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Observability and Confidence of Stability and Control Derivatives Determined in Real Time

Alfonso Noriega

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OBSERVABILITY AND CONFIDENCE OF STABILITY AND CONTROL DERIVATIVES DETERMINED IN REAL TIME

By

Alfonso Noriega

A Thesis Submitted to the Graduate Studies Office In Partial Fulfillment of the Requirements for the Degree of Master of Science in Aerospace Engineering

Embry-Riddle Aeronautical University Daytona Beach, Florida Fall 2013
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Alfonso Noriega

This thesis was prepared under the direction of the candidate’s thesis committee chairman, Dr. Richard “Pat” Anderson, Department of Aerospace Engineering, and has been approved by the members of his thesis committee. It was submitted to the Department of Aerospace Engineering and was accepted in partial fulfillment of the requirements for the degree of Master of Science in Aerospace Engineering.

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ABSTRACT

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Stability and control derivatives of an aircraft were estimated from real flight test data in real time. A higher language block diagram library was developed for this purpose. Parameter identification techniques and requirements were used to detect and rate maneuvers present in the data. These ratings were used to blend newly calculated derivatives with previously known values by means of a Kalman filter. The Kalman filter output was used to identify the health of control surfaces actuators. Statistical and measured data were used to predict the probability that an actuator failure has occurred at any given time during the flight. Sweeps of all the tuning parameters of the system were performed, and it was demonstrated that these tuning parameters can be used to obtain the desired performance based on requirements.
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Chapter 1

Literature Review

1.1 Introduction

This investigation was an experiment to determine the viability of determining the observability and confidence in stability and control derivatives obtained in real time. The applications for this include real time health monitoring of aircraft where a precise knowledge of the current stability derivatives is necessary. An example of this are Unmanned Aerial Vehicles (UAV). In UAVs, the control surfaces are deflected by servos controlled by a flight computer. Due to weight and cost limitations, small UAVs often rely on small servos with high failure rates. An inoperative servo greatly affects the UAV’s ability to maneuver in flight. Knowledge of a failure, although unrepairable in flight, would be beneficial to the UAV operator since any maneuvers that would place the UAV at risk could be avoided.

To detect an actuator failure, parameter identification techniques can be used. The control power of each control surface is closely related to the ability of the servo to actuate the control surface normally. The values calculated using parameter identification can be compared to a nominal control power known for each surface, and a large deviation from that nominal value would indicate an actuator failure. However, control power also depends on other factors such as downwash from the wing, fuselage sidewash, inertia of the UAV, location in the flight envelope, and sensor noise. For
this reason, the comparison between calculated and nominal value must include a
tolerance to compensate for imperfect measurements as well as the fact that control
power is not strictly constant.

In addition, effective parameter identification requires that the regressors be linearly
independent. For this reason, specific maneuvers would have to be constantly per-
formed to be able to calculate control power continuously. Since this is not practical,
statistical methods were used to calculate a theoretical model to determine the per-
cent confidence that a servo is still functional. These theoretical values were then
blended with a measured determination of a failure.

A system was developed that can provide the aircraft with a signal that indicates a
confidence level on the current stability derivatives estimate. If the confidence is low,
it indicates that the aircraft may need to perform special maneuvers for parameter
identification purposes, but if the confidence is acceptable the aircraft can continue
to operate normally. In addition, a method was developed to predict the probability
that a failure has occurred.

Another application of the system developed is to provide real time feedback to a flight
test engineer to determine if a maneuver was performed correctly or if it needs to be
repeated. This would save a significant amount of time during flight test programs
since maneuvers can be analyzed immediately, instead of having to land the aircraft
to perform the analysis.

In this investigation, the method for estimating the stability and control derivatives
and the probability of an actuator failure was done using a higher language block
diagram software that can run in real time in an on-board computer inside a flight
test aircraft. Stability and control derivatives are computed continuously using linear
regression by using batches of real time data. Each batch is analyzed for covariance of
the regressors, frequency content, and signal-to-noise ratio to determine the quality of the estimate. Using these continuously calculated stability derivatives and the calculated quality of each estimate, a method was developed for blending new data with previous data using a Kalman filter. The devised method includes a set of user-defined parameters that allow the system to be tuned for any aircraft. These tuning parameters make the block diagram generic, so that the system can be reused by tuning the parameters once for every new airplane.

For this investigation, it was desired to use flight test data with representative sensor noise of the same quality that is available to a flight test aircraft. The data used was of a Diamond Aircraft DA42-L360, a light composite twin engine airplane, which was obtained by Embry-Riddle Aeronautical University during a flight test program in 2009. The purpose of the flight test program was to develop a Level 6 simulator of the airplane to train pilots.

This report is distributed into four chapters. Chapter 1 is a literature review that explains the concepts and theories used in this investigation. It provides the reader with an introduction to statistics and parameter identification. Chapter 2 explains the specific methods used to create and tune the tools developed. Chapter 3 contains the results obtained. Chapter 4 provides the conclusions drawn from the results in Chapter 3, along with recommendations for future work.

1.2 Statistics

When dealing with a process that involves uncertainty, a statistical analysis is necessary to make educated decisions about the results of said process. It is important to identify the sources of uncertainty and the degree to which they affect the process.
Statistics provide a toolbox to analyze these imperfect processes and to extract useful information from them. This tools take into account information about the uncertainties and help predict, based on previous results and the quality of the data, the validity of new results. In the study of aircraft, imperfect sensors are used to obtain information about the aircraft’s state, motion, and control inputs. Due to the uncertainties introduced by these sensors, statistical analysis is necessary to determine the quality of the information obtained. The following sections provide a brief summary of the basic concepts of statistics and how they aid in the mathematical modeling of an aircraft.

### 1.2.1 Random Variables

When results are obtained from a random process with a given probability, a random variable is used to denote the result. Random variables are usually denoted by a capital letter, for example X. Once the random variable has been assigned a value (result from the process), it is denoted by a lower case letter \([1]\). If the possible outcomes for a random variable are finite, then it is called a discrete random variable. On the other hand, if the random variable has an infinite number of probable outcomes, it is called a continuous random variable.

### 1.2.2 Probability Distribution

Each possible outcome of a random variable has a certain probability to occur. A function that describes the likelihood of each outcome is called a probability distribution. Probability distributions can be either continuous or discrete, depending on if the number of possible outcomes for the random variable are infinite or finite,
respectively.

In this investigation, the probability distribution of a normal random variable was used to calculate the theoretical probability that a servo failure has occurred. It was done by adding the probabilities of a failure occurring at all times lower than the current time. When dealing with a continuous random variable, like in this case, this is done by integrating the probability distribution from an initial time to the current time.

1.2.3 Expectation

The expected value of a random variable, μ is the weighted mean of each of its possible outcomes, \( f(x) \). The weight given to each particular outcome is the probability of that outcome to occur. For a discrete random variable, the expected value is the sum of the product of each outcome times its probability. For a continuous random variable, the expected value is the integral of the product of each outcome times the probability distribution function. Eq. (1.1) shows how to compute the expected value for both cases.

\[
\mu = E(X) = \sum_x x f(x) \quad \text{for discrete variables, and}
\]
\[
\mu = E(X) = \int_{-\infty}^{\infty} x f(x) \, dx \quad \text{for continuous variables} \tag{1.1}
\]

This principle was used to determine when a servo failure is expected to occur, based on multiple previous tests on same models of a servo.
1.2.4 Variance

The variance of a random variable, $\sigma^2$, is the indication of its variability [1]. It is computed by finding the expected value of the square of the difference between all possible outcomes of the random variable and its mean, as shown in Eq (1.2). A low variance indicate that the possible outcomes of the random variable are close to the mean, while a large variance indicates the opposite.

$$\sigma^2 = E[(X - \mu)^2] \quad (1.2)$$

In a flight test program, variance is introduced by the fact that the sensors used to determine the states and control inputs of an aircraft are not perfect. Using an known true value of the quantity being measured, the variance can be calculated by recording what the sensors are measuring. The discrepancies between the known value and the measured ones is used to compute the variance of each sensor.

1.2.5 Covariance

Covariance is a measurement of the association between two random variables. A large value of covariance indicates a significant dependence, while the sign indicates if the relationship between the two random variables is positive or negative. The covariance between the random variables $X$ and $Y$ can be calculated as:

$$\sigma_{XY} = E[(X - \mu_X)(Y - \mu_Y)] \quad (1.3)$$

In parameter identification, it is important that the variables used to model the
aircraft are linearly independent, as it will be explained in the following sections. Since covariance provides a measurement of how linearly dependent two variables are, it is very important that their covariances are low.

1.3 Parameter Identification

Parameter Identification (PID) is the process of extracting a mathematical model of a physical system from imperfect observations [2]. The mathematical model can then be used for simulating different scenarios than the ones observed. This is of great interest in the aviation industry for several reasons. An aircraft simulator can prevent the need for expensive and lengthy flight test programs in order to predict or verify performance data and train new pilots unfamiliar with the aircraft. Simulators can also be used in the design of new control laws, since it provides the means to perform safe and inexpensive iterations of new designs.

The main process behind PID is to observe the response of a system to a set of specified inputs. The measurements of both the inputs and the outputs are then used, along with a theoretical knowledge of the system, to extract the mathematical model. This, however, is only possible if the regressors chosen for modeling are linearly independent from each other. For this reason, careful consideration is given to input design. In the identification of aircraft, control surface deflection patterns and magnitudes are designed in order to stimulate the natural modes of the aircraft, making its dynamic response observable and, therefore, the model extractable. Once the data is available, several methods of PID exist in both the time and frequency domains to obtain the information needed.
1.3.1 History of Parameter Identification

Parameter identification started as a rudimentary process and has evolved to a consistent, mathematical process over the years. In 1809, the system identification problem was first discussed with a statistical approach to solve it [3]. Several contributions have been made since then that are still significant today, but it was not until the 1960s that the interest in this area increased significantly, as was made evident by a large increase in publications on the subject [3].

The National Advisory Committee on Aeronautics (NACA) has published reports on aircraft specific identification since the 1920s. These early methods were mostly aimed at obtaining frequency and damping ratio estimates from flight test data. The stability and control coefficients were then selected in such a way that the frequency response parameters were matched. Different methods, such as linear regression and time vector techniques, were tried at that time, but measurement noise made it very difficult to obtain accurate results [3].

The lack of a robust method for estimating the stability and control coefficients resulted in a decrease in general interest in aircraft parameter estimation. This interest was renewed in 1968, when studies on output error methods were published [3]. The most significant of these was the maximum likelihood estimator, which used a Gauss-Newton algorithm for minimization purposes. It was also at this point that methods for designing flight test inputs using preliminary information on the aircraft were developed. Methods were also developed that used a Kalman filter for estimating the stability and control coefficients.

The use of digital computers aided significantly in the process of parameter identification [4]. Automatic data processing capabilities made the analysis of data easier
and more efficient. This allowed for methods in the time domain to be used, which are more intuitive and computationally intensive than the frequency domain. In addition, studies in real-time parameter identification were published in the late 1990s, which remain of high interest due to their capabilities in supporting adaptive flight controls and health monitoring of the system. In 2001, NASA Langley released to the public a collection of algorithms for parameter identification in MATLAB [5]. Titled System IDentification Programs for AirCraft (SIDPAC), it provides algorithms for parameter identification in the time and frequency domain both for post-processing and real-time methods. In addition, it contains tools for designing inputs with and without a priori knowledge of the aircraft.

In recent years, parameter identification research has been aimed at optimizing the entire identification process. Morelli [6] has presented the use of multisine orthogonal inputs in the identification process. These inputs are orthogonal to each other, allowing longitudinal and lateral-directional parameters to be estimated simultaneously. This reduces the flight tests required for identification purposes. In addition, Morelli [7] has combined a use of multivariate orthogonal functions with multisine inputs in order to efficiently model an aircraft. By using orthogonal functions instead of Taylor series expansions, it is easier to determine the model structure that accurately represents the aircraft. A better model structure also leads to more accurate parameter estimation. Expanding on this work, Bryan and Morelli [8] presents a method to automate the entire process. Once a priori model of the aircraft is known (from wind tunnel tests), flight test maneuvers are performed at different locations in the flight envelope. Each maneuver is used to update the preliminary model at that particular section. Once a section has been updated, a Gaussian blending function is used to eliminate any discontinuities. The process is repeated for all of the maneuvers. This automated process leads to very efficient and accurate parameter identification.
In addition, Brandon [9] has presented a method in which piloted maneuvers are used. These maneuvers are based on the multisine inputs, but are performed by a pilot. Fuzzy logic is then used to generate a set of membership functions that contribute to the response of the aircraft at different locations of the flight envelope. This process was also automated and lead to accurate results.

This investigation focuses on real-time parameter identification as well as on the automated blending of newly acquired data, both of which are current topics in this field. The following sections described the most common methods for parameter identification and maneuver design.

### 1.3.2 Linear Regression

Regression is a statistical technique for investigating the relationship between two measured variables [2]. Since the independent variables are related to the unknown parameters in a linear fashion, this is considered a *linear* regression problem. To solve it, measurements of both the dependent and independent variables are made and a least squares method is performed to obtain the unknown parameters in such a way that they minimize the squared error between the measured dependent variable and the equation output. For this reason, this method is also called equation error [2].

The main advantage of linear regression is its simplicity. The parameter estimate can be done in a single matrix multiplication. However, each dependent variable regression must be done individually. In addition, the mathematics behind this method ignore the measurement noise, leading to inaccuracies. To overcome this issue, many data points are needed. Because of this, linear regression is often used as a quick way of determining a preliminary parameter estimate and for determining the final model structure [5].
1.3.3 Output-Error

The output-error method is an iterative process of parameter estimation. It consists of creating a model and varying the parameters until the error between the model output and the measured outputs is at a minimum (hence output-error). The model consists of the equations of motion of an aircraft. After a model is available, a cost function is developed and minimized by means of an optimization routine such as the Newton-Raphson method. The output-error method can be used to determine longitudinal, lateral-directional, or a combination of parameters. It is computationally intensive, but provides historically more accurate results than the linear regression. It is often used in practice as the final identification step [2].

1.3.4 Frequency Domain

Parameter identification can be performed in the frequency domain by transforming the measured time series to the frequency domain using Fourier transforms. The advantages of frequency domain analysis include physical insight of the frequency content, direct applicability to control design, and smaller number of data points [2]. However, the use of digital computers that provide automatic data processing capabilities has shifted the focus of flight data analysis from frequency domain to time domain methods [4].

1.3.5 Maneuver Design

In the identification of a dynamical system, it is necessary to stimulate the dynamic response of the system in order to make all of its states observable. In the case of
aircraft parameter identification, this translates to stimulating the natural modes of the aircraft. Excitation of different modes leads to the better identification of different parameters. Different sets of maneuver types exist, each having advantages and disadvantages, that help obtain the necessary information out of the aircraft states. Since the aircraft model is assumed linear for the identification of its parameters, the maneuvers must be perturbation maneuvers about a reference condition. This means that the maneuvers must begin and end at zero and the deflections magnitude should be chosen small enough as to not drive the aircraft too far away from the reference condition, but not too small so that there is no content in the output data. In addition, in order to excite the aircraft’s natural modes, the maneuvers must contain enough frequency content at and around the frequency of the natural mode that is to be excited. Some of the maneuver types most often used in aircraft parameter identification are explained below.

**Multistep Inputs:** are inputs that consist of a series of steps to opposite sides of the trim deflection. The simplest example of this type of inputs is a doublet, which consists of a squared pulse in one direction immediately followed by a squared pulse in the opposite direction. Because both the pulses of a doublet have the same width, the frequency contained is limited to a small band around the frequency of the doublet. Another type of multistep input is the 3-2-1-1. As its name indicates, the 3-2-1-1 corresponds of 4 pulses in alternating direction of varying width. The first one is three units long, the second one is two units long, and the last two pulses are one unit each. This type of maneuver is preferable over the doublet because its varying widths grant it a much wider range of frequency content [2]. For this reason, the 3-2-1-1 is often used to stimulate highly damped modes such as the short period of an aircraft [4]. The disadvantage of the 3-2-1-1 is that it is significantly longer than the doublet, and the 3 pulse can drive the aircraft too far from the reference condition [2]. Figure
1.1 shows a doublet and a 3-2-1-1 maneuver.

![Doublet Input](image1)

![3-2-1-1 Input](image2)

Fig. 1.1: Doublet (top) and 3-2-1-1 (bottom) inputs.

*Multisine Inputs:* are inputs that consist of a sum of harmonic sinusoids of different amplitudes, frequencies, and phase lags [10].

The design of multisine inputs is an iterative process with the goal of reducing the relative peak factor (RPF) for an input of specified frequency range and amplitudes. A detailed process for designing this type of inputs has been given by Morelli [11]. The main advantage of this type of input is that it requires little *a priori* knowledge of the aircraft’s natural modes, since it covers a specified range of frequencies. The main disadvantage is that it is significantly more complex than the multistep inputs. Figure 1.2 shows a multisine input.

### 1.3.6 Data Compatibility

As mentioned before, in the identification of aircraft, the data used comes from sensors on the aircraft and is, therefore, not perfect. Some of the noise in the sensor
measurements exists because of instrumentation errors, like position or calibration errors, while some of the noise comes from the nature of the sensors themselves. To correct some of the errors, a method known as data compatibility is used. The process involves using the equations of motion along with measured quantities to reproduce the measured outputs of the system. The model and measured outputs are then compared, and a scaling and bias factor can be obtained to make the measured outputs match the theoretical model outputs. The equations used are shown below:

\[
\begin{align*}
\dot{u} &= rv - qw - g\sin\theta + ga_x \\
\dot{v} &= pw - ru + gc\cos\theta \sin\phi + ga_y \\
\dot{w} &= qu - pv + gc\cos\theta \cos\phi + ga_z \\
\dot{\phi} &= p + \tan\theta(q\sin\phi + r\cos\phi) \\
\dot{\theta} &= q\cos\phi - r\sin\phi
\end{align*}
\] (1.4) (1.5) (1.6) (1.7) (1.8)
\[
\dot{\psi} = \frac{q \sin \phi + r \cos \phi}{\cos \theta}
\]  
(1.9)

These parameters can then be integrated using an integration routine. The initial conditions are found using smoothed values at the initial time [2]. The airflow parameters can then be found as:

\[
V = \sqrt{u^2 + v^2 + w^2}
\]  
(1.10)

\[
\alpha = \tan^{-1} \left( \frac{w}{u} \right)
\]  
(1.11)

\[
\beta = \sin^{-1} \left( \frac{v}{\sqrt{u^2 + v^2 + w^2}} \right)
\]  
(1.12)

### 1.3.7 Data Collinearity

When the measured inputs and outputs of the system linearly depend on each other, it is known as data collinearity. Data collinearity is harmful to parameter identification because linearly dependent signals lead to an infinite number of solutions available. For this reason, it is important to check the gathered flight test data for collinearity. The simplest way to check for data collinearity is looking at the correlation matrix. Even though this method is not completely robust for determining collinearity, it has been found in practice that is a good indicator if more analysis is needed. It has been found that a correlation coefficient greater than 0.9 between two measured signals can lead to identification problems [2].


1.4 Problem Statement

To develop a method of estimating the stability and control derivatives of an aircraft in real time with an associated level of confidence that decreases as a function of time and data quality, and to use these values for monitoring the health of a control surface actuator. The estimate must include previously found values and blend them with the newly calculated ones as they become available, as to provide an estimate of the stability and control derivatives and their confidence even when the aircraft states are not observable. The method must provide a probability that an actuator failure has occurred.
CHAPTER 2

Methods

2.1 Block Diagram

To perform the identification and obtain an estimate of the stability derivatives, a block diagram library was developed in Simulink. The library consists of a series of blocks that perform the necessary analysis to obtain stability derivatives of the aircraft, along with a confidence value for each estimate. It was written in terms of tuning parameters, so the blocks can be arranged in any way desired and are completely generic. This means that the same blocks can be used for longitudinal or lateral-directional identification of an aircraft, or they can be arranged to be used on any other system. The blocks can be tuned for different systems through a one-time selection of parameters and thresholds. A flow chart for the final block diagram used for longitudinal identification is shown in Figure 2.1. The following sections describe the methods used and each of the blocks shown in Figure 2.1.
Fig. 2.1: Block diagram flow chart for longitudinal identification.
2.1.1 Batch Creator

The Batch Creator block was designed to accept the current measured value from a sensor and convert it into a sliding window data batch. It can be tuned for different lengths and columns. The inputs, outputs, and parameters of the block are shown in Table 2.1.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensor Current Value</td>
<td>Data Batch</td>
<td>Width of Window</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Length of Window</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sample Time</td>
</tr>
</tbody>
</table>

2.1.2 Parameter Identification

The parameter identification was done using the equation-error method. In this method, the forces and moments are expressed as linear equations and the coefficients of those equations are found using a least squares error. An example of this is trying to model the pitching moment coefficient $C_m$ of an aircraft as a function of its angle of attack $\alpha$, non-dimensional pitch rate $\hat{q}$ and elevator deflection $\delta_e$ by means of unknown parameters $C_{m\alpha}$, $C_{m\hat{q}}$, and $C_{m\delta_e}$ like follows:

$$C_m = C_{m0} + C_{m\alpha} \alpha + C_{m\hat{q}} \hat{q} + C_{m\delta_e} \delta_e + \nu_m$$ (2.1)

where $C_{m0}$ is a bias term and $\nu_m$ is a random error term. $C_m$ is called the dependent variable, while $\alpha$, $\hat{q}$, and $\delta_e$ are called the independent variables, or regressors. Since the independent variables are related to the unknown parameters in a linear fashion, this is considered a linear regression problem. To solve it, measurements of both
the dependent and independent variables are made and a least squares method is performed to obtain the unknown parameters in such a way that they minimize the squared error between the measured dependent variable and the equation output. For this reason, this method is also called equation error [2].

In general form, the linear model can be expressed in matrix notation as

\[ y = X\theta \quad (2.2) \]

and

\[ z = X\theta + \nu \quad (2.3) \]

where \( y \) is the true output, \( z \) is the measured dependent variable, \( \theta \) is a vector of unknown parameters, \( \nu \) is a vector of measurement errors, and \( X \) is a matrix of vectors containing regressors and ones for the bias term. The terms in \( X \) can consist of known functions of the measured independent variables or be the independent variables themselves. In addition, \( \nu \) is assumed to be a zero mean, normally distributed random variable with standard deviation \( \sigma^2 \). To minimize the squared error between the measured output and the model prediction \( X\theta \), a cost function \( J(\theta) \) is defined as

\[ J(\theta) = \frac{1}{2}(z - X\theta)^T(z - X\theta) \quad (2.4) \]

To find a parameter estimate \( \hat{\theta} \), the partial derivative of (2.4) with respect to \( \hat{\theta} \) is set equal to zero as shown below:
\[ \frac{\partial J}{\partial \hat{\theta}} = -X^T z + X^T X \hat{\theta} = 0 \] (2.5)

Solving (2.5) for \( \hat{\theta} \) yields equations for each of the unknown parameters in the following matrix form

\[ \hat{\theta} = (X^T X)^{-1} X^T z \] (2.6)

In addition, if \( v \) meets the assumptions of having zero mean, the covariance of \( \hat{\theta} \) can be shown to be

\[ \text{Cov}(\hat{\theta}) = E[(\hat{\theta} - \theta)(\hat{\theta} - \theta)^T] = \sigma^2 (X^T X)^{-1} \] (2.7)

The variance of the \( j \)th estimated parameter in the vector \( \hat{\theta} \) is the \( j \)th diagonal element of the covariance matrix given in (2.7). Defining the model output \( \hat{y} \) in the form of (2.2) and using (2.6) to find the parameter estimate, the model prediction to a given \( X \) is given by

\[ \hat{y} = X \hat{\theta} \] (2.8)

and the difference between the predicted value and the measured dependent variable is the vector \( \nu \). It is important that these residuals have in fact zero mean. Otherwise, it could mean that there is an unmodeled dependency on another independent variable, which would make the model inaccurate.

The Linear Regression block performs linear regression to fit each of the dependent variables using the regressors. In addition, it calculates a bias term for each regressor.
The inputs, outputs, and parameters of the block are shown in Table 2.2

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regressors</td>
<td>Error</td>
<td>Number of Rows</td>
</tr>
<tr>
<td>Dependent Variables</td>
<td>Coefficients</td>
<td>Number of Regressors</td>
</tr>
<tr>
<td>Correlation</td>
<td>Number of Dependent Variables</td>
<td>Previous Values to Check</td>
</tr>
<tr>
<td>Correlation Rate</td>
<td>Sample Time</td>
<td></td>
</tr>
</tbody>
</table>

2.1.3 Maneuver Detection

While the parameter identification is being performed on every single batch of data that it receives, it is important to add requirements and restrictions as to what can be included in the actual stability derivatives estimate. To do this, several techniques for obtaining good parameter identification results were implemented as means to detect and rate maneuvers as they become evident in the data. Usually, these methods are performed on the data before the parameter identification takes place, but given that the location of the maneuvers in this case is unknown, the process was reversed. Linear regression is applied to every single batch of data, but if no good results can be obtained from the data, the results are not used. If the data meets the requirements of good parameter identification maneuvers (i.e. a low correlation between the regressors, a high signal-to-noise ratio, and contains the natural frequencies of the aircraft as discussed in Chapter 1), the results are blended to the current estimates of stability derivatives. The methods used to detect and rate maneuvers are described in the following sections.
Data Correlation

To ensure that the linear regression algorithm provides good results, it is important that the regressors are linearly independent from each other. This is due to the linear nature of the process. If two linearly dependent signals are used to model a third signal, there is an infinite number of combinations in which this can be done [2]. A way to verify that the signals are linearly independent is by looking at the correlation matrix of the regressors. Low absolute value of the correlation between two regressors indicates that they are linearly independent. The correlation between two independent random variables, \( \rho_{X,Y} \), is given by:

\[
\rho_{X,Y} = \frac{\text{cov}(X,Y)}{\sigma_X \sigma_Y} \tag{2.9}
\]

where \( X \) and \( Y \) are two independent random variables, \( \sigma_X \) and \( \sigma_Y \) are the standard deviation of \( X \) and \( Y \), respectively, and \( \text{cov}(X,Y) \) is the covariance of \( X \) and \( Y \) as defined by:

\[
\text{cov}(X,Y) = E[(X - \mu_X)(Y - \mu_Y)] \tag{2.10}
\]

where \( E \) is the expected value operator, and \( \mu_X \) and \( \mu_Y \) are the expected values of \( X \) and \( Y \), respectively. The \( n \times n \) correlation matrix between \( n \) independent random variables is then given as:

\[
\rho = \begin{pmatrix}
\rho_{1,1} & \rho_{1,2} & \cdots & \rho_{1,n} \\
\rho_{2,1} & \rho_{2,2} & \cdots & \rho_{2,n} \\
\vdots & \vdots & \ddots & \vdots \\
\rho_{n,1} & \rho_{n,2} & \cdots & \rho_{n,n}
\end{pmatrix} \tag{2.11}
\]
As mentioned before, a correlation absolute value of 0.9 or higher can lead to problems with the identification. Since the correlation matrix is symmetric about the diagonal, only one side was used in determining the correlation of the regressors. The sum of the correlation coefficients in this region was used as an indicator of linear independence. Because a larger matrix can lead to a higher value than a smaller matrix, selecting the threshold for maneuver identification was left as a tuning parameter.

It was discovered that the actual value of the correlation coefficients was not enough to detect a PID maneuver, because when the aircraft is in a trim condition the measured values correspond to the noise of the sensors. Since these noises are random, there is small correlation between all of the signals. To prevent this from triggering false maneuver detections, the rate at which the correlation coefficients was used to make sure that their low value was not random noise, but actual linearly independent signals. In addition to this, a signal-to-noise ratio was used to further verify that a maneuver was taking place.

**Signal-to-Noise Ratio**

As its name implies, the signal-to-noise ratio is the ratio of the actual signal being measured to the measuring sensor’s noise. If the signal is not significantly larger than the noise of the sensor, uncertainty in the measurement exists. Historically, a ratio of 10 is ideal, while a ratio of 3 is the minimum for usable results [2]. In this investigation, the angle of attack and the angle of sideslip of the aircraft were used as measurements of the signal-to-noise ratio in longitudinal and lateral-directional maneuvers, respectively.

The trim value of the signals was removed and then the maximum value of the signal away from the trim condition was used to compute the signal-to-noise ratio. The
threshold for signal to noise ratio was selected as 3, to allow any usable data to be rated and identified.

The Signal-to-Noise Ratio block removes the trim value of the input signal and calculates the signal-to-noise ratio of that batch. The inputs, outputs, and parameters of the block are shown in Table 2.3

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal Batch</td>
<td>Signal-to-Noise Ratio</td>
<td>Number of Rows</td>
</tr>
<tr>
<td>Signal Batch</td>
<td>Signal-to-Noise Rate</td>
<td>Previous Values to Check</td>
</tr>
<tr>
<td>Signal Noise</td>
<td></td>
<td>Signal Noise</td>
</tr>
</tbody>
</table>

Table 2.3: Signal-to-Noise Ratio block parameters.

**Fourier Transform**

If the aircraft is excited at its natural frequencies, its states become linearly independent. This is based on the principle that when a dynamic system is excited at its natural frequency, the response data will contain more information [2]. This was exploited for determining if the regressors were linearly independent. To check if the aircraft’s known natural frequencies are present in each batch, a Fast Fourier Transform (FFT) was applied to each batch of data. A range of frequencies around the aircraft’s natural frequencies was specified, and the FFT was used to look for frequency content inside that range. It was assumed that if the data contains the desired frequency, the aircraft states (the regressors) are likely to be linearly independent. The FFT was calculated using the following equation:

$$X_k = \sum_{n=0}^{N-1} x_n e^{-i2\pi kn \frac{2}{N}} \quad k = 0, ..., N - 1$$  \hspace{1cm} (2.12)
where $x$ is the input vector of length $N$. Since the FFT is constantly calculating the power spectrum of the specified frequencies, a power threshold was set to prevent arbitrary data getting passed through. If no frequency exceeds the threshold, the output of the FFT is set to zero.

The frequency content block takes a signal and computes the FFT. It checks to see if any frequency exceeds the specified power spectrum threshold, and if there is, it outputs that frequency. The inputs, outputs, and parameters of the block are shown in Table 2.4

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regressor</td>
<td>Frequency</td>
<td>Number of Rows</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Frequencies of Interest</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Power Spectrum Threshold</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sample Time</td>
</tr>
</tbody>
</table>

**Maneuver Detection**

The maneuver detection block checks all the input values against specified thresholds. If a maneuver is detected, it outputs a signal indicating that a maneuver exists in the data. The inputs, outputs, and parameters of the block are shown in Table 2.5

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correlation Rate</td>
<td>Maneuver Present?</td>
<td>Correlation Threshold</td>
</tr>
<tr>
<td>Signal-to-Noise Ratio</td>
<td></td>
<td>Signal-to-Noise Threshold</td>
</tr>
<tr>
<td>Signal-to-Noise Rate</td>
<td></td>
<td>Signal-to-Noise Rate</td>
</tr>
<tr>
<td>Frequency Content</td>
<td></td>
<td>Frequencies of Interest</td>
</tr>
</tbody>
</table>
2.1.4 Kalman Filter

When the state of a system needs to be determined utilizing noisy measurements, a Kalman filter can be used [12]. In this investigation, a Kalman filter was used to merge the new measurements to old ones based on the quality of the new measurements. The general equations for the discrete Kalman filter used are the following [12]:

\[
    z_k = Hx_k + v_k
\]  
(2.13)

\[
    \hat{x}_k = \Phi_k \hat{x}_{k-1} + w_k
\]  
(2.14)

\[
    \hat{x}_k = \Phi_k \hat{x}_{k-1} + K_k(z_k - H\Phi\hat{x}_{k-1})
\]  
(2.15)

where the subscript \( k \) identifies the current time step, \( z_k \) is a measurement, \( H \) is the state transition model, \( x_k \) is the state estimate, \( \Phi \) is the observation model, \( K_k \) is the Kalman gain. In addition, \( v_k \) is the measurement noise, which is assumed to be zero mean and with covariance \( R \) such that:

\[
    v_k \sim N(0, R_k)
\]  
(2.16)

and, similarly, \( w_k \) is the process noise with covariance \( Q \) such that:

\[
    w_k \sim N(0, Q_k)
\]  
(2.17)

The Kalman filter block is used to blend the data. It uses the input coefficients and the
covariance of each to blend the new measured data to the known coefficients. It takes into account how good the new and old estimates are to merge them appropriately. The inputs, outputs, and parameters of the block are shown in Table 2.6

<table>
<thead>
<tr>
<th>Table 2.6: Kalman filter block parameters.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Input</strong></td>
</tr>
<tr>
<td>Process Noise</td>
</tr>
<tr>
<td>Calculated Coefficients</td>
</tr>
<tr>
<td><strong>R</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

**Data Blending**

The data blending process was performed using a Kalman filter. It was desired that the filter blend the newly calculated stability and control derivatives with the previously known values. These derivatives were assumed to be constant, but a non-zero process noise was introduce so that the covariance of the estimate increases with time if no reliable measurement is made. The measurement covariance, \( R \), was set as a function of the frequency content in the batch. If the batch contains the frequencies of interest, a high confidence is placed on the stability and control derivatives obtained from that batch.

To allow a tolerance to the frequency of interest, the value of \( R \) was calculated as a normal distribution with a mean, \( \mu_f \), at the natural frequency of the aircraft and a tuning standard deviation, \( \sigma_f \). Using the equation for a normal distribution, the measurement covariance was chosen to be:
\( R = K_R \left( 1 - e^{-\frac{(f-\mu_f)^2}{2\sigma_f^2}} \right) \) \hspace{1cm} (2.18)

where \( f \) is the frequency with the highest power in the batch and \( K_R \) is a scaling factor that sets the maximum value of \( R \) when the frequency content of a batch is outside the region of interest. Figure 2.2 shows a normalized plot of \( R \) with mean frequency of 0.5 Hz and a standard deviation of 0.1 Hz.

![Normalized R as a function of frequency plot (K_R = 1).](image)

**Fig. 2.2:** Normalized \( R \) as a function of frequency plot \((K_R = 1)\).

## 2.2 Actuator Failure

The following sections detail the theoretical, empirical, and combined models for determination of the probability of an actuator failure.

### 2.2.1 Theoretical Model

The operational time of a servo failure is assumed to be normally distributed. If the mean time and standard deviation of a servo failure, \( \mu_s \) and \( \sigma_f \) respectively, are
known, the probability that the servo will fail at time $T$ can be calculated using the normal distribution equation as:

$$p_T(t) = \frac{1}{\sigma_s \sqrt{2\pi}} e^{-\frac{(T-\mu_s)^2}{2\sigma_s^2}} \quad (2.19)$$

The probability that the servo has failed after a time $t$ can be found using the cumulative distribution function of Eq. (2.19). Since the probability of a failure is a function of the time in service of the servo, $T_s$, the cumulative distribution is modified as:

$$P_T(t) = \frac{1}{2} \left[ 1 + \frac{1}{\pi} \int_{-\frac{T_s-\mu}{\sigma_s \sqrt{2}}}^{\frac{T_s-\mu}{\sigma_s \sqrt{2}}} e^{-\tau^2} \, d\tau \right] \quad (2.20)$$

Figure 2.3 shows a representative graph of this probability. For a mean servo failure time of 50 hours and a standard deviation of 1 hour.
2.2.2 Empirical Model

The empirical model consists of measuring the control power and comparing it with a nominal value. The measurement is done using parameter identification techniques. Since the measurements are imperfect, the empirical model of an actuator failure includes a confidence level based on how well the requirements to obtain a good estimate from flight test data are met.

The control power measurement is obtained from the Kalman filter blended values, and if it is within a specified range from the nominal value, a boolean decision is made if a servo failure has occurred. The Kalman filter covariance for the control power is used as an indication of the confidence of the measurement. This covariance depends on the time from the last estimate and how good that estimate was. A poor identification maneuver would yield a lower confidence in that maneuver, and as time passes without another good maneuver, the confidence decreases.

2.2.3 Combined Model

The combined model uses the empirical model as a very accurate, low frequency model and the theoretical model as a high frequency, less accurate one. The two of them are blended together to calculate the actual probability of a servo failure, $P$, using the following equation:

$$P = P_T + K_S(P_E - P_T)$$  \hspace{1cm} (2.21)

where $P_T$ is the probability of servo failure calculated using the theoretical model, $P_E$ is the probability calculated using the empirical model, and $K_S$ is a value such
that $0 \geq K_S \leq 1$. By inspection, it can be seen that when $K_S$ is equal to one, the empirical model is trusted completely, and when $K_S$ is equal to zero, the theoretical model is trusted completely. To take into account the accuracy of the measurements, $K_S$ was defined as:

\[
K_S = \begin{cases} 
0, & \text{for } 1 - k_s P_{KF} < 0 \\
1 - k_s P_{KF}, & \text{for } 0 \geq 1 - k_s P_{KF} \leq 1 \\
1, & \text{for } 1 - k_s P_{KF} > 1
\end{cases}
\]  

(2.22)

where $P_{KF}$ is the covariance of the Kalman filter and $k_s$ is a parameter that relates the Kalman filter covariance to $K_S$. This way, when the Kalman filter covariance is zero (perfect measurement), the empirical model is trusted and when its high, the opposite is true.

### 2.3 Algorithm

The algorithm that the block diagram follows is based on the following equations. It is repeated each time the window slides one sample time. It begins by estimating the stability derivatives as follows:

\[
\hat{\theta} = (X^T X)^{-1} X^T z
\]  

(2.23)

A column vector is then created from the elements of $\hat{\theta}$. This is then sent to the Kalman filter as the measurement $z_k$.

\[
z_k = I x_k + K_R \left( 1 - e^{-\left( \frac{(f - \mu f)^2}{2\sigma_f^2} \right)} \right) I
\]  

(2.24)
where \( I \) is the identity matrix and \( f \) is the frequency content found by taking the Fourier transform of the angle of attack and applying a threshold as follows:

\[
f = \begin{cases} 
    f, & \rho > \rho_{\text{min}} \\
    0, & \text{for } \rho \leq \rho_{\text{min}}
\end{cases}
\]  

(2.25)

and

\[
\rho = \frac{Y \text{conj}(Y)}{n}
\]  

(2.26)

where \( Y \) is the fast Fourier transform given as

\[
Y_k = \sum_{n=0}^{N-1} y_n e^{-i2\pi k \frac{n}{N}} \quad k = 0, \ldots, N - 1
\]  

(2.27)

and \( \text{conj}(\cdot) \) indicates the complex conjugate of \( y \) and \( n \) is the length of the transform.

\[
\hat{x}_k = I \hat{x}_{k-1} + QI
\]  

(2.28)

\[
\hat{x}_k = I \hat{x}_{k-1} + K_k (z_k - I^2 \hat{x}_{k-1})
\]  

(2.29)

and the Kalman filter variance is given by

\[
P_{kF} = (I - K_k I)M_k
\]  

(2.30)

The probability of a servo failure is then given as

\[
P = P_T + K_s (P_E - P_T)
\]  

(2.31)
where the theoretical probability, $P_T$ is given by

$$P_T = \frac{1}{2} \left[ 1 + \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{(t+T_s-\mu)}{\sigma f \sqrt{2}} e^{-\tau^2} d\tau \right]$$  \hspace{1cm} (2.32)

The empirical probability, $P_E$, is a measurement of whether or not the servo has failed. That means that if the control power estimate is within the tolerance bounds, the probability is zero, or else the probability is one as shown below:

$$P_E = \begin{cases} 
0, & \text{for } |C_{M_\delta,\text{actual}} - C_{M_\delta,\text{estimate}}| \leq Tol \\
1, & \text{for } |C_{M_\delta,\text{actual}} - C_{M_\delta,\text{estimate}}| > Tol 
\end{cases}$$  \hspace{1cm} (2.33)

and

$$K_S = \begin{cases} 
0, & \text{for } 1 - k_s P_{KF} < 0 \\
1 - k_s P_{KF}, & \text{for } 0 \geq 1 - k_s P_{KF} \leq 1 \\
1, & \text{for } 1 - k_s P_{KF} > 1 
\end{cases}$$  \hspace{1cm} (2.34)

The batch creator block then slides one sample time and the process is repeated.
2.4 Summary of Parameters

The resulting block diagram is composed of fixed and variable parameters. The user inputs include known information about the aircraft, such as the frequencies of the natural modes. The variable parameters, however, need to be adjusted to each specific problem based on the desired performance. The following section summarizes the tuning parameters of the system.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_{\text{min}}$</td>
<td>Power spectrum threshold</td>
</tr>
<tr>
<td>$\sigma_f$</td>
<td>Standard deviation for frequency content calculation</td>
</tr>
<tr>
<td>$K_R$</td>
<td>Scaling factor for calculating measurement covariance</td>
</tr>
<tr>
<td>$Q$</td>
<td>Kalman filter process noise</td>
</tr>
<tr>
<td>$k_s$</td>
<td>Measurement Correction Gain</td>
</tr>
<tr>
<td>$T_s$</td>
<td>Servo time in service</td>
</tr>
</tbody>
</table>
CHAPTER 3

Results

A block diagram was created to determine the stability and control derivatives of an aircraft in real time and blend newly calculated data with previously known values. It then computes the probability of an actuator failure based on theoretical and observed data. The code that runs the block diagram was written in MATLAB in the form of Level 2 sfunctions. The code is comprised of a combination of publicly available MATLAB code found in SIDPAC written inside sfunctions, MATLAB-included functions, and code developed specifically for this investigation. The pieces are tied together by means of tuning parameters that allow to detect and rate maneuvers based on performance requirements determined by the user. The following sections include a description of these parameters along with graphical representation of the effect that each of the tuning parameters has on the overall system.

3.1 Base Model

The block diagram was tuned to obtain a base system. The tuning parameters were then varied and compared to the base system to explain their effect on the output of the system. The base parameters are shown in Table 3.1. For demonstration purposes, the elevator servo failure mean time ($\mu_s$) was assumed to be 50 hours, with a standard deviation ($\sigma_s$) of 1 hour.
Table 3.1: Summary of tuning parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_{\text{min}} )</td>
<td>250</td>
</tr>
<tr>
<td>( \sigma_f )</td>
<td>0.1</td>
</tr>
<tr>
<td>( K_R )</td>
<td>( 5 \times 10^6 )</td>
</tr>
<tr>
<td>( Q )</td>
<td>0.001</td>
</tr>
<tr>
<td>( k_s )</td>
<td>0.05</td>
</tr>
<tr>
<td>( T_s )</td>
<td>0</td>
</tr>
</tbody>
</table>

The following sections include plots obtained by varying each of the parameters in Table 3.1 one at a time, while maintaining the other ones fixed at the base value. Figure 3.1 shows the maneuvers detected by the base model. Since the thresholds for detecting maneuvers was not changed, all of the cases detect all the maneuvers, but the way they are rated and used differs.

![Fig. 3.1: Maneuvers detected by the longitudinal system.](image)

In order to test the capabilities of the system to detect a servo failure, a seeded fault was deliberately injected at 100 seconds into the flight. The fault was simulated by reducing the control power by a factor of 10. To simulate the fault in the previously recorded data of a functioning aircraft, the control deflection was multiplied by 10, while the response of the aircraft remained unchanged. The control power estimates and probability of a servo failure are presented as a function of the tuning parameters in the following sections.
3.2 Power Spectrum Threshold $P$

The Fourier transform applied to the data compares the highest power value to a specified threshold. If the frequency’s power exceeds the threshold, then it outputs that frequency. A higher power threshold requires that the maneuver occurs at the precise frequency of the natural mode in order to be considered accurate. It is desired that the system requires a maneuver to be around the natural frequency of the aircraft, but it should have a tolerance to allow for imperfect maneuvers since the maneuvers are injected by the pilot and might vary slightly each time. The following plots show the effect that changing this threshold has on the outputs of the system.

It can be seen in Figure 3.2 that reducing the threshold results on a wider range of frequencies being passed by the Fourier transform block. Figure 3.3 shows how these extra frequencies yield lower values of $R$ before and after each maneuver. Figure 3.4 shows how the power threshold allows the stability derivatives estimate to be updated more frequently by not requiring such high precision on the frequency content. This also leads to the lower covariance shown in Figure 3.5. Since a relaxed tolerance leads to more frequent updates of the elevator control power, the probability of a servo failure is lower for cases with a lower power spectrum threshold, as seen in Figure 3.6.

Figure 3.7 shows that the system is able to detect the seeded fault in the next available maneuver for all values of power spectrum threshold. However, there was a very small time delay that varied with the threshold value. This is due to the fact that the frequency varies depending on what section of the maneuver is currently present in the data. It can be seen in Figure 3.8 that the probability of a servo failure reaches 100% as soon as the new estimate is available. The time for the failure to be detected does not vary significantly with threshold value.
Fig. 3.2: Frequency content as a function of power spectrum threshold.

Fig. 3.3: Measurement covariance varying with power spectrum threshold.
Fig. 3.4: Elevator control power estimate varying with power spectrum threshold.

Fig. 3.5: Elevator control power covariance varying with power spectrum threshold.
Fig. 3.6: Probability of an elevator servo failure varying with power spectrum threshold.

Fig. 3.7: Elevator control power estimate with a seeded fault varying with power spectrum threshold.
3.3 Measurement Covariance Gain $K_R$

The measurement covariance gain was applied to the inverted normal graph for calculating the $R$ value going into the Kalman filter as a function of the frequency content in the batch. A higher $R$ value makes the Kalman filter rely more on the model than on the measurement. Once again it is desired that frequency has a tolerance band to allow for maneuver imperfections due to the pilot performing the maneuvers slightly different each time. The following plots show the effect that changing this value had on key outputs of the system.

It can be seen on Figure 3.9 that varying $K_R$ shifts the maximum value of the measurement covariance as a function of frequency, but it retains its shape. Figure 3.10 shows the effect on the elevator control power. It can be seen that a lower $R$ value when there is no frequency content causes the elevator estimate to drift towards zero.
This happens because a linear regression performed when the elevator deflection is not observable will yield a control power of zero.

Figure 3.11 shows that the rate at which the estimate covariance is increasing remains constant. However, a higher $K_R$ means that even when there is a maneuver in the data, the noisier measurement does not give the Kalman filter enough confidence to return the covariance back to zero, as it happens when $K_R$ is relatively low. Figure 3.12 shows that increasing $K_R$ causes the system to expect a higher failure probability for the servo. This is due to the higher estimate covariance seen in Figure 3.11 that causes the servo failure model to follow the theoretical model more than the empirical one.

It can be seen on Figures 3.13 and 3.14 that with increasing $K_R$, the control power estimates after the seeded fault are higher. This is because a large $K_R$ leads to high values of $R$ even when the frequency of the maneuver is close to the natural frequency of the aircraft. This makes the Kalman filter not trust the measurements, and the previous estimates (before the failure) have more significance than new ones. When the value of $K_R$ is very high, the previous values of the control power make the system fail to trust the new measurements, leading to a significant delay in the detection of the fault. Extreme caution should be used to prevent this.
Fig. 3.9: Kalman filter measurement covariance varying with $K_R$.

Fig. 3.10: Estimated elevator control power varying with $K_R$.  
Fig. 3.11: Elevator control power covariance varying with $K_R$.

Fig. 3.12: Probability of an elevator servo failure varying with $K_R$. 

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Fig. 3.13: Elevator control power estimate with a seeded fault varying with $K_r$.

Fig. 3.14: Probability of an elevator servo failure with a seeded fault varying with $K_r$. 

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3.4 Measurement Covariance Width $\sigma_f$

The width of the function used to compute $R$ is specified by $\sigma_f$. Since this function follows a normal distribution, its width is determined by a standard deviation. This width makes the Kalman filter more tolerant towards a frequency deviation from the known value, while maintaining its magnitude. The effect that this has on the system is that the Kalman filter relies more on the measurements when a maneuver is present even when the maneuver is away from the natural frequency of the aircraft, while the system is not affected when there is no maneuver present in the data. Unlike with the measurement covariance gain, $K_R$, the maximum value of $R$ stays constant. However, the width varies. The difference between these two parameters can be thought as the covariance gain $K_R$ dictates the behavior when the system is off, while $\sigma_f$ dictates the behavior when the system is on. It is desired that the system considers maneuvers around the natural frequencies of the aircraft. However, a large value of $\sigma_f$ can lead to the system blending data when no maneuver is present.

Figure 3.15 shows a decrease in $R$ width for frequencies around the natural frequency of the aircraft. Increasing $\sigma_f$ decreases the width, and the $R$ value for frequencies away of the natural frequency increases. Figure 3.16 shows that the estimated coefficients do not reach the estimated value, because the $R$ value is high. For the same reason, a lower $\sigma_f$ also makes the estimates’ covariance higher, and the failure probability follows the theoretical model closely at low values of $\sigma_f$. This can be seen in Figures 3.17 and 3.18, respectively.

Figures 3.19 and 3.20 show that the effect of $\sigma_f$ on the system is opposite of the effect on $K_R$. In this case, a low value of $\sigma_f$ results in higher estimates of control power after the fault is injected. It can be seen that for the lowest value, the system does not
detect the seeded fault. This can be explained because \( \sigma_f \) determines when the system will update its estimates during a maneuver, and a very low value makes the system extremely demanding. Caution should be used when tuning this parameter, since making the system too demanding leads to the system not being activated during some maneuvers, failing to detect a failure.

Fig. 3.15: Measurement covariance changing with \( \sigma_f \).
Fig. 3.16: Elevator control power changing with $\sigma_f$.

Fig. 3.17: Elevator control power covariance changing with $\sigma_f$. 
Fig. 3.18: Probability of an elevator servo failure changing with $\sigma_f$.

Fig. 3.19: Elevator control power estimate with a seeded fault varying with $\sigma_f$
Fig. 3.20: Probability of an elevator servo failure with a seeded fault varying with $\sigma_f$.

### 3.5 Kalman Filter Process Noise $Q$

The process noise of the Kalman filter determines how well the mathematical model represents reality for the system being observed. For this investigation, it was assumed that the stability and control derivatives were constants. A process noise was introduced so that the confidence in estimates decreases over time. A process noise of zero would represent that the estimates are perfectly constant. Since it is necessary to allow for change of the estimates, the process noise was used. Figure 3.21 shows the estimates for elevator control power with varying $Q$. It can be seen that the greater the process noise, the less constant the estimate remains. In addition, the covariance of the estimate grows rapidly (Figure 3.22), making the servo failure probability follow the theoretical model instead of the empirical, while lower $Q$ values lead to the measurements being trusted more as shown in Figure 3.23.
It can be seen on Figure 3.24 that high values of $Q$ lead to the estimates of control power quickly converging on their unobservable value of zero. This leads to the system predicting false failures, as can be seen on Figure 3.25. It can also be seen that when the values of $Q$ are low, the Kalman filter holds the previously known values of the control power constant. This is undesirable because the system must then receive various measurements of the failure before deciding that the information is correct. This leads to a significant delay in the failure detection process.

Fig. 3.21: Elevator control power estimate changing with $Q$. 
Fig. 3.22: Elevator control power covariance changing with $Q$.

Fig. 3.23: Probability of an elevator servo failure changing with $Q$. 
Fig. 3.24: Elevator control power estimate with a seeded fault varying with $Q$.

Fig. 3.25: Probability of an elevator servo failure with a seeded fault varying with $Q$. 
3.6 Measurement Correction Gain $k_s$

The measurement correction gain relates the stability and control derivatives’ covariance to how much the measurement is trusted in the servo failure model. A high $k_s$ reduces the effects on the measurements have on the probability output. It can be seen in Figure 3.26 that increasing $k_s$ reduces the time that a measurement is valid, making the model follow the theoretical model more closely. Figure 3.27 shows that $k_s$ has no effect on the failure probability of a seeded fault. This is because this parameter is more closely related to the slope of the failure probability estimate than to its actual value. For this same reason, however, $k_s$ is critical in the determination of how much time without a maneuver is acceptable before the system relies solely on the theoretical model of a failure.

![Failure Probability vs Time](image)

Fig. 3.26: Probability of an elevator servo failure changing with $k_s$. 
3.7 Servo Time in Service $T_s$

Since the probability of a servo failure depends heavily on the time the servo has been in service at the beginning of each particular flight, the servo time in service was varied and the probability of a failure is shown in Figure 3.28, nominally, and in Figure 3.29 with a seeded fault. It can be seen that the initial probability of a failure increases with the previous time in service of the servo. However, when a good estimate is available, the probability decreases significantly. It can also be seen that the rate at which the failure probability increases with the time in service. Figure 3.29 shows that the failure is successfully detected. This is because the servo time in service only affects the theoretical model of the servo. The system’s ability to detect a failure remains unchanged. It can be seen, however, that the longest the servo has been in service, the quicker the probability increases when there is no maneuver in the data. While $k_s$ determines the slope of the graph when moving from the measured model to the theoretical one, $T_s$ determines the slope of the theoretical model.
Fig. 3.28: Probability of an elevator servo failure varying with $T_s$.

Fig. 3.29: Probability of an elevator servo failure with a seeded fault varying with $T_s$. 
3.8 Summary of Results

Table 3.2 shows a summary of the tuning parameters along with the effect that increasing each of them has on the system outputs.

Table 3.2: Summary of tuning parameters and their effect on the system.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_{min}$</td>
<td>Allows less dominant frequencies to change the system</td>
</tr>
<tr>
<td>$\sigma_f$</td>
<td>Turns the system on for more accurate maneuvers only. Increases the effect of new data.</td>
</tr>
<tr>
<td>$K_R$</td>
<td>Non-maneuver measurements have less effect on system. Decreases the effect of new data.</td>
</tr>
<tr>
<td>$Q$</td>
<td>Makes the estimates vary significantly when no measurement is made.</td>
</tr>
<tr>
<td>$k_s$</td>
<td>Makes the failure probability return faster from the empirical to the theoretical model after a measurement.</td>
</tr>
<tr>
<td>$T_s$</td>
<td>Determines the slope of the theoretical model of a failure probability.</td>
</tr>
</tbody>
</table>
A higher language block diagram was developed to constantly scan flight test data as it becomes available. The data is grouped in batches of a specified length. The system analyzes the content of each batch and determines if a good estimate of stability and control derivatives is possible based on frequency content, regressor covariance, and signal-to-noise ratio. When a maneuver is detected, it is rated using the same parameters. The stability and control derivatives are constantly being calculated and a Kalman filter was used to blend the data. When a maneuver is found to contain sufficient information to provide a good estimate, the Kalman filter blends it with previously known values. When the maneuver is not good, the Kalman filter gives less importance to it and instead uses a theoretical model of the stability derivatives. A constant process noise was introduced to the Kalman filter so that the confidence on the estimates decreases over time.

To detect a failure in one of the control surfaces actuators, a statistical model was combined with measurements of the actuator’s health. Once an estimate of the stability and control derivatives and their confidence was available, the calculated control power was compared to a previously known value. If the estimate was close to the known value, it was assumed that the actuator for that particular control surface is working properly. A deviation from the known value, on the other hand, indicates that the actuator has stopped working. The confidence in the control power estimate
was then used to calculate how much the estimate can be trusted. If the control power is known with a high confidence, then the probability of a servo failure was low. On the other hand, if the estimate was poor, the statistical method was preferred.

The block diagram contains several tuning parameters that allow the user to adjust the system to specific needs. Different requirements can be met by varying the tuning parameters. Sweeps of all the tuning parameters were performed. The effect on each of the system outputs was presented and described. These results can be used as a guideline for tuning the system to specific requirements.

From the presented plots, it was determined that the most efficient way to control the capability of the system to detect faults is by tuning the measurement covariance width, $\sigma_f$. This value provides how close to a standard each maneuver has to be in order to be considered in the estimates of stability and control derivatives. Choosing a high value might lead to not detecting a failure unless a maneuver is performed flawlessly, while a low value might trigger false positives due to turbulence disturbances. It was also determined that the Kalman filter process noise, $Q$, had high sensitivity in detecting faults. This parameter determines how much time without a maneuver is acceptable before the system determines that a fault exists. A high value leads to the control derivatives deviating quickly to zero (since the control power is unobservable during level flight), making a fault quicker to occur. A low value of $Q$ holds the estimates constant until the new good measurement is made.

The other tuning parameters have indirect effects on the determination of an actuator failure, and can be tuned to obtain the desired performance of the system. Since the block diagram generated in this investigation uses statistical tools, the performance of the system is tuned in accordance to minimum and maximum accepted tolerances. It provides a probability level based on measurement and historical data that allows
to predict values based on data even when no measurement is available.

Chapter 3 demonstrates that the problem, as stated on Section 1.4 has been solved. The system developed was tuned to successfully detect and rate maneuvers in real time. The system computes stability and control derivatives constantly. A Kalman filter is used to determine the confidence level that varies with time and to merge newly calculated values with previously known ones based on maneuver rating. The system provides a probability level of an actuator failure based on measurements of the control power and theoretical values of a failure.

4.1 Future Work

The future work on this topic includes

- Testing the system on a flight test aircraft
- Developing multisine inputs that the flight computer is capable of injecting to the control surface when the confidence in the stability and control derivatives is low
- This investigation deals only with detecting an actuator failure. However, there are more components that could fail between the flight computer and the control surface. Two of these components are the servo controller and the control position transducer. The system developed does not account for this, and more work is required to determine a method for dealing with those cases.
4.2 References


[8] Brian, G. and Morelli, E.A. "Rapid Automated Aircraft Simulation Model Updating From Flight Data," AIAC14, Session 5A, Fourteenth Australian Interna-
tional Aerospace Congress, Melbourne, Australia, March 2011. PDF


