A Modern Approach to Sundials

INTRODUCTION TO VERTICAL SUNDIALS

Gnomon: casts the shadow onto the sundial face

Nodus: the location along the gnomon that marks the time and date on the dial plate

Style Height: angular distance of the gnomon from the dial face

Substyle Line: line lying in the dial plane perpendicularly behind the style

Substyle Angle: angle that the substyle makes with the noon-line

Shadow Lines: The declination lines are the lines followed by the Sun’s shadow, as shown in red in Figure 3. They are hyperbolic dependent on the declination of the Sun, the style height, SH, and the nodus location N. Style height is a function of the dial declination R and geographical latitude L, as shown in Figure 3a).

\[
SH = \frac{R0.009 - (L + R) - 29.20 + 20.00}{46.8}
\]

The declination of the Sun ranges from -23.5˚ < \(\delta\) < +23.5˚ throughout the year due to the rotational axis tilt of the Earth. The Sun is highest in the sky at any given point in the Northern hemisphere when 0 < \(\delta\) < +23.5˚.

Greenwich than the zone longitudinal line so the hour angles of the dial must be shifted.

The hour lines were easily calculated for each daylight hour, denoted by \(t\), and \(t\) (2.34) were calculated to then plug into the hyperbolic equation (3) for all range of \(x\) and \(y\) along the hour lines.

\[
e = \sqrt{\frac{g^2}{2} + e^2}
\]

Declining vertical sundials do not face any of the cardinal compass points but they are rotated by an angle \(\beta\).

Declining Dials

Reclining Dials

Reclining dials are generally oriented along a north-south line, for example they face due south for a sundial in the Northern hemisphere. In such a case the dial surface would have no declination. Reclining sundials are at an angle from the vertical, and have gnomons directly parallel with Earth’s rotational axis which is visually represented in Figure 3a).

\[
\theta = \frac{180}{\pi} \tan^{-1}\left(\frac{\sin(SH) - \sin(\delta)}{\cos(SH) \tan \delta}\right)
\]

Declining Dials do not face any of the cardinal compass points but they are rotated by an angle \(\beta\). The substyle is displaced from the noon-line at a tilt 0 from the North-South plane which is visually represented in Figure 1b). For a south-facing dial, the substyle angle is measured positively towards the, repeating hour lines.

METHODS

Different options were considered as far as getting a physical model of the sundial plotted in MATLAB. The MATLAB program is capable of producing versions of the image output that allows the landscape software to easily convert them into a cutting path for the Graphic: FC1200-60A cutter plotter. It does this by making the hour and declination lines thicker, removing the labels, and adding a thick border to the picture to stabilize the cutout pieces, as shown in Figure 2a). Figure 2b) shows the cutter path and trace. The red lines indicate where the blade actually cuts. The blue line shows the lifted path the plotter will take to cut the image out. In this conversion, the negative of the first image is what is being physically cut out, leaving the shape of the first as a “negative.”

RESULTS

A small scale sundial model printed out and proved to be correct as far as the hour lines go, and only a minor adjustment to the gnomon length was necessary in order for the declination lines to be validated. This was possible since initial calibration fell on a date near the vernal equinox, therefore the tip of the shadow should have fell just below the equinox declination line.

Shown in Figure 3 is the final sundial corrected for the longitude of Daytona Beach, FL. If the dial were not corrected for longitude, the noon line would fall directly vertical from the gnomon base. For vertical dials, the shortest shadow will be will the sun has its lowest altitude in the sky. For the Northern hemispheres, this is the Winter Solstice declination line. Similarly, the longest shadow will fall throughout the day of the Summer solstice (the lowerrmost red line on Figure 3). On the equinox, both vernal and autumnal, the tip of the shadow will be casted onto a horizontal line across the dial.

REFERENCE TO VERTICAL SUNDIALS

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