Singular reaction diffusion equations where a parameter influences the reaction term and the boundary condition (I)

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Abstract

We analyse positive solutions to the steady state reaction diffusion equation:

\[
\begin{align*}
-u'' &= \lambda h(t) f(u); \quad (0, 1) \\
-du'(0) + \mu(\lambda) u(0) &= 0 \\
u'(1) + \mu(\lambda) u(1) &= 0
\end{align*}
\]

where \( \lambda > 0 \) is a parameter, \( d \geq 0 \) is a constant, \( f \) is a \( C^2 \) increasing function on \([0, \infty)\) such that \( f(0) = 0 \) and \( \lim_{s \to \infty} \frac{f(s)}{s} = 0 \), \( h \) is a \( C^1 \) nonincreasing function on \((0, 1]\) with \( h(1) > 0 \) and there exist constants \( d_0 > 0, \alpha \in [0, 1) \) such that \( h(t) \leq d_0 t^\alpha \) for all \( t \in (0, 1] \), and \( \mu \) is an increasing continuous function on \([0, \infty)\) such that \( \mu(0) \geq 0 \). We will discuss existence and multiplicity results via the method of sub-supersolutions. Further, we will discuss uniqueness results for \( \lambda \gg 1 \).

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