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Is Socrates dead [in Mathematics]?: The role of the Mathematics Teacher in Higher Education now and for the next ten years

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Abstract

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Socratic questioning is an important tool of the mathematical trade for mathematicians and students alike. Simply "turning the crank" is not mathematics. Without the willingness and/or ability for Socratic thought new worlds of mathematics for the future may not be invented nor discovered. Therefore, future students of mathematics need not only be taught to learn from the past but to question toward the future as well. Consequently, the teacher of mathematics must fill a dual role: a) that of purveyor of facts and methods for the application of those already established structures that comprise mathematical thought and b) that of instructor in detective-ability.

As new initiatives promote "knowledge that integrates and synthesizes the perspective of several disciplines into mathematics education is faced with the opportunity to present undergraduate mathematics in context; mathematics rising out of literature, art, and/or the sciences; meaningful mathematics demanding attention because of its source. Teachers of mathematics have analytical approaches and concepts that apply across pedagogical boundaries as both method and metaphor in understanding and explaining the world around us, as is exemplified in many literary works. The mathematics immersed in literature, such as Richard Powers' *Galatea 2.2*, gives rise to powerful language which, for understanding of the metaphor, requires investigation into the meaning of the mathematics. The internet can be incorporated into learning as well as use of the computer, itself, or the graphing programmable-calculator. The internet provides look-up of text, such as *Alice in Wonderland* or *Flatland*, for reading facts and detecting meaningful relationships for mathematical understanding and applications as well as search and communication capabilities. When students combine these components and others to produce their own results, thinking involvement on many levels is at work in the process, as the facts are combined with critical processes. Consequently, the role of the mathematics Teacher in Higher Education now and for the next ten years must be to prepare students for worlds yet

unseen and to face ideas that are unexpected and, perhaps, unwelcome.

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Analyzing the Question

When mathematicians consider Socrates, immediately critical thinking comes to mind. Socratic questioning is an important tool of the mathematical trade for mathematicians and students alike. Mathematicians, themselves, must engage in questioning. Moreover, if this questioning spirit is not conveyed to students of mathematics, their mathematics exercise is just that, an exercise. Mathematics becomes a labor in futility when Socratic questioning is omitted from the mathematical process. Simply "turning the crank" is not mathematics.

The mathematician builds worlds upon worlds through questioning and developing ideas that rest upon ideas already developed. Mathematics, in itself, has developed as a way to understand and explain the world around us. During the fourth and fifth centuries BCE, the Pythagoreans investigated the physical world using whole numbers--0, 1, 2, 3, . . .-- and ratios of whole numbers--1/2, 3/4, 5/2. When their "Golden Ratio" was shown to be something other than a ratio of whole numbers (Wheeler, p.308), the Pythagoreans dissolved, unable to investigate the implications of the unwelcomed discovery. Fortunately, others went forward, investigating for themselves. Without the willingness and/or ability for Socratic thought new worlds of mathematics for the future may not be invented nor discovered.

Teachers of mathematics cannot omit the questioning skills from the mathematics that they teach. Therefore, future students of mathematics need not only be taught to learn from the past but to question toward the future as well. The student may need to practice mathematics for a particular skill or in an application presented at the moment, but to use the skill in the future will require that students be taught Socratic critical-ability to question. Consequently, the teacher of mathematics must fill a dual role: a) that of purveyor of facts and methods for the application of those already established structures that comprise mathematical thought and b) that of instructor in detective-ability.

A purveyor of facts and methods simply is a "show-and-tell" supplier. The student sees, copies the supplier, practices, and becomes skilled at the level of practice: a) much practice, high skill level; b) not much practice, shaky ability level; and c) no practice, little skill unless overcome by an exceedingly high talent. On the other hand, the instructor who develops the student's detective-ability challenges the student in various ways to become one who questions effectively and follows the clues which the answers to the questioning bring. First, the student-detective must have the facts and methods to base probing upon. Then, the student can be directed to pursue thought. The teacher acting in these dual roles and using innovative, meaningful ways must reach beyond disciplinary bounds.

Teaching Mathematics In 2000+

<u>Background</u>

During the past few years a movement toward Interdisciplinary Studies has exploded (DeZure). In the past, interdisciplinary has been attempted without removing the disciplinary aspect. As new initiatives promote "knowledge that integrates and synthesizes the perspective of several disciplines into a construction that is greater than the sum of its disciplinary parts" (DeZure, p.1), mathematics education is faced with the opportunity to present undergraduate mathematics in context; mathematics rising out of literature, art, and/or the sciences; meaningful mathematics demanding attention because of its source; mathematics sans isolation. Although many Interdisciplinary Studies courses exclude mathematics from their purview, "movement of methods and methods and analytical approaches across disciplinary boundaries has become an important feature of knowledge production today." (Klein, p.3) Teachers of mathematics have analytical approaches and concepts that apply across pedagogical boundaries as both method and metaphor in understanding and explaining the world around us, as is exemplified in many literary works.

<u>*Galates 2.2*</u>

The mathematics immersed in literature, such as Richard Powers' *Galatea 2.2*, gives rise to powerful language which, for understanding of the metaphor, requires investigation into the meaning of the mathematics. Some mathematical ideas used by Powers in compelling new ways are: a) algorithms, b) sine curve, c) isomorphic contour map, and d) Turing machine. His words, "The universe will be its own index. The *isomorphic* contour map, the way the data get packed together." (p.85) are exiting, inviting mathematical discovery and discourse about functions especially those that are isomorphic mappings. Words like "Now, when B started to do its *premutational* thing, I learned to say no." (p.90) move one to investigation into what combinations and permutations are all about.

Other investigations are stimulated by words like "The brain, Lentz had it, was itself just a glorified, fudged-up *Turing machine*." (p.70) Turing machine investigations, as well as other investigations, may be done via of internet searches or using appropriate logic text books. Students are usually highly receptive to using the internet, and, if they are reluctant, they soon become excited concerning internet use when given minimum instruction on its use. http://obiwan.univ.edu/computing/turing/more.htm is one of many sites featuring Turing machine information.

The university site gives a clear explanation of the Turing machine operation. However, to enhance understanding, students, under the teacher's direction, will find that a simple model of the Turing machine may be made of paper. A paper "triangle" (this shape is not sacred, just easy to cut and to hold when using), which has a small square cut from the middle to serve as a viewing area, a slit cut on either side of the viewing area to hold the instruction tape, and a paper strip which is no wider than the machine's slits and which houses blanks, zeros, and ones. The machine's *algorithm* is given as a five-tuple of instructions: state of the machine, what is read, the direction (left or right) to move, what is printed, and the new state of the machine. Iteration of the algorithm constitutes the machine's function being carried out until it halts. (It should be noted as well that the halting problem is an issue as well.)

Galatea 2.2 is no stranger to Trigonometry either. "The man's long, accreting addiction that made every day a sine wave of new hope crushed." (p.59) are words that bring up the image of the periodic sine curve. The student can relate the graph of this curve to the metaphor it inspires. With a graphing calculator or by hand, graphing the sine curve which is presented in this context deeply impresses the student by intertwining the text with mathematics in an unforgettable liaison.

Alice in Wonderland

Many more pieces of literature serve to bond the relationship between mathematics and metaphors of our world. *Alice in Wonderland*, of recent television fame, holds number controversy high. During the time period in which it was written by Lewis Carroll, negative integers were suspect by many as non-numbers. Students that are steeped in the "number line" of more modern times cannot imagine negative-one missing from the hallowed line. Although movie versions rarely portray the text "to the letter', fame enhances the perceived value of a text. Interestingly, even in reading the February 27-March 5 *TVGuide* 's description of the current depiction of *Alice* the student would find: "'We've *amplified* everything.' The March Hare, with its freaky *asymmetrical* eyes, scared Majorino." (p.21-22). It is mathematics that amplifies sine curves and that speaks of symmetry in various venues. With magnification and dilation as well as other topological wonders all taking place on Alice's body and in her surroundings, mathematics awareness, interest, and investigations take on new depths and meanings.

Flatland

Not only is *Alice* and other mathematically useful literature, such as *Flatland*, available at libraries and in bookstores, but such materials are accessible on the internet from several web sites. For example, *Alice* may be found along with musical accompaniment at http://www.megabrands.com/alice/goalice.html?. Retrievable at http://www.msc.cornell.edu/~rick/flatland/flatland.html, *Flatland* is a small, captivating book featuring geometry. The author, Edwin Abbott, relates a story of life in two-dimensions, before and after the invasion of Flatland by one from a world of three-dimensions. The book sparks much conversation and is highly thought provoking. Additionally, it points to how reticent society is to accept new paradigms and concepts that shatter "holy cows", an issue faced by mathematicians and scientists such as Galileo, Einstein, and others who recognized truths that had before been unseen.

Including Interdisciplinary Approaches

When students become involved in mathematical truths from contexts of history or interesting literature, investigations into the history of the times and makeup of the mathematicians personalities and lives can be incorporated into the study of the actual mathematical concepts. When such an interdisciplinary approach is included in students' studies, meaning and retention is enhanced. Incorporating technology, history, literature, and concepts strengthens the approach to the material. Approaching a topic from several aspects allows for students with different learning strengths to participate more fully in the learning process. Afterall, mathematics does not result from a vacuum but develops in a context of personality, the times, and history--that which has proceeded the new development. Another feature of the general education approach is that it highlights the truth that mathematics is an evolving body of knowledge, a discipline that is neither stagnate nor completed.

The interdisciplinary approach may be used to go beyond logical skills and pointedly mirror the discipline and self (Newell,1992, p.220). Consequently, an important feature of the interdisciplinary approach is that, when the world view and underlying assumptions of the discipline are made explicit (p.506), a strong sense of critical thinking is promoted and mathematics is revealed as more than computation and calculation that can be carried out with computing devices.

Learning with Technology

Technology and mathematical thought are not mutually exclusive either. Technology has become an increasingly important part of our daily life. In the mathematics classroom, students experience and understanding of the concepts can be enhanced by using technology at an integral part of the learning experience. The graphing programmable-calculator is useful but cannot replace certain aspects of mathematical accomplishment. Assuredly, mathematics is not *answers*. Rather, mathematics is procedures and thought processes which require critical thinking. However, as a tool the graphing programmable-calculator may be used in many settings. One application would be graphing complicated functions once a student has learned to graph for himself. Calculator skills, combined with previous facts, knowledge, and skills of how to graph, speed other processes which would be the focus of the learning. The facts are the basis for the ability to make meaning of the use of technology.

The internet can be incorporated into learning as well as use of the computer, itself, or the graphing programmable-calculator. The internet provides, look-up of text, such as *Alice* or *Flatland*, for reading the facts and detecting meaningful relationships for mathematical understanding and applications. It also provides the ability to search for facts and information from many sources around the world, as in the case of Turing-machine facts. To only use the computer as a source of tutorials or as a wordprocessor omits much of its power from use for learning. It takes critical thinking to use known facts and information to decide what search is needed, what information to retrieve, and what use of the information is appropriate. When students combine these components to produce their own results, thinking involvement on many levels is at work in the process, as the facts are combined with critical processes.

Searching for Socrates

If the answer to the question "Is Socrates dead [in Mathematics]? ",is "Yes", indeed, the discipline has lost its future and becomes dead as well. If the caution of A.N. Whitehead, that it is a "common error to assume that, because prolonged and accurate mathematical calculations have been made, the application of the result to some fact of nature is absolutely certain." (TCU, p.1), becomes a stumbling block, the doors to tomorrow's new worlds close. In applying mathematics, it is mandatory that Socratic questioning live and that the facts are put together

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