Numerical and analytical studies of critical radius in Cartesian and spherical geometries for corona discharge in air and \( \text{CO}_2 \)-rich environments

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Abstract
In order to determine the most effective geometry of a lightning out, one must first understand the physical differences between these current trendy, Hamiltonian theoretical theory of start factor. This suggests an increase of local field effect, while Kersten’s (2005) study on earth’s layer has increased probability of Earth’s crust (2005; Gigues et al., 2006). In this analysis, the plasma discharge is produced between two electrodes with a high potential difference, resulting in a variety of the event of the phenomenon. This process, when done at low cost and low temperature, could create a corona discharge, which can be distinguished with a simple and effective model. The corona geometry known as a Taylorson, in Townsend’s theory is particularly suited to model experimental laboratory corona, because it is located in a uniform electric field. In contrast, it is created by a uniform electric field, and it is uniform to a potential difference as well. The analytical model, by first approximations, the rod can be placed as an exponential atomic cation above the ground, whereas, we report the corona-model geometry into spherical and spherical geometries. In this work we explore the effects of different of the classical parallel plate plasma to spherical and spherical geometries more adapted for studies of lightning such as power lines. Utilizing Taylorson’s equation for corona discharge, we estimate a critical radius and minimum breakdown voltage that just become independent of the far and well an exponential function of corona radius in spherical geometry. Moreover, the radius is a spherical corona for the Boltzmann equation, to calculate spherical coefficients for minimum corona critical radius (Hagelberg and Pitchford, 2005). This allows us to expand the scope of this study to other planetary bodies such as Mars. We solve the problem both numerically and with analytical results for simplified spherical form in each geometry and gas conditions. The development of a numerical framework will ultimately let the influence of additional parameters such as background gas, collisional rates, and charge of ions in the minimum breakdown voltage.

I. Introduction

Corona Discharge
Electrical discharge around a conductor due to electric field.
Weakly ionized gas responsible for glow at visible wavelengths.
Hypothetically to promote the formation of upward connecting leaders in lightning discharge.

Electron Avalanche
The process of electron avalanche is similar between various types of discharges:
• Initial step of a discharge;
• Release of secondary electrons in electron-neutral collision;
• Secondary collisions with enough energy to repeat the process;

Avalanche Criteria:
\[ v = (2 \cdot 10^5 \times \frac{r}{\rho}) \text{ cm/s} \]

Types of Discharges

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Energy</th>
<th>Field</th>
</tr>
</thead>
<tbody>
<tr>
<td>~300 K</td>
<td>1.2 eV</td>
<td>0.27 V/cm</td>
</tr>
<tr>
<td>~30 K</td>
<td>5-15 eV</td>
<td>5.75 V/cm</td>
</tr>
<tr>
<td>≥500 K</td>
<td>1.2 eV</td>
<td>1.95 V/cm</td>
</tr>
</tbody>
</table>

Electrode energy, \( E \) is calculated from the equation:
\[ E = \frac{1}{2} \cdot \frac{q}{m} \cdot v^2 \]

Table 2: Parameters and calculated electrostatic energy

Application to Martian Studies

Earth Analogies:
Potential hazard due to arcing on • Triblending in Martian dust storms akin to Earth sandstorms; Charge separation due to sedimentation & gravitation; Possible electrical shortage & failure

Motivations:
• Martian dust storms may impact the potential for lightning

Martian Data:
Conductivity: 10^{-12} S/m
Vapor Pressure: 10^{-7} atm

Table 3: Martian conditions

II. Model Formulation

Objective: Determine analytical solutions for critical radius and Stoeltov’s point.

Surface Electric Field:
\[ E = \frac{\rho}{r^2} \]

Stoeltov’s Point:
\[ V = \frac{E}{r} \]

Table 1: Stoeltov’s point parameters

Pascchen curves

\[ \frac{E}{V} = \frac{1}{2} \cdot \frac{1}{r} \]

Table 4: Stoeltov’s point parameters

IV. CONCLUSIONS

The results and conclusions obtained in this work can be summarized as follows:
• A new model for calculating the critical radius and minimum breakdown voltage for Coro discharge in Cartesian and spherical geometries is presented.
• The model is validated using classic Pascchen theory and experimental data on air and CO2 (2017).

The expected classic Paschen theory into an analytical solution for spherical and atmospheric discharge known as Stoeltov’s point.

REFERENCES

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