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A Modern Approach to Sundials

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INTRODUCTION TO VERTICAL SUNDIALS

Gnomon: casts the shadow onto the sundial face

Nodus: the location along the gnomon that marks the time and date on the dial plate

Style Height: angular distance of the gnomon from the dial face

Substyle Line: line lying in the dial plane perpendicularly behind the style

Substyle Angle: angle that the substyle makes with the noon-line

Shadow Lines:

The declination lines are the lines followed by the Sun's shadow, as shown in red in Figure 3. They are hyperbolas dependent on the declination δ of the Sun, the style height *SH*, and the nodus location *N*. Style height is a function of the dial reclination *R* and geographical latitude *L*, as shown in ***Figure 1a)***.

$$SH = 90.0^o - (L + R) = 90.0^o - (29.2^o + 20.0^o) = 40.8^o$$

The declination of the Sun ranges from -23.5° < δ < +23.5° throughout the year due to the rotational axis tilt of the Earth. The Sun is highest in the sky at any given point in the Northern hemisphere when the δ = +23.5°. For the case of our sundial, the nodus point *N* was the tip of our gnomon and equivalent in magnitude as the length of the gnomon and the length of the style. The hyperbolic asymptotes *a*, *b*, and *c* (2,3,4) were calculated to then plug into the hyperbolic equation (1) for all range of *x* and *y* along the hour lines.

$$x = a\sqrt{\left(\frac{y^2}{b^2} + 1\right)} + c \quad (1)$$

$$a = \frac{N\sin(SH)\tan\delta}{\cos^2(SH) - \sin^2(SH)\tan^2\delta} \quad (2)$$

$$b = \frac{N\sin(SH)}{\sqrt{\cos^2(SH) - \sin^2(SH)\tan^2\delta}} \quad (3)$$

$$c = \frac{N\cos(SH)}{\cos^2(SH) - \sin^2(SH)\tan^2\delta} \quad (4)$$

Hour Lines:

The spacing between the hour lines is 15° between each hour on the clock. Since the Earth has a total of 360° in circumference, it is then divided into 24 hours around the globe. This is true for all sundials no matter the orientation of the dial plane.

$$\frac{360^o}{24h} = \frac{15^o}{hour}$$

The position of these hour lines is however dependent on the latitude *L* and reclination *R* of the dial, as well as what hour *t* that each line is corresponding to. To compute the daylight hour angles *HA* for a sundial in Daytona Beach, FL, the following equation is used:

$$\tan HA = \cos(L + R)\tan(15t) - 6^o$$

The hour lines were easily calculated for each daylight hour, denoted by *t*. This trigonometric expression has been adjusted for longitude to fit local mean time of specifically Daytona Beach, FL. However, if one would rather have local apparent time instead of local mean time, the extra difference of -6° could be taken out of the scheme. The 6 degree longitude correction is derived from

$$\text{Longitude of Daytona Beach} = 81.0^o\text{W}$$
$$\text{Zone Longitude} = 75.0^o\text{W}$$
$$\text{Correction} = \text{Zone - Actual} = 75.0^o\text{W} - 81.0^o\text{W} = -6.0^o\text{W}$$

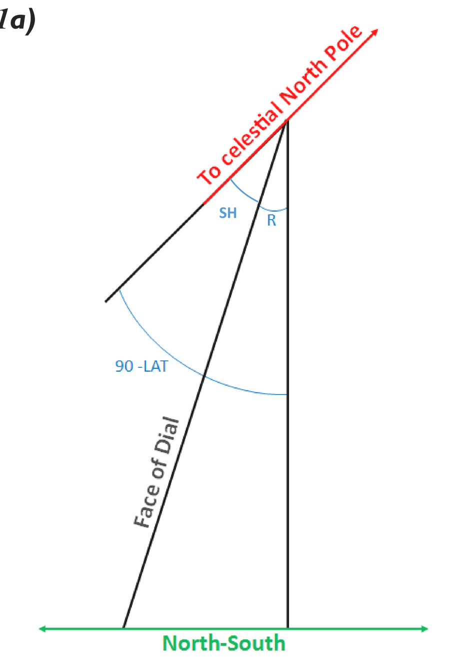
where the longitude is based on the geographical location of the dial, and zone longitude is a measure in degrees from the prime meridian in Greenwich, England. Daytona Beach is slightly further West from Greenwich than the zone longitudinal line so the hour angles of the dial must be shifted. ***Figure 3*** (blue) displays the graphical output from correcting for longitude using MATLAB. When choosing the physical location of a sundial, it is most important to account for the surface orientation in an accurate manner. This alone will define the overall sundial arrangement, namely the display of the hour lines and position of the declination lines. The angle of recline is really a matter of opinion of the sundial's creator. A reclination in the range 0° < *R* < 90° is ideal when compiling a vertical sundial facing due North or South.

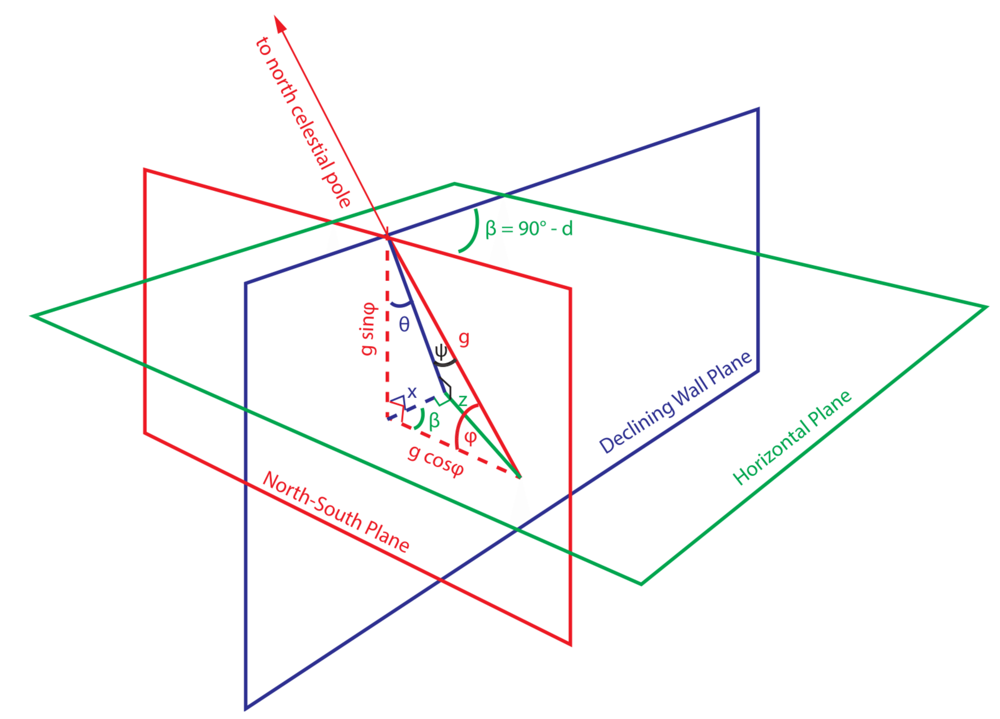
TYPES OF VERTICAL SUNDIALS

Reclining Dials

Reclining dials are generally oriented along a north-south line, for example they face due south for a sundial in the Northern hemisphere. In such a case the dial surface would have no declination. Reclining sundials are at an angle from the vertical, and have gnomons directly parallel with Earth's rotational axis which is visually represented in ***Figure 1a)***.

1a)





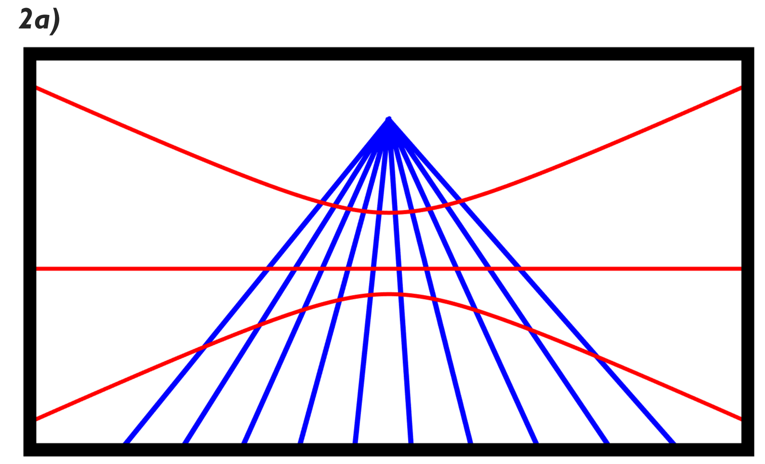
Declining Dials

Declining vertical dials do not face any of the cardinal compass points but they are rotated by an angle β. The substyle is displaced from the noon-line at a tilt θ from the North-South plane which is visually represented in ***Figure 1b)***. For a south-facing dial, the substyle angle is measured positively towards the evening hour lines.

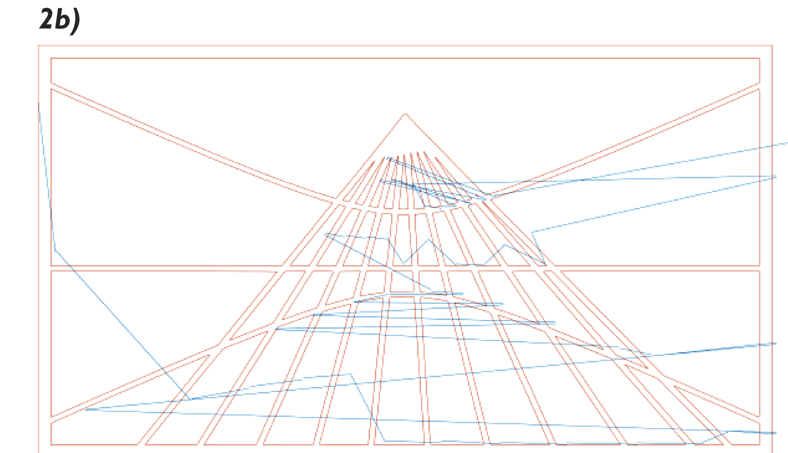
METHODS

Different options were considered as far as getting a physical model of the sundial plotted in MATLAB. The MATLAB program is capable of producing variations of the image output that allows the Inkscape software to easily convert it into a cutting path for the Graphtec FC2100-60A cutter plotter. It does this by making the hour and declination lines thicker, removing the labels, and adding a thick border to the picture to stabilize the cutout pieces, as shown in ***Figure 2a)***. ***Figure 2b)*** shows the cutter path and trace. The red lines indicate where the blade actually cuts, and the blue line shows the lifted path the plotter will take to cut the image out. In this conversion, the negative of the first image is what is being physically cut out, leaving the shape of the first as a "negative".

2a)

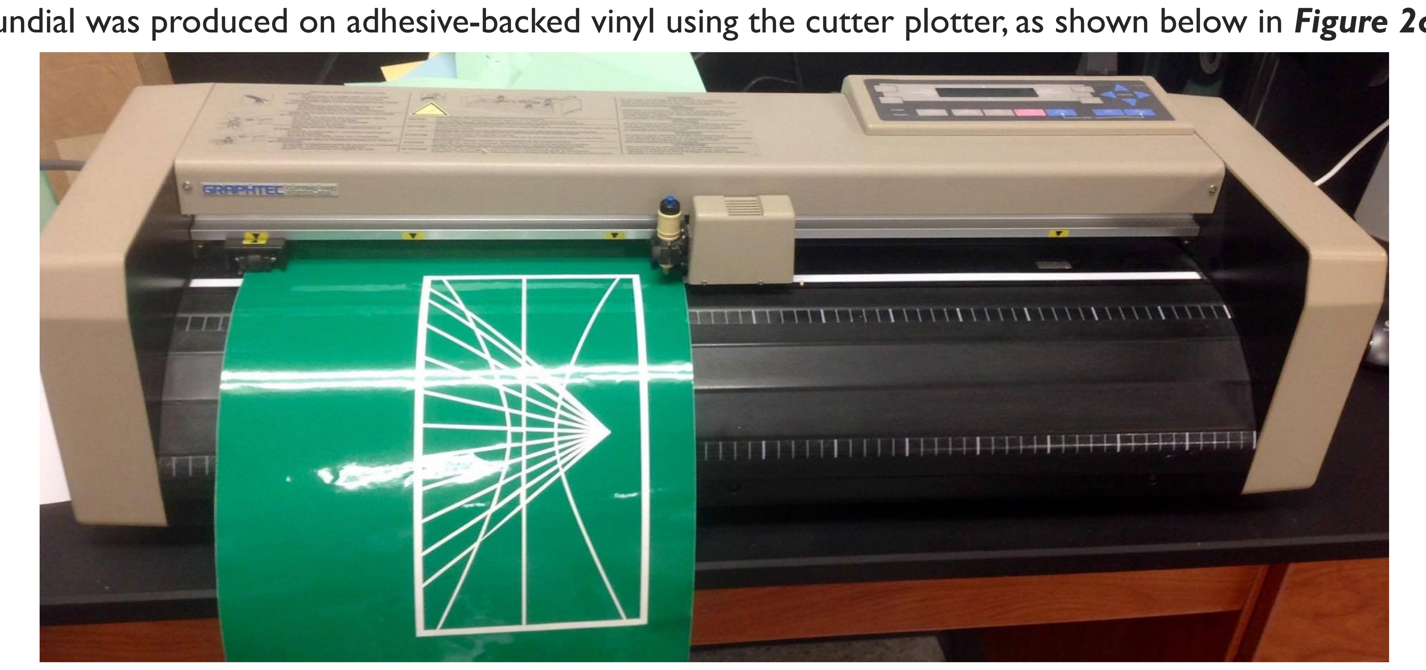


2b)



A sundial was produced on adhesive-backed vinyl using the cutter plotter, as shown below in ***Figure 2c)***.

2c)



RESULTS

A small scale sundial model printed out and proved to be correct as far as the hour lines go, and only a minor adjustment to the gnomon length was necessary in order for the declination lines to be validated. This was possible since initial calibration fell on a date near the vernal equinox, therefore the tip of the shadow should have fell just below the equinox declination line.

Shown in ***Figure 3*** is the final sundial corrected for the longitude of Daytona Beach, FL. If the dial were not corrected for longitude, the noon line would fall directly vertical from the gnomon base. For vertical dials, the shortest shadow will be will the sun has its lowest altitude in the sky. For the Northern hemisphere, this is the Winter Solstice declination line. Similarly, the longest shadow will fall throughout the day of the Summer solstice (the lowermost red line on ***Figure 3***). On the equinox, both vernal and autumnal, the tip of the shadow will be casted onto a horizontal line across the dial.

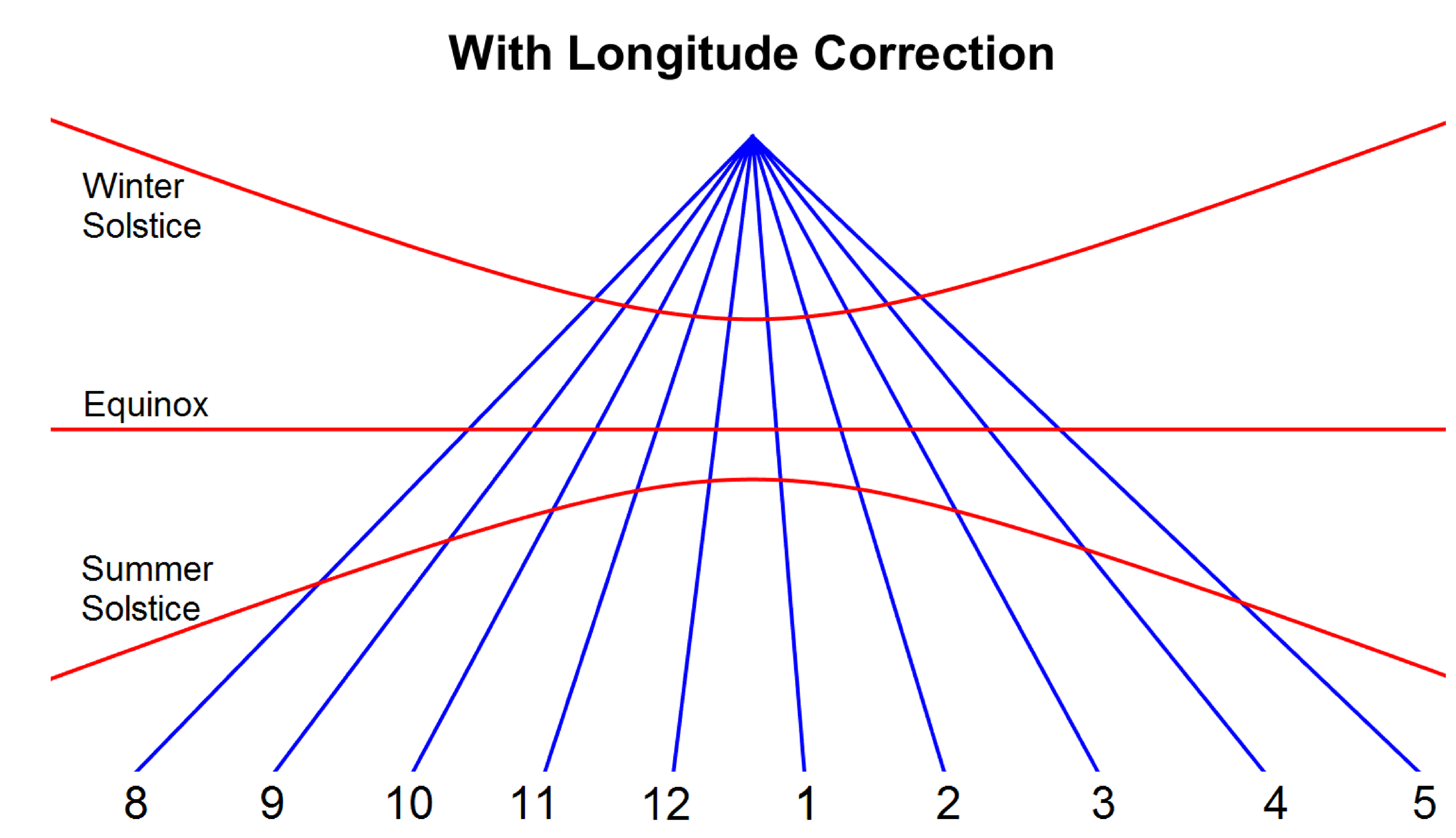


Figure 3

CONCLUSIONS

Engineering a sundial for a particular location and position must take many different parameters into account. Not only is the overall design dependent on the location of the Sun, but also the orientation of the dial plane. The simplest dial to make is a horizontal dial on a flat surface. When making a vertical dial that reclines and declines, the math can get more complex. Using plotting software such as MATLAB, the process becomes much simpler as far as making adjustments to scaling, dial orientation, and dial components. A physical model of the graphical sundial can be produced by the Graphtec cutter and tested which simplifies the process once calibration between paper size, MATLAB plots, and Inkcut software was determined. If scaling was slightly off, the length of the gnomon could easily be adjusted to make the shadow fall in the right places as far as time of the year. Difficulties arose in correctly positioning the declination shadow lines and in orientating the sundial pattern within Inkcut for correct orientation on Graphtec cutter. The end goal is to have a sundial that is reclining from the vertical and declining from the local meridian.

ACKNOWLEDGEMENTS

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