We consider the boundary value problem
\[
\begin{aligned}
-\Delta u + c(x)u &= \alpha m(x) u^+ - \beta m(x) u^- + f(x, u); \quad x \in \Omega, \\
\frac{\partial u}{\partial \eta} + \sigma(x)u &= \alpha \rho(x) u^+ - \beta \rho(x) u^- + g(x, u); \quad x \in \partial \Omega,
\end{aligned}
\]
where \((\alpha, \beta) \in \mathbb{R}^2, c, m \in L^\infty(\Omega), \sigma, \rho \in L^\infty(\partial \Omega),\) and the nonlinearities \(f\) and \(g\) are bounded continuous functions. We prove existence theorems for both the resonance and nonresonance cases relative to the asymmetric Spectrum with weights. For the resonance case, we provide a sufficient condition, the so-called generalized Landesman-Lazer condition, for the solvability. The proofs are based on variational methods and rely strongly on the variational characterization of the Spectrum.