

**VANISHING RELAXATION TIME DYNAMICS OF THE  
JORDAN MOORE-GIBSON-THOMPSON (JMGT) EQUATION  
ARISING IN HIGH FREQUENCY ULTRASOUND (HFU)**

The (third-order in time) JMGT equation is a nonlinear (quasilinear) Partial Differential Equation (PDE) model introduced to describe the acoustic velocity potential in ultrasound wave propagation. One begins with the parabolic Westervelt equation governing the dynamics of the pressure in nonlinear acoustic waves. In its derivation from constitutive laws, one then replaces the Fourier law with the Maxwell-Cattaneo law, to avoid *the paradox of the infinite speed of propagation*. This process then gives rise to a new third time derivative term, with a small constant coefficient  $\tau$ , referred to as relaxation time. As a consequence, the mathematical structure of the underlying model changes drastically from the parabolic character of the Westervelt model (whose linear part generates a s.c, analytic semigroup) to the hyperbolic-like character of the JMGT model (whose linear part generates a s.c, group on a suitable function space). It is therefore of both mathematical and physical interest to analyze the asymptotic behaviour of hyperbolic solutions of the JMGT model as the relaxation parameter  $\tau \geq 0$  tends to zero. In particular, it will be shown that for suitably calibrated initial data one obtains at the limit exponentially time-decaying solutions. The rate of convergence allows one then to estimate the relaxation time needed for the signal to reach the target. The interest in studying this type of problems is motivated by a large array of applications arising in engineering and medical sciences. These include applications to welding, lithotripsy, ultrasound technology, noninvasive treatment of kidney stones.