Distributed Delay Effects on Coupled van der Pol Oscillators, and a Chaotic van der Pol-Rayleigh System with Parametric Forcing

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Abstract

Distributed delays modeled by ‘weak generic kernels’ are introduced in two well-known coupled van der Pol systems, as well as a chaotic van der Pol-Rayleigh system with parametric forcing. The systems are closed via the ‘linear chain trick’. Linear stability analysis of the systems and conditions for Hopf bifurcation that initiates oscillations are investigated, including deriving the normal form at bifurcation, and deducing the stability of the resulting limit cycle attractor. The value of the delay parameter $a = a_{Hopf}$ at Hopf bifurcation picks out the onset of Amplitude Death (AD) in all three systems, with oscillations at larger values (corresponding to weaker delay). In both van der Pol systems, the Hopf-generated limit cycles for $a > a_{Hopf}$ turn out to be remarkably stable under very large variations of all other system parameters beyond the Hopf bifurcation point, and do not undergo further symmetry breaking, cyclic-fold, flip, transcritical or Neimark-Sacker bifurcations. This is to be expected as the corresponding undelayed van der Pol systems are robust oscillators over very wide ranges of their respective parameters. Numerical simulations reveal strong distortion and rotation of the limit cycles in phase space as the parameters are pushed far into the post-Hopf regime, and also reveal other features, such as how the oscillation amplitudes and time periods of the physical variables on the limit cycle attractor change as the delay and other parameters are varied. For the chaotic system, very strong delays may still lead to the cessation of oscillations and the onset of AD (even for relatively large values of the system forcing which tends to oppose this phenomenon). Varying of the other important system parameter, the parametric excitation, leads to a rich sequence of dynamical behaviors, with the bifurcations leading from one regime (or type of attractor) into the next being carefully tracked.
Keywords: amplitude death; distributed delays; bifurcation analysis; chaotic attractor