no longer stable and unstable manifolds attached to the equilibrium. We study the evolution of these manifolds as the parameter is varied.

If the sign of a certain term in the normal form is positive then for small positive \( \nu \) the stable and unstable manifolds of the system are either identical or must have transverse intersection. Thus, either the system is totally degenerate or the system admits a suspended Smale horseshoe as an invariant set. This happens at the Lagrange equilibrium point \( L_4 \) of the restricted three-body problem at the Routh critical value \( \mu_1 \).

On the other hand if the sign of this term in the normal form is negative then for \( \nu = 0 \) the stable and unstable manifolds persists and then as \( \nu \) decreases from zero they detach from the equilibrium to follow a hyperbolic periodic solution.

Joint work with Dieter Schmidt.