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Parametric Viscous Analysis of Gust Interaction with Stationary and Elastically Mounted Airfoil

Timothy M. Hollenshade Jr.

Embry-Riddle Aeronautical University - Daytona Beach

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Parametric Viscous Analysis of Gust Interaction with Stationary and Elastically Mounted Airfoil

By

Timothy M. Hollenshade Jr.

A Thesis Submitted to the Graduate Studies Office in Partial Fulfillment of the Requirements for the Degree of Master of Science in Aerospace Engineering

Embry-Riddle Aeronautical University
Daytona Beach, Florida

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This thesis was prepared under the direction of the candidate's thesis committee chairman, Dr. Vladimir Golubev, Department of Aerospace Engineering, and has been approved by the members of his committee. It was submitted to the Aerospace Engineering Department and was accepted in partial fulfillment of the requirements for the degree of Master of Science in Aerospace Engineering.

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ABSTRACT

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This thesis details the development and implementation of a module within a high-accuracy numerical viscous analysis code to simulate the nonlinear interaction of SD7003 airfoil with non-uniform, unsteady incoming flow. The study is focused on the low-Re number unsteady flows typical of MAV applications in which a gust encounter can induce a particularly significant aerodynamic and aeroelastic response. Efficient source models are developed to introduce sharp-edge and time-harmonic gust perturbations with specified amplitude, frequency and duration inside the computational domain through the source terms in the governing momentum equations. Parametric analysis of gust-airfoil interactions for different steady airfoil loads is conducted in comparison with equivalent pitch-ramp and time-harmonic pitching simulations. In addition, all obtained solutions are compared with corresponding predictions based on the inviscid, incompressible unsteady aerodynamic theory. The study reveals complex interaction of inviscid and viscous unsteady forces observed for different gust and pitching excitations, and identifies the degree of similarity between the corresponding gust and pitching airfoil responses. The latter part of this work utilizes an implemented iterative procedure in which a set of governing Navier-Stokes equations is solved
simultaneously with the nonlinear equations of motion for the structure, so that the fluid and structure are treated as a coupled dynamic system. The numerical procedure employs a high-order low-pass filter operator which selectively damps the poorly resolved high-frequency content to retain numerical accuracy and stability over a wide range of flow regimes. The strongly coupled, nonlinear unsteady aerodynamic and structural responses of an elastically mounted airfoil subject to harmonic, high-amplitude vortical gust are examined in a test study, with emphasis on the wing section transition to limit cycle oscillations (LCO).
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CHAPTER 1

INTRODUCTION

Modern UAV systems such as HALE-type vehicles employ both high aspect ratio wings and highly flexible structures for reduced weight for overall improved system efficiencies. Thus, in terms of system controllability, it is highly desirable to understand the gust-airfoil interaction so that impact to the system can be minimized. This study is not limited to fixed-wing aircraft systems but also has significant relevance to both turbines and rotorcraft based systems. Examples include but are not limited to unsteady rotor-stator interaction with high amplitudes of rotor-wake induced flow disturbances or blade-vortex interactions (BVI) in the rotorcraft operation environment.

This study explores three different high-intensity upstream flow turbulence models for evaluation of airfoil nonlinear responses of a flexible UAV wing. The viscous nature of the UAV/HALE operating environment makes the aircraft especially sensitive to upstream flow disturbances. Critical nonlinear effects stemming from viscous flow separations and an extreme response sensitivity to high-amplitude flow disturbances must be accounted for via the CFD approach. This puts the Euler and Navier-Stokes solvers described in Refs. [1-4] which employ fully nonlinear time-marching CFD approaches for unsteady flow prediction capabilities in high demand.

Despite a certain degree of maturity reached by low-Re aerodynamics research in recent years (as summarized, e.g., by Mueller and DeLaurier [5] and Shyy et al [6]), relatively few studies, in fact, have been focusing on the effects of gust impact on MAV wing aerodynamic performance. Recent experimental efforts [7-11] considered canonical pitch-ramp and plunge
airfoil maneuvers to model gust airfoil response in order to systematically examine patterns of
separated vortical flow dynamics especially including the aerodynamically critical process of
laminar separation bubble (LSB) formation and burst and related transition phenomena. A similar
approach was employed in corresponding computational studies [12-16]. On the other hand, the
actual unsteady, non-uniform upstream flow conditions have not been thoroughly taken into
consideration, and a few most recent studies [17, 18] have just reported the development of low-Re
wind tunnel facilities employing various means to generate gusts and shears in the test
sections. The few existing experimental [19] and numerical [20-22] studies only consider time-
harmonic oscillations of the free-stream velocity which primarily impacts the dynamics of the
airfoil boundary layer transition [23] and causes hysteresis in airfoil aerodynamic characteristics
at low Re numbers [21].

In contrast, the current study examines the airfoil response induced by the convected
upwash component of the unsteady, non-uniform upstream flowfield which directly affects the
unsteady aerodynamic loading. In particular, the gust interaction models producing either abrupt
(sharp-edge) or time-harmonic gust-induced variations of the effective airfoil angle-of-attack are
examined. In the classical linearized inviscid, incompressible unsteady aerodynamic theory
(reviewed, e.g., in Ref. [24]), the corresponding flat-plate responses to such disturbances are
analytically described by Kussner and Sears functions. The same theory provides the
aerodynamic response to a step change in the flat-plate angle-of-attack in terms of Wagner
function which asymptotically matches Kussner’s solution for sufficiently long durations of the
Corresponding pitch and sharp-edge gust excitations. A similar correlation exists between the
inviscid Sears’ solution for time-harmonic gust response and Theodorsen’s solution for airfoil
response to time-harmonic pitching oscillations, with both matching in the limit of low excitation
frequencies.

The latter part of this work is motivated by the need for an accurate, robust prediction
tool for analysis of nonlinear responses of a flexible aircraft wing to high-intensity upstream flow
turbulence. Aeroelastic analyses involving coupling of the nonlinear aerodynamic equations with
the linear structural equations are particularly costly to carry out [49]; therefore, standard practice in industry still relies heavily on panel methods [50]. While the reduced-order methods of aerodynamic analysis may be efficient computationally and thus better suited for multidisciplinary design studies, their reliability clearly depends on ability to take into account all physical mechanisms critical for the considered aeroelastic interaction process, including all relevant linear and nonlinear aspects of aerodynamics loads, structural responses, and couplings. Thus, they may not be adequate for prediction of inherently nonlinear aerodynamic phenomena unless a specific CFD or experimental data has been previously extracted to develop a proper set of basis functions.

A high-fidelity analysis is crucial for light-weight MAVs particularly sensitive to the upstream flow disturbances whose impact may compromise the MAV flight stability and performance. Such a tool should be able to accurately account for both structural and aerodynamic nonlinearities in aeroelastic systems that could lead, e.g., to a premature transition to flutter and/or LCO of the structure. Additionally, by using a robust analysis tool, a smart flexible wing structure may actually be designed to alleviate the severity of the gust impact, a fixture so wide-spread in natural flyers (Shyy et al, 2008 [6]). This work addresses the development of such a unified prediction tool on the basis of a high-accuracy Navier-Stokes solver (Visbal and Gaitonde, 2002 [34]) that has been successfully applied to a variety of steady and unsteady flow problems [40, 41, 44].
CHAPTER 2
PROBLEM FORMULATION

2.1 Physical Model

The current work considers the interaction of unsteady, 2-D, vortical flow disturbances with a stationary or elastically mounted airfoil. Such prediction capability is implemented within the structure of high-accuracy, unsteady, compressible Navier-Stokes (N-S) solver FDL3DI [57], first discussed in some detail below.

2.2 Governing Fluid Dynamic Equations

The FDL3DI code solves the governing N-S equations used unchanged in the flow domain for all laminar, transitional, and fully turbulent regions; all are represented in the original unfiltered form. Standard large-eddy-simulation (LES) techniques involve the addition of sub-grid stress (SGS) and heat flux terms; however, the proprietary Air-Force code, FDL3DI, uses an Implicit large-eddy-simulation (ILES) procedure. This procedure employs a high-order low-pass filter responsible for filtering poorly resolved high-frequency content from the solution. The filter is applied to the dependent variables during the solution process and is further described in Refs. [30-32].

The code’s governing equations are discretized via a finite-difference approach in which all the spatial derivatives are obtained via the high-order compact-differencing schemes from Ref. [33]. With respect to the current airfoil-gust interaction study a sixth-order scheme is used. Higher-order one-sided formulas are used at boundary points; this retains the tridiagonal form of the scheme. Time metric terms are evaluated in a manner as to ensure Geometric Conservation
Law (GCL) is satisfied. This was accomplished through the implementation of procedures described in detail in Ref. [34] in which the code was extensively validated for a variety of complex unsteady flows. Lastly, the time marching is accomplished via incorporating a second-order iterative, implicit approximately-factored procedure as described in Refs. [30, 31].

The compressible Navier-Stokes equations are employed in strong, conservative, time-dependent form as shown in (2.1). The physical coordinates (x, y, z, t) are transformed into generalized curvilinear computational coordinates (ξ, η, ζ, τ) and solved via the FDL3DI code:

\[
\frac{\partial}{\partial \tau} \left( \hat{Q} \right) + \frac{\partial \hat{F}}{\partial \xi} + \frac{\partial \hat{G}}{\partial \eta} + \frac{\partial \hat{H}}{\partial \zeta} + \frac{1}{Re} \left[ \frac{\partial \hat{F}_x}{\partial \xi} + \frac{\partial \hat{G}_y}{\partial \eta} + \frac{\partial \hat{H}_z}{\partial \zeta} \right] = S
\]  

(2.1)

The solution vector \( \hat{Q} = (\rho, \rho u, \rho v, \rho w, \rho e) \) is defined in terms of the flow density \( \rho \), Cartesian flow velocity components \( (u, v, w) \), and flow specific energy,

\[
e = \frac{T}{\gamma(\gamma - 1)M^2_w} + \frac{1}{2} (u^2 + v^2 + w^2),
\]  

(2.2)

where the flow pressure \( p \), temperature \( T \), and freestream Mach number \( M_w \), are connected via the perfect gas relationship \( p = \rho T / \gamma M^2_w \) which is assumed along with ratio of specific heats, \( \gamma \).

The other variables in (1) include the inviscid flux vectors defined by

\[
\hat{F}_i = \begin{bmatrix}
\rho \dot{u} \\
\rho u \dot{u} + \dot{\xi}_x \rho \\
\rho v \dot{u} + \dot{\xi}_y \rho \\
\rho w \dot{u} + \dot{\xi}_z \rho \\
(\rho e + p) \dot{u} - \dot{\xi}_i \rho
\end{bmatrix}, \quad \hat{G}_i = \begin{bmatrix}
\rho \dot{v} \\
\rho u \dot{v} + \dot{\eta}_x \rho \\
\rho v \dot{v} + \dot{\eta}_y \rho \\
\rho w \dot{v} + \dot{\eta}_z \rho \\
(\rho e + p) \dot{v} - \dot{\eta}_i \rho
\end{bmatrix}, \quad \hat{H}_i = \begin{bmatrix}
\rho \dot{w} \\
\rho u \dot{w} + \dot{\zeta}_x \rho \\
\rho v \dot{w} + \dot{\zeta}_y \rho \\
\rho w \dot{w} + \dot{\zeta}_z \rho \\
(\rho e + p) \dot{w} - \dot{\zeta}_i \rho
\end{bmatrix},
\]

(2.3)
And the viscous flux vectors, defined in Ref. [35], are

\[
\vec{F}_v = \begin{bmatrix}
0 \\
\xi_y \tau_{i1} \\
\xi_y \tau_{i2} \\
\xi_y \tau_{i3} \\
\xi_y \cdot b_i
\end{bmatrix}, \quad \vec{G}_v = \begin{bmatrix}
0 \\
\eta_x \tau_{i1} \\
\eta_x \tau_{i2} \\
\eta_x \tau_{i3} \\
\eta_x \cdot b_i
\end{bmatrix}, \quad \vec{H}_v = \begin{bmatrix}
0 \\
\zeta_x \tau_{i1} \\
\zeta_x \tau_{i2} \\
\zeta_x \tau_{i3} \\
\zeta_x \cdot b_i
\end{bmatrix}.
\]

(2.4)

\( \chi, i = 1, 2, 3 \) and similarly \( \xi, \eta, \) and \( \zeta \) are used in the following equations for the compact notation of the x, y, and z coordinates.

\[
\tau_y = \mu \left( \frac{\partial \xi_y}{\partial x_j} \frac{\partial u_x}{\partial \xi_k} + \frac{\partial \xi_y}{\partial x_k} \frac{\partial u_x}{\partial \xi_j} \right) - \frac{2}{3} \mu^2 \frac{\partial \xi_y}{\partial x_k} \frac{\partial u_x}{\partial \xi_j} \\
\tau_y = \mu \left( \frac{\partial \xi_y}{\partial x_j} \frac{\partial u_x}{\partial \xi_k} + \frac{\partial \xi_y}{\partial x_k} \frac{\partial u_x}{\partial \xi_j} \right) - \frac{2}{3} \mu^2 \frac{\partial \xi_y}{\partial x_k} \frac{\partial u_x}{\partial \xi_j}
\]

(2.5)

The right hand side, \( \vec{S} \), of (2.1) represents the source term through which the current work generates an incompressible unsteady vortical perturbation upstream of the wing section. This process is further explained in subsequent sections. All flow variables are normalized by their respective reference freestream values except for pressure which is non-dimensionalized by \( \rho_u \vec{u} \).
2.3 Implemented Fluid-Structure Interaction Models

2.3.1 Time-Harmonic Gust Model

The basic gust model employed in this study involves a fairly simple sinusoidal source perturbation equation. For numerical simulations, a single harmonic of the two-dimensional vortical perturbation velocity is selected, described in the form,

\[
\begin{align*}
    u_g &= \varepsilon_u \cos(\alpha x + \beta y - \omega_g t) \\
    v_g &= \varepsilon_v \cos(\alpha x + \beta y - \omega_g t)
\end{align*}
\]

(2.7)

where the gust component amplitudes are,

\[
\begin{align*}
    \varepsilon_u &= \frac{\varepsilon_g \beta u_\infty}{\sqrt{\alpha^2 + \beta^2}} \\
    \varepsilon_v &= \frac{\varepsilon_g \alpha u_\infty}{\sqrt{\alpha^2 + \beta^2}}
\end{align*}
\]

(2.8)

\(\varepsilon_g\) is the gust intensity relative to the mean flow, \(\alpha\) and \(\beta\) are the gust wave numbers in the \(x\) and \(y\) directions, \(\omega_g\) is the imposed gust frequency, and \(u_\infty\) is the convective freestream velocity. Note that \(\alpha = \omega_g / u_\infty\) and \(\beta = \alpha \tan \chi\), where \(\chi\) is the angle between the normal vector of the gust phase front and the \(x\)-axis.

![Figure 2.1: Time-harmonic gust-airfoil interaction model](image)
The 2D analytical gust source model utilized in this work is an extension of [51] in which a 1D gust was developed. In order to generate the gust of the form (2.7) downstream of the source region $|x - x_s| \leq \pi/b$, one should impose the following source components in the flow momentum equations,

$$
\begin{align*}
& s_u(x, y, t) = \beta K g(x) \hat{\lambda}(y) \cos(\omega_s t - \beta y - \alpha x) \\
& s_v(x, y, t) = K g'(x) \hat{\lambda}(y) \sin(\omega_s t - \beta y - \alpha x)
\end{align*}
$$

(2.9)

where the function $g(x)$ and constant $K$ are described in detail by Ref. [40], and the function $\hat{\lambda}(y)$ is selected to provide a smooth transition in the $y$-direction to provide a compact region of the uniform gust distribution,

$$
\hat{\lambda}(y) = \frac{1}{2} \{\tanh[3(y + y_c)] - \tanh[3(y - y_c)]\}
$$

(2.10)

Note that a shear layer is generated in the region where $\hat{\lambda}(y)$ varies, but the resulting pressure waves generally have moderate amplitudes.

This configuration serves as one of the benchmarks in computational fluid dynamics (e.g., Ref. [36]) and is intended to evaluate the unsteady response of a lifting surface. The perturbations to the flow field simulate incident flow turbulence to a body or upstream-generated flow unsteadiness of which the velocity field may be described in terms of the following Fourier spectrum containing various perturbation frequencies and wave numbers,

$$
u'(\tilde{x}, t) = \text{Re}\{\sum_{\omega} A_{\omega}(\tilde{x}) \exp[i(\tilde{k} \cdot \tilde{x} - \omega t)]\}
$$

(2.11)
2.3.2 Sharp-Edge Gust

![Image of sharp-edge gust models](image)

Figure 2.2: Sharp-edge uniform (top) and 1-Cos (bottom) gust-airfoil interaction models.

The sharp-edge convected gust models are commonly used (e.g., Ref. [15]) to investigate transient unsteady aerodynamic and aeroelastic responses of lifting surfaces to impinging short-duration flow perturbations. Such models can be described in terms of the upwash velocity profile,

\[
v_g = \begin{cases} 
\varepsilon_g f(t - x / u_\infty), & u_\infty(t - T_g) \leq x \leq u_\infty t \\
0, & \text{otherwise}
\end{cases}
\]  

where the classical form commonly used in the unsteady aerodynamic theories corresponds to \(f(t-x/\omega_u) = 1, T_g \to \infty\). Particularly for short gust durations \(T_g\), the natural gust behavior more closely resembles the so-called 1-Cos model [15] represented by

\[
f(t - x / u_\infty) = \frac{1}{2} \varepsilon_g \left[ 1 - \cos \left( \frac{2\pi}{T_g} (t - x / u_\infty) \right) \right]
\]  

The two gust configurations with fixed gust durations are illustrated in Figure 2.2. Note that the actual gust source implementation is developed by forcing the momentum equations with selected constant-amplitude velocity perturbations. Due to the fluid inertia, no special effort is required to obtain gusts similar to 1-Cos configurations at short gust durations since those are realized by
means of a natural ramping of the gust velocities. The gust amplitude $\varepsilon_g$ and duration $T_g$ are the parameters varied in the current study.

### 2.3.3 Aeroelastic Response Model

The FDL3DI aeroelastic response model has been developed to investigate, in-particular, limit cycle oscillation (LCO) phenomenon occurring for elastically mounted wings interacting with upstream flow at velocities above the critical flutter speed. Its incorporation to the fluid solver essentially involves coupling the equations governing the fluid motion and those governing the wing motion. The resulting single dynamic system is developed and represented following Ref. [47].

![Image of 2-DOF aerelastic airfoil model](image)

**Figure 2.3: Adapted 2-DOF aerelastic airfoil model.**

The two-degrees-of-freedom (2-DOF) plunging/pitching structural response of elastically mounted airfoil is governed by a set of non-linear equations (Appendix I) reducible to the following classical quasilinear form:
where the airfoil displacement in both plunging and pitching is given by the vector
\[ \nu(t) = [h(t), \alpha(t)] \]. \( L(t) \) and \( M(t) \) represent the lift and pitching moment about the rotation axis.

Wing section structural properties are introduced via the linear mass matrix, \( M_s \), and the damping matrix \( C_s \) defined as follows:

\[
M_s = \begin{bmatrix}
m & S_s \\
S_s & I_s
\end{bmatrix}, \quad C_s = 2 \begin{bmatrix}
\zeta_x \sqrt{k_3 m} & 0 \\
0 & \zeta_\alpha \sqrt{k_\alpha I_\alpha}
\end{bmatrix}
\]

(2.15)

Here \( m \) denotes the mass of the wing section, \( \zeta_x \) and \( \zeta_\alpha \) represent the damping logarithmic decrements, and \( S_s \) and \( I_s \) are the static and mass moments respectively. The non-linear, in general, spring forces introduced into the aeroelastic system through the non-linear stiffness function, \( F(\nu) \), as follows:

\[
F(\nu) = \begin{bmatrix}
k_h h(t) \\
k_{\alpha^3}(t) \alpha(t) + k_{\alpha^1} \alpha^3(t)
\end{bmatrix}
\]

(2.16)

Where \( k_h, k_{\alpha^1} \) and \( k_{\alpha^3} \) are the spring constants.

The lift and moment forces acting on the wing are calculated via a summation of all viscous and non-viscous forces as follows:

\[
L = f_\nu \sin \alpha + f_\nu \cos \alpha
\]

\[
M = f_\nu(Y Y_0) + f_\nu(X X_0)
\]

(2.17)
where,

\[
f_x = \frac{J}{\sqrt{\gamma}} \left[ x_\xi \mu \left( u_\xi \xi_x + u_\eta \eta_x + v_\xi \xi_x + v_\eta \eta_x \right) \right]
\]

\[
y_\xi \left( \frac{\rho}{J} \left( \lambda + 2 \mu \right) \left( u_\xi \xi_x + u_\eta \eta_x \right) \right)
\]

\[
f_y = \frac{J}{\sqrt{\gamma}} \left[ y_\xi \mu \left( u_\xi \xi_x + u_\eta \eta_x + v_\xi \xi_x + v_\eta \eta_x \right) \right]
\]

\[
x_\xi \left( \frac{\rho}{J} \left( \lambda + 2 \mu \right) \left( v_\xi \xi_x + v_\eta \eta_x \right) \right) \lambda \left( u_\xi \xi_x + v_\eta \eta_x \right)
\]

(2.18)

The numerical implementation reduces the structural problem to a set of ordinary differential equations (ODE) governing the plunging and pitching oscillations of the flexible wing section, and has been shown to be robust and insightful for analysis of effects of structural nonlinearities on the wing nonlinear aeroelastic response. In particular, Refs [25-28, 29] used the 2-DOF model to investigate the presence of internal resonances as well as for identification and control of LCO development in nonlinear aeroelastic systems. The elastic mounting capabilities of the FDL3DI code have been rigorously examined and utilized in many works including Ref. [41] and [44] conducted on the ERAU Zeus cluster. This representation is throughly documented by Dreyer [38]. This work further examines a nonlinear form for the equations of motion defined and discussed further in Appendix I.
CHAPTER 3

Numerical Implementation

This section encompasses the numerical implementation of the new FDL3DI solver’s capabilities employed in the current study.

3.1 Moving Grid Capability

The numerical FDL3DI CFD code has been developed to accommodate an arbitrary grid motion via four time-dependent coordinate variation equations. A grid point’s position in the X and Y plane is given via,

\[
X(t) = X_{pc} + (X_0 - X_{pc}) \cos \alpha \cdot (Y_0 - Y_{pc}) \sin \alpha \\
Y(t) = Y_{pc} + (X_0 - X_{pc}) \sin \alpha \cdot (Y_0 - Y_{pc}) \cos \alpha
\]

where, \((X_0,Y_0)\) corresponds to the airfoil starting position at \(\alpha_0\). Taking the derivative of (3.1) with respect to time yields the corresponding nodal velocity,

\[
\dot{X}(t) = (Y(t) - Y_{pc}) \dot{\alpha} \\
\dot{Y}(t) = (X(t) - X_{pc}) \dot{\alpha} \cdot \dot{Y}
\]

A dissipation region is employed to increase computational efficiency and reduce cell skewness. This is accomplished by driving the gridpoint velocities to zero over a period of space at a distance great enough away from the airfoil as to not induce any artificial effects in the flowfield.
3.2 2-DOF Structural Response Model

Development of the coupled structural, aerodynamic response was complex due to the inherent lagging of the response due to the built-in implicit solver of the FDL3DI code. This is accomplished via an elastic response model which utilizes implicit Beam-Warming [37] for the fluid solver, and a 4th order Runge-Kutta integrator for the structure. Its implementation required alterations to several aspects of the code’s operation. Within the code’s time marching procedure, the computed aerodynamic loads are used to determine the displacement vector which is, in turn, used to define the grid motion components \( (\dot{X}(t), \dot{Y}(t)) \). At each time step, a sub-iterative procedure ensures a properly resolved unsteady flowfield corresponding to the new airfoil position. The details of the module development are thoroughly documented in [38], in which it is validated to the experimental results of [39], and is utilized for parametric studies [40, 41].
3.3 Boundary Conditions

At interior points a centered formula is employed with a five-point stencil as shown below. Because of the centered stencil, the error is exclusively dispersive. Up to sixth order accuracy can be obtained though proper choice of coefficients.

\[
\phi'_1 + \alpha \phi'_2 = a_i \phi_1 + b_i \phi_2 + c_i \phi_3 + d_i \phi_4 + e_i \phi_5 + f_i \phi_6 + g_i \phi_7
\]

Equation 3.3 is employed at boundary point 1 and is responsible for the maintaining of the scheme’s tridiagonal nature.

The basic procedure of Lele [54] is used with interpolation formulas for obtaining the function values at mid-points. The use of function and derivative values at mid-points greatly facilitates the formation of some viscous terms. Additionally, the formation of the stress tensor at mid-points also requires certain derivative values at midpoints; the formulas for which may be derived in essentially the same manner as for interpolation. Due to the fact that node derivatives are restricted to 6th order, only up to sixth order accuracy for mid-point derivatives is sought.

The interior schemes summarized above address accuracy; however, the equally important aspect of stability must also be taken into consideration. Vichnevetsky [56] shows that central-difference based schemes exhibit spurious reflections at interfaces where a step-jump in the spacing is encountered. In practice, stability can be achieved via the introduction of dampening and/or filtering [55]. In order to reduce reflections in the computational space a filter, mentioned earlier, was developed and documented by Visbal and Gaitonde [57]. Employed on the surface of the airfoil is a no-slip, 4th-order, explicit zero pressure boundary. The far-field boundary is prescribed with fixed dependent variables set to free-stream conditions with the
the exception of the wake region downstream of the airfoil where a zero velocity gradient, 
\( \frac{\partial u}{\partial x} = 0 \), is imposed (figure 3.2).

3.4 Numerical Mesh and MPI Parallelization

The work described within utilized two geometries. References [38] and [44] employed a 649 x 
395 x 3 grid generated around a SD7003 airfoil for use with the FDL3DI code. In this work the 
original (fine mesh) resolution was maintained around a symmetric, 12%-thick Joukowski airfoil. 
In order to improve the computational efficiency a 327 x 198 x 3 mesh about an SD7003 airfoil 
was carefully tested against the original fine mesh. Good predictions agreement was obtained for 
flow parameters within the ranges of interest. Thus, in this work the latter geometry was utilized 
for stationary airfoil-gust versus pitching airfoil comparison. Later simulations in this work 
examining the full-motion capabilities of the solver utilize the original fine mesh about the 
symmetric airfoil geometry.
For all geometries a rounded trailing edge (figure 3.4) was utilized to avoid numerical instabilities in the region.

The FDL3DI solver incorporates Chimera (overset) capabilities which requires a five-point stencil. Thus, a standard O-style grid was employed with a five cell overlap (Figure 3.5). This characteristic of the solver had to be addressed when reducing the resolution of the mesh. The grid coordinates are oriented in the following manner. $\eta$ denotes the normal surface vector and $\xi$ traverses clockwise around the airfoil while $\zeta$ denotes the span-wise direction. This is expressed as follows: $(\xi, \eta, \zeta) = 649 \times 395 \times 3$. 
In the process of developing the overset mesh PEGASUS [45] software is used to compute the domain connectivity database as well as blanking grid points contained within solid boundaries. Afterwards, the mesh is efficiently partitioned into 32 overlapped blocks using the BELLERO [46] software package. This software makes generation of the decomposition layout nearly automatic while still maintaining high order connectivity patterns and interpolation weights. The resulting decomposed mesh provides input to the FDL3DI code in the current study. All simulations are conducted using ERAU’s 262-processor Beowulf Zeus cluster (64-bit, 3.2 GHz Intel Xeon, 4GB RAM systems).
CHAPTER 4

Validation

4.1 Sharp-Edge Gust Model

To validate the sharp-edge gust source model, the gust evolution downstream of the source region is first illustrated in Figure 4.1 (a, b). The gust upwash variations at selected moments of time, obtained along $y=0$ and $x=-1$ lines, are compared for gust amplitudes $\varepsilon_g = 0.07$ (a) and $\varepsilon_g = 0.35$ (b), respectively. These amplitudes correspond to $\Delta \alpha_m = 4^\circ$ and $20^\circ$ and are used in comparison to an equivalent prescribed pitching motion for the airfoil. Note a rather pronounced overshoot of the gust amplitude carried by the gust frontal wave, further settling close to the specified value. In simulations, the source region continuously “feeds” the gust for a prescribed duration $T_g$ which becomes overall slightly prolonged, because of the extension of the source region somewhat compensated by the natural source ramp-up and ramp-down periods caused by the fluid inertia. For comparison with the airfoil pitching motion, the best effort is thus made to synchronize the two events. Further commenting on Figure 4.1, the effect of the loaded airfoil’s potential velocity field is evident approaching the leading edge, as the gusts appear seemingly “absorbed” by the increasing corresponding mean flow velocity components. The comparison of the two gust amplitude cases confirms the actual superposition of the perturbation and the mean upwash components.
Figure 4.1: Evolution of sharp-edge gust upwash at $y=0$ (top) and $x=-1$ (bottom), for $T_g=3$ (left) and $T_g=10$ (right); $a_\infty=4^\circ$: (a) $\varepsilon_g=0.07$, (b) $\varepsilon_g=0.35$.

4.2 Time-Harmonic Gust Model

The time-harmonic gust discussed in Chapter 2 has been extensively validated though numerous works [38, 40, 41, 44, 47] and is employed by this study in a 1-D transverse manner (i.e., $\beta=0$ in Eq. 2.7). For further validation of the time-harmonic gust source model, including generated gust velocity perturbations, the reader is referred to Ref. [44].
4.3 Prescribed Grid Motion

4.3.1. Pitch-Ramp Motion

To compare with the sharp-edge gust cases, the airfoil pitch-up-down maneuver is prescribed with corresponding durations and pitching amplitudes equivalent to those produced by the gust encounters with maximum upwash-induced angles of attack $\Delta \alpha_m = \epsilon_g$ (rad). In addition, the effect of the gust duration on the unsteady airfoil response are examined for $T_g = 1, 3, 5$ and 10. The corresponding pitch-up and pitch-down airfoil motions are specified by prescribing the following variation of the angle of attack with the coefficient $k$ controlling the pitch ramping period.

$$\Delta \alpha(t) = \frac{1}{2} \Delta \alpha_m \{ \tanh[k(t' + T_g / 2)] - \tanh[k(t' - T_g / 2)] \}$$

$$t' = t - t_0 - T_g / 2 - 0.25$$

The $k$ in equation 4.1 controls the pitch ramping; its effect on the unsteady airfoil response is as follows.
For $k=20$, Fig. 4.3 illustrates the resulting pitch-up-down maneuvers obtained for $\Delta \alpha_m=20^\circ$ corresponding to the selected range of gust durations. Here it is worthwhile to note the airfoil begins pitching after $t=15$, which is when the gust source is initiated. This is due to the fact that the gust is generated in a source region upstream and takes time to propagate downstream to the leading-edge of the airfoil. The source region for the time-harmonic gust is defined by $x_s=-1.5$ and $b=5$. For the sharp-edge gusts, the choice of the source region was made for $x_s=-1.5$ and $b=20$. In the latter case, e.g., it takes $\Delta t=1.3$ for the gust front to reach the airfoil leading edge. For proper comparison, this determines the timing of the start of the pitching ramp ($t_0=16.3$) in respective moving-airfoil simulations.

The produced ramping period is selected to approximate the observed ramping of the gust upwash at the airfoil leading edge. For $\alpha_0=4^\circ$ and two amplitudes corresponding to $\Delta \alpha = 4^\circ$ and $20^\circ$, Figure 4.4 (a, b) compare the effect of different ramping periods corresponding to $k=20$ and $k=10$. Note a significant variation in the resulting peak amplitudes of the airfoil unsteady responses. For the small-amplitude pitch, the time histories are practically unaffected during the ramped-up airfoil position but the settling responses (after returning to the initial position) appear slightly more pronounced. For the high-amplitude pitch, a more noticeable difference in
fluctuations of the aerodynamic coefficients during the ramped-up position is observed. Below, the airfoil pitch-up-down motion with $k=20$ is selected for comparison with sharp-edge gust responses.

![Figure 4.4: Effect of airfoil pitching ramp on unsteady response for $T_g=3, \alpha_0=4^\circ$; $k=20$ (solid line), $k=10$ (dashed line). (a) $\Delta\alpha_m=4^\circ$, (b) $\Delta\alpha_m=20^\circ$](image)

### 4.3.2 Time-Harmonic Pitching

For comparison with time-harmonic gust responses, the pitching motion is prescribed in terms of the harmonic variation of the airfoil angle of attack with frequency $\omega_{pt}$ relative to the pitching center $x_p$,

$$\alpha(t) = \alpha_m \sin \omega_{pt} t + \alpha_0$$  \hspace{1cm} (4.2)
CHAPTER 5

Results

5.1 Sharp Edge Gust Response, $\alpha_0=4^\circ$

As indicated in the Introduction; The gusty urban environments characterized by flow disturbances can induce particularly significant aerodynamic and aeroelastic responses for MAVs. To further simulate this environment all durations and both amplitudes are studied for freestream conditions $M_{\infty}=0.1$ and $Re = \rho_{\infty}c/\mu = 10,000$ corresponding to $\alpha_0 = 4^\circ$ and $\alpha_0 = 8^\circ$. The “loaded” nature of the airfoil more closely resembles that typical of the UAV/MAV(s)’s operational environment. All variables are non-dimensionalized by the airfoil chord $c$, flow density $\rho_{\infty}$, and flow velocity $u_{\infty}$. A fixed time step with $\Delta t = 2 \times 10^{-4}$ is chosen for implicit time marching.

![Figure 5.1 Unsteady airfoil response to pitch-up-down maneuver for $T_g = 1, 3, 5, 10$; $\alpha_0=4^\circ$, $\Delta \alpha_m = 4^\circ$](image)

As a reference point for further comparison, Fig. 5.1 first shows the airfoil unsteady lift and moment responses to pitch-up-down maneuvers with different effective gust durations obtained for $\Delta \alpha_m = 4^\circ$ (for $\Delta \alpha_m = 20^\circ$ see Appendix VII, Fig. 5.2). The plots coincide for the extension of the specific duration followed by the previously discussed pitch-down transition.
peak and settling to the original steady-state fluctuations. In Fig. 5.1, note that the unsteady responses to small-amplitude short-duration pitches remain unsettled and keep increasing in amplitude as the duration increases. However, for the longest duration of $T_g = 10$, the pitched-up airfoil response eventually tends to settle down adjusting to the new steady-state airfoil position.

In contrast, for the high-amplitude pitch, Fig. 5.2 (Appendix VII), the airfoil responses appear (on average) at similar levels with violent fluctuations dominated by dynamics of stalled flows. However, in all high-amplitude pitch cases, the unsteady airfoil responses appear to settle relatively quickly following the pitch-down motion, approaching the original steady-state condition.

![Figure 5.3: Unsteady airfoil response to sharp-edge gust for $T_g = 3, 5, 10, 13$; $a_o = 4^\circ$, $e_g = 0.35$](image)

Fig. 5.3 summarizes similar results obtained for the high-amplitude gusts (for low-amplitude see Appendix VII, Fig. 5.4). Examination reveals drastically different airfoil responses observed between the low- and high-amplitude cases. Indeed, for the low-amplitude gusts, the lift and moment fluctuations continue to be dominated by von Karman vortex shedding which frequency $f \approx 0.2/\alpha$ remains practically unaltered relative to that of the steady-state fluctuations ($f \approx 2.86$). In contrast, the response to the high-amplitude gusts appears completely dominated by the stalled flow dynamics. These findings are further confirmed by a direct comparison of unsteady lift and moment responses to sharp-edge gust and correspondingly synchronized pitch-up-down maneuvers. The aforementioned is presented below.
In Fig. 5.5, the obtained sharp-edge gust and pitch-ramp airfoil lift responses are compared against corresponding Kussner’s and Wagner’s analytical results. Doing so helped further examine the character of dominant unsteady forces. The analytical results are shown in R.T. Jones’ [48] approximation for the flat-plate inviscid, incompressible initial lift responses. For the complete collection of gust versus pitching plots for $\Delta \alpha_m = 4^\circ$ see Appendix VII (Fig. 5.6). The analytical results are adjusted for the steady-state lift values at the time of the impact. Overall, the initial response in both cases appears to correlate well with inviscid predictions but soon viscous separation phenomena take over.

Additional insight is made possible via Appendix VIII which provides the side-by-side comparison of instantaneous contour plots of the flow vorticity, $\vec{\omega}_v = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$, obtained for $T_g = 10$ at the moments of time corresponding to the effective pitch ramp for $t=16.4...16.8$ (note the gust’s convected front), ramp-down for $t=26.4...26.8$ (note the gust’s convected tail), and at a later time of $t=32.8$. Except for the shortest-duration case, the pitch-up-down motion produces a much larger impact on the unsteady airfoil response thanks to the formation of the dynamic-stall vortices convecting along the airfoil’s suction side ($t=26.4$ in Appendix VIII). As noted before, for $\Delta \alpha_m = 4^\circ$, Fig. 5.5(left), the von Karman vortex street starts to re-establish its control of the unsteady aerodynamic response, with the new shedding frequency $f = \approx 1.43$ (compared to its double for the gust response) determined by the airfoil’s new shaded area in the
Clearly, from Fig. 5.6, it also takes a much longer time for the pitched-down airfoil to transition back to the original steady-state response (compare differences still existing at $t=32.8$ in Appendix VIII).

Finally, Fig. 5.7 (Appendix VII) shows a similar comparison of the unsteady lift and moment responses obtained for $\Delta \alpha_m = 20^\circ$, with further illustrations in Appendix IX showing the vorticity contours at the same moments of time as in Fig. 5.7. One can immediately note similarities between airfoil responses to the sharp-edge gusts and the pitch-up-down maneuvers, evident for all durations. The differences are primarily determined by the initial impact, with the dynamic stall effects abruptly taking over the unsteady response following the pitch-up motion and contrasting with a much smoother transition for the impinging gust ($t=16.8$ in Appendix IX). In the end, the flow patterns characterized by massive separation and stall vortex dynamics appear very similar ($t=26.4$). Interestingly, the transition back to the original state is much faster and smoother following the gust passage compared to the pitch-down motion, as observed for all durations in Fig. 5.7 and confirmed by clear differences still present at $t=32.8$ in Appendix IX. The comparison with analytical inviscid predictions in Fig. 5.5(right) generally confirms the previous findings.

5.2 Sharp Edge Gust Response, $\alpha_0=8^\circ$

![Figure 5.8: Comparison of airfoil response to sharp-edge gust vs. pitch-up-down maneuver; $\alpha_0=8^\circ$, $T_g=10$, $\Delta \alpha_m = 4^\circ$ (Left) $\Delta \alpha_m = 20^\circ$ (Right)](image)
All parametric simulations of gust durations and amplitudes were repeated with the airfoil installed at $\alpha_0=8^\circ$. Conducting a complete analysis similar to that of section 5.1 it is observed that the overall patterns of unsteady lift and moment time histories and the near-surface vortical flow dynamics appear similar to the previous study for both airfoil pitch-ramp and gust cases, with a few notable differences. To illustrate the latter, Figs. 5.9 and 5.10 (Appendix X) only present plots produced, respectively, for $\Delta\alpha_m=4^\circ$ and $20^\circ$ which are matched against the corresponding solutions shown in Figs. 5.6 and 5.7 (Appendix VII).

First, notice the significantly higher amplitudes of steady-state fluctuations of aerodynamic coefficients for the undisturbed airfoil which almost absorb the gust response for $\Delta\alpha_m=4^\circ$ thus making the latter barely visible in Fig. 5.9. As a result, the contrast with the pitch-ramp response appears more striking compared to the cases with $\alpha_0=4^\circ$ in Fig. 5.6. The only exclusion is noted for $T_g=10$ in Fig. 5.9(d) where the gust response eventually starts to develop large fluctuations signaling transition to the stalled flow dynamics.

On the other hand, similar to the previous study with the lower steady-state angle of attack, the pitch-ramp unsteady circulatory response in Fig. 5.9 becomes quickly overwhelmed by the stalled flow dynamics, this time persisting for all durations. However, the main difference appears in the long-term responses which, in contrast to results in Fig. 5.6, do not exhibit a clear pattern of settling to the original steady-state response after the pitch-down maneuver. In fact, the unsteady simulations produce conflicting results for different pitch-ramp durations. Although the unexpectedly
probabilistic character of the observed phenomenon should be examined in the further studies, the spikes in the aerodynamic response observed later in the time histories are associated with the delayed convection of the induced boundary-layer vorticity and its subsequent separation. This issue will be re-visited in the analysis of time-harmonic responses.

For the large induced angle of attack ($\Delta \alpha_m=20^\circ$), the comparison of Figs. 5.10 and 5.7 reveals an overall similarity of the unsteady gust and airfoil pitch-ramp responses for all considered cases. Remarkably, all of them eventually settle to the original steady-state fluctuations (in contrast to the above-noted lower-amplitude pitching cases in Fig. 5.9(b, d)). Furthermore, comparing with analytical inviscid predictions shown above in Fig. 5.8, the inviscid forces appear to dominate for much longer durations in the high-amplitude perturbation cases. Eventually, the stalled flow dynamics takes over the control of the amplitude of aerodynamic oscillations for the longest gust durations in Fig. 5.10(c, d). Finally, the inviscid forces again appear to dominate the transient aerodynamic response during the airfoil return to the original unperturbed state. Moreover, such dominance appears to regularize the settling process to the original steady-state flow conditions.

5.3 Time-Harmonic Gust Response

As studied previously for sharp-edge simulations, here both gust and pitching time-harmonic disturbances are examined for airfoil with steady loading corresponding to $\alpha_0=4^\circ$ and $\alpha_0=8^\circ$. The time-periodic character of the airfoil unsteady lift and moment responses is clearly observed in Figs. 5.11 and 5.12 (Appendix XI) presenting, respectively, the summary of pitching and gust simulations conducted with low and high excitation amplitudes. All responses eventually
transition to the steady-state oscillations following the gust passage or the airfoil return to the original position. Note that the gust responses are not in phase; no effort was made to synchronize, e.g., the phase of the initial upwash at the leading edge since such adjustments would not change the time-periodic response amplitudes.

Due to the considerable amount of work already conducted with the time-harmonic gust model it was this study’s intention to explore what forces dominate the flowfield solution. To accomplish this four cases of gust frequencies corresponding to wavelengths $\lambda_g = 1, 1.5, 2, 2.5$ (non-dimensionalized by the chord) are examined alongside the equivalent airfoil oscillation frequencies corresponding to $\omega_{pt}=2\pi/\lambda_g = 2\pi, 4\pi/3, \pi, 4\pi/5$ in Eq. 3.4. Both gust and pitching are conducted for period, $T_g = 10$. In order to draw conclusions as to the specific fluid forces at work the time-history of the airfoil response is presented with superimposed analytical results. The analytical results include Sears’ solution for thin-airfoil unsteady lift response to impinging sinusoidal, transverse gust, and the pitching-airfoil unsteady lift obtained from Theodorsen’s theory for oscillating airfoils (both are summarized, e.g., in Ref. [24]). The coefficient time histories for $\alpha_0=4^\circ$ and $\Delta\alpha_m=20^\circ$ with $\lambda_g = 1.5$ are shown below in Fig. 5.13. The remarkably close comparison for obtained lift/moment time histories against predictions of the inviscid, incompressible unsteady aerodynamic theory alludes to the dominance of inviscid forces present. The complete collection of time-history coefficients with superimposed inviscid solutions are shown in Appendixes III - VI.

Figure 5.13: Time-harmonic airfoil response with Inviscid theory for $\alpha_0=4^\circ$, $\Delta\alpha_m=20^\circ$ and $\lambda_g = 1.5$
In contrast to results from the previous sharp-edge sections, the viscous separation effects appear no longer dominant for the entire duration of the airfoil excitations. Such effects are mainly manifested through “wiggles” in the gust response curves especially noticed for low-amplitude cases in Fig. 5.12(a) (Appendix XI) where the induced lift fluctuations are comparable to steady-state levels. Interestingly, the viscous effects show a more significant effect on moment oscillations, particularly prominent for the high-amplitude gust perturbation in Fig. 5.12(b). As expected, the lift deviations from the linearized inviscid theory are more noticeable for the low-amplitude gust response where the viscous effects are more prominent (Appendix III). An intriguing departure from the linear response is also observed for the high-amplitude case and appears most pronounced for $X_g = 2.5$ in appendix IV (d). Note that the gust moment responses reveal a more impulsive and “jerky” behavior compared to the lift curves, which may present a challenge for effective vehicle control in gusty environment.

In general, the differences in amplitudes of the airfoil time-harmonic responses to transverse gust vs. pitching can now be explained based on the arguments from the inviscid incompressible theory. For the higher excitation frequency ($X_g = 1$ in appendix III - IV (a)), the greater circulatory and dominant non-circulatory components in the pitching lift response superimpose too far exceed the gust-induced lift amplitudes. In contrast, for the low-frequency case with $X_g = 2.5$ in appendix III - IV (d), the gust and pitching circulatory terms are comparable while the non-circulatory pitching component becomes much smaller, with the cumulative effect explaining the much reduced differences in the resulting lift and moment response amplitudes.

Appendix V and VI present a similar comparison for $\alpha_0 = 8^\circ$ of gust and pitching unsteady aerodynamic responses including results from the inviscid unsteady aerodynamic theory. With the airfoil approaching stall conditions, the major difference from the less-loaded airfoil cases appears in the more pronounced viscous effects observed in the airfoil response to the low-amplitude excitations in appendix V. All cases there reveal a less satisfactory comparison with corresponding inviscid predictions. Particularly notable are the results in appendix V (c) with $X_g = 2$ where, in contradiction to the inviscid analysis, the gust response for the only time exceeds the
one induced by the pitching oscillations. Moreover, the examined low-amplitude cases take longer time to transition back to the original steady-state oscillations. Similar to the previous cases with \( \alpha_0 = 4^\circ \), the inviscid theory shows good agreement with predicted high-amplitude lift responses in appendix VI, although it somewhat deteriorates for the low-frequency pitching in (d) where viscous effects become more pronounced.

5.4 Response of Elastically-Mounted Airfoil

The flexible capabilities described within this work are presented here for a 2D gust with intensity \( c_g = 0.1 \) convecting with \( \chi = 45^\circ \) and oscillating with reduced frequency \( k_g = 1 \) (\( \omega_g = 2 \)).

The gust is generated using the momentum source components (2.9) in a region specified by \( x_s = -3.5, \ y_s = 2, \) and \( b = 5 \) in equations (2.9) and (2.10). For these simulations the symmetric Joukowski airfoil is installed with the fine mesh at zero angle-of-attack and is evaluated for flow velocities \( V = 15...19 \text{m/s} \) and \( \text{Re} = 5 \times 10^4 \). A fixed time step of \( \Delta t \approx 10^{-4} \) is used in all flexible simulations and is carried out for \( 3 \times 10^6 \) time steps to let the structure establish its long-term response.
Figure 5.14 provides the comparison of time histories and corresponding FFT frequency spectra for the airfoil angle of attack and plunging amplitude. The corresponding unsteady lift and moment are shown via Fig. 5.15 (Appendix XI). The gust imposes a continuous forced excitation of the structure which responds with oscillations at the gust frequency clearly observed in all cases, except for plunging amplitude which in fact starts to develop an LCO-type behavior even at 15m/s, indicating a noticeable sensitivity of the aeroelastic response to the airfoil shape. Other parameters clearly indicate a contrast between responses for the two flow velocities, with the case of 18m/s showing a distinct superposition of the gust and LCO frequencies (ωLCO ~ 0.3) and the latter practically subdued in the case of 15m/s. A rather wide peak at the LCO frequency is attributed to the transitional airfoil behavior characterized by frequency shifts towards establishing the long-term structural response. Interestingly, the aerodynamic characteristics are still dominated by the gust response and appear similar in both cases, while the structural response clearly shows the difference between the two cases. Note that the amplitude of the
structural LCO response and its effect on the airfoil aerodynamic performance will significantly increase for larger flow velocities. In the present study, the maximum amplitudes of structural oscillations do not exceed 3° for pitching and 4% of chord for plunging oscillations, for which the linearized form of the equations of motion (2.14) is adequate.

Focus is now placed on the airfoil post-flutter response with different amplitudes of LCO, and compare linear and nonlinear structural response models for the highest examined flow velocity of 19m/s. Figure 5.16 first compares resulting pitching motion and unsteady lift time histories respectively. The corresponding plunging motion and unsteady moment are shown via Fig. 5.17 (Appendix XI). These simulations were conducted with the linearized equations of motion (2.14). Note a remarkable shift in the frequency and amplitude of the structural response observed for the case of 19m/s. The observed time histories can be roughly subdivided into three periods corresponding to the initial transient development, followed by a period characterized by high amplitude and higher LCO frequency of the unsteady structural and aerodynamic responses, and then the final period with established, lower LCO frequency and amplitude.
Figure 5.16: Time histories and FFT spectra for airfoil response to gust for 18 and 19m/s, linear structural model.

The LCO frequency shift for 19m/s is clearly observed in the FFT spectra, with two distinct peaks at $\omega_{LCO} \approx 0.6$ and 0.2. Note that the established LCO frequencies are different for the cases of 18 and 19m/s. At the same time, the gust excitation frequency is visible only in the aerodynamic response for 19m/s, with its amplitude now dominated by the structural response. The gust frequency response shows similar amplitudes for all cases, but also reveals an earlier onset of flow instability at 19m/s.

With the amplitudes of pitching and plunging oscillations reaching high values of $23^\circ$ for pitching and 15% of chord for plunging oscillations at 19m/s, the linearized structural motion model may not be adequate for such a flow regime. To examine the resulting discrepancies, the linear (2.14) and nonlinear (A.1) models are compared in Figures 5.18 and 5.19 (Appendix XI) respectively.
Note that the long-term responses appear similar (except for phase shifts) in Figure 5.17 both for LCO frequency and amplitudes. However, the transient processes observed in Fig. 5.18 are different, and particularly the high-amplitude oscillations around $\omega_{LCO} \sim 0.6$ are absent in results based on the nonlinear model that rather exhibit rich spectra of lower-amplitude transient modes before reaching the final LCOs. Finally, to illustrate the resolved flow patterns, Figure 5.20 shows a snapshot of the flow vorticity contours obtained at $t=253$ during the established airfoil LCO cycle using the nonlinear structural response model. Both the gust and separated flow vortical structures are evident, with resolution quickly deteriorating in the downstream wake region due to grid stretching.
Figure 5.20: Vorticity contour plot at $t=253$ for 19m/s, nonlinear response model.
Figure 3.2: Plots investigating the effect on the total integrated power over a cusp crossing when leaving out the lowest frequencies. First row shows the integrated total power over an entire cusp crossing. The second row shows a sample total power spectrum of the cusp crossing. The area under the total power spectrum curve is the total integrated power for the windowed data. The data has been windowed with a Hanning window of $\alpha = 0.5, 0.6, 0.7, 0.8, 0.9, 0.99, \text{ and } 1$. 
theoretical inviscid solutions revealing the overall dominance of inviscid effects both in gust and pitching responses.

In agreement with the inviscid theory, the time-harmonic pitching responses far exceeded the gust responses for high excitation frequencies, while the differences were significantly reduced for lower frequencies. The exception was observed in the case of the low-amplitude gust interacting with highly-loaded airfoil where the developed dynamic-stall vortices convecting along the airfoil’s suction side manifested the presence of nonlinear viscous forces.

The airfoil flexible response of the elastically mounted, symmetrical, unloaded Joukowski airfoil to the 2-D gust was obtained for viscous flow velocities corresponding to pre-flutter and post-flutter regimes and typical of MAV wing flow conditions. The gust imposed a continuous forced excitation of the structure which responded with oscillations at the gust frequency. The post-flutter regimes showed a distinct superposition of the gust and LCO frequencies, with the latter practically subdued in the case of 15 m/s. The comparison of the airfoil surface pressure revealed the unsteady response almost entirely dominated by the gust impact at 15 m/s, while above the flutter boundary the effect of the structural motion appeared increasingly more significant. Wide peaks at the LCO frequencies were attributed to the transitional airfoil behavior characterized by frequency shifts towards establishing the long-term structural response. For the highest examined post-flutter flow velocity of 19 m/s, the results revealed remarkable LCO frequency and amplitude shifts in the structural response. The comparison of quasilinear and nonlinear structural response models indicated different transient processes towards establishing similar final LCO patterns with a phase shift. In particular, the quasilinear model revealed a transient region characterized by a high-amplitude LCO at $\omega LCO \sim 0.6$ that abruptly switched to the lower-amplitude LCO at $\omega LCO \sim 0.2$, whereas the nonlinear model reached such long-term response through continuous spectrum shift with lower-amplitude transient modes.

Through this work potential limitations and consequent improvements to the employed process were identified. Extrusion of the grid away from the airfoil solid boundary with a cell growth rate causes a subsequent degradation of the numerical mesh resolution. This was observed and noted in the current study with respect to the phase shift of the time-harmonic gust disturbance. Unless resolved this impediment will affect future developments of upstream flow disturbances such as a Rankin-Vortex model being developed at the present time. To address this a overset mesh system, Fig 6.1, was constructed to exploit the Chimera.
and noted in the current study with respect to the phase shift of the time-harmonic gust

disturbance. Unless resolved this impediment will affect future developments of upstream flow
disturbances such as a Rankine-Vortex model being developed at the present time. To address this

an overset mesh system, Fig. 6.1, was constructed to exploit the Chimera (overset) capabilities of

the FDL3DI solver. To aid in the construction of the multi-domain mesh, OVERGRID [53], a

graphical user interface (GUI) was used. This software package allows for the interactive

visualization, manipulation, and diagnostics of overset surface and volume grids for input into the

PEGASUS [46] domain connectivity code.

![Figure 6.1: Overset Mesh and Subsequent Free-stream Solution](image)

The overset block (red block in Fig. 6.1) is uniform with respect to the cell spacing in all

coordinate directions. This may account for the numerical instabilities that exist on the edge of

the block further upstream of the airfoil. This is most likely due to the large discrepancy in cell

size between the overset block and underlying O-mesh. Proper implementation of the codes

overset capabilities as they apply to this problem will undoubtedly allow for more numerical

mesh resolution in the gust-source region upstream of the wing.
LITERATURE CITED


42. D. Hixon and B. Li, University of Toledo, Toledo, OH “External Verification Analysis (EVA) Method for Time Marching Computational Aeroacoustics Codes.”, AIAA-2008-2920


45 Suhs, Norman E and Rogers, Stuart E and Dietz, W E , “PEGASUS 5 An Automatic Pre-Processor for Overset-Grid CFD,” AIAA Paper 2002-3186


54 S K Lele Compact Finite Difference Schemes with Spectral-like Resolution Journal of Computational Physics, 103 16-42, 1992


In Ref [52], the general 2-DOF equations of motion for the center of mass of a wing section are derived from Lagrange’s equation, as briefly outline below. With respect to the wing section’s elastic axis, the position and velocity of the center of mass can be represented by

\[
\begin{align*}
 r &= r \cos(\alpha + \phi) \hat{i} - (y + r \sin(\alpha + \phi)) \hat{j} \\
 \dot{r} &= r\dot{\alpha} \sin(\alpha + \phi) \hat{i} + (\ddot{y} + r\dot{\alpha} \sin(\alpha + \phi)) \hat{j}
\end{align*}
\] (A 1)

so the total kinetic energy of the system is

\[
T = \frac{1}{2} m_r (\dot{r}^2 + \dot{\alpha}^2) + \frac{l_a \dot{\alpha}^2}{2} = \frac{m_r}{2} (\ddot{y}^2 + (\dot{r})^2 + 2 \dot{y} \dot{r} \cos(\alpha + \phi)) + \frac{l_a \dot{\alpha}^2}{2}
\] (A.2)

Using the kinetic energy terms of Lagrange’s equations, \( \frac{\partial T}{\partial \dot{y}} / \partial t \) and \( \frac{\partial T}{\partial \dot{\alpha}} / \partial t \), the following equations of motion can be derived from these contributions as well as the potential energy and the work resulting from the internal damping and external aerodynamic forces,

\[
\begin{align*}
 m_r \ddot{y} + S_a \cos(\alpha + \phi) \ddot{\alpha} - S_a \sin(\alpha + \phi) \dot{\alpha}^2 + C_y \dot{y} + F(y) &= -L \\
 S_a \cos(\alpha + \phi) \ddot{\alpha} + I_a \ddot{\alpha} + C_a \dot{\alpha} + F(\alpha) &= M
\end{align*}
\] (A.3)

In these equations, the Coulomb-type damping is neglected, but the higher-order terms including the centripetal acceleration, stiffness nonlinearities, and the transcendental terms are retained. Lastly, using the small angle theorem and assuming the affects of gravity to be minor with respect to aerodynamic forces, equation 3.6 can be produced.
APPENDIX II

All.1 FDL3DI Aerodynamics input file

&DATA
XM1=0.1,
RE=10000.0,
TW=1.002,
S1=0.38,
ALFA=4.0,
NDTAU=99999,
CFL=1.0,
IBETA=0,
DTVIS=0.0002,
DFIX=0.0002,
IDMPFIL=3,
ES4=0.01,
ES2=0.0,
FES4=20.0,
FES2=0.0,
OMGAV=1.0,
SRCONST=50.0,
IMOVE=0,
IV1ON=1,
IV2ON=1,
IV3ON=1,
IVMON=1,
ISGSMODEL=0,
TFWRATIO=2.0,
TURB=0.005,
EDINF=1.0,
AKE=0.0,
FACTKELIMIT=1.0,
IDIAG=1,
ITIMEINT=2,
INTERP=0,
isCHME=15,
JSCHME=15,
KSCHME=15,
IJKSCHMEKE=0,
IHYBRID=0,
IHYIROE=4,
IHYJROE=4,
IHYKROE=4,
XMHYBCUTOFF=0.0,
IHBCUTOFF=0,
JHBCUTOFF=0,
KHBCUTOFF=0,
IVISC=1,
IMETRC=1,
ILIMIT=1,
ICUT1=0,
ICUT2=0,
ICUT3=0,
JCUT1=0,
JCUT2=0,
JCUT3=0,
KCUT1=0,
KCUT2=0,
KCUT3=0,
XICUTOFF=0.05,
ETACUTOFF=0.05,
ZETACUTOFF=0.05,
IXIIS0=1,
IETAIISO=1,
IZETAISO=1,
NCONV=4,
ISUBON=1,
NSUBMX=3,
INMAX=1000,
RSTMAX=100,
RSTPRINT=10,
/&END

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INFORMATION ABOUT COMPACT DIFFERENCING SCHEME
****GRID****
C4-AC5-C6-AC5-C4
****X DIRECTION****
C4-AC5-C6-AC5-C4
****Y DIRECTION****
C4-AC5-C6-AC5-C4
****Z DIRECTION****
C4-AC5-C6-AC5-C4

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PERIODIC BC INFO: IPERDC, JPERDC, KPERDC
0 0 0

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RK4 INFORMATION
KSTAGES, COEFFICIENTS AS IN CODE
4 1. 6. 1. 3. 1. 3. 1. 6. 1. 2. 1. 2. 1. 1. 1. 2. 1. 1. 1.

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FILTER INFORMATION
IGFILTER,JGFILTER,KGFILTER
1 1 1
INOFIL,JNOFIL,KNOFIL
1 1 1
FOR I DIRECTION: INTERIOR FILTER ORDER, ALPHA, SPECIAL PARAMETERS (NUMBER AND VALUES)
8 0.4 0
POINT 1 ORDER, GAMMA, NOOPT, OPTPARMS
0 0.0
POINT 2
4 0.45 0
POINT 3
4 0.45 0
POINT 4
6 0.4 0
POINT 5
8 0.4 0
FOR J DIRECTION: INTERIOR FILTER ORDER, ALPHA, SPECIAL PARAMETERS (NUMBER AND VALUES)
8 0.4 0
POINT 1 ORDER, GAMMA, NOOPT, OPTPARMS
0 0.0
POINT 2
4 0.45 0
POINT 3
4 0.45 0
POINT 4
6 0.4 0
POINT 5
8 0.4 0
FOR K DIRECTION: INTERIOR FILTER ORDER, ALPHA, SPECIAL PARAMETERS (NUMBER AND VALUES)
8 0.4 0
POINT 1 ORDER, GAMMA, NOOPT, OPTPARMS
0 0.0
POINT 2
4 0.45 0
POINT 3
4 0.45 0
POINT 4
6 0.4 0
POINT 5
8 0.3 0

! THIS LINE BLANK
MULTIPLE BLOCKS
NOBLKS,((IJKBLK(IDMY,JDMY),JDMY=1,6),IDMY=1,NOBLKS)
0 7 24 1 19 36 37
   ! THIS LINE BLANK
Plunging Wing parameters: TOMG0,RFREQ,ALF1,OMEG0, XC, YC, ALF0
0.2 1.0 0.14324 -0.2 0.5 0.0 0.0
   ! THIS LINE BLANK
Gust source parameters: IGUST, GOMEGA, GEPS, GTANGLE, XGS, YGS, BGS, TAUGS, DTAUGS
1 3.141592653 0.35 0.0 -1.5 2.0 5.0 12.0 10.0

All.2  FDL3DI Structural input file

&structs
printdebug=1
useRK =0
AFY_doti =0.000
alpha_doti=0.000
mass =2.55E0
ass =-0.024
bss =0.2
C0 =340
S_alpha =10.4E-3
l_alpha =2.52E-3
ki_h =5.5E-3
ki_alpha =1.8E-2
k_h =450
k_alpha1 =9.3
k_alpha3 =55
useresume =0
/end
Time-harmonic airfoil response for $\alpha_0 = 4^\circ$ and $\Delta \alpha_m = 4^\circ$ with Inviscid theory superimposed:

(A) $\lambda_g = 1$,  (B) $\lambda_g = 1.5$,  (C) $\lambda_g = 2$,  (D) $\lambda_g = 2.5$
Time-harmonic airfoil response for $\alpha_0=4^\circ$ and $\Delta \alpha_m=20^\circ$ with Inviscid theory superimposed:

(A) $\lambda_g = 1$, (B) $\lambda_g = 1.5$, (C) $\lambda_g = 2$, (D) $\lambda_g = 2.5$
APPENDIX V

Time-harmonic airfoil response for $\omega_0=8^\circ$ and $\Delta \alpha_m=4^\circ$ with Inviscid theory superimposed:

(A) $\lambda_g = 1$,  (B) $\lambda_g = 1.5$,  (C) $\lambda_g = 2$,  (D) $\lambda_g = 2.5$
Time-harmonic airfoil response for $\alpha_0=8^\circ$ and $\Delta\alpha_m=20^\circ$ with Inviscid theory superimposed:

(A) $\lambda_g = 1$,  (B) $\lambda_g = 1.5$,  (C) $\lambda_g = 2$,  (D) $\lambda_g = 2.5$
APPENDIX VII

Figure 5.2 Unsteady airfoil response to pitch-up-down maneuver for $T_g = 1, 3, 5, 10; \alpha_0 = 4^\circ, \Delta \alpha_m = 20^\circ$

Figure 5.4: Unsteady airfoil response to sharp-edge gust for $T_g = 3, 5, 10, 13; \alpha_0 = 4^\circ, \epsilon_g = 0.07$
Figure 5.6: Comparison of airfoil response to sharp-edge gust vs. pitch-up-down maneuver; $\Delta \alpha_m = 4^\circ$, $\alpha_m = 4^\circ$.
(a) $T_g = 1$, (b) $T_g = 3$, (c) $T_g = 5$, (d) $T_g = 10$. 

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Figure 5.7: Comparison of airfoil response to sharp-edge gust vs. pitch-up-down maneuver; \( \Delta \alpha_m = 20^\circ \), \( \alpha_0 = 4^\circ \).

(a) \( T_g = 1 \), (b) \( T_g = 3 \), (c) \( T_g = 5 \), (d) \( T_g = 10 \).
APPENDIX VIII

Vorticity contours for sharp-edge gust (left) vs. pitch-up-down motion (right); $T_g = 10, \alpha_o = 4^\circ, \Delta \alpha_m = 4^\circ$.

\[ t = 16.4 \]

\[ t = 16.6 \]

\[ t = 16.8 \]
APPENDIX IX

Vorticity contours for sharp-edge gust (left) vs. pitch-up-down motion (right); $T_g=10$, $\alpha_0=4^\circ$,
$\Delta \alpha_w=20^\circ$. 

$t=16.4$

$t=16.6$

$t=16.8$
APPENDIX X

Figure 5.9: Comparison of airfoil response to sharp-edge gust vs. pitch-up-down maneuver; $\Delta \alpha_{m} = 4^\circ$, $\alpha_{o} = 8^\circ$.

(a) $T_e = 1$, (b) $T_e = 3$, (c) $T_e = 5$, (d) $T_e = 10$. 
Figure 5.10: Comparison of airfoil response to sharp-edge gust vs. pitch-up-down maneuver;\n\[\Delta \alpha_m = 20^\circ, \alpha_0 = 8^\circ.\]
(a) \(T_g = 1\), (b) \(T_g = 3\), (c) \(T_g = 5\), (d) \(T_g = 10\).
Figure 5.11: Unsteady airfoil response to time-harmonic pitching with $\omega_p$, corresponding to $\lambda_e = 1$, 1.5, 2, 2.5; $\alpha_0 = 4^\circ$. (a) $\Delta \alpha_m = 4^\circ$, (b) $\Delta \alpha_m = 20^\circ$. 
Figure 5.12: Unsteady airfoil response to time-harmonic gust with $\omega pt$ corresponding to $\lambda_\xi = 1, 1.5, 2, 2.5; \alpha_0 = 4^\circ$. (a) $\xi_\sigma = 0.07$, (b) $\xi_\sigma = 0.35$. 
Figure 5.15: Time histories and FFT spectra for airfoil response to gust for 15 and 18m/s, linear structural model.

Figure 5.17: Time histories and FFT spectra for airfoil response to gust for 18 and 19m/s, linear structural model.
Figure 5.19: Time histories and FFT spectra for airfoil response to gust for 19m/s, linear vs. nonlinear structural models.