INTRODUCTION TO VERTICAL SUNDIALS

Gnomon: casts the shadow onto the sundial face

Nodus: the location along the gnomon that marks the time and date on the dial plate

Style Height: angular distance of the gnomon from the dial face

Substyle Line: line lying in the dial plane perpendicularly behind the style

Substyle Angle: angle that the substyle makes with the noon-line

Shadow Lines: The declination lines are the lines followed by the Sun’s shadow, as shown in red in Figure 3. They are hyperbolic in nature due to the declination of the Sun, the style height SH, and the nodus location N. Style height is a function of the dial declination R and geographical latitude L, as shown in Figure 3a).

\[ SH = 90^\circ - (L + R) = 90^\circ - (29.5^\circ + 20.5^\circ) = 40^\circ \]

The declination of the Sun ranges from \(-23.5^\circ < \delta < +23.5^\circ\) throughout the year due to the rotational axis tilt of the Earth. This is highest in the sky at any given point in the Northern hemisphere when \(\delta = +23.5^\circ\). For the case of our sundial, the nodus point N was at the tip of our gnomon and equivalent in magnitude as the length of the gnomon and the length of the style. The hyperbolic asymptotes a, b, and c (2.3.4) were calculated to then plug into the hyperbolic equation (1) for all range of x and y along the hour lines.

\[ x = a \sqrt{\frac{y^2}{b^2} + 1} + c \]

\[ \delta = \frac{SH \tan{t}}{\cos{SH} \tan{\delta}} \cos{(SH \tan{\delta})} \tan{\delta} \]

\[ \theta = \frac{N \cos{(SH \tan{\delta})}}{\cos{SH} \tan{\delta}} \tan{\delta} \]

\[ t = \frac{\theta}{\cos{(SH \tan{\delta})}} - 6^\circ \]

Hour Lines: The spacing between the hour lines is 15’ between each hour on the clock. Since the Earth has a total of 360’ in 24 hours, it is then divided into 24 hours around the globe. This is true for all sundials no matter the orientation of the dial plane.

METHODS

Different options were considered as far as getting a physical model of the sundial plotted in MATLAB. The MATLAB program is capable of producing variations of the image output that allows the landscape software to easily convert it into a cutting path for the Graphtec FC1200-60A cutter plotter. It does this by making the hour and declination lines thicker, removing the labels, and adding a circuit board like border to the picture to stabilize the cutout pieces, as shown in Figure 2a). Figure 2b) shows the cutter path and trajectory lines indicate where the blade actually cuts, and the blue line shows the lifted path the tool will take to cut the image out. In this conversion, the negative of the first image is what is being cut. A small scale sundial model printed out and proved to be correct as far as the hour lines go, and only a minor adjustment to the gnomon length was necessary for the declination lines to be valid. This was possible since initial calibration fell on a date near the vernal equinox, therefore the tip of the shadow should have fell just below the equinox declination line.

RESULTS

A small scale sundial model printed out and proved to be correct as far as the hour lines go, and only a minor adjustment to the gnomon length was necessary for the declination lines to be valid. This was possible since initial calibration fell on a date near the vernal equinox, therefore the tip of the shadow should have fell just below the equinox declination line.

CONCLUSIONS

Engineering a sundial for a particular location and position must take many different parameters into account. Not only is the overall design dependent on the location of the Sun, but also the orientation of the dial plane. The simplest dial to make is a horizontal dial on a flat surface. When making a vertical dial that reclines and declines, the math can get more complex. Using plotting software such as MATLAB, the process becomes much simpler as far as making adjustments to scaling, dial orientation, and dial components. A physical model of the graphical sundial can be produced by the Graphtec cutter and tested which amplifies the process once calibration between paper size, MATLAB plans, and Inkscape software was determined. If scaling was slightly off, the length of the gnomon could easily be adjusted to make the shadow fall in the right places as far as time of the year. Difficulties arose in correctly positioning the declination shadow lines and in orienting the sundial pattern within Inkscape for correct orientation on Graphtec cutter. The end goal is to have a sundial that is reclining from the vertical and declining from the local meridian.

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REFERENCES