Stress Intensity Factor - Simple Solution For A Complex Problem

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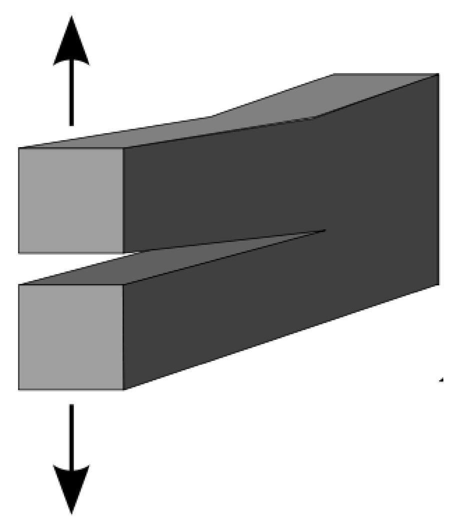
Abstract

This research explores the modeling of a cracked structure using a linear elastic finite element method (FEM) analysis with the objective of determining the stress intensity factor (SIF) for complex geometries. The analysis uses a mapping of the stress concentration area ahead of the crack tip to asymptotically determine the SIF. For model calibration, Irwin's approximation method was used in two known crack case solutions subjected to a uniform tensile loading (Mode I – Opening) with fixed geometric dimensions. With focus on aircraft airworthiness application, two more complex geometries with irregular stress distributions were analyzed and the results were extracted for future use in crack growth simulation.

Background

Stress intensity factor, SIF or K, is defined as the driver parameter characterizing the state of stress near a crack tip on a linear elastic material specimen. This region of stress can be related to the remote loading condition. The most critical loading condition is found to be a Mode I-opening, which can be represented as tensile stress normal to the crack as shown in *Fig.1*. K_I is dependent on the geometric parameters, including the size of the crack. The geometric factor, β, is a correction applied to the general SIF equation, when compared to Westergaard's solution for a center crack in an infinite plate, to include asymmetry and the finite geometric dimensions of a specimen. When considering a specimen and crack of arbitrary shape and size, the stress values ahead of the crack tip can be determined using *Eqn.1*, where r is the distance away from the crack tip and θ is the angle relative to the plane of the cracked section. In this study, θ = 0 for convention making *Eqn.1* simplify to *Eqn.2*. In *Eqn.3*, σ is defined as the nominal or refences stress value for a specimen under tensile loading, and a is the size of the crack. *Eqn.4* is used to asymptotically obtain the stress intensity factor (SIF) from the mapped stress ahead of crack tip, and consequently the geometric factor for a given reference stress and crack size by using *Eqn.3*.



(Fig.1) Mode I: Opening

$$\sigma = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2} - \sin\frac{3\theta}{2}\right) \quad \text{(Eqn.1)}$$

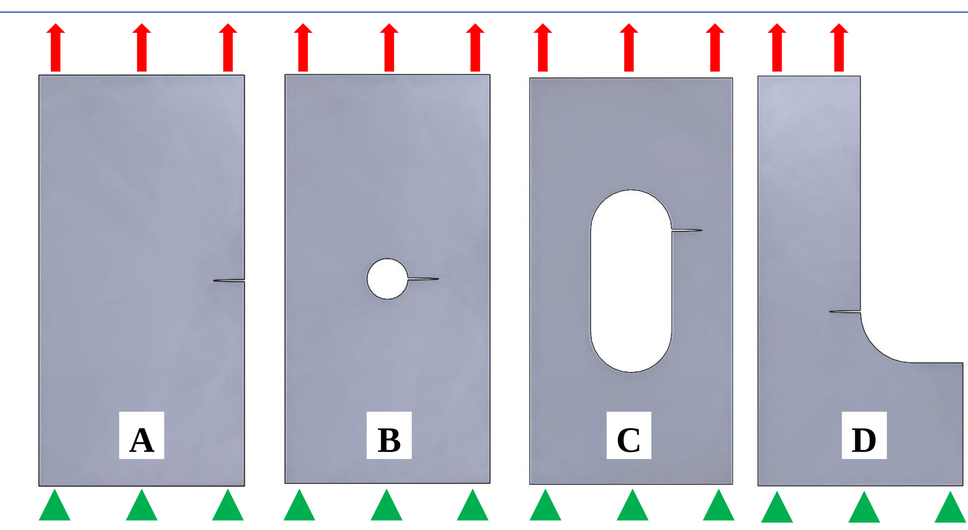
$$\sigma = \frac{K_I}{\sqrt{2\pi r}} \quad \text{(Eqn.2)}$$

$$K_I = \beta\sigma\sqrt{\pi a} \quad \text{(Eqn.3)}$$

$$K_I = \lim_{r \to 0} \sigma\sqrt{2\pi r} \quad \text{(Eqn.4)}$$

Methods

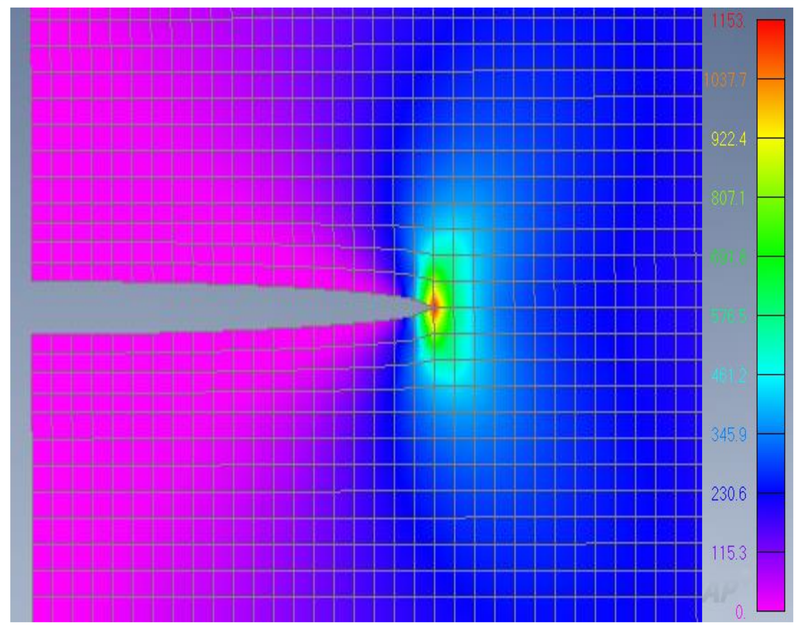
To determine the SIF's, Finite Element Models containing cracks were created using FEMAP. The models contained a mesh with 2D planer elements which has an element size of 0.5mm x 0.5mm. The crack was geometrically formed from an ellipse shape which was then simplified to tangential quad element shapes. For the simulations we considered Al 6061-T651 plate measuring 100mm x 200mm x 6.35 mm. The stress ahead of crack tip was then mapped and K_I was determined as explained before, known as Irwin's approximation. As proof of concept, a simple structure was initially considered with crack in the edge of a plate, as shown in *Fig.2A*. Once the FEM sensitivity analysis was completed and the result matched the expected SIF, the model was considered calibrated. In sequence, the same procedure was applied to a more complex geometry with a crack emerging from a hole (*Fig.2B*) for reproducibility check. In both cases, the results were within 10% of the expected values, what is the claimed accuracy of SIF equations available in the literature. The model was then applied to even more complex geometries, that could represent structure cutouts and stress concentrations in aerospace structures, depicted in *Fig.2C-D*.



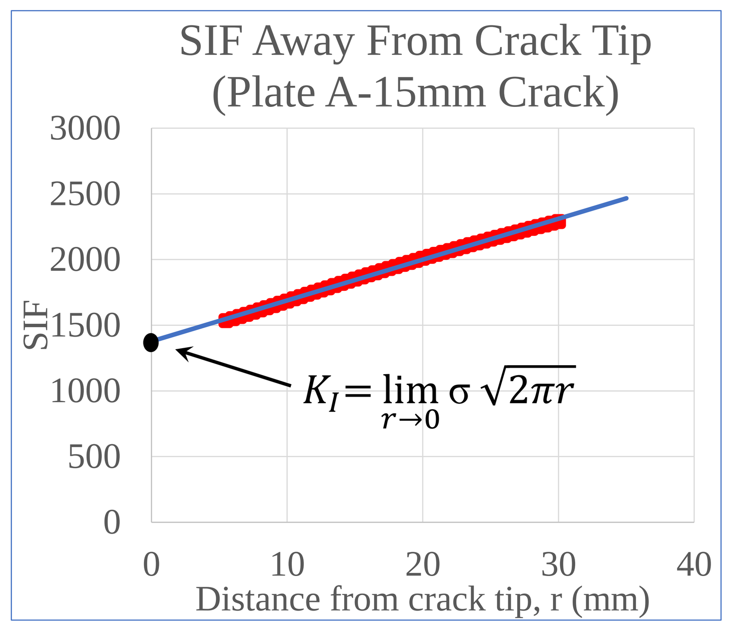
(Fig.2)

Results

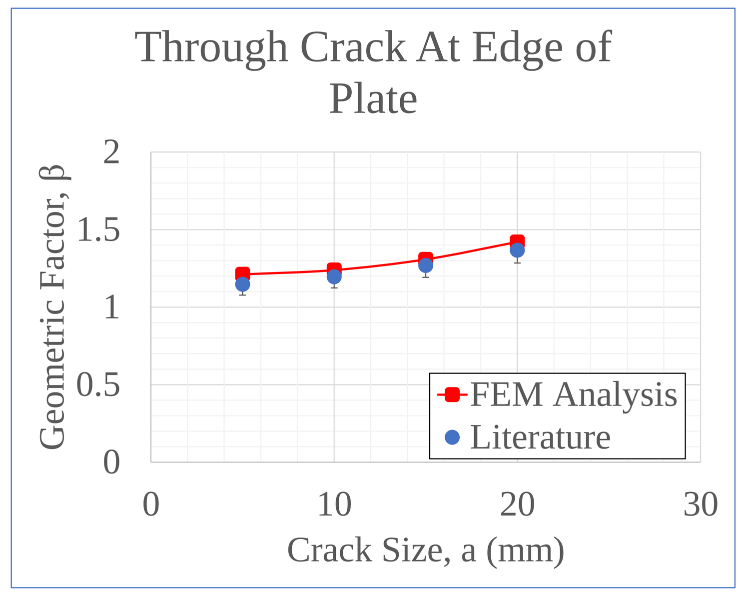
Fig.3 shows an FEM stress map near the crack tip. Fig.4 shows an example of SIF obtained from the stress values ahead of the crack tip (θ=0), for a given crack size. *Fig.5* and *Fig.6* show the values of calculated geometric factors, β's, as function of crack size. Both figures display the error bars to better visualize the expected accuracy for the model. *Fig.7* and *Fig.8* depict the geometric factors for a crack emerging from a cutout (*Fig.2C*) and for a crack emerging from a stress raiser position due to geometry variation (*Fig.2D*).



(Fig.3)



(Fig.4)



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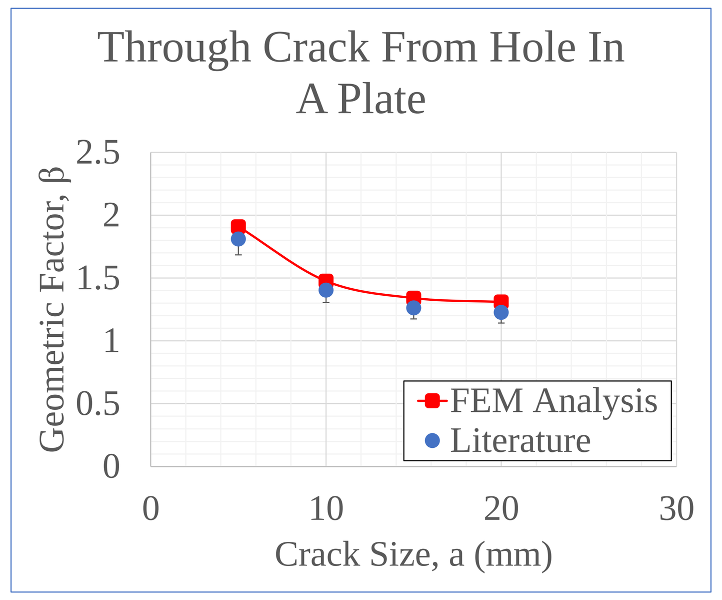
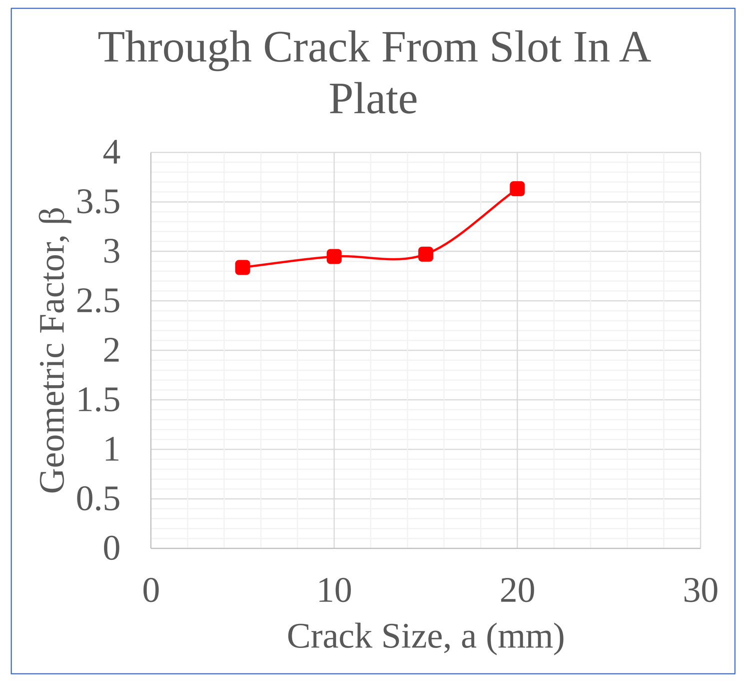
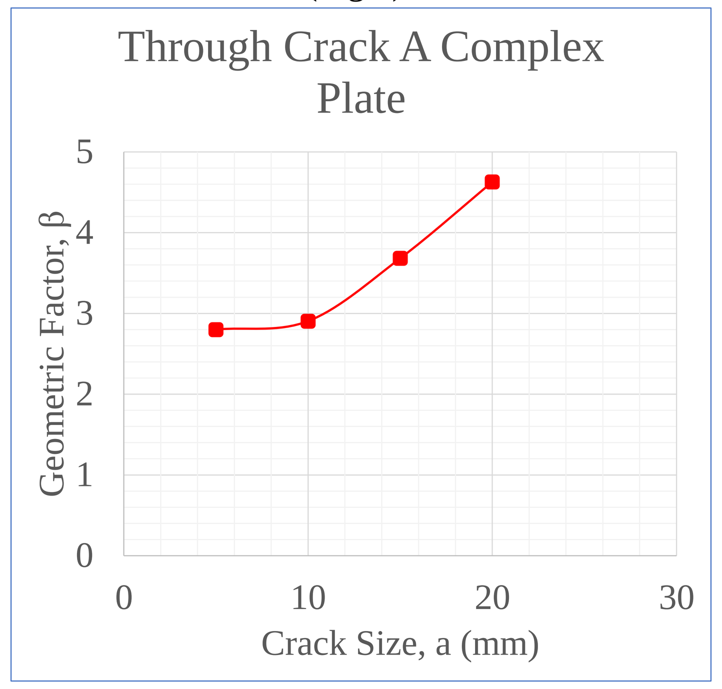


(Fig.6)



(Fig.7)



(Fig.8)

Conclusion

The use of an engineering software which performs linear static analysis calculations such as FEMAP showed good potential in the application of crack modeling, stress mapping, and stress intensity factor - SIF determination. Complex geometries are present in real aerospace structures, and the SIF solutions are rarely available for prompt application. This methodology defines a simple procedure to find SIF solutions for any geometry and it is applicable to any material. The geometries shown in *Fig.2C* and *Fig.2D* were selected as example because the lack of analytical solution and their common use within aerospace structures. This study is especially helpful in the application of damage tolerance analysis to comply with aircraft airworthiness. This study can be furthered by researching other methods of finding the SIF of a cracked components such as using energy equations to be compared to that of Irwin's approximation and determining which method is more accurate using FEA.

References

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