Title: Invariant Manifolds in the Hamiltonian–Hopf Bifurcation

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Abstract We study the evolution of the stable and unstable manifolds of an equilibrium point of a Hamiltonian system of two degrees of freedom which depends on a parameter ν. The eigenvalues of the linearized system are pure imaginary for $\nu < 0$ and complex with nonzero real part for $\nu > 0$. (These are the same basic assumptions as found in the Hamiltonian-Hopf bifurcation theorem of the authors.)

For $\nu > 0$ the equilibrium has a two-dimensional stable manifold and a two-dimensional unstable manifold, but for $\nu < 0$ there are no longer stable and unstable manifolds attached to the equilibrium. We study the evolution of these manifolds as the parameter is varied.

If the sign of a certain term in the normal form is positive then for small positive ν the stable and unstable manifolds of the system are either identical or must have transverse intersection. Thus, either the system is totally degenerate or the system admits a suspended Smale horseshoe as an invariant set. This happens at the Lagrange equilibrium point $\mathcal{L}_4$ of the restricted three-body problem at the Routh critical value μ_1.

On the other hand if the sign of this term in the normal form is negative then for $\nu = 0$ the stable and unstable manifolds persists and then as ν decreases from zero they detach from the equilibrium to follow a hyperbolic periodic solution.

Joint work with Dieter Schmidt.