Numerical and analytical studies of critical radius in new geometries for corona discharge in air and CO$_2$-rich environments

Jacob A. Engle, Jeremy A. Riousset
Department of Physical Sciences, Center for Space and Atmospheric Research (CSTAR), Embry-Riddle Aeronautical University, Daytona Beach, FL (englej@myr2u.edu)

Abstract
In this work, we focus on planar discharges produced between two electrodes with a high potential difference, resulting in ionization of the neutral gas particles and creating a current in the gap between the electrodes. Planar discharges can be observed as a brush corona, or glow, emission. The parallel plate geometry used in laboratory studies is particularly well suited to model experimental laboratory scenarios. However, it is limited in its applicability to lightning rods and power lines (Bello et al., 2003). In this work, we explore the effects of shifting the classical parallel plate model to spherical and cylindrical geometries more adapted to models of lightning rod and power transmission lines, respectively. Utilizing Townsend’s equation for corona discharge, we estimate a critical radius and minimum breakdown voltage that allows initiation of corona and formation of a glow corona around an electrode. Additionally, we explore the influence of the gas in which the discharge develops. We use a Boltzmann solver for the Boltzmann equation, to calculate Townsend coefficients for CO$_2$-rich atmospheric conditions (Bello et al., 2003). This allows us to expand the feasibility of a glow corona on other planetary bodies such as Mars, to calculate numerically and analytically to predict simplified formulae for planetary and gas variables.

I. Introduction

Corona Discharge
- Electrical discharge around a conductor due to electric field
- Weakly ionized gas responsible for glow at visible wavelengths
- Hypothesized to promote the formation of upward connecting leaders in lightning discharges

Electron Avalanche
The process of electron avalanche is similar across different types of discharges:
- Initiation of a discharge
- Release of secondary electrons in electron-neutral collision
- Secondary electrons with enough energy to repeat the process
- Avalanche criteria (Raizer, 1991), $E_{\text{th}} = 2.5 \times 10^6 \text{V/cm}$

Types of Discharges
- Temperature: $-500 \text{ K} \rightarrow 500 \text{ K}$
- Electric field: 1-12 V/cm
- Electric field: 0.2-27 V/cm
- Electric field: 0.2-27 V/cm
- Electric field: 0.2-27 V/cm

Application to Martian Studies
Motivations:
- Earth Analogy: Potential hazard due to arcing on Titanizing in Martian dust storms and landers
- Interference with sensitive external: Charge separation due to systems and data measurements
- Geostrophic of electrical discharge and integration in the Martian global electric circuit

II. Model Formulation
Corona Discharge
- Critical electric field: $\mathcal{E}(d) = \frac{\ln(d)}{d}$
- Minimum breakdown voltage: $V(d) = \frac{4 \ln(2) \mathcal{E}(d) d^2}{\pi}$
- Stoletov’s point: $V_{\text{sto}} = \frac{4 \ln(2) \mathcal{E}(d) d^2}{\pi}$
- CO$_2$ and air solutions taken at STP
- Boltzmann solver (Bolz)
- Comparison with experimental data
- Convergence of solutions near Stoletov’s points

Spherical solutions
Critical electric field: $\mathcal{E} = \frac{4 \ln(2) \mathcal{E}(d) d^2}{\pi}$
- Minimum breakdown voltage: $V(r) = \frac{4 \ln(2) \mathcal{E}(d) r^2}{\pi}$
- Stoletov’s point: $V_{\text{sto}} = \frac{4 \ln(2) \mathcal{E}(d) d^2}{\pi}$
- Largest error due to Taylor expansion of Gauss error function
- Boltzmann equation solver (Bolz)
- Highest minimum breakdown voltage

Cylindrical solutions
Critical electric field: $\mathcal{E}(R) = \frac{4 \ln(2) \mathcal{E}(d) d^2}{\pi R^2}$
- Minimum breakdown voltage: $V(R) = \frac{4 \ln(2) \mathcal{E}(d) R^2}{\pi}$
- Simplification using the LambertW function
- Solutions not valid for large radii
- Boltzmann equation solver (Bolz)

III. Results
Carson solutions
- Critical electric field: $\mathcal{E}(d) = \frac{\ln(d)}{d}$
- Minimum breakdown voltage: $V(d) = \frac{4 \ln(2) \mathcal{E}(d) d^2}{\pi}$
- Stoletov’s point: $V_{\text{sto}} = \frac{4 \ln(2) \mathcal{E}(d) d^2}{\pi}$
- CO$_2$ and air solutions taken at STP
- Boltzmann solver (Bolz)
- Comparison with experimental data
- Convergence of solutions near Stoletov’s points

IV. Conclusions
The results and conclusions obtained in this work can be summarized as follows:
- A new model for calculating the critical radius and minimum breakdown voltage for Corona Discharge in Cartesian, spherical, and cylindrical geometries
- The model is validated using classic Paschen theory and experimental data in air from Bello and Crabb (2013) and CO$_2$ from Dales (2003)
- We expand classic Paschen theory into analytic solutions for spherical and cylindrical geometry
- Our numerical model and the analytical solution show excellent agreement with experimental data
- The significantly lower pressure on Mars compared to Earth lowers the minimum breakdown voltage required to achieve corona discharge

Acknowledgments
This work is supported by the Enderby-Riddle Aeronautical University Office of Undergraduate Research (OUGR) and the NASA Aeronautics Research (NAR) program.

REFERENCES