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Design and Optimization of Aircraft Configuration for Minimum Drag

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Cover Page Footnote

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Introduction

A complex aircraft design process consisting of numerous disciplines has been developed over many years. These disciplines are integrated and blended together to generate an optimum configuration that satisfies the given requirements (Nguyen, 2011).

There are three phases of aircraft design; conceptual, preliminary, and detail phases. Among them, the conceptual design phase is characterized by the initial definitions that come from requirements established by market needs. Thus, this phase is the most interactive in the whole aircraft design process. The aircraft geometry will change several times driven by optimizations done in order to achieve mission requirements (de Paula, 2017).

Raymer established an aircraft conceptual design process characterized by a large number of design alternatives and trade-off studies, as well as a continuous change in the aircraft concepts under consideration (1999). Howe proposed a systematic and logical approach for several types of aircraft such as two-seat, aerobatic, short- and medium-haul airliners or short take-off landing (STOL) aircraft (2000). Corke proposed an optimization approach to conceptual design of a supersonic business jet (SSBJ; 2003).

Above traditional optimization methods rely on empirical data, which are readily available for common configuration aircraft. Multidisciplinary design optimization is a field of research that studies the application of numerical optimization techniques to the design of engineering systems involving multiple disciplines or components. Since the inception of multidisciplinary design optimization, various methods (architectures) have been developed and applied to solve multidisciplinary design-optimization problems (Martins & Lambe, 2013). In addition, Boone and Striz optimized aircraft configuration for minimum Drag and maximum Range (2010). Nhu Van Nguyen, Daniel Neufeld, Sang Ho Kim, and Jae-Woo Lee optimized Multidisciplinary Configuration Design for Advanced Very Light Aircraft (VLA) by using SQP algorithm (Nguyen, 2011). Ashraf and Abbas presented conceptual design and used genetic algorithm to optimize maximum Range of Supersonic Business Jet (SSBJ) (2014). Jaeger proposed Aircraft Multidisciplinary Design Optimization under both model and design variables uncertainty (2013). Furthermore, many optimization techniques are also implemented during the aircraft conceptual design (Hoburg & Abbeel, 2014; Perez, 2006; Zuo & Chen, 2015).

Drag is wasted energy, the generation of cross flow kinetic energy is an inherent byproduct of the generation of lift over a finite wing span (Takahashi & Donovan, 2011). Optimization for drag results in maximum L/D, which can improve the performance (Boone & Striz, 2010). To get the optimum lift distribution and optimum wing span, Hunsaker, with assistance from Phillips, minimize the induced drag (2017). McGeer designed wing for minimum drag with

practical constraints (1984). Rakshith proposed optimal low-drag wing planforms for tractor-configuration propeller-driven aircraft (2015). Pate and German presented wing optimization for minimum induced drag with generalized bending moment constraints (2013).

In this study, conceptual design code of Single Seat Aerobatic Airplane (SSA) was developed and validated. The previous researches study on the aerodynamics of aircraft that are mainly focus on the Drag of aircrafts. The results shown that the reduction of drag can directly improve the performances such as Range. The optimization of SSA focus on higher performance, better stability, therefore, the validated SSA was used as baseline for optimum configuration for minimum total drag to improve the performance.

Conceptual Design Code Development Steps

The role of the conceptual aircraft design is to propose aircraft configurations that can best meet a set of needs, then to identify several design alternatives. The conceptual design of a Single Seat Aerobatic aircraft (SSA) that present in (Raymer, 1999), will be considered, which allows us to define the main features of the aircraft.

Requirements

As mention above, the requirements which is the main part of the conceptual design. The SSA need to design cruise Range $\geq 280\text{nm}$ at 115kts, and maximum velocity of 130kts, and a stall velocity of 50kts.

Take-off distance $\leq 1000\text{ft}$

Rate of climb $\geq 1500\text{ ft/min}$

Crew weight = 220 lb

The engine (LYCOMING O-320-A2B) having C_{bhp} , specific fuel consumption is assumed 0.5 at cruise speed, revolution of 2700RPM, and horse power of 150Hp. The SSA has to fly with above requirements, and the related mission profile is given in Figure 1.

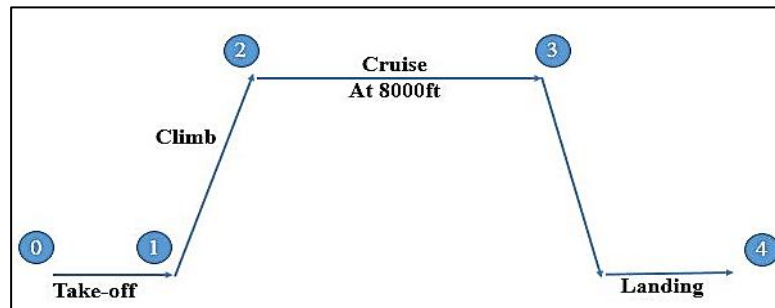


Figure 1. Mission Profile

Geometry Section

The selection of wing thickness ratio, aspect ratio, taper ratio, and sweep angle plays a vital role in determining the performance of a conceptual aircraft design (Raymer, 1999). Low wing configuration and aspect ratio of 6, taper ratio of 0.4, quarter chord sweep of 0, NACA 63₂015 as tip and 63₂012 as root were used to prevent tip stall. Horizontal and vertical tails were used with aspect ratios of 4, NACA 0012 and taper ratios of 0.4.

Power to Weight Ratio and Wing Loading

Hp/W of 1/8 was chosen by engine type and W/S was calculated for each mission segment: take-off, stall, cruise, and landing conditions. The lowest value was selected to ensure that the wing is large enough for all flight conditions.

Initial Sizing

Gross take-off had been calculated (1) using an iterative process and by using the fuel fraction for each mission segment along with an estimated weight of the same type of aerobatic aircraft from historical data. Gross take-off weight is the sum of Payload weight, Crew weight, Fuel weight, Empty weight of the aircraft. The SSA was designed one crew member.

$$W_0 = (W_{\text{crew}} + W_{\text{pay}}) / [1 - (W_f/W_0) - (W_e/W_0)] \quad (1)$$

$$(W_f/W_0) = 1.06 (1 - W_4/W_0) \quad (2)$$

$$(W_4/W_0) = W_1/W_0 * W_2/W_1 * W_3/W_2 * W_4/W_3 \quad (3)$$

$$(W_e/W_0) = 1.495 W_0^{-0.1} \quad (4)$$

Take-off weight fraction, climb weight fraction and landing weight fraction are selected 0.97, 0.985 and 0.995 respectively (Raymer, 1999).

Layout Design

Next, the actual sizes of wing, fuselage, tails, fuel tank, tire size, and propeller diameter were defined based upon the estimated gross take-off weight. Fuselage wetted area was defined by Sears-Haack (5), a symmetric revolution that also has relatively low wave Drag compared to other shapes (Howe, 2000). The wave drag is only concern with supersonic and transonic flight but in this paper, Sears-Haack's fuselage wetted area equation was used because it can easily and quickly define the fuselage wetted area.

$$S_{\text{wetf}} = 0.8083 \pi l r(0) \quad (5)$$

$r(0)$ is the maximum radius of fuselage and 'l' which is the quarter of the fuselage length. Fuselage fineness ratio of 6.38 was chosen.

Aerodynamics

Lift curve slope.

The lift curve is needed during the conceptual design for the following reasons. First, it is used to properly set the wing incidence angle. Secondly, it is important for longitudinal stability analysis (Raymer, 1999). The lift curve slope of SSA was done by (6).

$$C_{L\alpha} = \frac{2 \pi AR}{2 + \sqrt{4 + \frac{AR^2 \beta^2}{\eta^2} \left(1 + \frac{\tan \Lambda_{max}}{\beta^2} \right)}} \left(\frac{S_{exp}}{S} \right) F \quad (6)$$

$$\beta^2 = 1 - M^2 \quad (7)$$

Airfoil efficiency η is approximately as about in the 0.95 if the airfoil lift curve slope is as a function of Mach number (Raymer, 1999). S_{exp} is the exposed wing planform, i.e., the wing reference area less the part of the wing covered by the fuselage. F is the fuselage lift factor which is done by (8)

$$F = 1.07 (1 + d/b)^2 \quad (8)$$

Total drag.

The total drag highly effects on the performance parameters and minimization of total Drag directly improves the Range (Nguyen, 2011). Total drag is the sum of parasite Drag (C_{D0}) and lift induced Drag ($K C_L^2$).

$$D = 1/2 \rho_{cr} V_{cr}^2 S C_D \quad (9)$$

$$C_D = C_{D0} + K C_L^2 \quad (10)$$

$$C_{D0} = \Sigma [C_f F Q (S_{wet}/S_{ref})] + C_{Dmisc} + C_{DL\&P} \quad (11)$$

Parasite Drag (C_{D0}) is the total sum of wing, tail, fuselage Drag ($\Sigma [C_f F Q (S_{wet}/S_{ref})]$), leakage and protuberance Drag ($C_{DL\&P}$), engine cooling Drag, landing gear Drag and miscellaneous Drag (C_{Dmisc}).

$$C_f = f(M, R_e) \quad (12)$$

C_f , skin friction coefficient. Assuming SSA has fully turbulent flow at sea level, Mach number of 0.15 and viscosity of 0.37×10^{-6} are used. To get the actual Oswald span efficiency, equation (13) was used for straight wing aircraft that described in (Raymer, 1999).

$$K = 1 / (\pi e AR) \quad (13)$$

$$e = 1.78 [1 - 0.045(AR)^{0.68}] - 0.64 \quad (14)$$

$$C_L = (W/S) / (1/2 \rho_{cr} V_{cr}^2) \quad (15)$$

Stability and Control

Longitudinal static stability.

The longitudinal stability is the measure of response of the aircraft due to a changing pitch angel condition (Howe, 2000). The coefficient of longitudinal stability is done by (16)

$$C_{m\alpha} = - (SM) C_{La} \quad (16)$$

$$\overline{xnp} = \frac{(C_{La} \overline{x_{acw}}) - C_{m\alpha f} + \eta_{ht} \frac{S_{ht}}{S_w} C_{Lah} \frac{\partial a_h}{\partial a} \overline{x_{ach}}}{C_{La} + \eta_{ht} \frac{S_{ht}}{S_w} C_{Lah} \frac{\partial a_h}{\partial a}} \quad (17)$$

$$\overline{x_{acw}} = X_{acw} / C_{mac} = X_{c/4w} / C_{mac} \quad (18)$$

$$\overline{x_{ach}} = X_{ach} / C_{mac} = X_{c/4h} / C_{mac} \quad (19)$$

$$\frac{\partial a_h}{\partial a} = f(I_t, Z_t) = 0.62 \quad (20)$$

The horizontal distance between x_{acw} and x_{ach} is I_t and the vertical distance between the horizontal reference line and horizontal tail position is Z_t . The downwash, $\frac{\partial a_h}{\partial a}$ of 0.62 is set from the downwash estimation that described in (Raymer, 1999).

$$xnp = \frac{\overline{xnp}}{C_{mac}} \quad (21)$$

$$SM = (xnp - xcg) / C_{mac} \quad (22)$$

xcg is done by calculating each components' weights and using Statical Group Weights Method (Raymer, 1999). An appropriate range of $C_{m\alpha}$ is between - 1.5 and -0.16 (Howe, 2000).

Lateral-directional static stability.

Directional stability is the stability about vertical axis. The most important factor in directional stability is the vertical stabilizer.

$$C_{n\beta} = (C_{n\beta})_F + (C_{n\beta})_W + (C_{n\beta})_{VS} \quad (23)$$

A directional stability reasonable range is between 0.05 and 0.1 (Nguyen, 2011).

Performance

A performance parameter is a quantitative indicator representing how a vehicle operates in a specific flight condition. Typical performance parameters are weights, speeds, aerodynamic loads, engine thrust and power, range and endurance, accelerations, emission indexes (noise, exhaust gases) and many more. At least 60 different parameters can be taken into account in a full aircraft performance analysis (Filippone, 2006). The performance main parameters of Range, Take-off distance, climb was selected because the baseline aircraft SSA was only considered that three parameters. For the validation of design code, the authors also selected these parameters to know the accuracy of design code.

Range.

The Range is maximized by Breguet Range equation for propeller-power aircraft (24) at a cruise speed of 115kts. The maximum range is that the aircraft is flying at Lift to Drag Ratio Maximum while the specific fuel consumption and engine efficiency are constant.

$$\text{Range} = 550 (\eta_p / C_{bhp}) (L/D) \ln (W_3/W_2) \quad (24)$$

The maximum Range for L/D_{\max} in (26)

$$C_{L_{\max \text{ Range}}} = (C_{D0}/K)^{1/2} \quad (25)$$

$$C_{D_{\max \text{ Range}}} = 2C_{D0} \quad (26)$$

$$L/D_{\max} = (C_L/C_D)_{\max \text{ Range}} \quad (27)$$

Take-off distance.

Take-off distance was done by calculating the take-off parameter (TOP).

$$\text{TOP} = \frac{W/S}{\sigma C_{L_{TO}} T/W} \quad (28)$$

σ , density ratio is assumed 1. The aircraft take-off at 1.1 times the stall speed so the $C_{L_{TO}}$, take-off lift coefficient is the maximum lift coefficient divided by 1.21 (Raymer, 1999).

Climb.

Rate of climb, or vertical velocity, is the velocity times the sine of the climb angle (r) (Raymer, 1999).

$$V_v = V \sin(r) \quad (29)$$

$$\sin(r) = (T/W - D/W) \quad (30)$$

T/W was done by the using H_p/W and D/W which is the ratio of total drag and maximum take-off weight.

Design Code Validation

The design code generates all of the aircraft configurations, weights, Lift, Drag, Static Stabilities, and Performance parameters that are satisfied for design requirements by the conceptual design steps that was shown in each above section. And the design code was validated with existing SSA in (Raymer, 1999) as shown in Table 1 for wing, tails configurations and some specifications. Generally, the results from the code agree well with existing data. Maximum error of 1.6% at wing span and horizontal tail area was found.

Table 1
Design Code Validation

	SSA	Code	Units
W_0	1200	1200	lb
W_e	883	883	lb
W/S	10.2	10.3115	lb/ft ²
S	116	116.3747	ft ²
bw	26	26.4244	ft
AR	6	6	ft ²
λ	0.4	0.4	
Sht	25.3547	24.9487	ft ²
bht	10.0707	9.9897	ft
Svt	11.46481	11.2845	ft ²
bvt	4.1476	4.1142	ft
C_{L_α}	4.8547	4.8557	per radian
C_D	0.0337	0.0336	
$C_{m\alpha}$	-0.58	-0.5809	Stick fixed
$C_{n\beta}$	-0.0717	-0.0717	
Range	207.204	207.204	nm
Max Range	289.9744	289.9744	nm
TOP	120	120	
ROC	1500	1500	

Optimization

Sequential Quadratic Programming (SQP) has become the method for solving nonlinearly constrained optimization problems. In the form of non-linear program (NLP), includes as special cases linear and quadratic programs in which the constraint functions \mathbf{h} and \mathbf{g} are affine and \mathbf{f} is linear or quadratic. While these problems are important and numerous the great strength of the SQP method is its ability to solve problems with nonlinear constraints. For this reason, it is assumed that NLP contains at least one nonlinear constraint function. The SQP is to model at a given approximation solution, say x^k , by a quadratic programming subproblem, and then to use the solution to this subproblem to construct a better approximation x^{k+1} . This process is iterated to create a sequence of approximations that, it is hoped, will converge to a solution x^* (Boggs & Tolle, 1995).

The aircraft configuration is optimized by using SQP algorithm for minimum drag within the desired constraints and design variables. While considering an optimization problem, the number of variables, constraints, challenging objectives and time tend to increase the complexity of design space searching. An optimization tool must be flexible enough to include a high number of design variables to reach better design results. At this point, selecting strong variables, meaningful limits and assigning efficient penalties are crucial to obtain better results on behalf of the consumed time and design effort (Cavus, 2016).

The above SSA airplane was chosen as a baseline model for optimization. The objective function, variables and design constraints are considered as follows. The optimization equations can be mathematically written as below:

$$\text{Minimize: Total Drag Coefficient} \quad (31)$$

$$\text{Minimize: } f = C_D(x_i) \quad i = 1 \text{ to } 10 \quad (32)$$

$$\text{Subject to: } h(x) = 0 \quad (33)$$

$$g_j(x) \leq 0 \quad j = 1 \text{ to } 6 \quad (34)$$

The ten design variables are listed in Table 2. Wing loading, wing and tails configurations are considered as variables. The objective function is to minimize the total Drag coefficient. The airfoil of wing, horizontal and vertical tail, and thickness ratio are selected, hence the gear Drag, cockpit Drag, engine cooling Drag, miscellaneous Drag are fixed during the optimization.

Table 2
Design Variables

	Baseline	Bounds		Units
C_D	0.03346	lower	upper	
W/S	10.3115	8.5	23	lb/ ft ²
bw	26.6	23	30.8	ft
Cr	6.25	2.4	6.25	ft
Ct	2.533	2.4	3.18897	ft
bvt	4.1	3.2	4.895	ft
Crvt	4	1.6	4.9212	ft
Cvt	1.6	1.1017	2.4475	ft
bht	10.1	8.79	10.1	ft
Crht	3.6	1.32221	3.6	ft
Ctht	1.4	0.88156	1.5748	ft

Baseline performance is used as constraints for better performance and static stability is also considered in Table 3.

Table 3
Constraints

Range	\geq	280nm
Stall Speed	\leq	50 kts
Take-off Distance	\leq	1000ft
Rate of Climb	\geq	1500 ft/min
$C_{m\alpha}$	\geq	-1.5
$C_{m\alpha}$	\leq	-0.16
$C_{n\beta}$	\geq	0.05
$C_{n\beta}$	\leq	0.1

Optimizer

The overall architecture of the code can be seen in Figure 2 (waterfall diagram). Since the objective function and the constraints require some of the same subroutines, they are nested in another subroutine called 'Physics' which is called by both. The 'Atmosphere' subroutine uses the geometric altitude to determine the static pressure, temperature and density at the operational altitude. The 'Geometries' subroutine calculates the areas of the wing, tails, fuselage and tail location. The 'Weights' function takes the areas and design variables as input. It then uses a Statical Group Weights Method (Raymer, 1999) to calculate each components' weights and to estimate cg location of the aircraft. The 'lifts' function

takes its input and calculates first $C_{L\alpha}$ from the wing airfoil lift curve slope and aspect ratio for estimation of stability constraint.

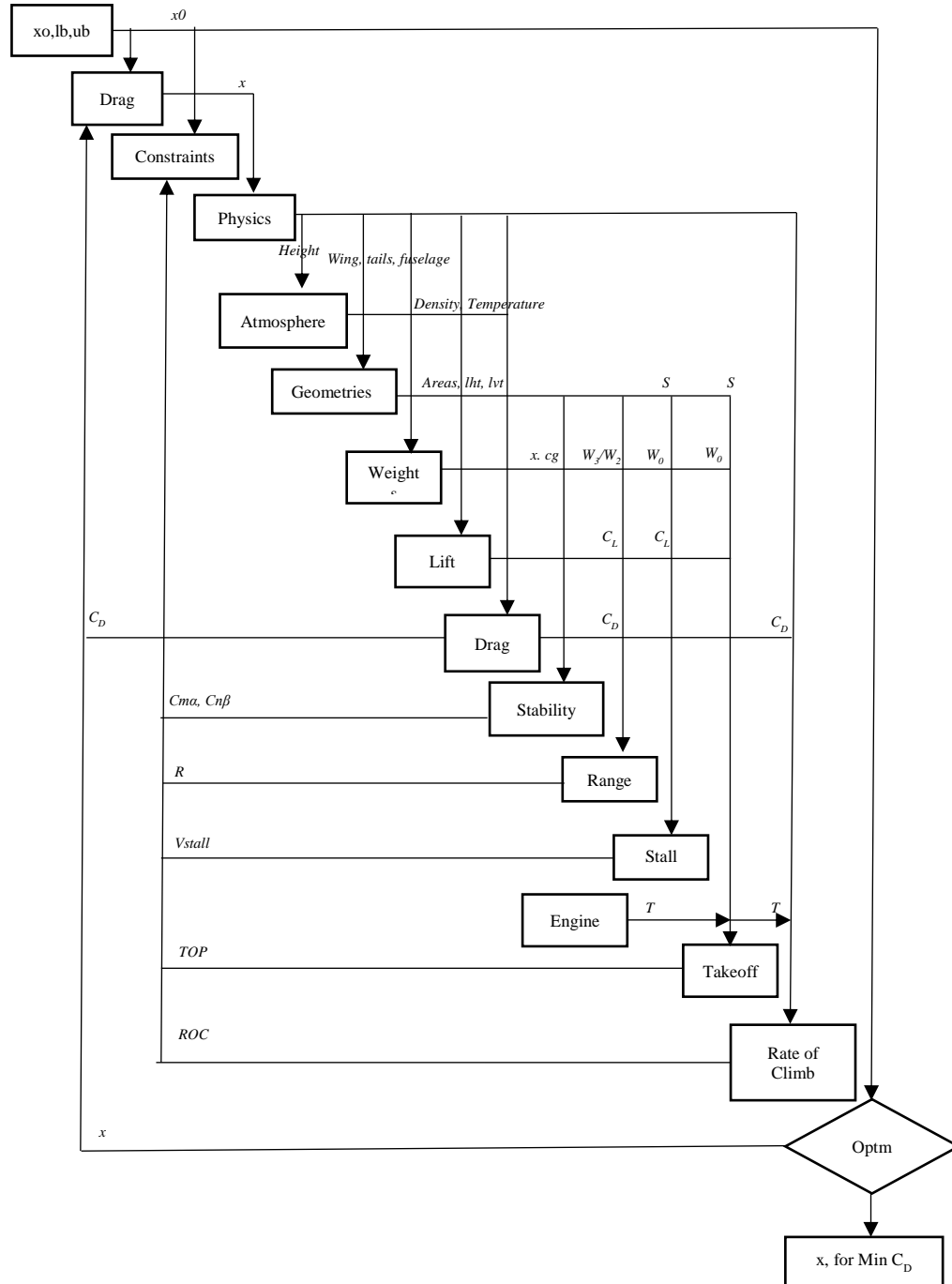


Figure 2. SQP optimizer

And the lift calculate by (14). ‘Drag’ function was estimated from the ‘Geometries’ function that includes wing areas, fuselage, tails, and landing gear was used as input for Drag coefficient. ‘Drag’ simply executes the ‘Physics’ function and passes the C_D value obtained from ‘Drag’ back to ‘Optm’.

The nonlinear constraint function, ‘Constraints’ simply calls ‘Physics’, then passes the results to a series of function detailed below.

‘Stability’ was done using ‘Geometries’ and ‘Weights’ as inputs and then the desired constraints of $C_{m\alpha}$, longitudinal stability and $C_{n\beta}$, directional stability are estimated. The range constraint value is obtained from the subroutine ‘Range’. This is calculated via an implementation of the Breguet range equation (18). The range value is then subtracted from the desired range to derive a constraint value consisted with the SQP convention that satisfied constraints are negative. ‘Stall Speed’ was estimated by using wing loading as input and then the desired stall speed (50kts) was subtracted from the $V_{stall}(x)$ model. The ‘Take-off’ constraint value was done by the wing loading to get the take-off parameter (TOP). The desired TOP was also subtracted from the TOP which was got from the (x) model. The ‘Rate of Climb’ constraint was calculated using thrust to weight ratio (T/W), from engine and drag to weight ratio (D/W) as inputs. (ROC) was subtracted from the desired rate of climb of 1500ft/min. The implementation of the optimizer will find the aircraft configuration (x) for minimum Drag coefficient at a given flight condition that satisfied the constraints.

Optimization Results and Discussion

The convergence history is shown for total Drag coefficient demonstrating that the design optimization formulation is successfully converged by using the Sequential Quadratic Programming (SQP) algorithm in Figure 3.

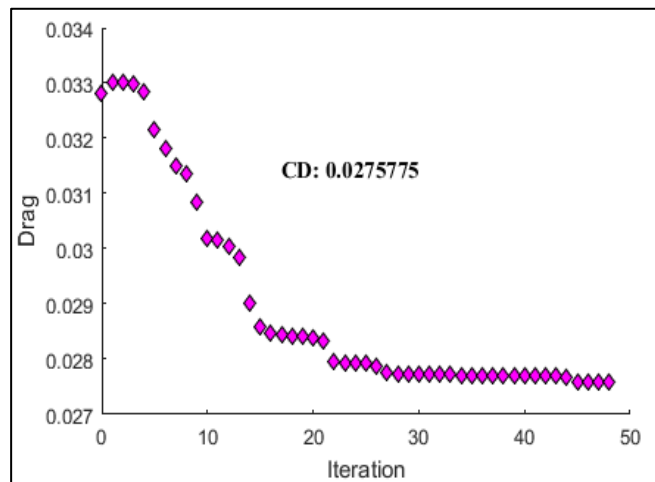


Figure 3. Convergence history of Objective Function

And optimized SSA configuration is shown in Table 4 for wing, tails configuration and some specifications. Also, the optimum configuration is shown in Figure 4 and Figure 5.

Table 4
Optimum Configuration

OPTIMUM CONFIGURATION			
	Baseline	Optimum	Units
W_0	1200	1200	lb
W_e	883	843.373	lb
W/S	10.2	8.5	lb/ft ²
Sw	116	141.1765	ft ²
bw	26	26.817	ft
AR	6	5.0939	
λ	0.4	0.5102	
bht	10.0707	8.79	ft
Crvt	3.9183	1.723	ft
bvt	4.1476	4.8	ft
Cvt	1.4271	1.127	ft
Crht	3.6	1.322	ft
Ctht	1.4	0.882	ft
$C_{L\alpha}$	4.8547	4.7385	per radian
C_D	0.0336	0.02757	Obj fun
Range	207.204	280	nm
Max Range	289.9714	395.67	nm
CONSTRAINTS			
Range	229	280	nm
Stall Speed	50	39.16933	kts
Take-off Dist.	1000	500	ft
Rate of Climb	1500	2487.192	ft/min
$C_{m\alpha}$	-0.58	-0.3786	Stick Fixed
$C_{n\beta}$	0.0717	0.06	

Optimum results show that the total Drag coefficient is reduced by 17.9% and the lower wing loading give more aerobatic performance and highly reduce lift induced drag. By the reduction of drag coefficient and higher lift from larger wing, the subsonic L/D of SSA rises from 12.1509 to 12.35. The increase in L/D causes Drag reduction and lower wing loading, giving less weight friction at cruise condition and improving maximum Range. Maximum Range is increased about

26.71% by the reduction of Drag coefficient. The minimization of total Drag directly improves the performance parameter such as Range at cruise speed. The stability constraints are also in the stable region and the aircraft empty weight is also reduced by the optimum configuration.

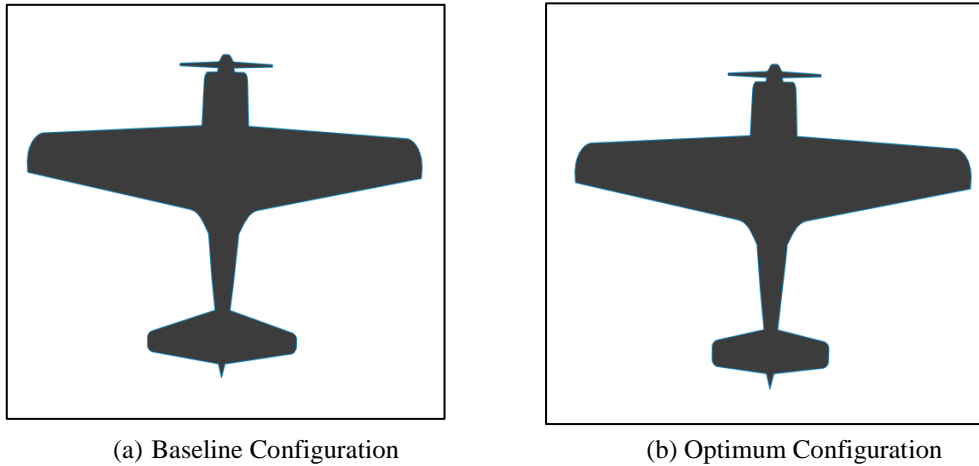


Figure 4. Comparison for Baseline and Optimum Configuration from Top View.

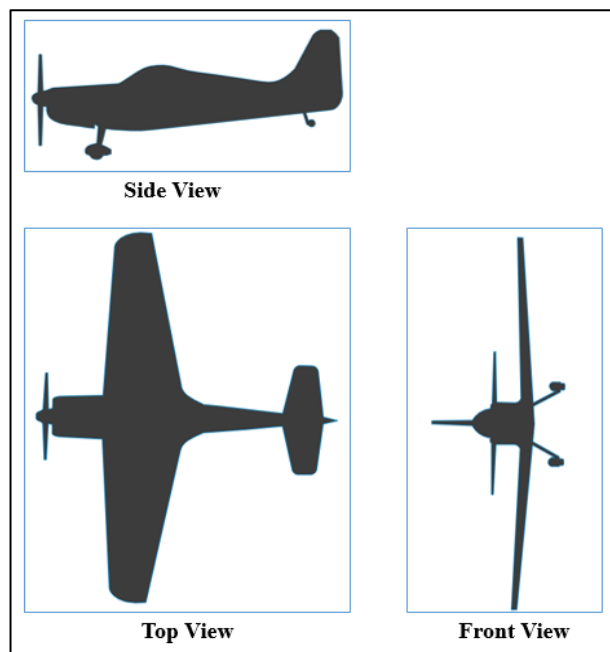


Figure 5. Optimum Configuration in Orthographic View.

Conclusions

The conceptual design code for SSA was successfully developed and validated with existing SSA. The validated SSA was used as a baseline model to optimize for minimum total Drag coefficient for better performance. Optimization of SSA was formulated using SQP algorithm to get optimum configuration. The optimum result shows that wing loading is lower than the baseline model's, which gives better aerodynamic performance that makes reduction of induced Drag. The wing area is higher from 116 to 141. Although the Drag coefficient was minimized, the wing span and wing area were larger than the existing aircrafts. By the higher wing area, the wing Drag can be larger than the baseline. Also, the higher wing span does not experience as much as a loss of lift and increase of Drag due to tip effects as a low aspect ratio wing. In addition, the longitudinal and directional stability are also in the range of historical data. Overall, the optimization not only minimizes total Drag but also gives shorter take-off distance, increases maximum Range, and also increases Rate of Climb.

Nomenclature

AR	= wing aspect ratio	S	= wing area
Aht	= horizontal tail aspect ratio	Sht	= horizontal tail area
Avt	= vertical tail aspect ratio	Svt	= vertical tail area
bw	= wing span	S_{wet}/S_{ref}	= wetted area ratio
Crvt	= vertical tail root chord	$S_{exposed}$	= exposed wing area
Ctvt	= vertical tail tip chord	S_{wetf}	= fuselage wetted area
C_{bhp}	= specific fuel consumption	ub	= upper bounds
C_D	= total Drag coefficient	V_{cr}	= cruise velocity
C_{D0}	= parasite Drag	W/S	= wing loading
C_L	= lift coefficient	W_0	= Gross take-off weight
C_f	= skin- friction coefficient	W_{crew}	= crew weight
C_{Dmisc}	= miscellaneous Drag	W_{pay}	= payload weight
$C_{DL\&P}$	= leakages and protuberances Drag	W_f	= fuel weight
$C_{L\alpha}$	= lift curve slope	W_e	= empty weight
C_{ma}	= longitudinal static stability	W_3/W_2	= cruise end and start weight ratio
$C_{m\alpha f}$	= fuselage pitching moment	x0	= baseline model
$C_{n\beta}$	= directional static stability	x	= optimized model
d	= fuselage diameter	ρ_{cr}	= density at cruise
D	= total Drag	η_p	= propeller efficiency
e	= Oswald efficiency	η	= airfoil efficiency
F	= Form Factor, fuselage lift factor	Λ_{maxt}	= wing sweep angle at maximum airfoil thickness
Hp/W	= horse power to weight ratio	λ	= wing taper ratio
K	= lift induced factor	η_{ht}	= dynamic pressure ratio at tail
lb	= lower bounds	x_{acw}	= wing aerodynamic center location
L/D	= lift to Drag ratio	x_{ach}	= horizontal tail aerodynamic center location
L/D_{max}	= maximum lift to Drag ratio	x_{np}	= neural point location
M	= Mach number	x_{cg}	= c.g location
Q	= interference factor =1	$\frac{\partial a_h}{\partial a}$	= downwash effect
Re	= Reynolds number		
R	= Range		
ROC	= rate of climb		
SSA	= Single Seat Aerobatic		

References

- Ashraf, H., & Abbas, A. (2014). Conceptual design and optimization of supersonic business jet. *1st AIAA Student Research Paper Conference (SRPC), Institute of Space Technology, Islamabad, 1(3)*, 1-4.
- Boggs, P. T., & Tolle, J. W. (1995). Sequential quadratic programming. *Acta Numerica*, 4, 1-51. doi:10.1017/S0962492900002518
- Boone IV, T. R., & Striz, A. G. (2010). Optimization of aircraft configuration for minimum drag. *In the proceedings of the 51st AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, Orlando, FL*. doi:10.2514/6.2010-3000
- Cavus, N. (2016). Aircraft design optimization with artificial intelligence. *AIAA SciTech Forum, In the proceedings of the 54th AIAA Aerospace Sciences Meeting, San Diego, CA, 4-8*. doi:10.2514/6.2016-2001
- Corke, T. C. (2003). *Design of aircraft*. (2003). Upper Saddle River, NJ: Pearson-Prentice Hall.
- de Paula, A. A. (2017). Drag polar prediction methodologies during aircraft design phases. *AIAA SciTech Forum In the proceedings of the 55th AIAA Aerospace Sciences Meeting, Grapevine, TX, 9-13*. doi:10.2514/6.2017-1409.
- Filippone, A. (2006). *Flight performance of fixed and rotary wing aircraft*. Burlington, MA: Elsevier.
- Hoburg, W., & Abbeel, P. (2014). Geometric programming for aircraft design optimization. *AIAA Journal*, 52(11), 2414-2426. doi:10.2514/1.J052732
- Howe, D. (2000). *Aircraft conceptual design synthesis*. Suffolk, UK: Professional Engineering Publishing. <https://doi.org/10.1017/S0001924000092095>
- Hunsaker, D. F. (2017). Designing wings with fixed twist for minimum induced drag. *AIAA SciTech Forum, In the proceedings of the 55th AIAA Aerospace Sciences Meeting, Grapevine, TX, 9-13*. doi:10.2514/6.2017-1419
- Jaeger, L. (2013). Aircraft multidisciplinary design optimization under both model and design variables uncertainty. *Journal of Aircraft*, 50(2), 528-538. doi:10.2514/1.C031914
- Martins, J. R. R., & Lambe, A. B. (2013). Multidisciplinary design optimization: A survey of architectures. *AIAA Journal*, 51(9), 2049-2075. doi:10.2514/1.J051895
- McGeer, T. (1984). Wing design for minimum drag with practical constraints. *Journal of Aircraft*, 21(11), 879-886. doi:10.2514/3.45058
- Nguyen, N. V. (2011). Multidisciplinary configuration design optimization for advanced very light aircraft. *The Korean Society for Aeronautical and Space Sciences*, 18-23. <http://www.dbpia.co.kr/Article/NODE01839444>.

- Pate, J. D., & German, B. J. (2013). Minimum induced drag with generalized bending moment constraints. *Journal of Aircraft*, 50(3), 936-946. doi:10.2514/1.C032074
- Perez, R. E. (2006). Multidisciplinary optimization framework for control-configuration integration in aircraft conceptual design. *Journal of Aircraft*, 43(6), 1937-1948. doi:10.2514/1.22263
- Rakshith, B. R. (2015). Optimal low-drag wing planforms for tractor configuration propeller-driven aircraft. *Journal of Aircraft*, 52(6), 1791-1801. doi:10.2514/1.C032997
- Raymer, D. P. (1999). *Aircraft design: A conceptual approach* (3rd ed.). Washington, DC: American Institute of Aeronautics and Astronautics, Inc.
- Takahashi, T. T., & Donovan, S. (2011). Non planar span loads for minimum induced drag. *In the proceedings of the 49th AIAA Aerospace Sciences Meeting including the New Horizons Forum and Aerospace Exposition Orlando, Florida, 4 - 7*. doi:10.2514/6.2011-639
- Zuo, Y., & Chen, P. (2015). Advanced aerostructural optimization techniques for aircraft design. *Hindawi Publishing Corporation, Mathematical Problems in Engineering, Article ID 753042*. <http://dx.doi.org/10.1155/2015/753042>