We focus on fluid-structure interactions (FSI) described by incompressible, viscous fluids coupled with elastic structures that move and deform inside them. Fluid-structure interactions have various medical and engineering applications, ranging from blood flow in stenosed arteries to the design of small-scale unmanned aircrafts. In most of the applications, the goal is the optimization or optimal control of the considered process. Specific examples include: minimizing turbulence, achieving given targets for fluid velocity or pressure, and minimizing the hydro-elastic pressure on the interface between the two environments. Therefore we consider PDE-constrained optimization and optimal control problems governed by moving boundary fluid-elasticity interactions. These interactions are highly nonlinear couplings of parabolic-hyperbolic type, described by a mismatch of regularity of the two solutions at the common interface. We discuss main challenges and describe the results that we obtained so far regarding the existence of optimal controls and the derivation of necessary optimality conditions.