Recent Advancements in Mathematical Finance

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In recent years, major advancements in mathematical finance have grown out of the study of two parabolic partial differential equations, the Nobel Prize winning Black-Scholes stock option equation

$$\frac{\partial u}{\partial t} = \frac{\sigma^2}{2} x^2 \frac{\partial^2 u}{\partial x^2} + rx \frac{\partial u}{\partial x} - ru \tag{BS}$$

and the Cox-Ingersoll-Ross zero coupon bond equation

$$\frac{\partial u}{\partial t} = \frac{\sigma^2}{2} x \frac{\partial^2 u}{\partial x^2} + (\beta x + \gamma) \frac{\partial u}{\partial x} - xu \tag{CIR}$$

both on

$$(x, t) \in (0, \infty) \times [0, \infty)$$

with initial condition

$$u(x, 0) = u_0(x) \geq 0.$$

By studying these problems on suitable weighted sup norm Banach spaces, we show that the $(C_0)-$semigroup governing the Black-Scholes equation is chaotic, and the $(C_0)-$quasicontractive semigroup governing the Cox-Ingersoll-Ross equation admits a new type of Feyman-Kac representation formula (as a limit). We also study problems when the volatility, interest rate, etc. may be time dependent. This is joint work with various coauthors.