Results

This project is designed to derive a fast and stable algorithm to solve a system of linear equations in connection to cubic splines. The well accepted Thomas Algorithm has 8N flops which is small compared to a standard matrix inversion algorithm. Here, we propose a new algorithm with 5N-4 flops. The proposed algorithm also leads to stability as shown in the following table, where the machine precision is 2.23e-16.

<table>
<thead>
<tr>
<th>Size</th>
<th>Relative Error (random a, b, c)</th>
<th>Relative Error (a = 4 and b, c = 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40x40</td>
<td>1.40E-15</td>
<td>1.69E-15</td>
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<tr>
<td>80x80</td>
<td>8.55E-16</td>
<td>1.55E-15</td>
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<td>120x120</td>
<td>3.97E-15</td>
<td>2.92E-15</td>
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<td>160x160</td>
<td>6.78E-15</td>
<td>2.88E-15</td>
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<tr>
<td>200x200</td>
<td>3.33E-15</td>
<td>3.96E-15</td>
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<td>240x240</td>
<td>2.63E-15</td>
<td>2.11E-15</td>
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<tr>
<td>280x280</td>
<td>9.31E-16</td>
<td>1.24E-15</td>
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<tr>
<td>320x320</td>
<td>6.88E-15</td>
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<tr>
<td>360x360</td>
<td>6.66E-15</td>
<td>4.78E-15</td>
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<tr>
<td>400x400</td>
<td>8.97E-16</td>
<td>2.41E-16</td>
</tr>
</tbody>
</table>

Table displays the first column holding dimensions of tridiagonal matrices, the second column with the relative error populated by a random tridiagonal matrix, and the last column with the tridiagonal matrix in cubic splines having a=4 and b=c=1.

Conclusion

The proposed derivation leads to:
- **Fast O(n) Algorithm**
- **Sparse Factorization**
- **Recursive Algorithm**
- **Stable Algorithm**
- **Accurate Algorithm**

References