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# An Optimal Airline Revenue Management Seat Pricing Plan Model

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Airline seat pricing is divided into different fare classes. The goal of an airline is to sell tickets at the highest fare possible, thus yielding a maximum profit for the airline. Managing optimal ticket sales is a science, with substantial potential payoffs, although it is time-consuming and expensive as the revenue management system is developed (Groves & Gini, 2015). Once developed, the revenue management system is updated on a frequent basis to account for existing reservations as well as expected future reservations, as part of a continuous improvement optimization effort. A revenue management system identifies opportunity costs where the airline may sell available full fare seats to high spending customers, and available discounted fare seats to customers with low-spending habits. As an example, a customer may request a discounted fare for a specific fare flight. This request is contingent, given seat number limitations, and the possibility of the airline selling the same seat at the full fare rate (Collins & Thomas, 2013).

The problem becomes evident as airlines must decide whether or not to accept a reservation offer at either a discounted fare or delay the acceptance of a reservation until later when the seat will change from discounted to full fare and sell the ticket at a much higher price. If the airline manages to sell the seat at the full fare price, they generate extra revenue, but if for some reason the seat does not sell before departure, the airline generates no revenue and loses on the opportunity to at least have been able to generate some revenue at the discounted fare class. Hence the need for airlines to be able to accurately forecast demand for the high paying customers who will purchase the seats at the full fare price, thus realizing potentially substantially higher payoffs (Szopinski & Nowacki, 2015).

## Review of Literature

Revenue management (RM) began within the airline industry in the 1970s using manual acceptance or rejection of booking requests. This mode of operation continued until computerized reservation systems automated the booking process. Littlewood (1972) describes the early work he performed in applying mathematical models to the development of revenue management in the airline industry. The author stresses that non-constrained demands for fare classes are independent, that the initial share of the total market non-constrained demand by the airline is the same for all fare classes and finally, that lower fare class booking limits are reached (Littelwood, 1972). The Flight Transaction History File (FTHF) used in this research recorded several variables and attributed data for a flight, such as:

a) The passenger name records (PNRs) logging the number of passengers booked

b) The number of days the reservation was made prior to the scheduled flight departure
c) The time at which the transaction was made in minutes past midnight
d) Several codes; 10, 20, 40, 12, and 70 indicating Booking, Cancellation, No show, Stand-by, and Departed load, respectively
e) Number of tickets sold per Code
f) Boarding point (departure airport)
g) Off point (arrival airport)
h) Class of fare
i) The PNR number indicating the number of bookings per number of days before departure
j) Ticket type, indicating whether ticker was a youth or reduced rate

Even though several statistical interpretations could be extracted from the FTHF such as demand forecasting, several factors prevented the accurate interpretation of data. First, the FTHF data were too large to store in any available system at the time. Second, passengers' booking lead times varied considerably from year to year and could be misleading. Thirdly, the flight numbers and flight times were often changed from year to year with no apparent flight to use for comparison (Littlewood, 2005). The author saw the need to develop a mathematical model to forecast demand by day and by sector given prior knowledge of forward bookings. The model that was derived provided an estimate $\widehat{D}$ of the demand for the sector if $B$ passengers are booked on a given sector at a certain time before departure.

$$\widehat{D} = B(1 - \widehat{c}) + \hat{S} \qquad \text{[Eq. 1]}$$

Where $\hat{c}$ and $\hat{S}$ are estimates of the passengers' cancellation rate and the number of subsequent arriving passengers respectively (who book in the period between the time considered and departure, and fly). Furthermore, Littlewood (2005) used additional information derived from the FTHF as well as calculated information to modify his forecasting model for a particular day. The information gathered with day of the week and the seasonal indices normalized, included:

a) Mean cancelation rate
b) Smoothed error of the cancelation rates
c) Smoothed absolute error of the cancelation rates
d) Seven weekly indices
e) Smoothed error of each of the day of week indices
f) Mean subsequent passengers
g) Trend in subsequent passengers

h) Smoothed error or subsequent passengers
i) Smoothed absolute error of subsequent passengers
j) Smoothed absolute error of subsequent passengers
k) 2-period indices showing the seasonal variation in subsequent passengers

As a result, the modified estimated daily demand model is indicated as

$$\widehat{D} = B(1 - \widehat{c}) + \hat{S}\hat{d}_i \qquad \text{[Eq. 2]}$$

where $\widehat{D}$ is the demand for a sector in a particular day $i$, $\hat{c}$ is the estimated cancelation rate (calculated by interpolation if necessary), $\hat{d}_i$ is the estimated day of the week index for day $i$, and $\hat{S}$ is an estimate of subsequent passengers (also calculated by interpolation if necessary), based on the estimated trend and seasonal indices (Littlewood, 2005). The author goes on to further clarify that one limitation as a result of these calculations lie in the extrapolation of data where a future manual intervention in response to necessary changes may render extrapolated data as inaccurate. As such, human input was still required for a decision making process that entailed accepting or rejecting a passenger reservation according to the fare price at booking time, in order to minimize the possibility of overbooking a flight. Littlewood (2005) discussed the probability of turning away a high yield customer (overbooking) as well as the probability of turning away a low yield customer. For example, he discusses a useful method of controlling fares with a long booking lead time if the sole objective is to maximize revenue by flight, and the mean revenue obtained from a high-yield passenger is R and from a low-yield customer is r, and P is the maximum probability losing a high-yield passenger, then low-yield passengers should continue to be accepted as long as

$$r \geq (1 - P)R \qquad \text{[Eq. 3]}$$

or

$$(1 - P) \leq \frac{r}{P} \qquad \text{[Eq. 4]}$$

Interpreting equations 2-4, to maximize revenue, low-yield passengers should continue to be accepted until $(1 - P)$ reaches the value of the ratio of the mean revenues from low-yield and high-yield passengers. It is therefore understood that if the acceptance of low-yield passengers is stopped sooner, a higher standard of service will be offered to high-yield passengers. Similarly, if the acceptance of low-yield passengers is stopped later, a lower standard of service will be offered to high-yield passengers.

A demand constraint was noted in a study by Khoo & Teoh (2014), to ensure that travelers' demand could be met satisfactorily. The demand constraint could be expressed as

$$\sum_{i=1}^{n}(SEAT_{n,OD}^{t})\left(f_{n,OD}\left(D_t^S, A_t^i\right)\right) \geq (1-\propto)D_t^S \; for \; t = 1,2,\ldots,T, \; S = s_1, s_2, \ldots s_k$$

[Eq. 5]

for a particular seat in an origin-destination flight of a specific period. In this case, a particular seat $1-\propto$ is the confidence level or service level, to meet stochastic demand. A dynamic programming model was adopted to solve as simpler, smaller, sub-problems for each operating period, and determine the optimal solution. When stochastic demand is considered, the probabilistic component captures demand uncertainty, providing a more accurate solution (Khoo and Teoh, 2014).

In a study by Jorge-Calderon (1997) the demand model for scheduled airline services for the entire network of European international routes in 1989 concluded that, overall, demand is price inelastic concerning the unrestricted economy fare. The study indicates that in short distance routes, airlines have made their highly discounted fares more widely available, probably to counter competition by other modes of transportation. As distance increases, discounted fares are used less, probably due to a lesser availability, which results in a higher proportion of price-sensitive traffic paying the unrestricted economy fare, thus making demand more elastic (Jorge-Calderon, 1997).

Airline seat allocation is contingent upon the demand for a particular fare. The demands for a fare class are allocated as the lowest fare class arrive first, and seats are booked for this class until a fixed time limit, or the demand is exhausted. Sales to this fare class are then closed, and sales to the class with the next lowest fare begin and this process repeats until the fares sell out. One other notation is that some fare classes may not open at all, depending on the airplane capacity, fares, and demand distributions. Further complications are introduced by factors such as multiple-flight passenger itineraries, interactions with other flights, cancellation and overbooking considerations, and the dynamic nature of the booking process in the lead-time before flight departure. At any time during the booking process, the observed demands in the fare class currently being booked and in lower classes, convey no information about future demands for higher fare classes. This excludes the possibility of basing a decision to close a fare class on such factors as the time remaining before the flight.

One of the first optimization methods to calculate booking limits was the expected marginal seat revenue heuristic approach of Belobaba (1987) which was an extension Littlewood's (2005) rule. A seller wishes to sell various goods by a deadline, for example, the end of a season. Further potential buyers enter over time and can strategically time their purchases. Within each period, the profit-maximizing mechanism awards units to the buyers with the highest valuations exceeding a sequence of cutoffs (Board, & Skrzypacz, 2016). Similarly, airlines wish to sell out the remaining available seats at the highest possible price. The optimal allocation of seat inventory is usually carried out among fare classes with a known projected demand forecasted distribution for each class with the aim at increasing the efficiency of revenue systems and enhancing customer satisfaction (Vardi et al., 2016, 20-37). Aside from demand forecasts, the optimization model also required fare inputs at leg and booking-class level (Poelt, 2016).

## Assumptions and Limitations

Our model is comprised of a seat pricing plan for an Economy fare consisting of two types of fare classes suitable for low-cost airlines. The Economy fare can be purchased either as a discounted fare or full fare value. The method that is used in this model illustration is the same as the standard nomenclature airlines use. We will abbreviate departures as *D* and arrivals as *A*. For illustration and calculation purposes in our model; we will assume that the airline sells two types of main cabin fares, a discounted ticket *O,* and a full fare ticket *Y*. All ticket fares are assumed to be roundtrip flights. We will abbreviate Departure/Arrival/Fare-type as DAF. Also, we will provide an actual ticket fare price. The discounted fares were priced more than 60 days in advance, and the full fares were priced as late as a few days within departure. All flights are assumed to be booked as round-trip fares returning to their origin within a few days' time. This assumption in the model allows for the more holistic itinerary originating and ending at the same airport.

The forecasted demand was developed to complete setting up the model. We will assume that the same type of aircraft is used for all legs of the trip for simplicity of the model constraints, in this case assuming an Airbus A320 with a 126 seat capacity in the Economy class. It is more than likely that a passenger will travel through a hub enroute to the final destination, and for this reason, we will also declare Atlanta as a hub in our model for completeness. The realistic addition of a hub will add to the complexity of seat allocations for each leg of the flight in our model. The model includes a list of ticket price fares for each leg, class, and a seat demand forecast for each leg of the trip.

Another assumption of relevance in the formulation of the model is the inevitable fact that, for one reason or another, some passengers will cancel their planned flight. Some passengers will fall into the category of no-show and thus miss their flight. Airlines anticipate expected cancellations and no-shows and make an effort to fill these anticipated, empty seats, with some oversold seats for each flight. This model considers the forecasted demand that may be modified to match actual demand when validating the model with actual data.

Additional revenue may be generated from customers using the discounted fare as many of these low-cost airlines enforce stricter carry-on and luggage weight restrictions on discounted fare class ticket holders. This model is mathematically solved using a linear programming method, from a seat-sale perspective only. For example, a full fare ticket may allow an extra carry-on bag. Most low-cost airlines allow discounted pricing on added luggage, meals, and seat selection capability, if prepaid.

For simplicity of understanding the basic model and the dynamics involved in structuring a linear program that optimizes seat allocation for maximum revenue, free fare upgrades and loyalty program fares were excluded. Rewards type parameters can be embedded into this simple model and refined to include sales timelines when considering rewards and loyalty programs. These types of revenue generators may be added as part of a more complex model that includes more revenue-generating possibilities.

## Methods

For an airline reservation system to operate optimally, an airline must determine how many discounted fare class seats and how many full fare class seats to make available for purchase in the Economy section's main cabin. This model is especially suitable for low-cost airlines offering only economy fare tickets. In Figure 1 we are depicting a route with four possible final destinations for an airline. A passenger may depart, arrive, and terminate a flight, from any one of the three airports in our model. The airport abbreviations for Phoenix, Atlanta, and Daytona Beach will be P, A, and D respectively.

The discounted fare will be denoted as O and the full fare as Y. Possible passenger itineraries departing from Phoenix (P), could terminate in Atlanta (A) or Daytona Beach (D). These itineraries will be denoted as PAO, PAY, PDY, and PDO, respectively. Possible passenger itineraries from Atlanta could terminate in Phoenix or Daytona Beach. These itineraries will be denoted as APO, APY, ADO, and ADY. Possible passenger itineraries from Daytona Beach could terminate in Atlanta or Phoenix. These itineraries will be denoted as DAO, DAY, DPO, and DPY.


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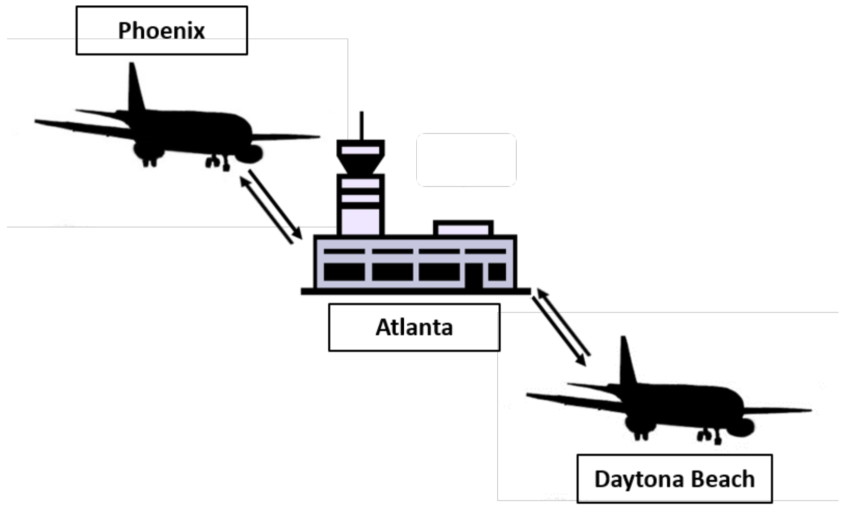


*Figure 1*. Revenue Management Model with Four Possible Legs
Source: Images licensed under Creative Commons
https://commons.wikimedia.org/wiki/File:Airport_symbol.svg
http://www.stockpicturesforeveryone.com/2011/08/aircraft-sketches-and-silhouettes.html

Table 1 depicts the different types of Departures and Arrivals. There are six possible discounted fare classes and six possible full fare classes. Thus, a possible of 12 departure-arrival fare legs is necessary to include all of the possible legs originating from the three airports. For example, a flight departing from Phoenix and arriving in Atlanta with a discounted fare class will have DAF code of PAO, as shown in DAF 1. The cost of each fare, along with the projected seat Demand Forecast is also shown for completeness. The Demand Forecast data represent the baseline or expected demand of passengers in each one of the twelve possible flight itineraries. Naturally, the demand forecast is higher for discounted-fare seats compared to full-fare priced seats as the sale of the available seats becomes available at a considerable timeframe before the actual flight takes place. From Table 1 we can generate a complete list of equations to construct our mathematical model as a linear programming problem.

We compose our model as a sub-problem to maximize revenue, in this case, by selling a seat at the highest possible price. In order to construct our linear programming mathematical problem, we must formulate the objective function so that we maximize its value with the cost of each seat in each of the four possible legs of a trip, thus yielding the highest possible revenue with the sale of each seat, until all seats are sold. The constraints account for all departing flights covering all possible legs of a trip having no more than 126 available seats at any given flight. We must also construct the projected demand forecast for each possible leg of a trip. Finally, we must impose a non-negativity restriction for all of our values as they must be greater than or equal to a zero possible value.

Table 1
*Itinerary Fares and Projected Demand Forecast*

| DAF | Departure | Arrival | Fare Class | DAF Code | Fare Cost | Demand Forecast |
|---|---|---|---|---|---|---|
| 1 | Phoenix | Atlanta | O | PAO | $330 | 72 |
| 2 | Phoenix | Daytona Beach | O | PDO | $314 | 56 |
| 3 | Atlanta | Phoenix | O | APO | $330 | 68 |
| 4 | Atlanta | Daytona Beach | O | ADO | $257 | 45 |
| 5 | Daytona Beach | Atlanta | O | DAO | $257 | 40 |
| 6 | Daytona Beach | Phoenix | O | DPO | $338 | 50 |
| 7 | Phoenix | Atlanta | Y | PAY | $611 | 29 |
| 8 | Phoenix | Daytona Beach | Y | PDY | $617 | 22 |
| 9 | Atlanta | Phoenix | Y | APY | $611 | 34 |
| 10 | Atlanta | Daytona Beach | Y | ADY | $597 | 12 |
| 11 | Daytona Beach | Atlanta | Y | DAY | $597 | 32 |
| 12 | Daytona Beach | Phoenix | Y | DPY | $678 | 9 |

*Note: sourced from Data Adapted from Koursaris & Marion, 2018*

The mathematical model is solved as a system of linear equations using the simplex method, an algorithm that derives an optimal solution using a finite number of steps, devised by the American mathematician George Dantzig (Cottle, 2006). The simplex method uses a large number of iterations to find possible, feasible, essential solutions until an optimal solution is found, whenever it exists. As the given configuration is solved, a transformation is applied by using Gaussian elimination, and the process repeats as many times as necessary until an optimal solution is found. The formulation for this model can be found in the appendix.

## Results and Discussion

The revenue management problem was solved using POM-QM Linear Programming Decision Science software for the optimal solution as shown in Figure 2. The optimal solution results show that the maximum revenue the airline can generate is $160,558 and should allocate the following number of Economy discounted fare, O, and full fare, Y, seats for each one of these legs:

- 72 O seats to PHX-ATL
- 3 O seats to PHX-DAB
- 68 O seats to ATL-PHX
- 12 O seats to ATL-DAB
- 35 O seats to DAB-ATL


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- ➢ 50 O seats to DAB-PHX
- ➢ 29 Y seats to PHX-ATL
- ➢ 22 Y seats to PHX-DAB
- ➢ 34 Y seats to ATL-PHX
- ➢ 12 Y seats to ATL-DAB
- ➢ 32 Y seats to DAB-ATL
- ➢ 9 Y seats to DAB-PHX

For example, as our computerized solution in Figure 2 depicts, to generate a maximum revenue of $160,558, taken into consideration the flight departing from Phoenix, the airline should allocate 75 (72+3) discounted fare seats and 51 (29+22) full fare seats, for a total of 126 available seats. Similarly, all Atlanta outbound flights will have 80 discounted fare seats and 46 full fare seats allocated for a total of 126 available seats. Lastly, our optimal solution indicates that all outbound flights from Daytona Beach should have 85 discounted fare seats and 41 full fare seats allocated for a total of 126 available seats.

Linear Programming Results

Airline Revenue Management Seat Pricing Plan Model Solution

| | PAO | PDO | APO | ADO | DAO | DPO | PAY | PDY | APY | ADY | DAY | DPY | | RHS | Dual |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Maximize | 330 | 314 | 330 | 257 | 257 | 338 | 611 | 617 | 611 | 597 | 597 | 678 | | | |
| Constraint 1 Departing PHX | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | <= | 126 | 314 |
| Constraint 2 Departing ATL | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | <= | 126 | 257 |
| Constraint 3 Departing DAB | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | <= | 126 | 257 |
| Constraint 4 PAO Demand Forecast | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | <= | 72 | 16 |
| Constraint 5 PDO Demand Forecast | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | <= | 56 | 0 |
| Constraint 6 APO Demand Forecast | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | <= | 68 | 73 |
| Constraint 7 ADO Demand Forecast | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | <= | 45 | 0 |
| Constraint 8 DAO Demand Forecast | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | <= | 40 | 0 |
| Constraint 9 DPO Demand Forecast | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | <= | 50 | 81 |
| Constraint 10 PAY Demand Forecast | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | <= | 29 | 297 |
| Constraint 11 PDY Demand Forecast | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | <= | 22 | 303 |
| Constraint 12 APY Demand Forecast | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | <= | 34 | 354 |
| Constraint 13 ADY Demand Forecast | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | <= | 12 | 340 |
| Constraint 14 DAY Demand Forecast | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | <= | 32 | 340 |
| Constraint 15 DPY Demand Forecast | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | <= | 9 | 421 |
| Solution-> | 72 | 3 | 68 | 12 | 35 | 50 | 29 | 22 | 34 | 12 | 32 | 9 | | 160558 | |

*Figure 2*. Revenue Management Seat Pricing Plan Model Optimal Solution
*Note: POM-QM (2010) Decision Sciences software to complete our work.*

The results also reveal other essential decision-making pieces of information in the calculated dual values. The binary value conveys the additional revenue that can be generated should an additional seat of a specific class become available after all projected demand seats have been sold out. Examining the binary value results from our model solution, the most revenue that can be generated is $421 should an additional DAB-PHX full fare class seat becomes available after all nine projected demand seats have been sold out. The next most desirable revenue-generated value would be $354 should an additional ATL-PHX full fare class seat becomes available. The least desirable revenue-generated values would be those with $0 binary values, in the case of our model results the three legs, PHX-DAB, ATL-DAB, and DAB-ATL, all discounted fare class values.

## Summary and Conclusions

This study investigated a linear programming problem to depict the optimal revenue management seat pricing and allocation plan model for a low-cost airline offering full fare and discounted fare economy class seats using a set of given constraints to construct the mathematical set of equations affecting revenue generation. The revenue management plan's objective was to maximize the airline's potential revenue in the Economy class section given a full fare and discounted fare economy class seats for a low-cost airline company. In order to calculate the maximum possible revenue that an airline can generate from the sale of the available seats, several constraints had to be taken into account. One constraint was the seating capacity of the type of airplane flown for each leg. For simplicity of the model, we used the same type aircraft for all possible legs, an Airbus A320 with a seating capacity of 126 in the Economy section. Other constraints were the fare costs for each seat in both the discounted fare and full fare classes. A third constraint taken into account was the projected demand forecast.

Our results concluded the maximum revenue that can be generated from our model, given the fare cost and demand forecast, is $160,558. The exact number of recommended seats allocated for each specific fare-type was calculated. For generating additional revenue, the optimal solution contained the type of seat to target with the highest binary value, as they become available, or in anticipation of cancellations and no-shows. Additional data may be imputed into the model as more criteria and constraints add to the complexity of the model. Finally, our recommendation is to revise the model frequently for currency and up-to-date optimal value calculations.

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## Appendix

$$Max\ 330PAQ + 314\ PDQ + 330APQ + 257ADQ + 257DAQ + 338DPQ + 611PAY + 617PDY + 611APY + 597ADY + 597DAY + 678DPY$$

Subject to:

| | |
|---|---|
| $PAQ + PDQ + PAY + PDY \leq 126$ | Departing Flights from Phoenix |
| $ADQ + APQ + ADY + ADY \leq 126$ | Departing Flights from Atlanta |
| $DAQ + DPQ + DAY + DPY \leq 126$ | Departing Flights from Daytona Beach |
| $PAQ \leq 72$ | Projected Demand Forecast PAQ |
| $PDQ \leq 56$ | Projected Demand Forecast PDQ |
| $APQ \leq 68$ | Projected Demand Forecast APQ |
| $ADQ \leq 45$ | Projected Demand Forecast ADQ |
| $DAQ \leq 40$ | Projected Demand Forecast DAQ |
| $DPQ \leq 50$ | Projected Demand Forecast DPQ |
| $PAY \leq 29$ | Projected Demand Forecast PAY |
| $PDY \leq 22$ | Projected Demand Forecast PDY |
| $APY \leq 34$ | Projected Demand Forecast APY |
| $ADY \leq 12$ | Projected Demand Forecast ADY |
| $DAY \leq 32$ | Projected Demand Forecast DAY |
| $DPY \leq 9$ | Projected Demand Forecast DPY |

$PAQ, PDQ, PAY, PDY, ADQ, APQ, APY, ADY, DAQ, DPQ, DAY, DPY \geq 0$ Non-negativity constraints