MA-220

Applied Calculus for Business & Life Sciences

Supplement

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MA220 Supplement

Table of Contents

Sections:

Directions and suggestions for this supplement:

a) Please use this booklet to improve your math skills.

b) Write in this booklet or on separate sheets of paper.

c) Check your answers in the back of this supplement.

d) Try not to use notes or ask for help until after completing and checking your work.

H. P. Lippert & J. S. Collins

Tips for Success in Mathematics
(Section 0)

- **<u>Attitude</u>** – How you approach a task is just as important as completing the task. Your attitude determines whether you can make math tougher than it needs to be or make it a positive learning experience. A negative attitude usually generates low success. However, if you approach math with a positive attitude, despite pass or fail, you can find the experience easier.

- **<u>Attendance</u>** – You will find that if you miss class, there is no one that will re-teach material missed and you are on your own about learning it. As an adult, you are responsible for any material missed, and your instructor and the tutoring lab will not teach you the material again. Suggestions to acquire the material on missed class days are: (a) get copies of notes from your peers (do *not* ask your professor for a copy of their notes- they usually go from memory), (b) read your book, (c) meet with study groups, (d) rent videos from the AA Center (tutoring), or (e) research the topic online.

- **<u>Pay Attention</u>**: This means you should be active in your learning. Write down what your professor says or puts on the board (also star problems that they stress more than others because it might be on the test). It's okay if you don't understand the details of the problem because you can ask (a peer, your professor, a tutor, and/or study group). Suggestion: Try to rework the problem and see if it's an algebra step that you missed.

- **<u>Prepare for Class</u>**: Keep up with the course schedule. Your professor is more likely to give you deadlines in class. Skim the material that's going to be covered in class ahead of time. Make comments or notes on topics that you might need to pay attention to in class.

- **<u>Keep Up</u>**: Don't procrastinate! It's easier said than done but realize that most professors will not accept late homework or projects. **Study for exams starting *at least* 3 days before the exam.** This is the time to put formulas together and study key problems/concepts that may have been stressed in class.

- **<u>Work in Groups</u>**: This is one of your best assets as a student. Choose peers in your class that appear motivated to pass. Be a leader and help others.

- **<u>Homework</u>**: Start it that day. <u>Don't wait until the night before it's due.</u> Try to do as much as possible without assistance (this includes solution manuals). Good homework scores and low test scores are the result of you relying on others to complete your work.

- **<u>Help</u>**: Check out the following: your professor's office hours; Tutoring Center (daytime and evening hours); Supplemental Instruction (course specific in the evenings); Online resources like YouTube; Private tutoring.

- **<u>Enjoy</u>**: College can be the best part of your life and yes, it is hard and, yes, a lot of time needs to be spent, but you need time to let your body relax. So find time to relax, bond with your classmates, and enjoy this great experience that has been presented to you.

Algebra Review
(Section 1)

A. Factor completely.

1. $16x^4 - 81y^4$

2. $49s^2 - 70st + 25t^2$

3. $50x^2y - 72y^3$

4. $12x^2 + 23x + 10$

B. Solve for "x".

1. $x(3x-2)(x-7) = 0$

2. $5x^3 - 10x^2 = 0$

3. $x^3 + 2x^2 - 15x = 0$

4. $x^3 - 2x = 0$

C. Simplify.

1. $\dfrac{27a^2 - 6a - 1}{27a^2 + 12a + 1}$

2. $\dfrac{2}{3} + \dfrac{3}{5} - \dfrac{4}{7}$

3. $\dfrac{x^{-1} + y^{-1}}{x^{-1} - y^{-1}}$

4. $\left(\dfrac{64}{27}\right)^{-\frac{2}{3}}$

5. $\dfrac{3}{\sqrt{5} - \sqrt{7}}$

6. $\dfrac{y + \dfrac{x^2}{y}}{y^2}$

Algebra Review (cont.)

(Section 1)

7. $\dfrac{-3x}{x^2-9}+\dfrac{2}{x-3}$

8. $\dfrac{x^{\frac{3}{5}}\,y^{-\frac{3}{2}}}{x^{-\frac{4}{3}}\,y^{\frac{1}{2}}}$

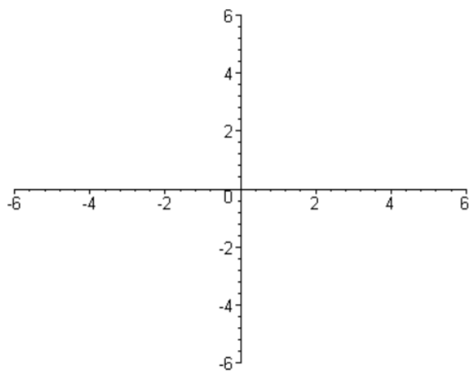
D. Accurately graph, indicate domain and range in <u>interval notation</u>.

1. $y=-\dfrac{5}{4}x+2$

Domain: ___________

Range: _____________

Is the graph a function? _________

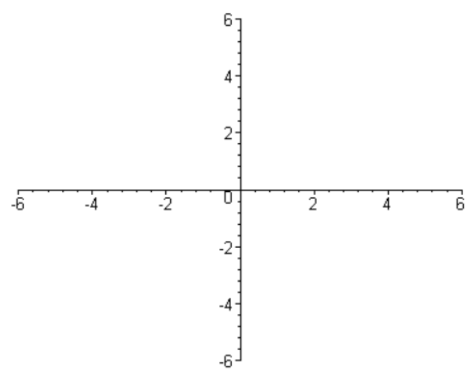


2. $y=-2x^2$

Domain: ___________

Range: _____________

Is the graph a function? _________



E. Given the two points (-5, -3) and (7, 2) Find the following:

1. The distance between the points

2. The slope of a line through the points

3. The equation of a line through the points

4. The slope of a line perpendicular (normal)

Linear Functions
(Section 2)

Hint: *You will need these formulas:* $y - y_1 = m(x - x_1)$, $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$, $d^2 = \Delta x^2 + \Delta y^2$

1. Find the slope of a line passing thru the points: (5, -2) and (-3, 7)

2. Find the equation of the line passing thru the two points: (5, -2) and (-3, 7)

3. Find the distance between the two points in the previous problem.

4. Given $5x - 2y + 8 = 0$. Find the equation of a line passing thru (1, -5) parallel to the given line.

5. In the previous problem what would be the equation of the line thru the same point and normal (perpendicular) to the given line?

Linear Functions (cont.)
(Section 2)

6. *Application*. Hypoxemia is a condition created when the intake of oxygen decreases due to the increase of altitude. Nausea, fatigue, dizziness, and headaches are only a few of the symptoms of hypoxemia. When a pilot ascends, the barometric pressure goes down and can trigger hypoxemia. The following table gives the percent of oxygen in the air at specific altitudes (in meters).

Percent, x	20.9	18.6	16	12.3	10.5
Altitude, $A(x)$	0	914	2133	4261	5486

a. Plot a graph of the altitude, $A(x)$, with respect to percent, x.

b. Draw a linear line, L = $A(x)$, through the points (20.9, 0) and (10.5, 5486).

c. Find the equation of the line for $A(x)$.

d. Assuming the trend continues, estimate the percent of oxygen at 6400m.

Linear Functions (cont.)
(Section 2)

7. *Application*. In the United States, businesses are projected to spend more on software and computer equipment. The following table gives the percentage change in computer equipment and software (seasonally adjusted), in 2013. *Note: $x = 0$ corresponds to 2013.*

Year, x	0	1	2	3
Percent Change, y	1.5	3.2	5.8	7.5

a. Plot a graph of the percentage, y, with respect to time, x.

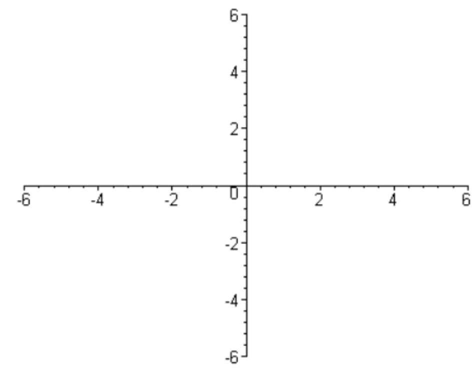
b. Draw a linear line, $f(x)$, through the point (0, 1.5) and (3, 7.5).

c. Find the equation of the line for $f(x)$.

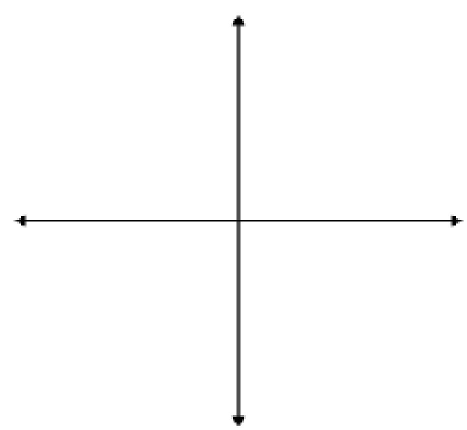
d. Assuming the trend continues, estimate the percentage change in spending for $x = 5$.

Functions and Models
(Section 3)

1. Given the function $f(x) = \frac{5}{x+5}$, find the domain and range, sketch the graph.



2. Given the profit function $P(x) = -5x^2 + 1000x$, graph and find the quantity that yields maximum profit and the maximum profit.



3. Given the two functions, determine which is consumer (demand) and which is supplier (supply) oriented.
   $p = -\frac{3}{4}x + 81,\;\; p = \frac{3}{2}x + 54$
   a. Discuss the slope in each case in class with your peers and professor..

   b. What is market equilibrium price and quantity?

Functions and Models (cont.)
(Section 3)

4. *Application*. An aircraft instrument distributer sells GPS units for $350.00 each. If the units cost the distributer $200.00 each and the fixed cost is $50,000, find the Revenue, Cost and Profit equations.
   a. Revenue equation

   b. Cost equation

   c. Profit equation

   d. How many units should the manufacturer sell to breakeven?

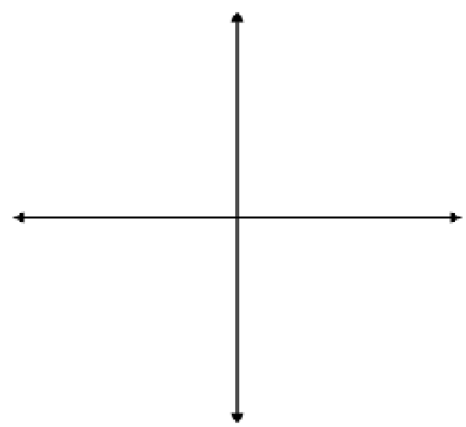
   e. If they sell 500 units, what would be the profit?

   f. How many units to get a profit of $100,000?

Functions and Models (cont.)
(Section 3)

5. *Application*. The height of a person can be predicted by the length of the femur bone. The starting height (in inches) of a child is 32.01 and increases by 1.880 per length (in inches) until maturity.
   a. Find a linear function that represents the height (H) as a function of length (L).

   b. Graph a representation of the information provided.



   c. If the height of a person is 67.166 inches, estimate the length of his or her femur bone.

Limits
(Section 4)

Evaluate the following limits:

> Hint:
> *Remember to start with direct substitution, determine the form, and then follow the procedures from class lecture.*

1. $\lim_{x\to 4}(x^2 - 3x + 2)$

2. $\lim_{x\to\infty}\left(\frac{2}{x} - 15\right)$

3. $\lim_{x\to -3}\left(\frac{5x^2 + 14x - 3}{x + 3}\right)$

4. $\lim_{x\to 1}\left(\frac{7}{x - 1}\right)$

5. $\lim_{x\to\infty}\left(\frac{6x^2 - 11}{3 - 7x^2}\right)$

6. $\lim_{x\to -1}\left(\frac{(x-2)^2 - 9}{x + 1}\right)$

7. $\lim_{t\to -2}\left(\frac{t^3 + 8}{t + 2}\right)$

8. List the functions that are **not** continuous and show (explain) why.

Limits (cont.)
(Section 4)

9. *Application*. The concentration of a drug in a patient's bloodstream *t* hours after injection is given by

$$c(t) = \frac{-0.5t}{t^2 + 25}$$

Evaluate $\lim_{t \to \infty} c(t)$ and interpret your result.

10. *Application*. The average cost per disc in dollars incurred by Hawk's Nuts and Bolts in making *x* discs is given by the average cost function

$$\bar{C}(x) = 75 + \frac{3000}{x}$$

Evaluate $\lim_{x \to \infty} \bar{C}(t)$ and interpret your result.

Slope of a Tangent Line
(Section 5)

Hint:
These are delta Δ process problems. Follow the steps from class, then plug in the x or y value

1. $y = x^2 - 5x$ at the point (1, -4)

2. $y = 5 - 3x^2$ at the point (2, -7)

3. $y = 7x - 2$ when x = 2

4. $y = x^3 - 3x + 5$ at (-1, 7)

5. $y = 3 - x^2$ when x = 5

Slope of a Tangent Line (cont.)
(Section 5)

More advanced, possibly after a second lecture on delta process. Same steps, just more algebra.

6. $y = \frac{-3}{x^2}$ at (1, -3)

7. $y = \sqrt{3x}$ when x = 3

8. $y = \sqrt{5x - 1}$ at the point (2, 3)

Slope of a Tangent Line (cont.)
(Section 5)

9. *Application*. The quarterly profit (in thousands of dollars) of Avion Flight School is given by

$$P(x) = -\frac{1}{3}x^2 + 7x + 30 \quad (0 \leq x \leq 50)$$

where *x* (in thousands of dollars) is the amount of money Avion spends on recruiting per quarter.

a. Find $P'(x)$.

b. What is the rate of change for Avion Flight School's quarterly profit if the amount spent on recruiting is $10,000 per quarter (x = 10)?

10. *Application*. Under controlled laboratory conditions, the population size of a certain bacteria culture at time t (in hours) is given by

$$R = f(t) = 5t^2 + 10t + 3$$

Find the rate of population growth at t = 5 hours.

Short Form of the Derivative
(Section 6)

Find the derivative of each of the following:

Hint:
The derivative of x^n *is* nx^{n-1}

1. $y = -3x^4 + 5x^3 - 2x^2 + 3x - 80$

2. $y = \sqrt[3]{x}$

3. $y = \dfrac{1}{\sqrt{x}}$

4. $y = \dfrac{5}{x^7}$

5. $y = \dfrac{5x-3}{\sqrt{x}}$

Short Form of the Derivative (cont.)
(Section 6)

6. *Application*. The velocity (in centimeters per second) of blood r cm from the central axis of an artery for a certain individual is given by

$$v(r) = 1000(0.2^2 - r^2)$$

Find $v(0.1)$ and $v'(0.1)$. Interpret your results.

7. *Application*. The relationship between the amount of money x that Avion Flight School spends on advertising and the company's total sales S(x) is given by the function

$$S(x) = -0.0002x^3 + 6x^2 + x + 500 \quad (0 \leq x \leq 200)$$

where x is measured in thousands of dollars. Find the rate of change of the sales with respect to the amount of money spent on advertising. Is Avion Flight School's total sales increasing faster than the amount of money spent on advertising if they have spent $10,000?

8. Find the slope of a line tangent to $y = 2\sqrt{x}$ at the point (9, 6) and ***write the equation of that line.***

Hint:
This is a review question, use:
$y - y_1 = m(x - x_1)$

Products and Quotients
(Section 7)

Take the derivative $\left(\frac{dy}{dx}\right)$ of each of the following and simplify as shown in class.

Hint:
Memorize the product and quotient rules

1. $y = \frac{7}{\sqrt[3]{x}}$

2. $y = \frac{5x}{3x^2 - 7}$

3. $y = (x^3 - 2x)(4x - 5)$

4. $y = x^4(3x^2 - 5x + 17)$

5. $y = \frac{11x + 4}{-5x + 9}$

Products and Quotients (cont.)
(Section 7)

6. Find the slope of a line tangent to $y = \frac{6}{x}$ at the point (2,3).

> Hint:
> *Slope of tangent line and instantaneous rate of change just means derivative.*

7. Find the instantaneous rate of change of y with respect to x for the following: $y = \frac{-3x+7}{5x}$

8. *Application*. The concentration of a certain drug in a patient's bloodstream *t* hours after injection is given by
$C(t) = \frac{-0.5t}{t^2 + 25}$
Find the rate at which the concentration of the drug is changing at t = 2 hours.

9. *Application*. From experience, Avion Flight School knows the average student pilot will progress according to the rule
$F(t) = \frac{60t + 180}{t + 6} \quad (t \geq 0)$
where F(t) measures percent of error after *t* number of weeks spent flying on a specific maneuver. Find the rate of the percent of error after a student spends 3 weeks ($\approx$ 13 hours of flight time) flying on a specific maneuver.

Chain Rule
(Section 8)

Review Questions: Find the derivative for each of the following:

1. $y = x^3 - \frac{1}{4}x^2 + 10x - 120$

2. $y = \frac{2}{x} + x$

3. Find the slope of a line tangent to $y = \frac{4}{\sqrt{x^3}}$ at the point (4,1/2).

Find the derivative for each of the following:

4. $y = (3x^2 - 5x)^7$

5. $y = \sqrt[4]{x^5 - 2x}$

> Hint:
>
> *Use the chain rule for* u^n
>
> $n \cdot u^{n-1}\left(\frac{du}{dx}\right)$
>
> *Don't forget the derivative of the inside!*

Chain Rule (cont.)
(Section 8)

6. $y = 5x^3\left(7 - 2x^2\right)^6$

7. $y = \dfrac{3x^3 - 5x}{\sqrt{9 - 7x}}$

Hint:
Numbers 6 and 7 are combination rules

8. *Application*. According to a joint study, carbon dioxide emissions of an airplane is similar to an automobile. If the emissions (in parts per million) is given by

$E(t) = \frac{1}{100}\left(t^2 + 4t + 45\right)^{2/5}$, where "t" time is in hours

a. Find the rate at which the emissions level is changing with respect to time.

b. Find the rate at which the emissions level will be changing in 5 hours.

Marginal Functions
(Section 9)

1. A company sells an A/C part for $200 per unit. If the total productions cost in dollars for x units produced is: $C(x) = 500{,}000 + 80x + .003x^2$ (production capacity is 30,000 units). Find:
   a. Revenue equation

   b. Profit equation

   c. Number of units to maximize profit

   d. Maximum profit

   e. Marginal cost to produce the 10,000th unit.

2. If the total cost to produce a product is given by $C(x) = .0015x^3 - .9x^2 + 200x + 60{,}000$. Find marginal cost for:
   a. 100

   b. 200

   c. 300

Marginal Functions (cont.)
(Section 9)

3. *Application*. Aviation Commuter Air Service receives a monthly revenue of $R(x) = 8500x - 100x^2$ dollars when the price charged per passenger is "x" dollars.

   a. Find the marginal revenue, $R'(x)$.

   b. Compute $R'(41.5), R'(42.5),$ and $R'(43.5)$.

   c. Based on results from part (b), what price should the airline charge in order to maximize their revenue?

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4. An aircraft moves according to the distance (s), time (t), equation $s = \frac{1}{3}t^3 - \frac{11}{2}t^2 + 24t + 500$. Find the aircraft's time to maximum height and the maximum height (s).

Higher Order Derivatives
(Section 10)

1. Find y''' (third derivative) for $y = x^5 - 4x^3 + 10x - 6$.

2. How many derivatives would you have to take to get the equation $y = x^7$ to become zero?

3. Find $y^{(4)}$ (fourth derivative) for the equation $y = \frac{2}{x}$.

> Hint:
> *Start with a negative exponent and stay in that form til all derivatives are done, then simplify to a positive exponent*

4. Given $s = -5t^3 + 400t + 200$ find the acceleration (s'').

5. Find the second derivative for $y = x^2(5x-7)^3$

Higher Order Derivatives (cont.)
(Section 10)

6. Find the second derivative for $x^2 + y^2 = 16$

FYI:
In this class we really only have a need for the first and second derivatives

7. Find the third derivative y''' for $y = \sqrt[3]{x}$.

8. *Application*. An aircraft moves according to the distance (s), time (t), equation $s = \frac{1}{3}t^3 - \frac{11}{2}t^2 + 24t + 500$. Compute $s'(10)$ and $s''(10)$ and interpret the results.

Implicit Differentiation
(Section 11)

Find $\dfrac{dy}{dx}$ for each of the following:

> Hint:
> *Don't forget to keep the differential notation*
> $\left({}^{dy}\!/\!{}_{dx}\right)$
> *when you take the derivative of the dependent variable (y)*

1. $5x-7y+12=0$

2. $3x^2-2y^3=50$

3. $6y-3x+7x^3y^2=20-y^3$

4. $4(3x-7y)^5=10$

5. $\dfrac{5}{x}-\dfrac{3}{y}=7$

Implicit Differentiation (cont.)
(Section 11)

6. *Application*. A propeller moves in circular motion such that the tips follow the equation $x^2 + y^2 = 25$. If a tip were to break off when located at the point (-4, 3), find the equation of a line that the tip would follow.

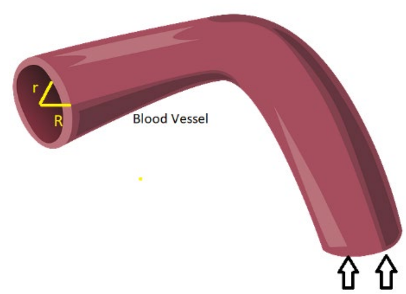
> *Again you will need the point slope equation*
> $y - y_1 = m(x - x_1)$

7. *Application*. Suppose a certain company has an output, Q, given by $Q = 20x^{3/5}y^{2/5}$ in millions of dollars. If "x" millions of dollars are spent on labor and "y" millions of dollars are spent on capital, find how fast the output of the company is if it spends \$32 million on labor, \$243 million on capital, and the dollars spent on capital and labor is increasing at a rate of \$1 million.

8. *Application*. The velocity (in centimeters per second) of blood r cm from the central axis of an artery and R cm radius of the artery for a certain individual is given by

$$v = 1000(R^2 - r^2)$$

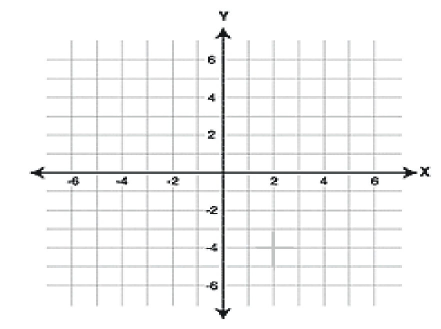
Find how fast the velocity of blood is moving when $R = 0.05\ cm, R' = 1.2\ cm/s\ , r = 0.04cm,$ and $r' = 1.2\ cm/s$.



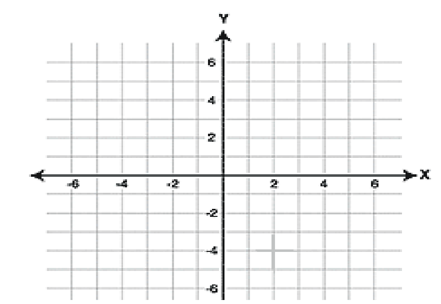
Curve Sketching
(Section 12)

Sketch the graph of each of the following, label the max/min (stationary) points, and the inflection point(s).

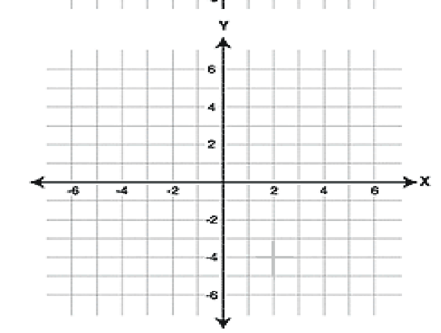
1. $y = 2x^3 + 3x^2 - 12x$



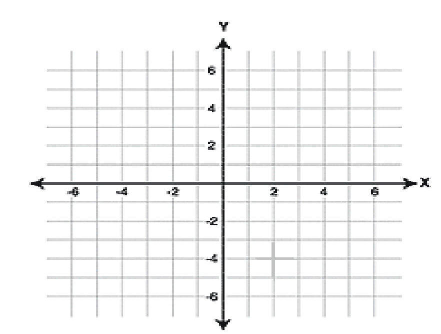
2. $y = x^5 - 5x$



3. $y = 3x^5 - 5x^3$

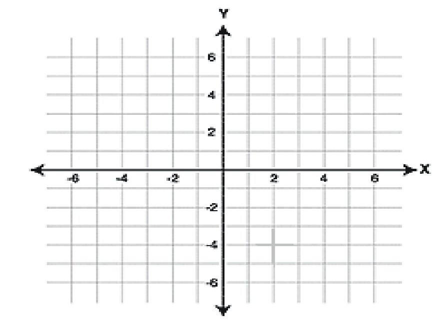


4. $y = 4x^3 - 3x^4$

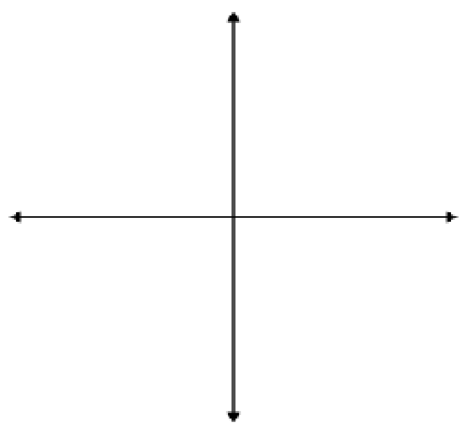


Curve Sketching (cont.)
(Section 12)

5. $y = x^4 - 6x^2$



6. *Application*. The traffic flow (in miles per hour) into a major city between 6 a.m. and 10 a.m. on a typical day can be approximated by the function $f(t) = .25t^4 - 2t^2 + 56$ $(0 \leq t \leq 3)$ where t = 0 corresponds to 6 a.m. Sketch a graph using curve sketching rules and interpret the results.



Applied Max/Min, Optimization
(Section 13)

Review Question:

1. An aircraft moves according to the distance (s), time (t), equation $s = \frac{1}{3}t^3 - \frac{11}{2}t^2 + 24t + 500$. Find the aircraft's time to maximum height and the maximum height (s).

Traditional Optimization Questions:

2. A rectangle has a perimeter of 160 ft. What length and width would yield the maximum area?

3. A rectangular storage area is to be constructed on one side of a commercial building. If no fencing is needed on the building side and there is 200 ft. of fencing available, what would be the dimensions to maximize area?

4. An open box is to be made from a square piece of cardboard whose sides are 10 cm long by cutting equal squares from each corner and folding. Find the size of the cutouts that yield the box of maximum volume.

Applied Max/Min, Optimization (cont.)
(Section 13)

<u>Course Related Optimization Questions:</u>

5. The quantity demanded each month for a certain product from Aviation Supply Industries is given by $D = \frac{50x}{0.01x^2+1}$ $(0 \leq x \leq 20)$ where "D" is measured in dollars and "x" is measured in units of a thousand. To yield a maximum revenue, how many units of that certain product must be sold?

6. Aviation Charter charges \$300 per person if exactly 200 people show up for a trip to the Bahamas. If more than 200 people schedule a charter flight, then the fare is reduced by \$1 for each additional person. Assuming that more than 200 people schedule a charter flight, answer the following:

   a. Discuss with your peers and professor why the revenue equation is $R(x) = (300 - x)(200 + x)$.

   b. How many additional passengers will generate a maximum profit? What is the total number of passengers that will yield a maximum profit? What is the maximum profit?

Exponential Functions
(Section 14)

1. Simplify:

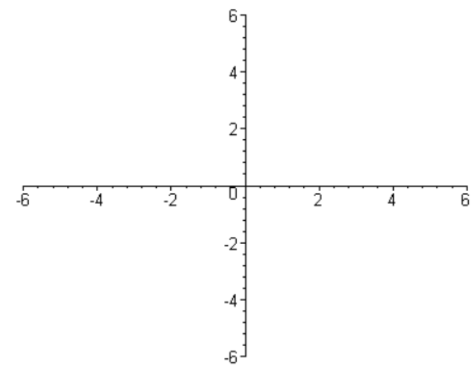
a. $\left(x^{\frac{2}{3}}\right)\left(x^{\frac{3}{5}}\right)$

b. $\dfrac{y^{\frac{4}{3}}}{y^{\frac{5}{8}}}$

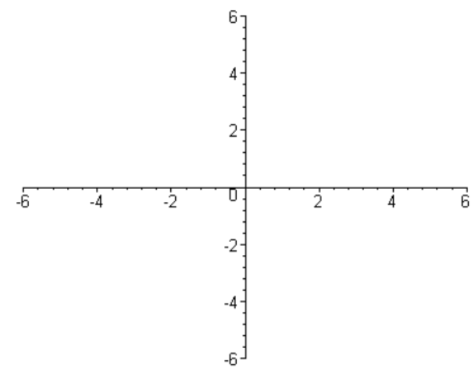
c. $\dfrac{x^{\frac{3}{4}}y^{\frac{1}{5}}}{x^{3}y^{\frac{2}{3}}}$

d. $e^{5x}e^{3x}$

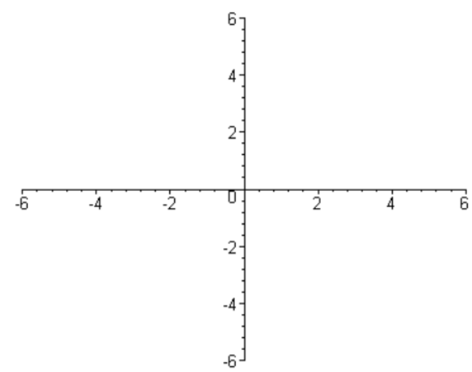
2. Graph: $y = 2^x$



3. Graph: $y = e^x$



4. Graph: $y = \frac{1}{e^x}$



Exponential Functions (cont.)
(Section 14)

5. *Application*. A 60kg skydiver jumps out of a plane with no initial velocity and air resistance of 0.8v. Assuming that downward direction is positive, the equation of the skydiver's velocity is given by

$v(t) = 735 - 735e^{-t/75}$ $(0 \leq t \leq 3)$ where "t" time is in minutes.

a. Find the velocity of the skydiver by completing the following table:

Minutes, t	0	1	2	3
Velocity, v				

Round results to two decimal places.

b. Use the information in part (a) to sketch a graph of *f(t)*.

Compound Interest
(Section 15)

1. Given \$5000 invested for one year at 7.5 % interest, find the final amount if:

    a. Compounded quarterly

    b. Compounded monthly

$$A = P\left[1 + \frac{r}{n}\right]^{nt}$$

$$A = Pe^{rt}$$

    c. Compounded daily

    d. Compounded continuously

2. A person invests \$10,000 for 5 years compounded monthly at a rate of 4%, find the final amount.

Compound Interest (cont.)
(Section 15)

3. If \$7000 were invested for 10 years at 4.5% compounded continuously what would be the final amount?

4. Approximately how long would it take to double your money at an interest rate of 7%?

5. *Application*. Juan purchased 1500 shares of Avion Mechanics Supply stock for \$30,000 (including commissions). He sold the shares 2 years later and received \$42,100 after deducting commissions. Find the effective annual rate of return on his investment over the 2-year period.

Derivatives of Exponential Functions
(Section 16)

Find $\frac{dy}{dx}$.

1. $y = 5e^{7x}$

2. $y = \frac{e^x}{2x}$

3. $y = \sqrt{1 - e^x}$

4. $y = \frac{x}{e^x+1}$

5. $y = e^{1+x^3}$

6. $y = e^{\sqrt[3]{x}}$

7. $y = xe^{\frac{1}{x}}$

Derivatives of Exponential Functions (cont.)
(Section 16)

8. *Application*. A local aviation gift shop found that "t" days after a sales promotion the volume of sales (in dollars) declined. The given function $S(t) = 10000(1 + e^{-0.4t}), (0 \leq t \leq 6)$ represents the trend.

a. Find the rate of change for the volume of sales when t = 1, t = 3, and t = 6.

b. How many days will the sales volume drop below $12,000?

9. *Application*. In a flu epidemic, the total number of students on a state university campus who had contracted influenza by the *x*th day was given by $f(x) = \frac{2000}{1+99e^{-x}}$ $(x \geq 0)$.

a. Derive an expression for the rate at which the flu was being spread.

b. Sketch a graph of *f(x)* using a graphical tool (e.g., calculator, Desmos, or free online graphing). Discuss and interpret the graph with your peers and/or professor.

Indefinite Integral
(Section 17)

Integrate each of the following, don't forget the "C". Simplify whenever possible.

1. $\int 5x^3 dx$

Hint:
This is the reverse of the derivative. The integral of
$u^n \mathrm{du}$, is $\dfrac{\mathrm{u}^{\mathrm{n+1}}}{n+1}+c$

2. $\int 3dx$

3. $\int \sqrt{x}\ dx$

4. $\int \left(-2x^3+7x^2-x+9\right)\ dx$

5. $\int \frac{1}{x^3}\ dx$

6. $\int \frac{x^2-x}{\sqrt{x}}\ dx$

7. $\int x^2\left(x^3-7\right)^4 dx$

8. $\int \sqrt[5]{8x+5}\ dx$

Indefinite Integral (cont.)
(Section 17)

9. $\int \frac{3x}{\left(5x^2 - 2\right)^7}\, dx$

10. *Application*. As a part of a quality-control program, bolts and screws manufactured by Aviation Nuts and Bolts are subjected to a final temperature fluctuation inspection before packing. The rate of increase in the number of sets of bolts and screws checked per hour by an inspector "t" hours in the 8 a.m. to 10 a.m. shift is approximated by the function $N'(t) = -3t^2 + 12t + 45 \quad (0 \le t \le 2)$.

    a. Find an expression, *N(t),* that approximates the number of sets of bolts and nuts inspected at the end of *t* hours.

    b. How many sets does the average inspector check during the two-hour period?

11. *Application*. The rate of change of the level of an invisible gas that is an irritant and impairs breathing is present in the atmosphere on a certain summer day in the city of Riverndale and is given by

    $R(t) = 4.2388t^2 - 0.422t^3$, measured in pollutant standard index per hour.

    a. Find the level, *A(t)*, at any time "t" given the following information: $A'(t) = R(t)$.

    b. Suppose "t" is measured in hours where t = 0 corresponds to 7 a.m. Determine the pollutant standard index at 10 am.

Definite Integral and Area Under the Curve
(Section 18)

1. Find the exact area under the curve represented by $y = -x^2 + 4x$, between x=1 and x=4.

2. Integrate each of the following:

A. $\int_1^3 (3x-1)^2\,dx$

> Hint:
> *This is the same process as the indefinite integral except when done integrating just evaluate the answer with the upper limit and subtract the value at the lower limit.*

B. $\int_{-2}^{2} 2x(x-3)dx$

C. $\int_0^4 \sqrt{x}dx$

> Remember:
> *You don't need a "c" in these. You are looking for a numerical answer.*

D. $\int_1^3 5dx$

E. $\int_0^4 \frac{1}{\sqrt{2x+1}}dx$

Definite Integral and Area Under the Curve (cont.)
(Section 18)

3. *Application*. A division of Aviation Supply Industries manufactures a certain product. Management has determined that the daily marginal cost function associated with producing this product is given by $C'(x) = 0.0003x^2 - 0.12x + 20$ where $C'(x)$ is measured in dollars per unit and "x" denotes the number of units produced. Management has recently determined that the daily fixed cost incurred in the production is \$800. Find the total cost incurred by Aviation Supply Industries in producing the first 300 units of their product per day.

4. *Application*. The price of a certain commodity in dollars per unit at time "t" (measured in weeks) is given by $p = 6 + 2t^2 - 2t^{1/2}$. What is the average price of the commodity over a 4-week period from

t = 0 to t = 4?

5. *Application*. The concentration of an antiemetic in the body "t" hours after ingestion is given by

$c(t) = \frac{t}{10\sqrt{t^2+1}}\ mg/cm^3$. Determine the average concentration of the drug in the patient's bloodstream over the first 2 hours after the drug is ingested.

Answers to MA220 Supplement
(Section 19)

1. Algebra Review

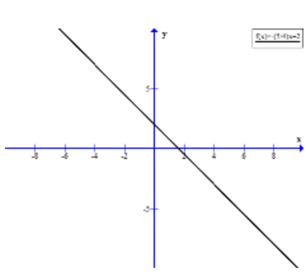
$A1)\ (4x^2+9y^2)(2x+3y)(2x-3y)\qquad A2)\ (7s-5t)^2\qquad A3)\ 2y(5x-6y)(5x+6y)$

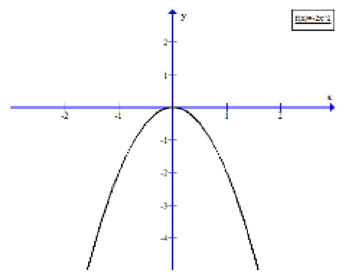
$A4)\ (4x+5)(3x+2)\qquad B1)\ x=0,\frac{2}{3},7\qquad B2)\ x=0,2\qquad B3)\ x=0,3,-5$

$B4)\ x=0,\sqrt{2},-\sqrt{2}\qquad C1)\ \frac{3a-1}{3a+1}\qquad C2)\ \frac{73}{105}\qquad C3)\ \frac{y+x}{y-x}\qquad C4)\ \frac{9}{16}$

$C5)\ -\frac{3\left(\sqrt{5}-\sqrt{7}\right)}{2}\qquad C6)\ \frac{y^2+x^2}{y^3}\qquad C7)\ \frac{-x+6}{x^2-9}\qquad C8)\ \frac{x^{\frac{29}{15}}}{y^2}$

$D1)\ domain: (-\infty,\infty); range: (-\infty,\infty); function: yes$

D1 Graph: 

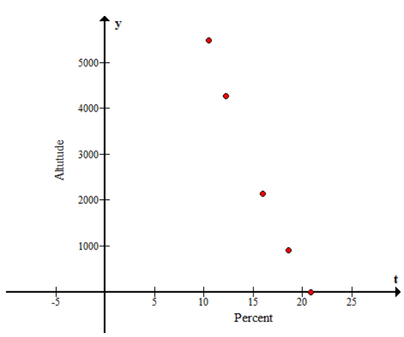
D2 Graph: 

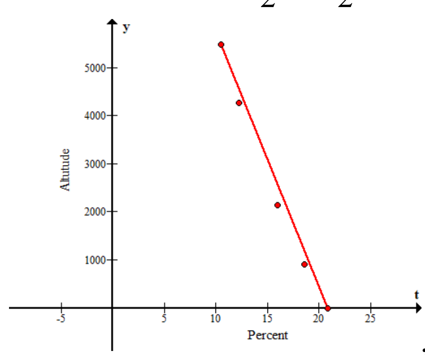
$D2)\ domain: (-\infty,\infty); range: (-\infty,0]; function: yes$

$E1)\ 13\qquad E2)\ m=\frac{5}{12}\qquad E3)\ y=\frac{5}{12}x-\frac{11}{12}\qquad E4)\ m_{norm}=-\frac{12}{5}$

2. Linear Functions:

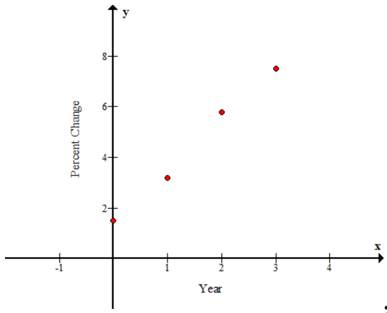
1) $m=-\frac{9}{8}$, 2) $y=\frac{-9}{8}x+\frac{29}{8}$, 3) $d=\sqrt{145}$, 4) $y=\frac{5}{2}x-\frac{15}{2}$, 5) $y=\frac{-2}{5}x-\frac{23}{5}$,

6a) 

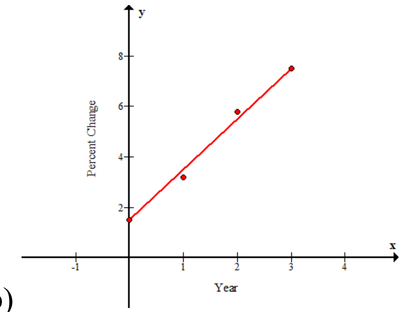
6b) 

, 6c) $A(x)=-527.5x+11024.75$,

6d) 8.8%, 7a)



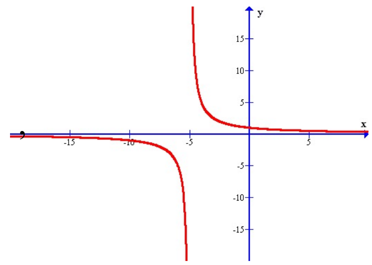
,7b)



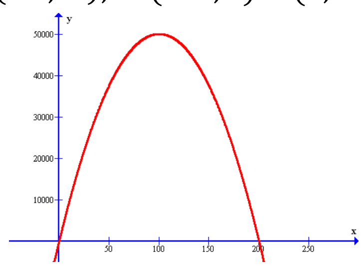
, 7c) f(x)= $2x + 1.5$, 7d) 11.5%

3. Functions and Models:

1) $D: (-\infty, -5) \cup (-5, \infty); R: (-\infty, 0) \cup (0, \infty)$

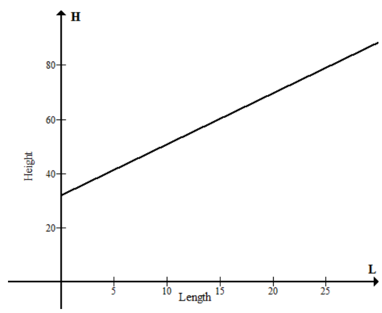


2) P(100) = 9500,



3a) class discussion, 3b) price= 12; quantity = 72, 4a) R(x) = 350x, 4b) C(x) = 200x + 50000, 4c) P(x) = 150x – 50000, 4d) 333.3 units, 4e) P(500) = \$25000, 4f) 1000 units,

5a) $H(L) = 1.880L + 32.01$, 5b)



, 5c) 18.7 inches

4. Limits:

1) 6, 2) –15, 3) –16, 4) DNE, 5) –6/7, 6) –6, 7) 12, 8) class discussion, 9) 0; the concentration of a certain drug will eventually wear off after so many hours, 10) 75

5. Slope of a Tangent Line:

1) –3, 2) -12, 3) 7, 4) 0, 5) -10, 6) 6, 7) ½, 8) 5/6 9a)$P' = -\frac{2}{3}x + 7$, 9b) 0.3333 = \$33.33 per \$100 spent on advertising, 10) 60

6. Short Form of the Derivative:

   1) $y' = -12x^3 + 15x^2 - 4x + 3$, 2) $y' = \frac{1}{3x^{2/3}}$, 3) $y' = \frac{-1}{2x^{3/2}}$, 4) $y' = \frac{-35}{x^8}$, 5) $y' = \frac{5}{2x^{\frac{1}{2}}} + \frac{3}{2x^{\frac{3}{2}}}$

   6) v(0.1) = 30cm/s; $v'(0.1) = -200\ cm/s/s$; the velocity of blood is 30cm/s and the rate at which the velocity of blood is decreasing at a rate of 200cm/s/s, 7) $S' = -0.0006x^2 + 12x + 1$; yes 8) $m = \frac{1}{3}, y = \frac{1}{3}x + 3$

7. Products and Quotients:

   1) $\frac{-7}{3x^{4/3}}$, 2) $\frac{-15x^2 - 35}{(3x^2 - 7)^2}$, 3) $16x^3 - 15x^2 - 16x + 10$, 4) $18x^5 - 25x^4 + 68x^3$, 5) $\frac{119}{(-5x + 9)^2}$,

   6) –3/2, 7) $\frac{-7}{5x^2}$, 8) -0.012 9) $f' = \frac{180}{(t+6)^2}$; 2.2%

8. Chain Rule:

   1) $3x^2 - \frac{1}{2}x + 10$, 2) $\frac{-2}{x^2} + 1$, 3) $m = \frac{-3}{16}$, 4) $(42x - 35)(3x^2 - 5x)^6$ 5) $\frac{5x^4 - 2}{4(x^5 - 2x)^{3/4}}$,

   6) $-120x^4(7 - 2x^2)^5 + 15x^2(7 - 2x^2)^6$ 7) $\frac{7(3x^3 - 5x)}{2(9 - 7x)^{3/2}} + \frac{9x^2 - 5}{(9 - 7x)^{1/2}}$,

   8) $C' = \frac{t+2}{125(t^2+4t+45)^{3/5}}$; 0.004 ppm/hr

9. Marginal Functions:

   1a) R(x) = 200x, 1b) $P(x) = -500000 + 120x - 0.003x^2$, 1c) 20000 units, 1d) \$700000, 1e) $c'(10000) = \$140$, 2a) $C'(100) = 65$, 2b) $C'(200) = 20$, 2c) $C'(300) = 65$, 3a) $R' = 8500 - 200x$, 3b) 200; 0; -200, 3c) \$42.5, 4) t = 3s; s = 531.5 ft

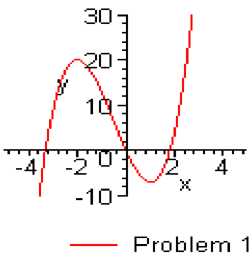
10. Higher Order Derivatives:

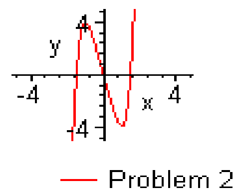
    1) $y''' = 60x^2 - 24$, 2) 8, 3) $y^{(4)} = \frac{48}{x^5}$, 4) $s'' = -30t$, 5) $y'' = 2(5x - 7)^3 + 60x(5x - 7)^2 + 150x^2(5x - 7)$, 6) $y'' = \frac{-y^2 - x^2}{y^3}$, 7) $\frac{10}{27x^{8/3}}$, 8) $s'(10) = 14$ means at time t = 10, the velocity is 14; $s''(10) = 9$ means at time t = 10, the acceleration is 9.

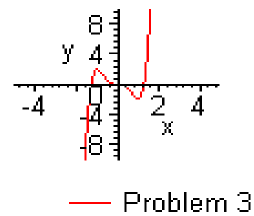
11. Implicit Differentiation:

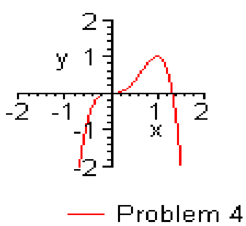
1) 5/7, 2) $\dfrac{x}{y^2}$, 3) $\dfrac{-21x^2y^2+3}{14x^3y+3y^2+6}$, 4) 3/7, 5) $\dfrac{5y^2}{3x^2}$, 6) $y=\dfrac{4}{3}x+\dfrac{25}{3}$, 7) 24 cm/s, 8) $29.4 million

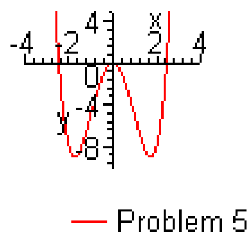
12. Curve Sketching:

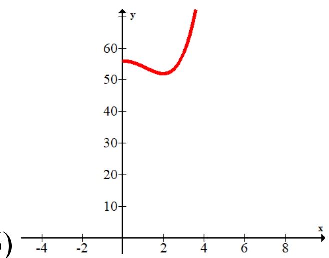
1) 

2) 

3) 

4) 

5) 

6) 

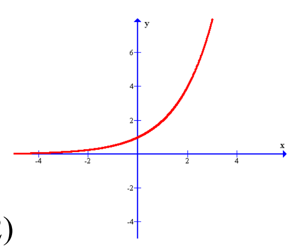
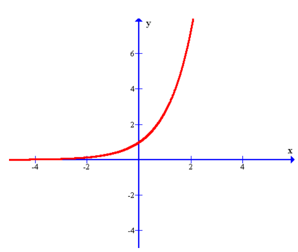
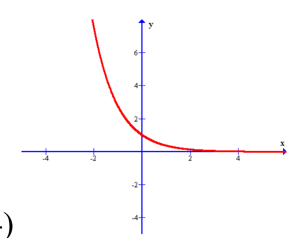
13. Applied Max/Min; Optimization:

1) time = 3 , height = 531.5, 2) L = 40, W = 40 , 3) 50 by 100, 4) cutout = 5/3 , 5) 10 units,

6a) class discussion, 6b) 50; 250; $62500

14. Exponential Functions:

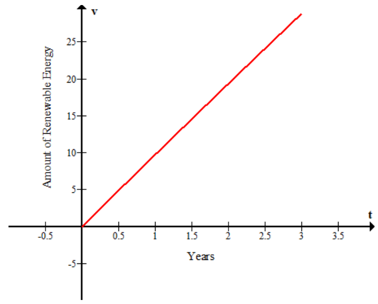
1a) $x^{19/15}$, 1b) $y^{17/24}$, 1c) $\dfrac{1}{x^{9/4}y^{7/15}}$, 1d) e^{8x},

2) 
3) 
4) 

5a)

Minutes, t	0	1	2	3
Velocity, v	0	9.73	19.34	28.82

5b)

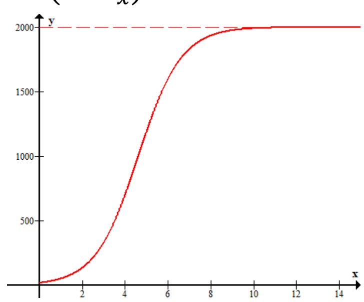


15. Compound Interest:
1a) 5385.68, 1b) 5388.16, 1c) 5389.38, 1d) 5389.42, 2) 12209.97, 3) 10978.19, 4) 9.9 yrs, 5) 18.46%

16. Derivatives of Exponential Functions:
1) $\frac{dy}{dx} = 35e^{7x}$, 2) $\frac{dy}{dx} = \frac{xe^x - e^x}{2x^2}$, 3) $\frac{dy}{dx} = \frac{-e^x}{2(1-e^x)^{1/2}}$, 4) $\frac{dy}{dx} = \frac{e^x + 1 - xe^x}{(e^x+1)^2}$, 5) $\frac{dy}{dx} = 3x^2 e^{1+x^3}$

6) $\frac{dy}{dx} = \frac{e^{\sqrt[3]{x}}}{3x^{2/3}}$, 7) $\frac{dy}{dx} = e^x\left(1 - \frac{1}{x}\right)$, 8a) S(1) = -2681; S(3) = -1205; S(6) = -363, 8b) 4 days,

9a) $f' = \frac{198000e^{-x}}{(1+99e^{-x})^2}$, 9b)



17. Indefinite Integral:

1) $\frac{5x^4}{4} + C$, 2) $3x + C$, 3) $\frac{2}{3}x^{3/2} + C$, 4) $-\frac{1}{2}x^4 + \frac{7}{3}x^3 - \frac{1}{2}x^2 + 9x + C$,

5) $\frac{-1}{2x^2} + C$, 6) $\frac{2}{5}x^{5/2} - \frac{2}{3}x^{3/2} + C$, 7) $\frac{1}{15}\left(x^3 - 7\right)^5 + C$, 8) $\frac{5}{48}(8x+5)^{6/5} + C$,

9) $\frac{-1}{20\left(5x^2 - 2\right)^6} + C$, 10a) $N(t) = -t^3 + 6t^2 + 45t$, 10b) N(2) = 106, 11a) $A = 1.41293t^3 - 0.1055t^4$,
11b) A(3) = 29.6

18. Definite Integral and Area Under the Curve:
1) 9, 2A) 56, 2B) 32/3, 2C) 16/3, 2D) 10, 2E) 2, 3) \$4100 , 4) 56 , 5) 0.12