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Compressible Flow Analysis of Thrust Augmenting Ejectors

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COMPRESSIBLE FLOW ANALYSIS OF THRUST AUGMENTING EJECTORS

by

Mohamed Moujahid

A Thesis Submitted to the
Office of Graduate Programs
in Partial Fulfillment of the Requirement for the Degree of
Master of Science in Aerospace Engineering

Embry-Riddle Aeronautical University
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COMPRESSIBLE FLOW ANALYSIS OF THRUST AUGMENTING EJECTORS

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Mohamed Moujahid

This thesis was prepared under the direction of the candidate's thesis committee chairman, Dr. L.L. Narayanaswami, Department of Aerospace Engineering, and has been approved by the members of his thesis committee. It was submitted to the Office of Graduate Studies and was accepted in partial fulfillment of the requirement for the degree of Master of Science in Aerospace Engineering.

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Abstract

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The present work was initiated due to the need for a method to understand and predict the thrust augmenting characteristics of jet ejectors. The mixing process in ejectors can be analyzed using either the control volume approach, or detailed models based on the Navier-Stokes Equations and the theory of turbulent jets. The control volume approach uses integrated forms of the conservation equations of mass, momentum and energy. It is chosen in the first part of the study since it affords the best vehicle for the parametric studies required to understand the potential of ejectors for a given application. Compressibility effects are taken into consideration. Losses, however, are not accounted for in the analysis. A more detailed approach, based on the turbulent mixing model derived by Abramovich, is presented in the second half of the study. The
model used proved to be very accurate in describing the turbulent mixing flow process taking place in the ejector chamber. The results from the two approaches are found to be in good agreement, although some discrepancies could be found in the case of supersonic ejectors.
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CHAPTER 1: INTRODUCTION

1.1 INTRODUCTORY REMARKS

This thesis is part of an ongoing research program directed toward understanding and analyzing the concept of thrust augmentation resulting from the use of ejectors. Ejectors have been used for many years in the exhaust systems of turbojet and rocket engines, primarily for thrust augmentation on V/STOL (vertical/short take off and landing) applications and for noise reduction. The latter was achieved by mixing the high temperature exhaust flow with the ambient air to provide lower jet noise and plume radiation. A thrust augmenting ejector is basically a mechanical device used to increase or "augment" the thrust of a primary propulsive nozzle through fluid dynamic means. It can be viewed as a fluid dynamic pump that uses the momentum of a fast jet (primary flow) from a primary nozzle to entrain and pressurize a suction stream (secondary flow).
A typical thrust augmenting ejector consists of a high pressure nozzle to accelerate the primary flow and an inlet section to capture the secondary or entrained flow. It also consists of an intermediate section (mixing duct) in which the primary and secondary flows mix and exchange momentum, and a diffuser section to match the pressure of the discharge with that of the surrounding atmosphere. Figure 1.1 highlights the main components of an ejector. Ejectors operate by inducing large amounts of air from the ambient fluid through the entrainment action of the primary nozzle shear layer. The key mechanism for this operation is turbulent fluid mixing. The primary nozzle flow is exhausted into a larger duct, usually called the ejector shroud, where it entrains and interacts with the secondary flow. The induced motion in the ambient fluid results in a local pressure less than ambient at the primary nozzle exit plane. It causes the primary jet to exhaust at a higher velocity and kinetic energy than it would otherwise have had. The two flows after entering the mixing duct start to interact with each other. This interaction is primarily due to a viscous shear mechanism, "mixing", and results in an energy transfer from the primary flow to the secondary flow. The mixed flow when exhausting to the ambient back pressure, provides a greater total thrust due to the energy exchange which has taken place than the primary propulsive nozzle alone. The ratio of this total device thrust to the ideal thrust of a primary propulsive nozzle exhausting to the same ambient back pressure is called the thrust augmentation ratio.
Figure 1.1 Schematic of a single stage ejector
1.2 PREVIOUS RESEARCH DONE ON EJECTORS

Unlike ejector pumps which were satisfactorily used for a variety of applications in the late 1800's, the first exploratory tests of ejector augmentors took place in 1927. These tests were oriented toward showing the feasibility of jet propulsion for airplanes. It was not until 1949 that the technical community was finally awakened to the full potential of these devices by Theodore Von Karman through his classical Reisner Anniversary Theoretical Treatment for incompressible, diffuserless ejectors. The paper explained the principle of the ejector, specifically the Coanda ejector. In the following years, numerous theories have been proposed and several experiments tried. Noteworthy among these are Bertin's experiments with multiple annular nozzle configurations, and Foa's invention of the non steady rotary jet flow augmentor. Both devices tried to improve the efficiency of the interaction between the ejector primary and secondary flows, and obtained reasonable success in achieving this goal. Still, it was not until 1972 when Quinn provided a "briefing to industry" on the Air Force Aerospace Research Laboratory (ARL) work on hypermixing nozzles through the use of mixer lobes, that the technical community started finally to show new interest in the possible application of ejectors to aircraft propulsion systems. Numerous attempts were made to establish analytical methods capable of predicting the effects of temperature, pressure and size on ejector performance. These theories were proposed over the years in order to improve the current understanding of ejector operation and performance, and overcome the lack of a precise and reliable theory of turbulent entrainment. The latter made it extremely difficult to improve the jet mixing process. The picture which emerged then was one of
fragmentation within the technical community. There was a big gap between those who believed that ejector augmentors had reached a stage of development which permitted a viable flight system application, and those who believed that there was still a need for continued research in the matter due to the discrepancies in the understanding of the interacting physical phenomena. The next section will summarize the previous work performed, and progress made in the analysis and experimentation of thrust augmenting ejectors.

1.3 Theories developed

1.3.1 Thermodynamic Cycle Analysis

The ejector flow field consists of interacting regions of turbulent flow. Due to this complex flow behavior, it was difficult to develop accurate methods capable of providing a detailed analysis and description of the flow process inside the ejector. In an early approach by Porter and Squyers, thermodynamic cycle analysis was used. The approach was based upon the assumption that the two flow streams, once in the ejector chamber, mixed. Therefore, without regard to whether it could physically happen, the mixing process was assumed to take place isentropically. The mixed flow resultant state was a function of the initial states, the primary nozzle discharge pressure at the inlet section and the entrainment ratio. This theory proved to be highly inaccurate due to the inherently non isentropic jet mixing process in the ejector. Other theories tried to provide an understanding of the fundamental physics of ejector augmentors, and were approached on two levels: (1) The so called "Control Volume Approach" which described the overall
and what occurred in terms of bulk changes in energy, momentum and enthalpy, and (2) The "Physical Phenomena Approach" which analyzed the individual physical processes in terms of the fundamental mechanisms of energy and momentum transfer.

1.3.2 Control Volume Approach:

The Control Volume Approach was based on a quasi-one dimensional analysis suggested by Von Karman in his classical Reissner Anniversary theoretical treatment for incompressible, diffuserless ejector augmentors. In this approach, the primary nozzle and secondary inlet processes were assumed to be isentropic, as was the exit diffuser process. The ejector geometry was specified by its inlet and diffuser area ratios. The values of the flow parameters at different locations were determined by the simultaneous solution of the mass, momentum and energy conservation equations. This analysis proved to be very useful for identifying basic trends and parametric studies, and determined the effects of different flow parameters on thrust augmentation. However, it should be noted that this approach is limited by the two major assumptions made in the analysis: (1) The flow was quasi-one dimensional, (2) the flow was incompressible and (3) the secondary stream remained parallel to the jet axis even in the absence of a shroud. The quasi-one dimensional assumption further limits the analysis to ejectors with inlet area ratios less than about 25. This number was obtained experimentally by Bevilaqua and Quinn as a limiting value beyond which the parallel flow assumption was not expected to remain valid).
The author of the present document used the last mentioned approach, combined with a one dimensional compressible flow analysis, in the first part of this thesis. The method proved to be very useful in predicting the ejector performance and thrust augmentation characteristics. However, it was unable to describe the interaction between the primary and secondary streams in the mixing region of the ejector.

As these methods lacked a detailed description of the mixing process taking place in the ejector chamber, new analytical methods capable of predicting the turbulent mixing within the ejector had to be developed. The development of such methods was one of the principal advances in ejector technology during the past fifteen years. These new methods were called physical-phenomena approaches as they provided detailed analyses of the turbulent mixing processes within the ejector.

### 1.3.3 Physical Phenomena Approach

Perhaps the best example of the physical phenomena approach is provided by the finite difference model of Gilbert and Hill which used a mixing length model for the turbulence to analyze two dimensional ejectors. In this model, the interaction/mixing zone was characterized by three distinct regions: (1) secondary and primary fluid potential flow "core" regions, (2) wall boundary layer and primary jet secondary shear layer region, and (3) a downstream region of developing flow. The model used the two-dimensional, steady, time averaged boundary layer forms of the continuity, momentum and energy equations. In order to solve these equations, various assumptions and relations were required. In particular, the Prandtl assumption for $\varepsilon$, the eddy momentum
diffusivity, was used. In addition, the mixing length in the jet shear region was assumed to be a function of the shear layer width only. The mixing layer in the wall boundary layer region was a function of the local boundary layer thickness \( \delta \), and two empirically derived constants. By using these approximations, the governing equations of mass, energy and momentum were reduced to a parabolic set, and solved by marching through the ejector in the streamwise direction. The volume of flow pumped through the ejector was also determined by iterating on the inlet velocity of the entrained stream until the computed exit pressure matched the ambient pressure.

Another method developed by Dejoode and Patankar used a three dimensional analysis to predict the entrainment of jets from multiple slots and nozzles. This analysis relied on the extensive use of numerical computations of turbulent flows, and used a streamwise marching procedure in order to determine the mean pressure gradient and streamwise velocity component.

The physical -phenomena methods attempted to overcome the limitations inherent in the control volume approach. These methods established flow models which described the turbulent jet mixing, phenomenon of major significance to the device performance. They, however, encountered the same limitations as did the previous methods, mainly because the state-of-the-art of fluid dynamics in general is such that flow models for the turbulent mixing of jets must rely on (usually limited) empirical bases.
CHAPTER 2:  COMPRESSIBLE FLOW ANALYSIS

The objective of the study in this chapter is to provide the thrust augmentation levels that could be obtained by a well designed ejector. An analytical model is developed in order to predict the performance and describe the flow process in a high entrainment ratio, compressible flow ejector with a constant area mixing chamber.

2.1 Background

A review of existing ejector literature brought the following to light: There were very few documents available on ejector flow theory and performance predictions. Some of the analyses found used an incompressible approach. This was inadequate for the high temperature and high pressure flows of the jet engine due to compressibility effects.
Others relied on the extensive use of semi-empirical methods. Still others used experimental data on the performance and application of ejectors for v/stol aircraft. Also, a large portion of the previous analyses was oriented towards applying the ejector as a pumping device, in order to increase the secondary flow's total pressure instead of its application as a thrust augmenting device.

These factors led the author of this present investigation to develop a one dimensional ejector flow theory coupled with a compressible flow analysis. For the present study, the control volume approach is reformulated in a way that can be simplified while getting detailed and reliable ejector performance characteristics. Whereas the flow parameters representing the design requirements can be assumed to be fixed, one may wish to study the effect of varying a set of variables of designer's choice. This is especially important in optimization studies. The computer programs that have been developed as a result of the present study are included in the appendices.

2.2 Formulation of the Mathematical Model

There are two principal applications of an ejector: (1) as a jet pump where the energy of the primary fluid is used to increase the stagnation pressure of the secondary fluid, (2) as a thrust augmentor where the momentum of the primary flow is increased by mixing with the secondary flow, thus increasing propulsive efficiency. In the following analysis, the thrust augmentation capability is of primary interest. In addition to the ability to increase thrust of a primary fluid, ejectors have other inherent advantages which make them highly desirable for thrust system applications. These are: (1) a simplicity of
the basic design, (2) no moving parts, (3) ease of conformation to geometric constraints and (4) the possibility of achieving these advantages with a minimum weight through the use of mixer lobes and vortex generators. Their implementation, however, in an effective system application has failed in the past mainly because of the lack of understanding of the details of the flow phenomena in the ejector. Analyses conducted to date have assumed constant area mixing owing to its simplicity. Constant pressure mixing may also be analyzed in a straightforward manner. However, no reliable methods are available for analysis of a general mixing process.

The main purpose of the analysis presented here is to provide a complete description of the important flow parameters at specific locations within the ejector, and to describe the overall operation of the ejector as an entrainment and thrust augmentation device. The analysis is intended mainly for air-to-air ejectors, but could be used with dissimilar fluids. The parameters used in the analysis are described in the nomenclature. The geometrical parameters are shown in Figure 2.1.

2.3 Assumptions used in the analysis

The following assumptions are made for the ideal flow in order to simplify the analysis:

1. the flow is compressible and calorically perfect. The specific heat ratio is constant (K=1.4),

2. the flow is one dimensional and steady,

3. the flow is inviscid,
Figure 2.1 Detailed description of a single stage ejector\(^6\)
4. wall shear forces creating skin friction losses are assumed to be negligible when compared to the pressure forces, and the momentum of the primary and secondary streams.

5. mixing is initiated in a constant area duct at the location where the primary flow is fully expanded (primary flow pressure is equal to the local secondary flow pressure),

6. no heat is transferred across the walls of the ejector,

7. complete mixing is achieved at the end of the mixing duct and

8. when the primary nozzle is operated at an off-design pressure ratio, the primary jet is assumed to expand or contract isentropically until the primary and secondary streams have equal static pressures. This adjustment process is assumed to take place in the accommodation region of the inlet of the ejector between sections 1 and 2, (see Figure 2.1), and is assumed to be completed before the two streams start to mix.

2.4 Analysis of the Primary Nozzle

The primary nozzle characteristics of major significance are the following: (1) The peripheral surface interaction area, (2) the Mach number of the primary jet at the beginning of the interaction region and (3) the angle of the primary jet relative to the incoming secondary flow. The peripheral surface area of the primary jet can be increased through the use of multiple primary nozzles. For example, the peripheral length (P) which comes into contact with the secondary flow for a single circular primary jet of area
A = \pi D^{3/4} \text{ is } \pi D. \text{ On the other hand, if the jet is divided into four smaller circular jets of overall area } A, \text{ the total peripheral contact length for the four jets is } 2\pi D, \text{ twice that of the single jet. Consequently, the primary and secondary streams will interact over a wider boundary.} \text{ The Mach number of the primary jet can be either subsonic or supersonic, depending on the primary flow stagnation conditions and the local primary nozzle exit static pressure.}

If the total pressure of the primary jet is greater than or equal to the value necessary to choke the primary flow, the exit static Mach number is defined by the exit to throat area ratio of the primary nozzle. This will determine the exit static pressure both for the primary and secondary flow at the entrance to the interaction zone, which is the pressure level at which the mixing process is started.

2.5. Analysis of the Secondary Inlet Section

It is the function of the inlet of the ejector to ingest fluid from the environment, and to accelerate or decelerate this ingested mass flow to the required inlet flow conditions at the entrance to the mixing region. During this process, the ingested fluid will encounter a loss of momentum as a result of skin friction, blockage or wave losses. These losses, however, will be neglected in this analysis for the purpose of simplicity. They could be introduced through the use of experimentally evaluated empirical factors. The geometrical parameters and flow conditions in the ejector are defined as shown in Figure 2.1. The primary stream enters the inlet section as a high velocity jet; its mach number may be as high as 3.5. The large momentum of the primary jet along with the
physical enclosure of the primary nozzle enables secondary flow to be induced by lowering the primary nozzle static back pressure below ambient due to local (secondary) velocity effects. In the inlet region, it is assumed that the primary and secondary jets do not mix, but the primary jet expands or is compressed until its static pressure matches that of the secondary stream. This process generally occurs through series of oblique expansion and compression waves. At the point where the static pressures are equal, denoted as section 2, the accomodation process is completed and the flows are parallel. The losses caused by the shock waves are quite small $^{4,1}$, and the accomodation process is assumed to be isentropic.

In a perfect gas the stagnation pressures are related to the local mach number by the following equation,

$$\frac{p_T}{p} = (1 + \frac{K-1}{2}M^2)^{\frac{K}{K-1}}$$

(2.1)

At the end of the inlet section (or accomodation region), the static pressure of the primary and secondary streams are equal. Therefore, the following relation must be satisfied,

$$\frac{p_{p0}}{(1+\frac{K-1}{2}M^2_{p2})^{\frac{K}{K-1}}} = \frac{p_{s0}}{(1+\frac{K-1}{2}M^2_{s2})^{\frac{K}{K-1}}}$$

(2.2)

Under the assumption of an adiabatic inlet, the stagnation temperature of the secondary stream at station 2 is equal to the free stream stagnation temperature. Also, for an
isentropic secondary stream, the stagnation pressure at station 2 is equal to the free stream stagnation pressure and, therefore, if $M_s$ is the desired Mach number at station 2, then

$$
\frac{P_s}{P_s} = \left( \frac{1 + \frac{1}{2} M_s^2}{1 + \frac{k+1}{k-1} M_s^2} \right)^{\frac{k-1}{k}} \tag{2.3}
$$

which expresses $P_s$ in terms of $M_s$.

Under the same assumptions, $M_p$ can be expressed in terms of $M_s$. Since $P_s$ is equal to $P_p$,

$$
M_p = \sqrt{\frac{2}{k-1} \left[ \left( \frac{P_m}{P_s} \right)^{\frac{k-1}{2}} - 1 \right]} \tag{2.4}
$$

These expressions relate the flow parameters at station 2 to the given properties of the free stream or flight conditions and the primary jet. The geometry of the constant area mixing duct requires that,

$$
A_{p2} + A_{s2} = A_m \tag{2.5}
$$

where $A_{p2}$ is the primary stream area at section 2, $A_{s2}$ is the secondary stream area at section 2 and $A_m$ is the mixed flow area at section m.

Even though the ejector inlet and exit area ratio are the same for every case, the inlet and exit geometries are different. For example, if the velocity of the secondary stream
increases as the stream enters the ejector, an accelerating inlet geometry is required. If the secondary velocity decreases, a decelerating inlet is required. Therefore, the inlet flow may require either a subsonic or supersonic nozzle or diffuser. Similarly, the exhaust flow may require either a subsonic or supersonic nozzle or diffuser.

Since both primary and secondary streams are assumed to flow isentropically in the inlet section from their stagnation conditions values of temperature and velocity can be obtained for each stream at location (2) by the following equations,

\[ T_{p2} = \frac{T_{po}}{(1 + \frac{k}{2} M_{p2}^2)} \]  

\[ T_{s2} = \frac{T_{so}}{(1 + \frac{k}{2} M_{s2}^2)} \]  

\[ V_{p2} = \sqrt{\frac{2K}{K-1} R(T_{po} - T_{p2})} \]  

\[ V_{s2} = \sqrt{\frac{2K}{K-1} R(T_{so} - T_{s2})} \]

2.6. Analysis of the Mixing Region

2.6.1 The Control Volume Approach

The primary and secondary streams enter the mixing region with equal static pressures and parallel velocities and start to interact. If the duct is of sufficient length and
if viscous effects are neglected, the mixing process will continue until a uniform flow with constant properties across the channel is obtained at section m (Figure 2.1). The governing equations are the bulk conservation equations (mass, momentum and energy) for the constant area mixing process, state equation for the mixed flow and the isentropic flow relations for the inlet and diffuser flows. In the case of zero shear forces at the walls, primary and secondary fluids with the same molecular weights, specific heat at constant pressure and ratio of specific heats, the bulk equations are obtained as follows:

The mass conservation equation for the ejector mixing region becomes

\[ \dot{m}_s + \dot{m}_p = \dot{m}_m \]

or

\[ ρ_s A_s U_s + ρ_p A_p U_p = ρ_m A_m U_m \]

in which \( ρ, U \) and \( A \) are the density, velocity and cross-sectional area of the streams, and the subscripts \( p, s \) and \( m \) refer to the primary, secondary and mixed flows. It is assumed that the static pressure is constant at each cross-section of the ejector and the velocity distributions are uniform. Similarly, the energy equation under the assumption of adiabatic ejector surfaces and calorically perfect gases becomes

\[ \dot{m}_p T_{op} + \dot{m}_s T_{os} = (\dot{m}_p + \dot{m}_s) T_{om} \]

or

\[ ρ_p U_p A_p (c_p T_{p2} + \frac{U_{p2}^2}{2}) + ρ_s U_s A_s (c_s T_{s2} + \frac{U_{s2}^2}{2}) = ρ_m U_m A_m (c_m T_m + \frac{U_{m2}^2}{2}) \]

and the momentum equation reduces to
The equations of state for the three streams take the form

\[ P_i = \rho_iRT_i \quad (2.13, 2.14, 2.15) \]

where the subscript \( i \) refers to primary (p), secondary (s) and mixed (m) flows.

The secondary stream is assumed to flow isentropically through the inlet and the mixed flow is assumed to exhaust isentropically through a nozzle or diffuser. Therefore, the energy conservation equation,

\[ c_p T_0 = c_p T + \frac{U_0^2}{2} \quad (2.16) \]

and the second law of thermodynamics,

\[ \frac{T}{T_0} = \left( \frac{P}{P_0} \right)^{\frac{k-1}{k}} \quad (2.17) \]

have the same form in both the inlet and exhaust flows. The pressure matching conditions at the inlet of the mixing section,

\[ P_{p_2} = P_{s_2} \quad (2.18) \]

and exit of the ejector,
\[ P_3 = P_\infty \]  

complete the set of twelve equations for the fourteen flow parameters. Since these equations constitute an indeterminate system of twelve equations and fourteen unknowns, it is necessary to specify two of the unknowns in order to obtain the solutions. The approach used here is to specify the ejector inlet area ratio \( \frac{A_{e1}}{A_{p2}} \), and the inlet static pressure \( P_{s2} \), at the entrance to the mixing region. The ejector geometry is defined by specifying values for the ejector inlet area ratio \( \frac{A_{e1}}{A_{p2}} \) and exit area ratio \( \frac{A_3}{A_m} \). In order to satisfy Equation (2.18), the design pressure ratio of the nozzle must match the pressure ratio of the solution. Since the nozzle exit area is constant, the nozzle throat area is changed to match the nozzle exit pressure to the ejector inlet pressure.

The inlet static pressure \( P_{s2} \) is assigned different values. Specification of the static pressure at the entrance to the mixing section is equivalent to specifying the primary and secondary mass flows, and by using equation (2.10) the total mass flow pumped through the ejector. As a result, solutions to the conservation equations are obtained as a function of the mixed flow Mach number at the end of mixing.

2.6.2 Method of Solution to the Control Volume Approach

Use of the equations of state (2.13, 2.14 and 2.15) and the continuity equation, Eq (2.10), lead to the following
Expressing the velocities in terms of Mach numbers and temperatures,

\[ U = M \sqrt{KT} \]  \hspace{1cm} (2.22)

and expressing the temperatures in terms of the stagnation temperature and Mach number, the Mach number at the end of mixing, \( M_m \), can be expressed in terms of the inlet conditions at station 2 by rearrangement of the continuity, energy and momentum equations (2.10), (2.11) and (2.12). The resulting relationship\(^{10}\) is

\[ A(KM_m^2)^2 + B(KM_m^2) + 1 = 0 \]  \hspace{1cm} (2.23)

where,

\[ A = 1 - J^2 \frac{(K-1)}{2} \]

\[ B = 2 - KJ^2 \]

and the quantity \( J \) is defined as
\[
J = \frac{\sqrt{\frac{f_{p2}}{T_{mp}} \left( \frac{p_{o1}}{p_{o2}} + M_{p2} \right) + \beta M_{m2}^2}}{\left( 1 + \beta \frac{p_{o1}}{p_{o2}} \right) (1 + \beta)}
\] (2.24)

with \( \beta \), the mass flow rate ratio, expressed by the relation

\[
\beta = \frac{m_1}{m_p} = A_{sp} \frac{M_p}{M_p} \sqrt{\frac{T_{p2}}{T_{m2}}} = A_{sp} \frac{M_p}{M_p} \sqrt{\frac{T_{np}}{T_{m2}}} \left( \frac{1 + K \frac{1}{2} M_{m2}^2}{1 + K \frac{1}{2} M_{p2}^2} \right)
\] (2.25)

If conditions at station 2 are specified, the Mach number \( M_m \) at the end of mixing can be expressed as the solution of the quadratic equation (2.23), under the form

\[
M_m = \sqrt{-\frac{-B \pm \sqrt{B^2 - 4A}}{2KA}}
\] (2.26)

Therefore, for any given set of flow properties, \((M_{p2}, M_{s2}, p_{os}, T_{np} / T_{m2})\) at the start of mixing, there are two possible flows after completion of the mixing process. The solutions to eq (2.26) will be referred to as \( M_{m(-)} \) when the negative sign in eq (2.26) is used and \( M_{m(+)} \) when the positive sign in eq (2.26) is used. Analysis of the flow properties indicates that the two solutions to equation (2.26) are related by the expression

\[
M_{m(-)}^2 = \frac{(K - 1) M_{m(+)2}}{2K M_{m(+)2} - (K - 1)}
\] (2.27)
which is the relationship between Mach numbers across a normal shock wave. As a result, the two solutions represent flows which, at the end of mixing, may be either subsonic or supersonic. Depending upon the initial properties of the primary and the secondary streams at station 2, either or both solutions may represent physically achievable flows. The two solutions, however, may also represent states not realistically achievable from the given initial conditions, even though they are consistent with the conservation laws represented by equations (2.10), (2.11), (2.12) and (2.13). The occurrence of each state has to be analyzed according to the thermodynamic laws, since both states satisfy the conservation laws. To determine the validity of each state the solution is investigated with the aid of the second law of thermodynamics. Each flow representing a state in which there is a net increase in entropy is considered as being physically achievable. On the other hand, end states with decreased entropies are discarded as impossible.

The solution representing a subsonic mixed flow (subscript (-)) always satisfies the second law of thermodynamics, and it is referred to as the first solution. The solution representing a supersonic mixed flow (subscript (+)) is referred to as the second solution, and satisfies the second law of thermodynamics only under certain inlet conditions, as will be discussed in later sections of this document. Once the Mach number ($M_m$) at the end of the mixing process is known, the pressure ratio is evaluated by using equations (2.20) and (2.21)
\[ \frac{p_m}{p_{\gamma 2}} = \frac{(1 + \beta)M_p}{(1 + A \phi)M_m} \sqrt{\left( \frac{T_m}{T_{\gamma p}} \right)^{1 + \frac{K - 1}{2} M_m^2} \left( \frac{1 + \frac{K - 1}{2} M_m^2}{1 + \frac{K - 1}{2} M_m'^2} \right)} \quad (2.28) \]

where \( \frac{T_m}{T_{\gamma p}} \) is given by

\[ \frac{T_m}{T_{\gamma p}} = \frac{1 + \beta}{1 + \beta} \quad (2.29) \]

The temperature at the end of mixing can also be calculated in terms of the temperature at station 2, as follows,

\[ \frac{T_m}{T_{\gamma 2}} = \frac{1 + \frac{K - 1}{2} M_m^2}{1 + \frac{K - 1}{2} M_m'^2} \left( \frac{T_m}{T_{\gamma 2}} \right) \quad (2.32) \]

and

\[ \frac{T_m}{T_{\gamma 2}} = \frac{1 + \frac{K - 1}{2} M_m^2}{1 + \frac{K - 1}{2} M_m'^2} \left( \frac{T_m}{T_{\gamma p}} \right) \quad (2.33) \]

2.7 Analysis of the Diffuser Section

In dealing with flow through the diffuser, considerations must be given to: (1) The satisfaction of external (ambient) boundary conditions, specifically, the exit static pressure, (2) boundary layer growth and flow separation and (3) possibility of continued primary/secondary interactions within the diffuser.

In the case of supersonic exhaust flow, the diffuser exit static pressure should be equal to the ambient pressure for maximum thrust augmentation. For subsonic exhaust flow, the static ambient pressure imposes this boundary condition. As a result, the static pressure gradient which is present throughout the diffuser establishes a match between
the static pressure at the entrance to the diffuser and the static pressure at the end of the interacting zone. Consequently, the diffuser provides a strong effect on the mixing process, the mass flow entrainment and the overall device performance.

The thrust augmentation, however, can be severely degraded if the diffuser is poorly designed. An inefficient diffuser increases the boundary layer growth in the presence of the adverse pressure gradient and may lead to flow separation.

The continuation of primary/secondary interactions within the diffuser is not addressed in this analysis. It is due to the assumption made in the control volume approach that the mixing chamber length of the ejector is sufficient to ensure complete mixing of primary and secondary flows, and provide a uniform mixed flow at the entrance to the diffuser.

Once the mixing process is completed at the end to the interaction region, and the flow properties are determined, it is essential to return the mixed flow to ambient pressure efficiently for maximum thrust augmentation. The required diffuser geometry, necessary for efficient discharge, is determined by evaluating the required pressure ratio for return to ambient pressure at the outlet. This can be done by the use of equations (2.30) and (2.3) for the static pressure ratio, and as follows for the stagnation pressure ratio

\[
\frac{P_{in}}{P_r} = \left(1 + \frac{k-1}{2} M_m^2 \right)^{\frac{k}{k-1}} \left( \frac{P_m}{P_r} \right)
\]  

(2.34)

Also, assuming isentropic discharge from station m to station 3, where the pressure is the ambient pressure, the exit Mach number is
The required area ratio for the diffuser is determined with the use of the continuity equation

\[ M_3 = \sqrt{\frac{2}{K-1}} \left( \left( \frac{P_m}{P_x} \right)^{\frac{K-1}{K}} - 1 \right) \]  

(2.35)

Thus, the outlet geometric requirements for the return of the mixed flow to ambient pressure are evaluated from the flow parameters \( M_m \) and \( \frac{P_m}{P_x} \) at the conclusion of the mixing process.
Chapter 3. Analysis of the solutions to the control volume approach

It has been established in the previous chapter that mixing of two streams of compressible flows having arbitrary initial properties results in one of two possible states upon conclusion of the process. These two final states are differentiated by the fact that the Mach number of the fully mixed flows are related in the same way as are the Mach numbers across a normal shock wave. One of these states corresponds to subsonic flow and the other to supersonic flow. Detailed examination of the solutions is provided in this chapter in terms of entropy production.
3.1 Ejector Cycle Analysis

Study of the solutions to the quadratic equation (2.23) according to Belivaqua\textsuperscript{12}, reveals that the ejector falls into two basic categories. These categories are dependent on whether the thrust augmentation results from the transfer of kinetic energy or thermal energy from the primary stream to the secondary stream. The different processes inherent to ejector operation in the aerodynamic cycle show the difference between these categories. Figure 3.1, which is a temperature-entropy diagram, shows the processes the secondary stream goes through in an ideal ejector. The compression and expansion processes in the ejector are assumed isentropic while the jet mixing process, which drives the ejector, is inherently non isentropic due to the irreversible production of turbulence and heat by viscosity.

At low speeds, secondary air at ambient pressure goes through an expansion process as it accelerates from station A to station 1 into the ejector. From stations 1 to 2 the two streams start to mix. This mixing process increases the static pressure and compresses the secondary stream. The entropy increases due to the turbulent mixing and exchange of heat between the two streams as they interact. From stations 2 to 3, the diffuser compresses the mixed flow isentropically back to ambient pressure. There is a net production of thrust because the expansion from A to 1 creates more energy than the compression from 2 to 3 requires. In this case, it is the kinetic energy delivered during mixing that increases jet thrust so that the low speed ejector works like a ducted fan. At high speeds, however, the secondary air is compressed as it flows into the ejector from A to B. Mixing of the two streams results in an increase in temperature and pressure of the
secondary stream as shown from B to C. There is an increase in entropy due to the production of turbulence. The nozzle expands the mixed flow back to ambient pressure from C to D. There is a net thrust production because the expansion from C to D creates more energy than the compression from A to B absorbs. In this case, it is the transfer of thermal energy during mixing that increases the jet thrust so that the ejector works like a ramjet.

![Figure 3.1 Comparison of Ejector and Ejector Ramjet Cycles](image)

3.2 Analysis of the solutions in terms of entropy production

Since both solutions to Equation (2.23) represent flows which satisfy the laws of mass, energy and momentum in a constant area mixing channel, the possibility of
physically achieving these end states must be examined in terms of their entropy production based on the second law of thermodynamics. The entropy of each flow at the inlet to the mixing region, denoted as station 2, with respect to an arbitrary reference value is expressed as

\[ S_2 - S_r = m'_p(s_p - s_r) + m'_s(S_S - S_r) \]

\[ = \dot{m}_p R \left[ \left( \frac{1}{\eta} \right) \ln \left( \frac{T_{p2}}{T_r} \right) + \left( \frac{\beta}{\eta} \right) \ln \left( \frac{T_{s2}}{T_r} \right) \right] - (1 + \beta) \ln \left( \frac{P_{s2}}{P_r} \right) \]  

(3.1)

where \( n = \frac{K}{K-1} \).

Similarly, at the outlet to the mixing chamber, denoted as station m, the flow entropy is

\[ S_m - S_r = \dot{m}_p R \left[ \left( \frac{1}{\eta} \right) \ln \left( \frac{T_{m}}{T_r} \right) - \ln \left( \frac{P_m}{P_r} \right) \right] (1 + \beta) \]  

(3.2)

The total change in entropy in the mixing chamber is given by

\[ \Delta S = S_m - S_2 = \dot{m}_p R \left[ \left( \frac{1}{\beta} \right) \ln \left( \frac{T_{m}}{T_{s2}} \right) + \left( \frac{\beta}{\eta} \right) \ln \left( \frac{T_{m}}{T_{s2}} \right) \right] - (1 + \beta) \ln \left( \frac{P_m}{P_{s2}} \right) \]  

(3.3)

Equation (3.3) determines the total entropy change of the flow from the initial to the final states of the mixing process. The possibility of achieving these end states depends on whether there is a positive entropy change or a negative entropy change.
3.3 Existence of the first and second solution

Examination of the solutions to the mixed flow Mach number $M_m$ reveals that when the determinant to the quadratic equation (2.23) is positive the two solutions represent flows at the end of the mixing process, which in one case is supersonic and in the other case is subsonic. It is also evident that the quadratic equation (2.23) has no real solution when its determinant is negative, and a single solution when the determinant reaches a value of zero. It can also be shown that when the determinant is zero, corresponding to $B^2=4A$, $M_m=1$ which can be considered as the choking limit of the mixed flow. Solution to the equation $B^2-4A=0$ yields an expression for the quantity $J$ defined in equation (2.24) as

$$J_c = \frac{1}{K} \sqrt{2(K+1)} = 1.565 \text{ for } K=1.4$$  \hspace{1cm} (3.4)

where subscript $c$ refers to the choking condition. As a result, the mixed flow will choke ($M_m=1$) when $B^2=4A$ or $J=J_c$ in a constant area mixing duct. Using equation (2.24) and the choking value of $J(=J_c)$, it can be shown that

$$\left(\frac{T_{np}}{T_{ma}}\right)_c = K \sqrt{\left(\frac{T_{np}}{T_{oa}}\right)_c} - 1$$  \hspace{1cm} (3.5)

or

$$\left(\frac{T_{np}}{T_{oa}}\right)_c = \left(\frac{K}{2} \pm \sqrt{\frac{K^2}{4} - 1}\right)^2$$  \hspace{1cm} (3.6)
where subscript c refers to the thermal choking phenomena and the quantity K is given by

\[ K = \frac{\frac{1}{J^2} \left[ \frac{a}{K M_p} + (\alpha - 1) \frac{M_s^2}{M_p} \right] (\alpha - 1)^2 \left( \frac{M_s}{M_p} \right)^2 \left( \frac{\nu_s}{\nu_p} \right) \left( \frac{t_{op}}{t_p} \right)}{(\alpha - 1) \left( \frac{M_s}{M_p} \right)^2 \left( \frac{\nu_s}{\nu_p} \right) \left( \frac{t_{op}}{t_p} \right)} \]  \hspace{1cm} (3.7)

where \( \alpha = \frac{a}{d_p} + 1 \).

The two values of \( \frac{T_{op}}{T_{os}} \) represented by equation (3.6) can be shown to be inversely proportional for \( \alpha \neq 1 \)

\[ \left( \frac{T_{op}}{T_{os}} \right)_{C^+} = \left( \frac{T_{os}}{T_{op}} \right)_{C^-} \]  \hspace{1cm} (3.8)

where in this case, the (+) and (-) signs refer to the positive and negative signs in the solutions to equation (3.6). Equation (3.8) illustrates that if \( M_s, M_p \) and \( \alpha \) are held fixed choking will occur at a given ratio of the larger to smaller stagnation enthalpies, regardless of which of the two flows contains the larger stagnation enthalpy.

The next sections of this chapter illustrate the existence of both solutions for specified inlet conditions and area ratios. The results are presented in Tables 3.2.1 and 3.2.2. In table 3.2.1 a computer printout of both supersonic and subsonic branch solutions is presented for a primary/secondary stagnation pressure ratio of 6, stagnation temperature ratio of 3.35 (corresponding to a primary stagnation temperature of 1000K) and a secondary to primary inlet area ratio of 10. These stagnation ratios are representative of the flow conditions possible with a modern jet engine. The first column of table 1 contains the secondary flow inlet static pressure at station 2, while the second column
contains the secondary flow Mach number which is taken as the independant variable. The value of the primary Mach number $M_p$ (column 3) is set by matching the static pressure of the primary to the secondary flow at the inlet to the ejector and by using the isentropic relations (Equations (2.2) and (2.3)).

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<th>$M_s$</th>
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Table 3.1.2 Subsonic and Supersonic Solutions for the mixed flow for the following inlet conditions ($P_{op}/P_{os}=6,T_{op}/T_{os}=3.35,A_s/A_p=10$)
Columns 4 and 5 give the mixed flow Mach number $M_m$ for both branches of the solution. $M_{\text{SUB}}$ is the mixed flow Mach number for the subsonic branch, while $M_{\text{SUP}}$ represents the supersonic branch solution.

Figure 3.2.1 represents the mixed flow Mach number $M_m$ for the supersonic and subsonic cases. From figure 3.2.1, it can be seen that the mixed flow Mach number on the subsonic branch is less than 0.8, while the supersonic mixed flow Mach number is greater than 1.3. This occurs except in the region where the secondary flow Mach number reaches 1 and where both supersonic and subsonic flows tend to a unique solution.

It is observed that choking of the mixed flow occurs at higher values of primary stagnation temperature. It takes place in a region near $M_s = 1$. It is believed to result from the injection into the ejector shroud of heated primary gas. As a consequence, the mixed flow Mach number has no real solution for a range of secondary inlet Mach number $M_s$. That range is found to be near 1 (as shown in Figure 3.2.2). To better illustrate the choking phenomenon table 3.2.2 represents another computer printout of both solutions with the same inlet flow conditions, except that the stagnation temperature ratio is taken to be 4 instead of 3.35.
Figure 3.2.1 Mixed Flow Solutions for Unchoked Flow.
Table 3.2.2 Subsonic and Supersonic Solutions for the mixed flow for the following inlet conditions ($P_{0P}/P_{0S}=6$, $T_{0P}/T_{0S}=4$, $A_S/A_P=10$)

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Figure 3.2.2 is generated from the data taken from table 3.2.2 to illustrate the choking phenomenon. $M_m$ has only imaginary solutions and neither branch is shown in Figure 3.2.2. It is also found that the corresponding secondary inlet Mach number lies within a particular region, referred to by Hoge $^{13}$ as the forbidden region (the region over which the solutions do not exist).
Figure 3.2.2 Mixed Flow Solutions for Choked Flow.
The choking phenomenon, as described in Figure 3.2.2 and Table 3.2.2, occurs for that specific region of secondary flow Mach which is approximately between the values of 0.8864 and 1.1320. This choking eliminates a section of the curve as indicated on figure 3.2.2. It results from an increase in stagnation temperature ratio of 4. Figure 3.2.3 shows the region of choked flow for a bypass ratio of 10, a pressure ratio of 6 and a temperature ratio of 5.7. According to Hoge\textsuperscript{13}, at the higher values of the inlet flows stagnation temperature ratios and mass flow rate ratios the mixed flow becomes choked. He has also indicated the boundaries of the region over which the solutions do not exist, and which he refers to as the forbidden region. He concluded that the shape of the forbidden region is independent of the pressure ratio, and depends only on the value of the bypass ratio and the temperature ratio. He was able to develop an equation for the boundaries of the forbidden region which takes the form,

\[
\left( M_p^{\text{RI}} + \frac{1}{M_p^{\text{RI}}} \right) + \beta \left( M_S^{\text{RI}} + \frac{1}{M_S^{\text{RI}}} \right) \frac{1}{\sqrt{TR}} = 2 \sqrt{\frac{(1+\beta)^2}{TR}} \quad (3.9)
\]

where the parameter $M^R$ was defined as the ratio of velocity to the speed of sound at Mach number of one. $\beta$ was the bypass ratio and $TR$ was the temperature ratio. The parameter $M^R$ was chosen in place of the Mach number since a finite range covers all Mach numbers from zero to infinity. Hoge\textsuperscript{13} also developed the equation of the curve for which the primary inlet pressure equaled the secondary from the isentropic relations, which yielded

\[
M_p^{\text{RI}} = \sqrt{\left( \frac{K+1}{K-1} \right) - PR^{-\left( \frac{K-1}{K} \right)} \left( \frac{K+1}{K-1} - M_S^{\text{RI}} \right)} \quad (3.10)
\]

where $PR$ was the primary to secondary stagnation pressure ratio.
Figure 3.2.3 Description of the choked flow region

13
3.4 Effect of back pressure on both solutions

Once the ejector design is selected, its operation will be determined by the boundary conditions imposed on the ejector. Therefore, in addition to the primary and secondary total pressures and total temperatures, the back pressure must also be known in order to determine the ejector operating point. At any value of $M_s$ the design is the same for a point on the supersonic or subsonic branch. As a result, if the back pressure, or in this case the effective back pressure due to the presence of the diffuser, is set at a value equal to the mixed flow static pressure on the subsonic branch, then the ejector will operate at that design point. Furthermore, $P_{2p}$ would be equal to $P_{2S}$. This follows since the exit flow is subsonic.

On the supersonic branch, the situation is quite different. If the back pressure at the entrance to the diffuser section is sufficiently reduced, the ejector will make a transition to the supersonic solution branch. Ejector operation becomes then independent of further reductions in diffuser pressure. In this case, the ejector will operate at only one point on the supersonic branch irrespective of the value of the back pressure. This operating point can be determined by the methods described by Fabri and Siestrunck. In 1958, they presented the results of an extensive study of air-to-air ejectors with high pressures ratios, in which the primary air flow was supersonic. Although they were primarily concerned with jet pumps, they presented a theory which was in good agreement with experimental results for the predicted rates of induced mass flows. For the case of supersonic mixed flows and a supersonic primary flow, Fabri and Siestrunck stated that the inlet flow pattern was similar to that shown on figure 3.3. This flow pattern represented the case
where the primary flow inlet pressure exceeded the inlet pressure of the secondary flow. Therefore, the primary flow had to undergo an additional expansion in the entrance region to the mixing tube. The case where the two inlet pressures matched was a limiting case, and therefore, could be determined from the analysis. Since the expansion took place very quickly in the entrance region, the flows remained unmixed and the slip line between the primary and secondary flow is shown as a double line emanating from the primary nozzle. In the case where the primary inlet pressure is less than the secondary inlet pressure there would be a shock in the primary fluid immediately at the entrance, and would increase the pressure in the primary fluid. This would require the slip line at the nozzle lip to turn inward. Therefore, the secondary flow would "see" a minimum area at the inlet, and its Mach number would reach one for the supersonic mixed flow case, due to the flow pressure in the mixing tube required for the supersonic branch. If this was not the case, the secondary stream pressure would increase in the mixing tube. This would have lead to a breakdown of the supersonic flow in the primary jet and a subsonic mixed flow. It is essential to notice that these arguments will not hold if a throat is placed in the secondary stream, ahead of the inlet, since the secondary flow could then be supersonic when the pressures are matched at the inlet.
Figure 3.3  Inlet Flow Pattern for an Ejector Operating with a Supersonic Mixed Flow and Having a Supersonic Primary Flow and a Subsonic Secondary Flow.14
Chapter 4: Ejector performance And Optimization

The scope of the present chapter is to determine reasonable estimates of thrust augmentation that could be achieved within an ejector over a range of secondary inlet Mach numbers from low subsonic to supersonic.

4.1 Ejector Performance

To evaluate the influence of any parameter on ejector performance, it is essential to first fix the ejector size in relation to the size of its reference jet. In order to accomplish this, it is convenient to define a reference jet as a free jet whose gas has the same stagnation properties and mass flow as those of the primary jet of the
ejector. Since the inlet section represents a key element in ejector design, the ejector cross section is related to the primary nozzle cross section. The discharge from the gas generator (primary flow) has known characteristics including its mass flow \( m_p \), its stagnation temperature and its stagnation pressure. The primary flow is fully expanded into a pressure \( P_{2S} \) different from its normal discharge pressure \( P_a \). Therefore, the primary nozzle discharge area must avoid any alteration of the mass flow from the gas generator. The ejector size is defined as the area ratio of the ejector mixing section to that of its reference jet when expanded isentropically to ambient pressure.

The ejector is assumed to ingest fluid from a given free stream Mach number \( M_0 \) without losses. The process is assumed to be one of isentropic expansion or compression depending on the value of the free stream Mach number. After complete mixing, the resulting uniform flow at station \( m \) is exhausted through a diffuser or nozzle. Selection of the particular outlet geometry necessary to return the flow to ambient pressure depends on whether the mixed flow is supersonic or subsonic. In the case of supersonic mixed flow a convergent divergent nozzle is required, while a diverging passage is needed for subsonic mixed flow. The primary fluid injected through the inner core of the ejector is considered to be the energized stream (high temperature, high velocity gas), and the secondary fluid at the outer region is considered to be the ingested stream. The net thrust of the ejector is compared to the net thrust of its reference jet in order to provide a meaningful indication of the ability of the ejector to augment the thrust of its reference jet. This thrust augmentation is described by the ratio of the momentum increment between the free stream and station 3, to the momentum of the primary mass flow.
exhausted isentropically to ambient pressure. Therefore, for an air breathing propulsive
system,

\[
\Phi = \frac{(m_p^e + m^i_e)(U_3 - U_\infty)}{m_p^e(U_3 - U_\infty)} = (1 + \beta) \frac{M_p \sqrt{T_3 - M_{\infty} \sqrt{T_{\infty}}}}{M_p \sqrt{T_p - M_{\infty} \sqrt{T_{\infty}}}} \tag{4.1}
\]

where \(M_{\infty}\) is the free stream Mach number, \(T_{\infty}\) is the free stream temperature and \(U_\infty\) is the free stream velocity. If the primary jet of the ejector is non air breathing (rocket), the expression for thrust augmentation must be modified by eliminating the so called "ram" drag terms associated with the mass flow of the ejector's primary jet.

Since the ingestion and injection into and the discharge from the ejector are assumed to be isentropic, the following relations are obtained

\[
P_{03} = P_{0m} \quad \text{(4.2)} \quad \text{and} \quad T_{03} = T_{0m} \quad \text{(4.3)}
\]

\[
P_0 = P_{01} \quad \text{(4.4)} \quad \text{and} \quad T_0 = T_{01} \quad \text{(4.5)}
\]

\[
\left(\frac{\rho^e}{\rho}\right) \left(\frac{K-1}{K}\right) = \frac{T^e}{T_{\infty}} = 1 + \frac{K-1}{2} M_p^{2\infty} \quad \text{(4.6)}
\]

where subscript \(o\) refers to stagnation conditions.

The thrust augmentation ratio \(\Phi\) can be evaluated for any given values of \(M_o\), \(M_s\), \(P_{op}/P_{os}\), \(T_{op}/T_{os}\) and \(\alpha\), knowing the conditions of the mixed flow at station \(m\) through equations (2.28), (2.29), (2.30) and (2.31).
4.2 Ejector Optimization

The concept of ejector design optimization, for both "positive" and "negative" solutions to the equations governing the flow through an ideal ejector, provides a means for achieving high ideal thrust augmentation for V/STOL applications.

Stationary Ejectors ( \( M_\infty = 0 \) )

The author of the present document will consider ejector performance for the stationary case in which the ejector is at rest with respect to the undisturbed medium. Under these conditions, the thrust augmentation ratio becomes,

\[
\Phi = \frac{(1+\beta)M_2 \sqrt{T_3}}{M_p \sqrt{T_x}} \tag{4.7}
\]

For specified injected gas conditions and area ratios, there exists only one free parameter for the determination of a unique solution to Equation (2.23). Using \( M_s \), the secondary flow Mach number at the start of mixing as that parameter, ejector thrust augmentation is evaluated as a function of \( M_s \) to determine whether a maximum or limit exists. For comparison purposes the values of the stagnation pressure for both inlet flows are assumed to be the same as those used for in Figure 3.2.1 and 3.2.2. The results are presented in table 4.2.1 and 4.2.2. The stagnation temperature ratios used are 3.35 for table 4.2.1 and 4 for table 4.2.2. The inlet area ratio \( A_3/A_p \) is held constant for both tables and has a value of 10. The results obtained are presented in both tables as follows.
It should be noticed that even though the secondary flow Mach number at the start of mixing is defined as the independent variable, it can be controlled by adjusting the effective back pressure at the entrance to the diffuser. The latter pressure is further determined by the isentropic expansion taking place in the diffuser and the back pressure at the outlet.

Table 4.2.1. Entropy Production and Thrust Augmentation Ratio for \( \frac{P_{op}}{P_{os}}=6, \frac{T_{op}}{T_{os}}=3.35, \frac{A_s}{A_p}=10 \)

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<th>T.A.R(_{SUB} )</th>
<th>( \Delta S_{SUP} )</th>
<th>T.A.R(_{SUP} )</th>
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Table 4.2.2. Entropy Production and Thrust Augmentation Ratio for ($P_{\text{OP}}/P_{\text{OS}}=6$, $T_{\text{OP}}/T_{\text{OS}}=4$, $A_{\text{S}}/A_{\text{P}}=10$)

It was shown earlier, through Equation (2.23), that there was a double solution to the conservation laws for a control volume encompassing the ejector mixing chamber. One solution corresponded to subsonic flow and the other to supersonic flow. The subsonic solution was referred to as the first solution and the supersonic as the second solution.
To illustrate the influence of inlet flow stagnation conditions on stationary ejector performance, the variation of thrust augmentation is plotted versus the secondary flow Mach number. This variation is generated for both first and second solutions at an arbitrary chosen primary nozzle pressure ratio and for two different primary nozzle stagnation temperatures.

4.3 Unchoked Ejector Flow Performance Under Both Solutions

4.3.1 Thrust Augmentation Levels Under the First Solution

Figure 4.2.1 uses the data taken from table 4.2.1. It illustrates typical ideal thrust augmentation and flow characteristics, resulting from the injection of low temperature gas into an ejector, as a function of the Mach number of the secondary flow at the start of mixing. It is seen from the computer printout (see Appendix A) and from figure 4.2.1, that under the first solution or subsonic branch, the flow at the end of mixing exists only between the limits of $0 < M_s < 1.6153$ for this example. Values of $M_s$ greater than 1.615 can still contribute to the existence of the subsonic branch at the end of mixing. However, they will be discarded in the analysis because $M_s$ is limited at its upper range by the physical restraint that the Mach number of the flow at the exit to the diffuser $M_3$ must be greater than 0.

Examination of the ejector flow under the first solution in Figure 4.2.1 leads to the following observations: The flow at the end of mixing under the first solution, (subsonic branch), exists only when the secondary flow reaches values between the limits of 0 and 1.6153. Specifically, for values of $M_s$ greater than 1.6153 the mixed flow experiences a
deficiency in kinetic energy and fails to overcome the adverse pressure gradient encountered in the exhaust diffuser. The first solution produces a maximum of 1.7132 in thrust augmentation at a value of $M_s = 1.097$, as illustrated in Figure 4.2.1. After reaching that peak value the thrust augmentation level decreases continuously. It can also be concluded from Figure 4.2.1 that: (1) The mixed flow Mach number at the end of the mixing process remains subsonic, (2) the entropy change during the mixing process is always positive, and therefore does not violate the second law of thermodynamics and (3) operating the ejector under the first solution requires a convergent subsonic inlet. It also requires a divergent subsonic diffuser at the outlet, as the mixed flow remains subsonic.

### 4.3.2 Thrust Augmentation Levels Under the Second Solution:

It is important to notice from figure 4.2.1 that ejectors operating under the second solution can generate thrust augmentation levels much higher than the ones generated under the first solution. This high performance occurs when the secondary flow Mach number is subsonic. However, the thrust augmentation obtained by the use of the second solution, with subsonic mixing, is limited to the region where the total entropy change during mixing is greater than zero. This eliminates a portion of the curve corresponding to a decrease in entropy during the mixing process as seen on figure 4.2.1.
4.4 Effects of Choking on Ejector Flow Performance Under Both Solutions

The injection of heated gas, as was mentioned in previous sections of this report, brings into focus the phenomenon of thermal choking as illustrated in Figure 4.2.2. The possibility of choking of the mixed flow occurs in a region where the secondary flow Mach number is near one. Specifically, the range of values of $M_s$ over which mixing cannot proceed to its conclusion due to thermal choking is noticed to lie between the values of 0.840 and 1.167. This range may change according to the inlet conditions, area ratios and entrained mass flow rates.

4.4.1 Thrust Augmentation Under the First Solution

It is seen from Figure 4.2.2 that under the first solution the curve of thrust augmentation is made of two sections. The first segment contains the lower choking point and in which the secondary flow Mach number is between 0 and 0.8240. The other segment contains the upper choking point and limits the value of $M_s$ from 1.1677 to 1.6451. This last segment is restricted at its upper end by the requirement that the exit Mach number at the diffuser $M_3$ must be positive. It is concluded that thrust augmentation levels are much higher with subsonic values of $M_s$ than supersonic. Moreover, the maximum thrust augmentation occurs at the lower choking point which varies with injected gas characteristics and inlet area ratios.

4.4.2 Thrust Augmentation Under the Second Solution
The second solution as seen in Figure 4.2.2 displays two segments for the given inlet conditions. A lower segment that corresponds to secondary flow Mach numbers values between 0.3128 and 0.840. This segment contains the lower choking point and the limit point (ΔS = 0). An upper segment over which the secondary flow Mach number is supersonic and lies between the values of 1.1677 and 1.6153. This second segment contains the second choking point and displays higher levels of thrust augmentation than the previous segment. It is bounded by an upper limit that results from the restriction that the diffuser Mach number M₃ must be positive in order to return the flow to ambient pressure. Thrust augmentation under the second solution usually displays a local maximum performance point with a supersonic value of M₂S which in this case occurs at the upper choking point. It also displays a limiting performance point at a subsonic value of M₂S limited by the second law of thermodynamics.
Figure 4.2.2 Ejector Performance for Choked Flow
4.5 Concluding Remarks

In conclusion, real solutions to the mixing process exist and thrust augmentation occurs only for a limited range of secondary inlet flow Mach numbers as a result of physical constraints. These constraints are:

- The mixing process can only contribute to an increase in the total entropy.
- Once the mixing process ends, the mixed flow must have sufficient kinetic energy to overcome the adverse pressure gradient encountered in the diffuser.
- The flow cannot exist within the choked region.

As a result, ejector performance is limited to a range of values of secondary inlet Mach numbers bounded on each end by one of the constraints mentioned above. Enforcement of these constraints for a specific range of secondary Mach number shows the existence of three distinct operating points that characterize optimal ejector performance. These points are defined as follows:

- Optimal performance under the first solution which occurs at subsonic values of $M_{2s}$ at the lower choking point.
- Optimal performance under the second solution where a local maximum point occurs in the absence of choking at transonic or supersonic values of $M_{2s}$. In the case of choking, the second solution exhibits two regions. One region containing the lower choking point and the other region the upper choking point, which in this case corresponds to the local optimum performance point.
- Limiting performance under the second solution is established by the second law of thermodynamics and always occurs at a subsonic value of $M_{2s}$. 
### 4.6 Summary of approaches to overall device performance

An understanding of the fundamental physics of ejectors can be approached on two levels: (1) The control volume approach or overall process and what occurs in terms of bulk changes in energy and enthalpy and (2) The physical phenomena approach which contributes to the overall process in terms of the fundamental mechanisms of energy and momentum transfer.

The control volume approach, which is used in the first chapters of this document, treats the ejector essentially as a "black box" by satisfying the bulk conservation equations between the device inlet and exit. It enables an understanding of "gross effects", such as area ratios and inlet stagnation condition trends on thrust augmentation. In doing so, the analysis overlooks the phenomena of major significance to the device performance and suffers from a lack of specification to the turbulent mixing taking place within the control volume considered.

The physical phenomena approach, which will be used in the next chapters of this report, attempts to overcome the limitations inherent in the control volume approach. It establishes flow models capable of predicting the turbulent mixing within the ejector. However, in doing so, the analysis encounters two major problems. These are: (1) The complexity of the flow interactions taking place in an ejector makes it difficult to include all the significant phenomena in the turbulent mixing model and (2) the flow model used in the analysis, due to the limitations of fluid dynamics, must rely on empirical data taken from the theory of free jet turbulent mixing. Nevertheless, this method has proven
to be successful in describing the requirements for efficient and complete mixing and in understanding the basic mechanisms of the mixing process.
Chapter 5: The Phenomenological Approach

This chapter describes the basic fluid mechanical processes involved in the operation of ejectors, in order to obtain a better understanding of their operation, and for the development of accurate analytical models. Specifically, it is concerned with the main characteristics of a computational model for axisymmetric jet mixing in a constant area mixing tube.

5.1 Introduction

The thrust augmentation of an ejector system is governed by the laws of fluid mechanics associated with the entrainment of surrounding ambient air by the primary jet
flow and the turbulent mixing of this entrained fluid with the primary jet. However, as mentioned in previous sections of this report, past researches have provided insufficient information related to the operation of ejectors. According to these investigations the ejector thrust augmentation results from the low pressure on the shroud entrance region caused by entrainment of secondary fluid. Pressure recovery is achieved by turbulent mixing between the primary jet and the secondary stream. Exhausting the already mixed flow through a diffuser further enhances the thrust augmentation by reducing the entrance pressure to the ejector shroud. The flow processes that relate entrainment, mixing and diffusion result in a pressure distribution on the shroud and primary nozzle surfaces. The integrated effect of the pressure forces over the surfaces provides a large contribution to the increase in thrust of the system.

5.2 Objective of the investigation

The specific objective of this part of the investigation is to apply a turbulent flow model to describe the mixing process and to predict the flow requirements for efficient mixing. This model should be able to determine the role of entrained fluid, and its mixing with the primary jet, on the shroud surface pressure distribution and on ejector performance. It should also be able to predict the variation of the various profiles, (static pressure, velocity and temperature), along the length of the mixing chamber. Knowledge of these profiles, and specifically the static pressure distribution, allows calculation of the thrust augmentation.
5.3 Physical Description of a Compressible Turbulent Jet Discharging into an outer stream

The propagation of a turbulent jet in an external steam is characterized by the thickness of the zone of turbulent mixing and by the profiles of velocity, temperature, pressure and other parameters of the gas in the cross section of the flow. The part of the jet in which there is a core of potential flow is called the initial region. One of the fundamental properties of this region is that the static pressure is constant throughout the flow. As a result, the velocity in the potential core of the jet remains constant. Beyond this region the velocity profile becomes "lower" and "wider" with increasing distance from the beginning of the jet. This region is termed as the main region and is characterized by the increase with downstream distance of the transverse dimensions of the jet.

In treating the velocity profiles for both regions of the jet, the following parameters are used as dimensionless coordinates characterizing the location of a point in the flow. In the initial region of the jet, the dimensionless coordinate is computed from the outer edge of the jet as follows,

\[ \eta = \frac{y_1 - y_2}{y_1 - y_2} = \frac{y - y_2}{b} \quad (5.1) \]

and for the main region of the jet, the following dimensionless coordinate is expressed by,

\[ \varepsilon = \frac{y}{r} \quad (5.2) \]

where \( y_1 \) and \( y_2 \) are ordinates of the internal and external boundaries of the turbulent border layer in the initial region of the jet. \( r \) is the radius of the jet or the jet boundary.
corresponding to zero velocity, and \( y \) is the ordinate of a point which corresponds to an arbitrary value of the dimensionless velocity (see Figure 5.3.1). Abramovich, Zhestkov and Al\(^1\) showed that the experimental values of dimensionless velocity, given for various conditions of jet discharge from the nozzle, were in good agreement with the results computed according to the formula\(^1\)

\[
\Delta U = \frac{U_1 - U}{U_1 - U_2} = F(\eta) = \left(1 - \eta^{3/2}\right)^2 \quad (5.3)
\]

for the initial region of the jet and

\[
\Delta U = \frac{U - U_H}{U_m - U_H} = \Delta U = F(\epsilon) = \left(1 - \epsilon^{3/2}\right)^2 \quad (5.4)
\]

for the main region. These equations are referred to as "the law of the 3/2" or the "schlichting formulas". The temperature profiles in the cross sections of compressible jets were also approximated by the following formulas\(^1\),

\[
\Delta T = \frac{T_1 - T}{T_1 - T_2} = \varphi(\eta) = 1 - \eta \quad (5.5)
\]

for the initial region, and

\[
\Delta T = \frac{T - T_H}{T_m - T_H} = \psi(\epsilon) = 1 - \epsilon^{3/2} \quad (5.6)
\]

for the main region.
5.4 General description of the analytical model

In this section the analytical turbulent flow model described in the last section is applied to predict the mechanisms of entrainment and turbulent mixing that takes place in the ejector. The configuration investigated is an axisymmetric single nozzle jet ejector with constant area mixing tubes. The turbulent flow model is based upon the classical steady two phase jet model of Abramovich. The validity of the Abramovich model for describing the turbulent mixing process was demonstrated by comparing the analytical results with numerous experimental data, relating to ejector flow measurements, gathered at the Air Force Flight Dynamics Research Laboratory. The analysis is based upon
the hypothesis that the mixing phenomenon in the ejector is fundamentally similar to the mixing of a free turbulent jet with the surrounding medium, given the restriction that the ejector inlet and mixing chamber areas are very large compared to the area of the primary jet nozzle.

It has been observed that the turbulent mixing process in high speed compressible shear layers is dominated by large scale coherent structures. It consists of an engulfment process that captures large quantities of unmixed fluid and transports them across the mixing layer. Many studies, using flow visualization and conditional sampling, have lead to a better understanding of the structure and role of these large scale motions. However, few turbulence models have been developed that make use of the importance of these structures. The present model assumes that the high speed mixing process continues to be dominated by large scale coherent motions.

5.5 Assumptions used in the turbulent model

The following initial assumptions are made for the analysis:

1. The primary flow may be subsonic or supersonic.
2. The primary and secondary flows are the same perfect gas.
3. No heat is transferred across the walls of the ejector.
4. The ejector consists of an axisymmetric, cylindrical constant area mixing chamber with a single primary nozzle located along the axis.
5. The secondary flow and the combined flows after mixing are assumed to remain subsonic throughout the ejector.
6. The static pressure is constant across any section perpendicular to the axis of the mixing chamber.

5.6 Formulation of the turbulent flow model

Abramovich 17, through his experimental investigations of the theory of a free jet, established an analogy between the velocity fields at the lateral cross sections of a mixing chamber of an ejector and at the cross sections of a free jet discharging into the surrounding medium. He found that the velocity profile at each cross section of the mixing chamber, bounded by the cylindrical walls, corresponded to the central part of the dimensionless velocity profile of a free jet at the same cross section. The existence of this analogy enabled calculation of the various flow profiles in terms of the initial parameters of the mixing streams at any arbitrary cross section in the mixing chamber. This was made possible by setting up integral equations which expressed the fundamental laws of conservation of mass, energy and momentum. These conservation laws defined the flow variables at any arbitrary point of the mixing chamber in terms of the initial parameters of the mixing streams. Using the momentum equation the pressure change between the initial and final cross sections of the mixing chamber is determined by

\[
(P_m - P_{2S})A_m = \rho_{2P} U_{2P}^2 A_P + \rho_{2S} U_{2S}^2 A_S - \rho_m U_m^2 A_m \quad (5.7)
\]

where \(A_m\) is the mixing chamber area, \(A_{2P}\) is the primary nozzle discharge area and \(A_{2S}\) is the entrained flow inlet area. This equation illustrates the fact that, in contrast with a free
jet in which the static pressure remains constant in the radial and longitudinal axis of the jet, the static pressure in the mixing chamber of the ejector increases along its longitudinal axis and reaches the value of back pressure at the exit to the diffuser. This increase in static pressure is due to the presence of a coflowing induced stream and to the imposed pressure gradient resulting from the turbulent mixing of the two streams.

The above equation can be reduced to the non dimensional form below

\[ \frac{P_m - P_{2h}}{\rho_{2h} U_p^2} = \frac{\alpha}{(\alpha + 1)^2} (1 - \alpha n)(1 - \alpha \theta n) \]  

(5.8)

where \( \alpha = \frac{A_p}{A_s} \), \( n = \frac{\rho_{2p}}{\rho_{2s}} \), and \( \theta = \frac{\beta_s}{\beta_p} \). This equation determines the static pressure of the flow after complete mixing which occurs at an appreciable distance (theoretically infinite) from the initial cross section. The non uniformity of the flow field in the ejector mixing chamber makes the calculations of the flow parameters after complete mixing inadequate. An accurate calculation of these flow parameters must take into account the non uniformity of the flow field, the determination of the optimal length of the mixing chamber and the knowledge of the theory of mixing of streams along the length of the mixing chamber. Because of the similarity found between the turbulent structure within the ejector and that found in free jets developing in a coflowing stream, the turbulent flow within the ejector is divided into two distinct flow regions. These regions are interdependent and play a critical role in ejector thrust augmentation.
5.7 Entrance Region

This region is defined as the part of the jet in which there is a core of potential flow immersed in an outer stream which may be accelerating or decelerating, depending on the shape of the duct and the rate of entrainment of mass into the jet. The entrance region begins at the primary nozzle exit plane and continues downstream where the potential core of the jet ends and mixing of the two streams starts. The static pressure in the supersonic primary flow at the nozzle exit plane may be different from the static pressure in the surrounding secondary flow. This forces the primary flow to expand or contract isentropically until its static pressure matches that of the secondary flow. Turbulent transport in this region is confined to the jet which does not interact with the ejector walls. The length of the entrance region is determined as follows

\[ X_h = \frac{x_h}{b_0} = \pm \frac{1+m}{\sqrt{c(1-m)(0.416+0.134m)}} \] (5.9)

where \( m = U_{2s}/U_{2p} \), and the half thickness of the jet at the end of the entrance region is

\[ \frac{b_h}{b_0} = \frac{1}{0.416+0.134m} \] (5.10)

There are two mixing zones in the initial region of a jet situated along both sides of the potential core of flow. These zones are symmetrical relative to the axis of the jet and develop independently of one another (see Figure 5.5.1).
5.8 Main Region

It consists of the region of the flow downstream of the section where the jet attaches to the walls of the mixing chamber. It is also the region in which no zone of undisturbed ejected flow (secondary) exists and in which turbulent transport towards the walls of the mixing chamber is the most significant phenomenon. Due to the presence of non uniform flow properties the two flows interact through turbulent mixing. A schematic of a typical mixing section process is shown in Figure 5.5.2. Although the figure illustrates the distribution of velocity in one plane the actual mixing process, regardless of section geometry, is a three dimensional process. Depending upon the initial flow parameters the mixing process is a function of the mixing length available. A zero mixing length section may occur when all mixing takes place within the ejector diffuser. In general, as the mixing length of an ejector is increased, for either subsonic or supersonic primary nozzle flows, the performance of the ejector will improve. However, when the mixing process is nearly complete and the mixing length is further increased, the viscous effects begin to accumulate and become dominant. Further increase in length will then degrade the augmentation performance and lead to flow separation. Various
investigations have determined the optimum mixing length to diameter ratio (L/D) for non diffusing flow to be between 4 and 12. Multiple primary nozzle arrays will in general require a smaller ratio, while single primary nozzles require more mixing length. The optimum mixing length ratio is further influenced by the amount of entrained flow and whether the primary is supersonic or subsonic.

The presence of the adverse longitudinal pressure gradient leads to a reduction of the ejected (secondary) flow velocity in proportion to the distance from the initial cross section of the flow, where the velocity equals $U_{2S}$. This results from the assumption that the ejected flow is a wakelike flow with respect to the jet issuing from the nozzle. Using the analogy between the flow in the ejector mixing chamber and in a free jet, the concept of nominal wake velocity of the ejected flow is introduced to the analysis. Neglecting losses, the nominal wake velocity of the ejected flow at cross sections where the pressure is $P_x$ is given by

$$U_{W}^2 = U_{2S}^2 - 2\frac{P_x - P_{2S}}{\rho_{2S}}$$  \hspace{1cm} (5.11)

In the main region of the mixing chamber the longitudinal pressure gradient is small. Therefore, the wake flow velocity is assumed to remain constant at all cross sections. This transforms the previous equation to

$$U_{W}^2 = U_{2S}^2 - 2\frac{P_1 - P_{2S}}{\rho_{2S}}$$  \hspace{1cm} (5.12)
where $U_H$ is the nominal velocity of wake flow in the main region of the mixing chamber and $P_3$ is the flow pressure at the final cross section after complete mixing. $U_H$ can be derived as

$$U^2_H = U^2_{2S} \left[ 1 - \frac{2l(1-\alpha)(1-\alpha\theta)}{\alpha^2\theta(\alpha+1)^2} \right]$$  \hspace{1cm} (5.13)

The concept of excess velocity $\Delta U$ is introduced in the analysis just as in the theory of a free jet$^{17}$

$$\Delta U = U - U_H$$  \hspace{1cm} (5.14)

where $U$ is defined as the absolute rate of flow at the given point. Using the analogy between the flow in a free jet and in the mixing chamber of the ejector, the rate of flow at any point of an arbitrary cross section of the main region of the cylindrical chamber is expressed as$^{17}$

$$\frac{\Delta U}{\Delta U_m} = \frac{U - U_H}{U_m - U_H} = f(\varepsilon) = (1 - \varepsilon^{1.5})^2$$  \hspace{1cm} (5.15)

where $U_m$ is the axial velocity at a given cross section and $\varepsilon$ is a dimensionless length relative to the free jet, defined as $y/r$. The quantity $y$ is the radius to some point and $r$ is the radius of the free jet at the same cross section. $\varepsilon_k$ is another dimensionless length relating the mixing chamber of the ejector to the free jet, and is defined as $R/r$. Following Abramovich$^{17}$, the temperature profile is taken to be the square root of the velocity profile.
\[ \Delta T = \frac{T - T_H}{T_m - T_H} = \Psi(\varepsilon) = 1 - \varepsilon^{3/2} \]  
(5.16)

Writing the equation of continuity between an arbitrary cross section of the main region of the chamber and the terminal cross section 3, at which complete uniformity of the flow field is assumed, leads to the following

\[ \rho_3 U_3 A_3 = \int_A \rho UdA \]  
(5.17)

Assuming the density to be constant across the cross section and subtracting the quantity \( U_H A_3 \) from both sides of the equation yields

\[ (U_3 - U_H)A_3 = \int_A (U - U_H) dA \]  
(5.18)

or

\[ \pi R^2 \Delta U_3 = 2\pi \int_R \Delta U YdY \]  
(5.19)

Using the dimensionless quantities \( \Delta U \) /\( \Delta U_m \) and \( y/r \) transforms the above equation to

\[ \Delta U_3 = 2\Delta U_m \frac{1}{e_k} \int \xi (1 - \xi^{1.5})^2 \xi d\xi \]  
(5.20)
After integration, the Equation becomes

\[ \Delta U_3 = \Delta U_m \frac{1}{\varepsilon_k} \left( \varepsilon_k^2 - 1.143 \varepsilon_k^{1.5} + 0.4 \varepsilon_k^3 \right) \]  

(5.21)

Using the quantity \( A_1(\varepsilon_k) = 1 - 1.143 \varepsilon_k^{1.5} + 0.4 \varepsilon_k^3 \) yields

\[ \Delta U_m = \Delta U_3 / A_1(\varepsilon_k) \]  

(5.22)

This equation determines, in terms of the quantity \( \varepsilon_k \), the excess velocity on the axis of an arbitrary cross section of the mixing chamber. The velocity at an arbitrary point of a given cross section of the mixing chamber is determined from the equation for the velocity field (5.15)

\[ \Delta U = \Delta U_m (1 - \varepsilon^{1.5})^2 = \Delta U_3 \frac{(1-\varepsilon_k^{1.5})^2}{(1 - 1.143 \varepsilon_k^{1.5} + 0.4 \varepsilon_k^3)} \]  

(5.23)

where \( \tau = y/R = \varepsilon/\varepsilon_k \). This last equation, along with equation (5.23), determines both the velocity at the axis of an arbitrary cross section and the variation of velocity along the radius at each cross section.

5.9 Pressure profile analysis

Unlike the free mixing of a turbulent jet discharging into a coflowing stream, the turbulent mixing of streams taking place in a cylindrical ejector is accompanied by a variation of pressure along the length of the chamber. This pressure gradient was observed experimentally \(^17\) to be high at the entrance to the mixing chamber due to suction, then to decrease gradually in the middle section and to increase towards the end.
of the mixing chamber. It is determined at an arbitrary cross section of the chamber by using the momentum equation, (neglecting the friction on the walls of the chamber),

\[ A_3 U_3 + P_3 A_3 = \int_{A_3} U dA + PA_3 \quad (5.24) \]

or

\[ (P_3 - P) A_3 = \int_{A_3} U dA - A_3 U_3 \quad (5.25) \]

Using the quantity \( U_H A_3 = |U_H| dA \) in the above equation and assuming the density to be constant along the cross section considered yields

\[ \rho_{3-p}^{\frac{1}{p}} = \frac{2}{R^2} \int_R U \Delta U Y \Delta Y - U_3 \Delta U_3 \quad (5.26) \]

The first term on the right hand side of the equation is calculated as follows

\[ \frac{2}{R^2} \int_R U \Delta U Y dY = \frac{2}{R^2} \int_R \Delta U (\Delta U + U_H) Y dY \]

\[ = 2 \Delta U \frac{r^2}{m} \int_r^R \left( \frac{\Delta U}{\Delta U_m} \right)^2 Y dY + 2 U_H \Delta U \frac{r^2}{m} \int_r^R \frac{\Delta U}{\Delta U_m} \frac{Y dY}{R^2} \]

\[ = \Delta U_m A_2(\varepsilon_K) + U_H \Delta U_m A_1(\varepsilon_K) \quad (5.27) \]

where

\[ A_2(\varepsilon_K) = \frac{2}{\varepsilon_K^2} \int_{\varepsilon_K}^{\varepsilon_K^*} (1 - \varepsilon^{1.5})^4 \varepsilon d\varepsilon = 1 - 2.286\varepsilon_K^{1.5} + 2.4\varepsilon_K^3 - 1.23\varepsilon_K^{4.5} + 0.25\varepsilon_K^6 \quad (5.28) \]

Also, using the quantity

\[ \Delta U_3 = \Delta U_m A_1(\varepsilon_K) \quad (5.29) \]
yields
\[ \frac{2}{R^2} \int U \Delta U Y dY = \Delta U_3^2 \left( \frac{A_2(\epsilon_K)}{A_1(\epsilon_K)} + \frac{U_H}{\Delta U_3} \right) \] (5.30)

and
\[ \Delta U_3 U_3 = \Delta U_3^2 + U_H \Delta U_3 = \Delta U_3^2 \left( 1 + \frac{U_H}{\Delta U_3} \right) \] (5.31)

Sustituting these results in the momentum equation yields
\[ \frac{P_3 - P}{\rho} = \Delta U_3^2 (\tau(\epsilon_K) - 1) \] (5.32)

where
\[ \tau(\epsilon_K) = \frac{A_2(\epsilon_K)}{A_1(\epsilon_K)} \] (5.33)

\( \tau(\epsilon_K) \) is approximated to be
\[ \tau(\epsilon_K) = 1 + 0.007\epsilon_K + 0.95\epsilon_K^4 \] (5.34)

It is observed from equation (5.32) that a decrease in \( \tau(\epsilon_K) \) leads to an increase in pressure in the mixing chamber. When \( \tau(\epsilon_K) \) reaches the value of 1, the pressure tends to the value of the final mixing chamber pressure \( P_3 \) which is the largest pressure in the chamber without taking friction into account. Equations (5.23) and (5.32) determine the velocity and the pressure profiles at any point in the mixing chamber when the dimensionless quantity \( \epsilon_K \) is known. Once these profiles are known, the theory of a free jet is used one more time to determine the quantity \( \epsilon_K \) at the location considered. The appropriate derivations will be shown in appendix (A) of this report.
5.10. Temperature Profile Analysis

Considering the analogy established between the velocity fields of a free jet and the mixing chamber of an ejector at the same cross section, the temperature is taken to be the square root of the velocity field

\[
\frac{\Delta T}{\Delta T_m} = \frac{T - T_2}{T_m - T_2} = \sqrt{\frac{\Delta U}{\Delta U_m}} = 1 - \varepsilon^{1.5} \quad (5.35)
\]

\(\Delta T\) is the difference between the temperature at a given point in the jet and in the surrounding flow

\[\Delta T = T - T_H = T - T_2 \quad (5.36)\]

\(\Delta T_m\) is the difference between the temperature on the jet axis and the surrounding fluid

\[\Delta T_m = T_m - T_H = T_m - T_2 \quad (5.37)\]

The equation of conservation of mass is used to determine the variation of temperature \(\Delta T_m\) along the axis of the mixing chamber. This equation is developed between a given cross section in the mixing chamber and the final cross section, and for streams of different densities it is written as

\[2 \int^R \frac{\Delta U}{T} Y dY + 2U_H \int^R \frac{Y dY}{T} = U_3 \frac{R^2}{T_3} \quad (5.38)\]
Substituting the quantity $\Delta U$ by equation (13.16) yields the following

$$\frac{\Delta U_m}{U_3} = \left[ \frac{1 - e^{-1.5}}{e^{-1.5}} \right] + \frac{U_H}{U_3} \frac{\Delta H}{\theta_{i+1}} = \frac{1}{2} e^{2 \alpha \theta_{n+1}} \theta (\theta - 1) \tag{5.39}$$

After a long process of integration, the equation reduces to

$$\frac{4}{\alpha \theta_{n+1}} \left[ \frac{\beta^2 - 2}{4} x^4 + (\beta^3 - 1)^2 \left( x + \frac{\beta}{6} N \right) \right] + \frac{4m(a+1)}{\alpha \theta_{n+1}} \left( x + \frac{\beta}{6} N \right) = \frac{x^4}{1 - \beta^4} \tag{5.40}$$

where

$$x = \sqrt{\varepsilon_k} \quad \beta = \frac{3}{\sqrt{\varepsilon_k}} \quad m = \frac{U_H}{U_{2p}} \, .$$

and

$$N = \frac{(x - \beta)^2}{x^2 + \beta^4} - 2\sqrt{3} \arctan \frac{2x + \beta}{\sqrt{3}} \, .$$

Equation (5.40) is highly non linear and determines in implicit form the quantity $T_m/T_{25}$.

It is inconvenient to use even though it establishes, in principle, the relation between the temperature on the axis of the flow with the location of the cross section under consideration.

In this thesis a subroutine from the IMSL MATH Library was chosen and was used to solve for the roots of the equation for a given cross section and a specified value of the quantity $\varepsilon_k$. The subroutine did converge and returned seven roots to the equation for specified inlet conditions and ejector mixing chamber length. Five of these roots were discarded as being physically unrealistic, and the two remaining roots were both valid in describing the temperature field at the given cross section of the flow.
CHAPTER 6: Conclusions and Recommendations

6.1 Concluding Remarks

An analytical method is developed to predict the performance characteristics of axisymmetric single nozzle compressible flow ejectors, with constant area mixing tubes. The analysis is based upon the two phase jet turbulent model of Abramovich. The primary flow is assumed to be either subsonic or supersonic, while the secondary and mixed flow are supposed to remain subsonic throughout the mixing process. In considering the relations among surface pressure distributions, velocity profiles and the
flow field inside the ejector, it is convenient to differentiate between two regions, the entrance region and the main region. These two regions are interdependent and play a critical role in ejector thrust augmentation. The entrance region describes the region between the primary nozzle exit plane and the point where the jet reaches the wall. Furthermore, the flow field analysis in this region is based upon the theory established for the initial region of a turbulent jet spreading into an external stream of fluid, and in which the potential core velocity of the jet remains constant.

The main or interaction region describes the region of the flow downstream of the point where the jet reaches the wall. The velocity profile in this region is allowed to vary similarly to the free jet profile. Integral techniques are then used in both regions to determine the various flow profiles along the mixing tube.

The analytical predictions of static pressure variations, velocity profiles and temperature profiles, for specified inlet primary and secondary flow conditions, agreed well with the theory behind the ejector mixing process and are included in the appendix. Some common features of the ejector flow fields investigated show a decay in centerline velocity, as it is shown in figure 6.1, indicated by a reduced growth rate of the jet in presence of the shroud. The presence of a coflowing induced flow and the imposed pressure gradient are the principal reasons for the reduced jet growth rate. The pressure within the ejector increases downstream. And although the expected effects of such an adverse pressure gradient is to increase the jet growth rate, experimental studies have indicated that the presence of a coflowing stream dominates over the pressure gradient in the evolution of the jet.
Figure 6.1 Decay in Centerline Velocity due to the Presence of a Coflowing Stream
The temperature profile is assumed to be similar to that of a free jet. Consequently, an approximate form of the mass equation is used in the analysis to determine the variations of temperature inside the ejector. However, a more accurate temperature profile coupled to the mass equation could lead to a better temperature profile prediction.

The results of the investigation also suggest that the mixing chamber length must be carefully selected to increase pressure recovery. The latter results from more complete mixing in a longer mixing tube against the increased wall friction losses. No simple length-to-diameter ratio relation is found to be applicable to mixing chamber design. The optimum mixing tube length is found to be dependent on various parameters such as primary flow conditions and entrainment ratios at the operating point. The results also show that once the optimum mixing length is selected, no further increase in thrust augmentation can be expected by increasing the length of the mixing section beyond the optimal value.

6.2. Recommendations

An important area of future improvement in ejector studies is the search for a relationship that incorporates the ideal chamber mixing length, the entrainment ratio, and the inlet flow conditions. Once a specified level of entrainment has been achieved by the primary nozzle and inlet section, the mixing length required to maintain the flow is set.
REFERENCES


APPENDIX (A)

Determination of the quantity $\varepsilon_K$

The determination of the quantity $\varepsilon_K$ is essential for calculating the velocity, pressure and temperature profiles at any point of the ejector mixing chamber. The relations previously derived for a free jet, (see refs), will be used to relate $\varepsilon_K$ to the cross section or the length of the mixing chamber. The quantity $r$ is defined as the radius of the free jet corresponding to the same initial ejector flow parameters, at the given cross section. $\varepsilon_K$ is determined by first calculating the radius of the transition cross section of the jet, $r_n$, as well as $r_c$, the radius of the cross section at which the excess velocity on the axis equals one half the initial excess velocity of the ejecting flow (primary) $\Delta u_1$.

\[
r_n^2 = r_1^2 = \frac{1}{A_2 + m(A_1 - A_2)} \quad \text{(A.1)}
\]

and

\[
r_c^2 = r_1^2 = \frac{4}{A_1 + m(2A_1 - A_2)} \quad \text{(A.2)}
\]

where

\[
A_1 = 2 \int_1^2 (1 - \varepsilon^{1.5})^2 d\varepsilon = 0.258 \quad \text{(for a free jet)}
\]
\[ A_2 = 2 \int^1 (1 - e^{-1.5})^4 e \, de = 0.134 \quad \text{(for a free jet)} \]

and

\[ m = \frac{U_w}{U_{2f}}. \]

The next quantity that needs to be determined is the distance between the beginning of the main region and the mean cross section \( x_n \) by using the derived formulas for a free jet.

\[ x_c - x_n = \frac{1}{r} (x_c - x_n) = \pm \frac{r_{c-r_n}}{\frac{a}{a} \ln \frac{2\pi}{2\pi}} \quad \text{(A.3)} \]

where

\[ a = \frac{4m}{1-m} \]

and

\[ c = 0.18 \text{to} 0.21 \quad \text{(from experimental data)} \]

The distance from the primary nozzle up to the beginning of the main region is approximated by the empirical expression derived for a turbulent free jet (see refs):

\[ x_c = 1.5x_H = \pm 1.5 \frac{1+m}{c_H(1-m)\sqrt{0.214+0.144m}} \quad \text{(A.4)} \]

where

\[ c_H = 0.23 \text{to} 0.25 \quad \text{(from empirical data)} \]

The quantities obtained above are characteristics of a free jet and are independent of the ejector's parameters. They are dependent only on the magnitude of \( m \). The next step is to evaluate \( x^* \), the dimensionless distance from the transition cross section of the jet. The
quantity $x^*$ is determined for a given cross section at a distance $x$ from the primary nozzle by

$$x^* = \frac{x_1 - x_n}{x_1 - x_n}$$

(A.5)

The quantity $x/r_1$ is related to the length of the mixing chamber $l$ by

$$\frac{x}{r_1} = 2l \frac{r}{r_1} = 2l \sqrt{\frac{d+1}{d}}$$

(A.6)

where

$$l = \frac{l}{d}$$

$l$ is the length of the mixing chamber, and $d$ is the mixing chamber diameter. In addition, the quantity $(r^*-r_n^*)$ which is defined as the radius of the free jet at the cross section considered in the analysis, is expressed by

$$r^* - r_n^* = \frac{r-r_n}{x-x_n} = \pm \frac{2e}{d} \ln \left[ \frac{2+e(x^*+1)}{2+e} \right]$$

(A.7)

Finally, the radius $r$ of the free jet at the given cross section is determined by

$$\frac{r}{r_1} = (r^* - r_n^*)(x_c - x_n) + r_n$$

(A.8)

Once the radius of the free jet is determined through the formulas derived in this section, the quantity $\varepsilon_k$ can be evaluated. This in turn enables the calculation of the velocity, pressure and temperature profiles, at a given cross section of the mixing chamber, as long
as that cross section is far enough away from the nozzle and does not fall in the initial region. Therefore, it should be kept in mind that the formulas derived for calculating the various profiles in the ejector mixing chamber are valid only for the main region in which a jet profile exists over the entire cross section of the chamber, and is described by

$$\frac{\Delta U}{\Delta U_m} = \left(1 - \varepsilon_{K}^{1.5}\right)^2$$  \hspace{1cm} (A.9)
APPENDIX (B)

The following program was developed for the compressible, one-dimensional, modified control volume approach. It was used to provide thrust augmentation levels for several inlet area ratios and primary and secondary inlet stagnation parameters.

```
* purpose : to solve the flow parameters at the end of mixing using the *
  control volume approach in a constant area ejector *
  *
  programmer : MOEAMED MOUJAHID *
  *
  variable key : P static pressure *
  T temperature *
  M MACH number *
  APT primary to total area ratio *
  RHO density *
  PP0 primary total pressure *
  PA secondary total pressure *
  TP0 primary total temperature *
  TS0 secondary stagnation temperature *
  ASP secondary to primary area ratio *
  C speed of sound *
  U speed of flow *
  MSUB subsonic flow MACH number at the end of mixing *
  MSUB supersonic flow MACH number at the end of mixing *
  M3ID flow MACH number for primary nozzle *
  M3SUB flow MACH number at exit to the diffuser for subsonic solution *
  M3SUP flow MACH number at exit to the diffuser for supersonic solution *
  TAR thrust augmentation ratio *
  subscriptP: primary flow conditions at ejector inlet *
  subscriptS: secondary flow conditions at ejector inlet *
  subscriptSUB: subsonic solution to the mixed flow *
  subscriptSUP: supersonic solution to the mixed flow *
  subscript3: flow conditions at exit to diffuser *
```
PROGRAM EJECTOR

C declaration of variables

DOUBLE PRECISION PA, PS, PP0, TS, TS0, TP, TP0, MS, MP, APT,
& FRS, TRP, TRS, PRP, R, K, CP, CS, US, UP, N, N1,
& YSUB, WSUB, YSUP, WSUP, M3ID, M3SUB, M3SUP,
& DSSUB, DSSUP, ZSUB, ZSUP, ASP, PSUB, PSUP, N2,
& REOS, RHOB, MFR, J, A, B, DET, MSUB, MSUP, N4,
& T3SUB, TSUB, XSUB, TSUP, XSUP, TARSUB, TARSUP,
& T3ID, M3ID, T3SUB, M3SUB, T3SUP, M3SUP, N3

PARAMETER (K=1.4, R=287)

C enter stagnation conditions for primary and secondary flows

PRINT*,'ENTER VALUES OF PP0, PA, TP0, TS0, ASP'
READ*, PP0, PA, TP0, TS0, ASP
PRINT 120

120 FORMAT(' PS MS MP MSUB MSUP DSSUB DSSUP MFR TARSUB TARSUP')

.C compute ideal flow mach number for primary nozzle
M3ID = ((PP0**((K-1)/K)-1)**2/(K-1))**0.5
T3ID = TP0/(1+((K-1)*M3ID**2)/2)

C vary stagnation inlet pressure form 0.1 atm to 1 atm
DO 100 PS = 0.2, 0.99, 0.02.

C calculate secondary flow mach number
MS = (((PA/PS)**((K-1)/K)-1)*2/(K-1))**0.5

C calculate primary flow mach number
MP = (((PP0/PS)**((K-1)/K)-1)*2/(K-1))**0.5

C calculate primary and secondary flow conditions
TS = TS0/(1+((K-1)/2)*MS**2)
TP = TP0/(1+((K-1)/2)*MP**2)
APT=(1/MP**2)*((2/(K+1))*(1+((K-1)/2)*MP**2))**((K+1)/(K-1))
CS = (K*R*TS)**0.5
CP = (K*R*TP)**0.5
US = MS*CS
UP = MP*CP
RHOS = (PS/(R*TS))*101300
RHOP = (PS/(R*TP))*101300

C determine mass flow rate ratio
MFR = ASP*(MS/MF)*((TP/TS)**0.5)

C calculate pressure and temperature ratios
PRP = PS/PP0
PRAS = PS/PA
TRP = TP/TP0
TRAS = TS/TSS

N = X/(X-1)
N1 = (k-1)/k
N2 = (X-1)/2

J = ((TRP**0.5)*(((ASP+1)/(X*MF))+MF)+MFR*MS*((TS/TP0)**0.5))/((1+MFR*(TS0/TP0))*(1+MFR)**0.5

A = 1-((J**2)*(X-1)/2
B = 2-X*(J**2)
DET = (B**2)-(4*A)

IF ( DET .LT. 0 ) THEN
   PRINT*, 'IMAGINARY SOLUTION
ELSE

C compute solutions to the flow at the end of mixing
MSUB = ((-B-(DET**0.5))/(2*X*A))**0.5
MSUP = ((-B+(DET**0.5))/(2*X*A))**0.5

C calculate flow conditions at end of mixing on the subsonic branch
TSUB0 = (TP0+MFR*TS0)/(1+MFR)
XSUB = (1+(N2*(MF**2)))/(1+(N2*(MSUB**2)))
ZSUB = (1+MFR)*MF*(((TSUB0*XSUB/TP0)**0.5)/((ASP+1)*MSUB)
TSUB = TS*XSUB*TSUB0/TS0
PSUB = PS*TSUB
PSUB0 = ((1+N2*(MSUB**2))**N)*PSUB
YSUB = TSUB/TP
WSUB = TSUB/TS
Y1 = YSUB
W1 = WSUB
Z1 = ZSUB
DSSUB = N*LOG(Y1)+N*MFR*LOG(W1)-(1+MFR)*LOG(Z1)

C calculate diffuser exit flow conditions for subsonic mixed flow
N3 = PSUB0**N1
N4 = 2*((PSUB0**N1)-1)
M3SUB = (2*(((PSUB0**N1)-1)/(X-1)))**0.5
T3SUB = TSUB0/(1+((X-1)*M3SUB**2)/2)

C evaluate thrust augmentation ratio for subsonic mixed flow
TARSUB = (1+MFR)*M3SUB*(T3SUB**0.5)/(M3ID*(T3ID**0.5))

C calculate flow conditions for supersonic mixed flow at end of mixing
XSUP = (1+N2*(M2**2))/((1+N2*(MSUP**2))
ZSUP = (1+MFR)*M2*(((PSUB0*XSUP/TS0)**0.5)/((ASP+1)*MSUP)
PSUP = PS*ZSUP
PSUP0 = (1+(X-1)*MSUP**2)/2*PSUP
TSUP = TS*XSUP*(TSUB0/TS0)
YSUP = TSUP/TP
WSUP = TSUP/TS
DSSUP = N*LOG(YSUP)+N*MFR*LOG(KSUP)-(1+MFR)*LOG(ZSUP)

C calculate flow conditions for supersonic mixed flow at exit to the diffuser
M3SUP = (2*(((PSUP0**((1/N)-1)))/(X-1)))**0.5
T3SUP = TSUB0/(1+((X-1)*M3SUP**2)/2)

C evaluate the thrust augmentation ratio for the supersonic solution
TARSUP = (1+MFR)*M3SUP*(T3SUB**0.5)/(M3ID*(T3ID**0.5))

ENDIF

C output results corresponding to subsonic and supersonic solution
PRINT 150, PS, MS, MFR, MSUB, MSUP, DSSUB, DSSUP, MFR, TARSUB, TARSUP
150 FORMAT(//,F5.3,2X,F6.4,2X,F6.4,2X,F7.4,2X,F6.4,1X,
& F8.4,1X,F8.4,1X,F7.4,2X,F6.4,2X,F6.4)
100 CONTINUE
END
APPENDIX (C)

The following program was developed to calculate the velocity and pressure profiles for the turbulent in the mixing chamber of the ejector. It used the equations derived for the 2-D phase turbulent jet model of Abramovich.

PROGRAM TUR3L

*****************************************************************************
* This program calculates the velocity profiles and pressure profiles using *
* the equations derived for the turbulent flow in the mixing chamber of the ejector based on the similarity found by ABRAMOVICH between the velocity profiles of a free jet and the velocity profile in the mixing chamber of the ejector. *
*****************************************************************************

C declaration of variables

DOUBLE PRECISION X,R,C,CH,P0,P0S,T0,T0S,ASP,
& ARATIO ,MS,MP,FS,TS,TP,CS,CP,US,UP,RHOS,REOP
& ,XFR,DENRT,RAU33,P3,DEP3,US,URATIO
& ,RN,A1,A2,RC,A,DELTX,XN,XR1,LSTR,XSTR
& ,RSTR,RAD,R1,EX,A1EX,UM,TEX,PDENRT

PARAMETER (X=1.4,R=287,C=0.18,CE=0.23,A1=0.258,A2=0.134)

C Input of stagnation conditions and area ratios

PRINT*,'ENTER VALUE OF PRIMARY NOZZLE RADIUS R1'
READ*,R1
PRINT*,'ENTER VALUES OF P0,P0S,T0,T0S'
READ*,P0,P0S,T0,T0S
PRINT*,'ENTER AREA RATIO ASP=AS/AP '
READ*,ASP

C Inlet stagnation pressure is varied from 0.1atm to 0.9atm

DO 100 PS=0.1,0.9,0.1
PRINT*," "
PRINT*,"PS= ',PS
PRINT*," "
C Calculation of stagnation conditions for primary flow and secondary flow at inlet to the ejector

C*******************************************************
C Calculation of secondary flow mach number
MS=((FOS/PS)**((X-1)/(X-1))-2/(X-1))*0.5
PRINT*, 'MS= ', MS
C Calculation of primary flow mach number
MP=((FOP/PS)**((X-1)/(X-1))-2/(X-1))*0.5
PRINT*, 'MP= ', MP
C Calculation of secondary inlet static temperature
TS=TOS/(1+((X-1)/2)*MS**2)
PRINT*, 'TS= ', TS
C Calculation of inlet static primary temperature
TP=TOP/(1+((X-1)/2)*MP**2)
PRINT*, 'TP= ', TP
C Calculation of secondary and primary speed of sound
CS=(X*R*TS)**0.5
CP=(X*R*TP)**0.5
C Calculation of secondary and primary velocities
US=MS*CS
PRINT*, 'US= ', US
UP=MP*CP
PRINT*, 'UP= ', UP
C Calculation of secondary and primary densities
REOS=(FOS/(R*TS))*101300
REOP=(FOP/(R*TP))*101300
C Calculation of the mass flow ratio
MFR=(MS/MP)*((TP/TS)**0.5)
PRINT*, 'MFR= ', MFR
DENSRT=REOP/REOS
ARATIO=1/ARATI0
RATU3P=ARATIO*(1+MFR*DENSRT)/(ARATIO+1)
PRINT*, 'RATU3P= ', RATU3P
C U3 is the mixed flow velocity after complete mixing
U3=RATU3P*UP
PRINT*, 'U3= ', U3
C density ratio between primary flow and completely mixed flow at station 3
DERTP3=(1+MFR*DENSRT)/(MFR+1)
C******************************************************************************
C This part of the program calculates EX
C******************************************************************************
C UH is the nominal velocity of wake flow
C
UH=(US**2*(1-2*(1-ARATIO*MFR)*(1-ARATIO*MFR*DENSRT))}
/(ARATIO*MFR**2*DENSRT*(ARATIO+1)**2))**0.5
`URATIO=UE/UP

The radius of the transition cross section of the jet

RN=(1/(A2+URATIO*(A1-A2)))**0.5

The radius of the cross section at which excess velocity is one half the initial excess velocity

RC=(4/(A2+URATIO*(2*A1-A2)))**0.5

Distance between begining of main region and mean cross section

A=4*URATIO/(1-URATIO)

DELTX=(RC-RN)/(2*C*LOG((2+2*A)/(2+A)))

Distance from nozzle up to begining of main region

XN=ABS((1.5*(1+URATIO))/(CR*(1-URATIO)*(0.1214+0.144*URATIO)**0.5))

Dimensionless distance from transition cross section of jet

LSTR=L/D, L=length of chamber, D=diameter of chamber

DO 200 LSTR=1,10,1

PRINT*,`LSTR=`,LSTR

XR1=2*LSTR*(((ARATIO-1)/ARATIO)**0.5)

XSTR=(XR1-XN)/DELTX

DRSTR=ABS((2*C/A)*LOG((2+A*(XSTR+1))/(2+A)))

Radius of free jet at cross section

RAD=(DRSTR+DELTX+RN)*R1

PRINT*,`RAD=`,RAD

EX=(R1/RAD)*(((ARATIO+1)/ARATIO)**0.5)

PRINT*,`EX=`,EX

AEX=1-1.143*EX**1.5+0.4*EX**3

UM is the center velocity at the cross section corresponding to the value of EX considered

UM=((U3-U)/AEX)+UB

PRINT*,`UM=`,UM

TEX=1+0.007*EX+0.95*EX**4

PDENRT is the pressure to density ratio at the cross section considered corresponding to the value of EX

PDENRT=(TEX-1)*(U3-U)**2

PRINT*,`PDENRT=`,PDENRT

200 CONTINUE

END`
PROGRAM TEMP

REAL EX, N, M, O, Y, I, A, B, AEX
INTEGER ITMAX, NROOT
REAL EPS, ERRABS, ERRREL, ETA, ZREAL, WRRRN
PARAMETER (NROOT=7)
INTEGER INFO(NROOT)
REAL F, X(NROOT), XGUESS(NROOT)
EXTERNAL F, ZREAL, WRRRN

C EXTERNAL F
PRINT*, 'ENTER VALUES OF EX, N, M, O, I '
READ*, EX, N, M, O, I
AEX = 1-1.143*(EX**1.5)+0.4*(EX**3)
PRINT*, 'AEX= ', AEX
Y = EX**0.5
A = 4*(I*(N*O+1)-M*(I+1))/(AEX*I*O*(N+1))
PRINT*, 'A= ', A
B = 4*M*(I+1)/(I*O*(N+1))
PRINT*, 'B= ', B
C DECLARE VARIABLES
C Set values of initial guess
C XGUESS = (1.2, 1.3, 1.4, 1.5, 1.6, 2)
C DATA XGUESS/1.0, 1.1, 1.2, 1.4, 1.8, 2.0, 2.4/
C
EPS = 1.0E-5
ERRABS = 1.0E-5
ERRREL = 1.0E-5
ETA = 1.0E-2
ITMAX = 100
C Find the zeroes
CALL ZREAL (F, ERRABS, ERRREL, EPS, ETA, NROOT, ITMAX, XGUESS, & X, INFO)
CALL WRRRN ('The zeroes are', 1, NROOT, X, 1, 0)
C WRITE (NOUT,200) (X(X), X=1,NROOT)
C200 FORMAT (' The solution to the system is ',X,' = (',7F8.4,')')
PRINT*, 'F(1.08)= ', F(1.08)

END

REAL FUNCTION F(X)

REAL X, Z

Z = LOG(((Y-X)**2)/((Y**2)+X*Y+(X**2)))
& -2*(3**0.5)*ATAN(((2*Y)+X)/(X*(3**0.5)))

F = A*(((Y**7)/7)+(((X**3)-2)*(Y**4)/4)+(((X**3)-1)**2)
& *(Y+X*Z/6)) + 3*(Y+X*Z/6) - ((Y**4)/(1-(X**3)))

RETURN

END