Numerical and analytical studies of critical radius in Cartesian and spherical geometries for corona discharge in air and CO₂-rich environments

Jacob A. Engle, Jeremy A. Riousset
Department of Physical Sciences, Center for Space and Atmospheric Research (CSAR), Embry-Riddle Aeronautical University, Daytona Beach, FL
(englej2@my.erau.edu)

Abstract
In order to determine the most effective geometry of a lightning out, one must first understand the mathematical relations between the current densities, i.e., numerical solutions of the governing differential equations. This work, the first study of its kind, focuses on the electron avalanche that is a continuous stream of fast-moving electrons that can be used to control the path of a stream of fast-moving protons. The electron avalanche is the primary cause of the current density, and the current density, in turn, is the primary cause of the electric field. The electric field is the primary cause of the corona discharge, and the corona discharge is the primary cause of the lightning out.

I. Introduction

Corona Discharge
Electrical discharge around a conductor due to electric field:
- Weakly ionized gas responsible for glow at visible wavelengths;
- Hypothesized to promote the formation of upward connecting leaders in lightning discharge.

II. Model Formulation

A. Electric field of a point charge in the Cartesian geometry:
\[ E = \frac{Q}{4\pi\varepsilon_0 r^2} \]

B. Electric field of a point charge in the spherical geometry:
\[ E = \frac{Q}{4\pi\varepsilon_0 r^2} \]

C. Electric field of a cylindrical charge in the Cartesian geometry:
\[ E = \frac{Q}{2\pi\varepsilon_0 r z} \]

D. Electric field of a cylindrical charge in the spherical geometry:
\[ E = \frac{Q}{2\pi\varepsilon_0 r z} \]

III. Results and Discussion

A. Analytical solutions for electric field in Earth and Mars as a function of r in spherical geometry:
\[ E_r = \frac{Q}{4\pi\varepsilon_0 r^2} \]
\[ E_\theta = \frac{Q}{4\pi\varepsilon_0 r^2} \]
\[ E_\phi = \frac{Q}{4\pi\varepsilon_0 r^2} \]

B. Analytical solutions for electric field in Earth and Mars as a function of r in Cartesian geometry:
\[ E_x = \frac{Q}{4\pi\varepsilon_0 r^2} \]
\[ E_y = \frac{Q}{4\pi\varepsilon_0 r^2} \]
\[ E_z = \frac{Q}{4\pi\varepsilon_0 r^2} \]

IV. Conclusions
The results and conclusions obtained in this work can be summarized as follows:
- A new model for calculating the critical radius and minimum voltage for corona discharge in Earth and Martian geometries is presented.
- The model is validated using classic Paschen theory and experimental data on air ionization and corona discharge.
- The Paschen curve for Martian atmosphere is shown to be excellent.
- Our numerical model and the analytical solution show excellent agreement.

REFERENCES

[Insert references here]