A Full-Wave Investigation of the Use of a 'Cancellation Factor' in GW-Airglow Interaction Studies

Yonghui Yu

Embry-Riddle Aeronautical University - Daytona Beach

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A Full-Wave Investigation of the Use of a ‘Cancellation Factor’
in GW-Airglow Interaction Studies

by

Yonghui Yu, B.Sc.

A Thesis Submitted to the
Department of Physical Sciences
in Partial Fulfillment of the Requirements for the Degree of
Master of Science in Space Science

Advisor: Dr. Michael P. Hickey

Embry-Riddle Aeronautical University
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A Full-Wave Investigation of the Use of a ‘Cancellation Factor’
in GW-Airglow Interaction Studies

by
Yonghui "Johnny" Yu

This thesis was prepared under the direction of the candidate's thesis committee chair, Dr. Michael P. Hickey, Department of Physical Sciences, and has been approved by the members of his thesis committee. The thesis was submitted to the Department of Physical Sciences and was accepted in partial fulfillment of the requirements for the degree of Master of Science in Space Science.

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ACKNOWLEDGMENTS

I would like to take this opportunity to thank my advisor, Dr. Michael P. Hickey, for all of his assistance and guidance from the very initial stages of this project. I remember as far as March 2001, when I decided to pursue a higher degree from United States, which up until that time there have been over ten years since I got my bachelor of applied physics in China. It is Dr. Hickey who gave me an offer as a research assistant to support my graduate study. He not only helped me settle down on campus, but also let me acquaint the culture of U.S. gradually. During the first year of course study, he also guided me reading papers, improving English speaking skills, even invited me to picnic, made me home away from home.

He gave me both an interest and an understanding in the subject of atmospheric dynamics. It is his course named geophysical fluid dynamics that started to buildup the basis of my research knowledge. From the book named An Introduction to Dynamic Meteorology (edited by James R. Holton), to Dynamics in Atmospheric Physics (edited by Richard S. Lindzen), Dr. Hickey step by step introduced to me the fundamentals of fluid mechanics in the middle and upper atmosphere. Furthermore, he emphasized atmospheric dynamics is not simply the derivation and application of equations, rather understanding the nature itself. Another important book recommended by Dr. Hickey is The Upper Atmosphere In Motion (edited by C.O. Hines), which opened up to me a totally new challenge and amazing subject — atmospheric gravity waves. This subject undoubtedly forms the basis of the theoretical material contained in this thesis. It is also this book that answered many of the fundamental questions relating to these waves, and became my most helpful and favorite source of the research information.

By the way, I would like to extend my enthusiastic gratitude to his wife Diane who helped me solve some graphic problems and scan the necessary figures. During numerical programming and thesis writing, Dr. Hickey provided valuable comments on passage arrangements and grammar correction. Their high expectations and continuous support are greatly appreciated. Thank you!
DEDICATION

I dedicate this work to my father, my mother, my wife, and my daughter who plan to celebrate her fourth birthday. I could finally overcome all difficulties because of their love and support.
Atmospheric gravity waves (GWs) perturb minor species involved in the chemical reactions of airglow emissions in the mesopause region of the earth’s atmosphere. The so-called ‘Cancellation Factor’ (CF) is defined as a transfer function relating the amplitude of airglow brightness fluctuation to the amplitude of GW-induced fluctuation in temperature [Swenson and Gardner, 1998]. This transfer factor can be used to determine GW fluxes and the forcing effects of GWs on the mean state through airglow observations, because GW fluxes are proportional to the square of GW amplitude.

Numerical models [Walterscheid et al., 1987; Schubert et al., 1991] have previously shown that the airglow relative brightness fluctuation can be much larger than the brightness-weighted relative temperature fluctuation (that is, Krassovsky’s ratio is much greater than 1). Analytical expressions of the CF in the OH nightglow were derived by Swenson and Gardner [1998] and later used by Swenson and Liu [1998]. We introduce the full-wave model [Hickey et al., 1997, 1998] describing GW propagation in a non-isothermal, windy, and viscous atmosphere (combined with the chemical reaction scheme for the OH (8, 3) Meinel emission) to derive fluctuations in the OH nightglow from which an equivalent CF is calculated. Extensive comparisons between our CF and that of Swenson and colleagues show under what atmospheric conditions and which range of GW parameters the CF would be expected to provide a good measure of GW amplitude.

This thesis consists of four chapters that deal with the calculations and comparisons of the CFs in the OH nightglow from both the analytical and numerical models under various atmospheric conditions.

In the first chapter the general subject of internal GWs is introduced for the non-specialist of this field. It reviews the historical theory and observation of atmospheric GWs, and also emphasizes the role of atmospheric GWs in...
producing the reversal of global temperature gradients at the mesopause. At the end of this chapter, the motivation for calculating the CF is introduced.

In the second chapter numerical models of GW-driven fluctuations in the OH nightglow are described in detailed in three developing stages. The Walterscheid et al. [1987] model incorporated a five-reaction photochemical scheme and the complete dynamics of linearized acoustic GWs in an isothermal and motionless atmosphere, but only calculated Krassovsky’s ratio for an infinitesimally thin airglow emission layer. Hickey’s [1988] model was extended to include the dynamical effects of internal GWs propagating in a viscous, thermally conducting, and rotating (though windless) isothermal atmosphere. The model of Schubert, Walterscheid & Hickey [1991] investigated how the characteristics of the OH nightglow from an extended emission region were modified by eddy momentum and eddy thermal diffusivities. In the rest of the second chapter the full-wave model [Hickey et al., 1997, 1998] along with the chemical reaction scheme for the OH (8, 3) Meinel emission as well as the analytical model of Swenson and Gardner [1998] are introduced.

The third chapter commences with a comparison of the CFs derived from the analytical model of Swenson and Gardner [1998] with the CFs calculated with the full-wave model numerically. Much of the work involves the development of computer programs and the plots of data outputs. The analysis and discussion begin with the assumption of an ideal atmosphere, which is isothermal, quasi-adiabatic, and motionless, and later continue to that of a more realistic atmosphere (non-isothermal, dissipative, and with meridional and zonal winds). In the case including the influence of mean winds, we employ wind profiles representative of December 15 and GWs traveling in the eastward direction. These comparisons allow us to determine the accuracy of the calculations and the validity of the assumptions used in the analytically derived CF of Swenson & Gardner [1998].
In the last chapter we summarize the advantage and disadvantage in both approaches. The more accurate calculation of the CF in the OH nightglow under a more realistic atmosphere provides a better understanding of GW effects on the mesospheric dynamics. The CF can be used by optical experimenters to relate their airglow observations to GW energy and momentum fluxes in the stated altitude region.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Title page</td>
<td>I</td>
</tr>
<tr>
<td></td>
<td>Approval page</td>
<td>II</td>
</tr>
<tr>
<td></td>
<td>Acknowledgements</td>
<td>III</td>
</tr>
<tr>
<td></td>
<td>Dedication</td>
<td>IV</td>
</tr>
<tr>
<td></td>
<td>Abstract</td>
<td>V</td>
</tr>
<tr>
<td></td>
<td>List of tables</td>
<td>X</td>
</tr>
<tr>
<td></td>
<td>List of figures</td>
<td>XI</td>
</tr>
<tr>
<td></td>
<td>Chapter</td>
<td></td>
</tr>
<tr>
<td>1.</td>
<td>Introduction of Atmospheric Gravity Waves</td>
<td>1</td>
</tr>
<tr>
<td>1.1</td>
<td>Historical background review</td>
<td>1</td>
</tr>
<tr>
<td>1.2</td>
<td>Atmospheric oscillation theory</td>
<td>3</td>
</tr>
<tr>
<td>1.3</td>
<td>Mathematical framework of Acoustic-Gravity Waves</td>
<td>5</td>
</tr>
<tr>
<td>1.4</td>
<td>Gravity Wave-driven refrigerator</td>
<td>8</td>
</tr>
<tr>
<td>1.5</td>
<td>Cancellation Factor used in the OH nightglow fluctuations</td>
<td>11</td>
</tr>
<tr>
<td>2.</td>
<td>Analytical and Numerical modelings for</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>the OH nightglow responses to various gravity waves</td>
<td></td>
</tr>
<tr>
<td>2.1</td>
<td>Gravity Wave-induced fluctuations in the OH nightglow</td>
<td>15</td>
</tr>
<tr>
<td>2.1a</td>
<td>Walterscheid et al. [1987] model</td>
<td>15</td>
</tr>
<tr>
<td>2.1b</td>
<td>Hickey [1988] model</td>
<td>18</td>
</tr>
<tr>
<td>2.1c</td>
<td>Schubert, Walterscheid &amp; Hickey [1991] model</td>
<td>20</td>
</tr>
<tr>
<td>2.2</td>
<td>Introduction of the full-wave model combined with</td>
<td></td>
</tr>
<tr>
<td></td>
<td>the chemical reaction scheme for the OH (8, 3) Meinel</td>
<td></td>
</tr>
</tbody>
</table>
emission

2.3 The Analytical Model of Swenson and Gardner [1998]

3. Comparisons between the Analytical and Numerical results under various atmospheric conditions

3.0 Linear Comparability
3.1 Isothermal, Quasi-adiabatic condition
3.2 Non-isothermal, Quasi-adiabatic condition
3.3 Isothermal, Non-adiabatic condition
3.4 Non-isothermal, Non-adiabatic condition
3.5 Non-isothermal, Non-adiabatic condition with Tidal winds

4. Conclusions and scope for future work

Appendix 1
Appendix 2
Appendix 3

References
## LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table 1.</td>
<td>The chemical reactions and rate constants for the OH nightglow in Walterscheid et al. [1987] model.</td>
<td>16</td>
</tr>
<tr>
<td>Table 2.</td>
<td>Rates of volumetric production $P$ and loss $L$ of minor constituents by the chemical reactions of Table 1.</td>
<td>17</td>
</tr>
<tr>
<td>Table 3.</td>
<td>The chemical reactions and rate constants for the OH (8,3) Meinel airglow in the full-wave model.</td>
<td>25</td>
</tr>
<tr>
<td>Table 4.</td>
<td>The relation between $\xi$ and $</td>
<td>T' - T</td>
</tr>
</tbody>
</table>
**LIST OF FIGURES**

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 1.</td>
<td>A complex wave pattern resulting from the intersection of two gravity waves progressing on approximately orthogonal headings over Arecibo on January 21, 1993.</td>
<td>2</td>
</tr>
<tr>
<td>Figure 2.</td>
<td>Mean atmospheric temperature of December 15 derived from the MSIS-90 model for a non-isothermal atmosphere.</td>
<td>4</td>
</tr>
<tr>
<td>Figure 3.</td>
<td>The Brunt-Väisälä period (solid curve) and Acoustic cut-off period (dashed-dotted curve) profiles.</td>
<td>5</td>
</tr>
<tr>
<td>Figure 4.</td>
<td>Schematic illustrating the allowed and prohibited phase speeds for gravity waves at Wallops Island for winter and summer [Lindzen 1981].</td>
<td>9</td>
</tr>
<tr>
<td>Figure 5.</td>
<td>The jet stream absorbs waves in one direction but allows them to pass in the opposite. Reproduced courtesy of John Cho, Department of Earth, Atmospheric, and Planetary Sciences, Massachusetts Institute of Technology.</td>
<td>10</td>
</tr>
<tr>
<td>Figure 6.</td>
<td>Linear &amp; nonlinear CF comparisons for a fixed horizontal wavelength of 500 km in the SG98 model. $\left</td>
<td>T''/T_{z_{ow}} \right</td>
</tr>
</tbody>
</table>
\[ \left| \frac{T'}{T_{z_{\text{OH}}}} \right| = 1\% \text{ (dashed-dotted curve)} \]
\[ \left| \frac{T'}{T_{z_{\text{OH}}}} \right| = 5\% \text{ (dashed-dotted-dotted curve)} \]
\[ \left| \frac{T'}{T_{z_{\text{OH}}}} \right| = 10\% \text{ (solid curve)} \]

Figure 7.
The background volume emission rate (VER) profile.

Figure 8.
The CF plotted with respect to the wave intrinsic phase velocity for a fixed horizontal wavelength of 100 km under an isothermal and quasi-adiabatic atmosphere. The CF derived from the SG98 model for chemical and dynamical processes (solid curve), for chemical process only (hairline). The CF derived from the full-wave model for chemical and dynamical processes (dashed-dotted curve), for chemical process only (dashed curve), for dynamical process only (dotted curve).

Figure 9.
Similar to the Figure 8 except for a fixed horizontal wavelength of 500 km.

Figure 10.
Similar to the Figure 8 except for a non-isothermal and quasi-adiabatic atmosphere. The CF derived from the full-wave model for complete processes in an isothermal atmosphere (dashed-dotted-dotted curve).

Figure 11.
Similar to the Figure 10 except for a fixed horizontal wavelength of 500 km.

Figure 12.
Similar to the Figure 8 except for an isothermal and non-adiabatic
atmosphere.

Figure 13.
Similar to the Figure 12 except for a fixed horizontal wavelength of 500 km.

Figure 14.
The CF plotted with respect to the wave intrinsic phase velocity for a fixed horizontal wavelength of 100 km under an isothermal atmosphere. The CF derived from the SG98 model for the adiabatic condition (solid curve). The CF derived from the full-wave model for the non-adiabatic condition (dashed-dotted curve), for the adiabatic condition (dotted curve).

Figure 15.
Similar to the Figure 14 except for a fixed horizontal wavelength of 500 km.

Figure 16.
Similar to the Figure 8 except for an non-isothermal and non-adiabatic atmosphere.

Figure 17.
Similar to the Figure 16 except for a fixed horizontal wavelength of 500 km.

Figure 18.
The Amplitude of temperature perturbation profile for a fixed horizontal wavelength of 500 km under a non-isothermal and non-adiabatic atmosphere. The amplitude of temperature perturbation derived from the SG98 model for $V_{ph} = 50 \text{ m s}^{-1}$ (solid curve), for $V_{ph} = 110 \text{ m s}^{-1}$ (dashed-dotted curve). The amplitude of temperature perturbation derived from the full-wave model...
for $V_{ph} = 50 \, \text{m s}^{-1}$ (dashed curve), for $V_{ph} = 110 \, \text{m s}^{-1}$ (dotted curve).

Figure 19.
The meridional (positive southward) wind profile of December 15 (dashed-dotted curve). The zonal (positive eastward) wind profile of December 15 (solid curve).

Figure 20.
The intrinsic velocity as a function of altitude. The extrinsic phase velocity of $50 \, \text{m s}^{-1}$ (solid curve), the extrinsic phase velocity of $70 \, \text{m s}^{-1}$ (dashed-dotted curve).

Figure 21.
The amplitude of temperature perturbation as a function of height for a fixed horizontal wavelength of 100 km under a non-isothermal and non-adiabatic atmosphere with tidal winds of December 15. The extrinsic phase velocity of $50 \, \text{m s}^{-1}$ (solid curve), the extrinsic phase velocity of $70 \, \text{m s}^{-1}$ (dashed-dotted curve).

Figure 22.
The real part of volume emission rate perturbation ($\Delta V$) profile for a fixed horizontal wavelength of 100 km. The extrinsic phase velocity of $50 \, \text{m s}^{-1}$ (solid curve), the extrinsic phase velocity of $70 \, \text{m s}^{-1}$ (dashed-dotted curve).

Figure 23.
The CF plotted with respect to the wave intrinsic phase velocity for a fixed horizontal wavelength of 100 km under a non-isothermal and non-adiabatic atmosphere. The CF derived from the SG98 model without wind (solid curve). The CF derived from the full-wave model without wind (dotted curve). The CF derived from the full-wave model with horizontal winds
(dashed-dotted curve) where the azimuth of gravity waves is in the eastward direction.

Figure 24.
Similar to the Figure 23 except for a fixed horizontal wavelength of 500 km.
Chapter 1.

Introduction of Atmospheric Gravity Waves

1.1 Historical background review

The upper mesosphere and lower thermosphere (~ 80 - 110 km altitude) exhibit a wide range of actively chemical and dynamical phenomena. Meteor trail distortion and animated aurora have been viewed by mankind for centuries [Hines, 1963]. In the aftermath of the volcanic explosion of Krakatau in 1885, buoyancy waves were first detected by instruments [Kelley, 1997]. Scientific considerations (e.g., Stewart 1882; Trowbridge 1907) were initiated and continued for several decades [Hines, 1963]. Substantial interests [Queney 1947, 1948; Scorer 1949] first applied them to the “lee waves” that arise from the wind flowing over mountains. The stream of observational results during the 1950’s [Martyn, 1950; Gossard and Munk, 1954; Eckart, 1960] added more evidences of cellular waves and traveling ionospheric disturbances (TIDs) [Hines, 1965a]. C. O. Hines was among the first to recognize the implications of those observations and hypothesize that the TIDs were simply the manifestations of internal gravity waves propagating in the ionospheric plasma. He also built up the mathematical foundations to provide quantitative descriptions of the atmospheric gravity waves, a universally accepted theory today [Hines, 1960; Kelley, 1997].

Gravity waves arise from a number of lower atmospheric sources like jet streams, tidal waves, earthquakes, volcanic eruptions, nuclear explosions, and thunderstorms, and from upper atmospheric sources associated with aurora. They are also believed to be an important mechanism for transporting energy and momentum to high altitudes. Their profound influence on the overall structure of the upper atmosphere has gained much attention among the atmospheric physics community. A remarkable fact of the mesopause region is that although in most regions of the atmosphere the summer polar regions are considerably warmer than their winter counterparts, the summer polar
mesopause is actually the coldest place in the terrestrial environment [Kelley, 1997]. The structure and dynamical state of the mesopause region is strongly influenced by gravity waves propagating upward from below. It has also been inferred that some gravity waves may heat the thermosphere at least as strongly as does solar radiation [Hines, 1965b]. Gravity waves exist within a wide range of spatial and temporal scales. Observations reveal a continuous spectrum of gravity waves with horizontal wavelengths of a few to several thousand kilometers, periods ranging from ~5 minutes to tens of hours (depend on altitudes and latitudes), and a general phase downward motion [Munro, 1953, 1958; Heisler, 1958]. They are predominantly vertically transverse waves, and have frequencies far below the audible frequencies of acoustic waves.

The chemiluminescent emissions in the upper atmosphere have been used for several decades to study atmospheric gravity waves. Atmospheric
gravity waves can set the local air parcels into oscillatory motions, further upsetting the chemical equilibrium, and thus modulate the airglow intensity. The image of the OI(557.7nm) emission displayed in Figure 1 consists of two freely propagating quasi-monochromatic gravity waves intersecting on approximately perpendicular headings and forming a distinctive cross-hatch pattern over Arecibo on January 21, 1993 [Taylor and Garcia, 1995a; Hickey et al., 1997].

1.2 Atmospheric oscillation theory

In the absence of atmospheric motions the gravity force must be exactly balanced by the vertical component of the pressure gradient force [Holton, 1992]:

$$\frac{dp_0}{dz} = -\rho_0 g,$$  \hspace{1cm} (1.1)

where $p_0$ and $\rho_0$ are unperturbed atmospheric pressure and density, respectively, and where $g$ is the acceleration of gravity and $z$ is the vertical coordinate (positive upward).

An air parcel that undergoes an adiabatic displacement from its equilibrium level will be positively (negatively) buoyant when displaced vertically downward (upward) so that it will tend to return to its equilibrium level and the atmosphere is said to be stably stratified [Holton, 1992].

The oscillation equation:

$$\frac{D^2}{D t^2}(\delta z) = -N^2 \delta z,$$  \hspace{1cm} (1.2)

where $N^2 = g \frac{d \ln \theta_0(z)}{dz}$.

(1.3)

The static stability criteria for dry air:

$$d \theta_0(z) / dz > 0.$$  \hspace{1cm} (1.4)

$D / D t$ is the substantial derivative (following a parcel), and $D / D t = \partial / \partial t + \mathbf{v} \cdot \nabla$ where $\partial / \partial t$ is the Eulerian time derivative, $\delta z$ is the vertically displaced small distance without disturbing its environment, $N$ is referred to as the Brunt-Väisälä frequency, $\theta_0(z)$ is the potential temperature of
the basic state, and $\theta_0(z) = T_0(z) \left( \frac{\bar{p}_{oo}}{p_0(z)} \right)^\kappa$ where $\bar{p}_{oo} = 1000$ mbar, $\kappa = R/c_p$, $R$ is the gas constant, $c_p$ is the specific heat at constant pressure, and $T_0$ & $p_0$ are unperturbed atmospheric temperature and pressure respectively, $\gamma$ is the ratio of the specific heats, and $C$ is the speed of sound.

Earth's atmosphere consists of multiple layers (e.g., troposphere, stratosphere, mesosphere, and thermosphere, each of them is separated by the tropopause, stratopause, or mesopause, respectively) that are distinguished on the basis of temperature stratification (Figure 2). The Brunt-Väisälä period ($\tau_{BV} = 2\pi / N$) for the Earth's atmosphere varies as a function of height and solar cycle conditions, and ranges from a few minutes in the lower atmosphere to about 15 minutes in the thermosphere (Figure 3). In general, atmospheric gravity waves have periods longer than the Brunt-Väisälä period; their oscillation and propagation are highly anisotropic [Holton, 1992].

Figure 2. Mean temperature derived from the MSIS-90 model [Hedin, 1991]
1.3 Mathematical framework of Acoustic-Gravity Waves

Under the influence of gravity, the background gas pressure $p_0$ decreases exponentially with increasing height, so that for an isothermal atmosphere [Lindzen, 1990]:

$$p_0(z) = p_0(0)\exp(-z/H), \quad (1.5)$$

$$H \equiv RT/g \equiv C^2/\gamma g, \quad (1.6)$$

$$C^2 = \gamma p_0 / \rho_0, \quad (1.7)$$

$$p_0 = \rho_0 RT. \quad (1.8)$$

$H$ is the local scale height, $R$ is the gas constant, and $T$ is the temperature.

The linear theory of acoustic-gravity waves assumes that the single fluid background atmosphere is isothermal, stationary, and horizontally stratified. Superimposed wave motions are assumed to have only small perturbation
magnitude and occur adiabatically. Forces due to pressure gradients, gravity, and inertia are treated explicitly. The oscillations are governed by the linearized momentum, adiabatic state, and continuity equations [Hines, 1960]:

\[
\rho_0 \frac{\partial U}{\partial t} = \rho g - \nabla p, \quad (1.9)
\]

\[
\frac{\partial p}{\partial t} + U \cdot \nabla p_0 = C^2 \left[ \frac{\partial \rho}{\partial t} + U \cdot \nabla \rho_0 \right], \quad (1.10)
\]

\[
\frac{\partial \rho}{\partial t} + U \cdot \nabla \rho_0 + \rho_0 \nabla \cdot U = 0. \quad (1.11)
\]

Those equations relate the perturbed velocity \( U (u, w) \), the perturbed atmospheric pressure \( p \) and density \( \rho \). By assuming that plane wave solutions exist for (1.9), (1.10) and (1.11) we can write [Hines, 1960]

\[
\frac{p - p_0}{p_0} = \frac{\rho - \rho_0}{\rho_0} = \frac{U_x}{X} = \frac{U_z}{Z} = A \exp \left( \omega t - K_x x - K_z z \right), \quad (1.12)
\]

where \( P, R, X \) and \( Z \) are all complex constant amplitudes, and \( A \) is the real constant amplitude. Substituting (1.12) back to the linearized (1.9), (1.10) and (1.11), one builds up the matrix equation:

\[
\begin{bmatrix}
 i\omega & 0 & -iK_x & -(1/H + iK_z) \\
 0 & -iK_x C^2 / \gamma & i\omega & 0 \\
 g & -(1/H + iK_z) C^2 / \gamma & 0 & i\omega \\
 -i\omega C^2 & i\omega C^2 / \gamma & 0 & (\gamma - 1) g \\
\end{bmatrix} \Phi = 0, \quad (1.13)
\]

where

\[
\Phi = \begin{bmatrix}
 \delta p \\
 \rho_0 \\
 \delta p \\
 \rho_0 \\
 u \\
 w
\end{bmatrix} \propto \exp \left( \omega t - K_x x - K_z z \right). \quad (1.14)
\]

The wave angular frequency \( \omega \) and the horizontal wavenumber \( K_x \) are both real and constant, \( K_x = k_x \), but the vertical wavenumber \( K_z \) is complex, allowing for a change with height \( z \) in effective wave amplitude. The plane wave
solutions require the determinant of the matrix in (1.13) to be zero, then the wave numbers appearing in (1.12) are related to the wave angular frequency by the dispersion equation [Hines, 1960]:

$$\omega^4 - \omega^2 C^2 \left( K_z^2 + K_x^2 \right) + \left( \gamma - 1 \right) g^2 K_x^2 + i \gamma g \omega^2 K_z = 0. \quad (1.15)$$

If $K_z$ is purely imaginary, the wave is termed 'evanescent' and only the amplitude varies with height. Since there is no phase variation with height, evanescent waves cannot transport energy in the vertical direction. The second alternative is to let [Hines, 1960]

$$K_z = k_z + i/2H = k_z + i \gamma g / 2C^2, \quad (1.16)$$

where $k_z$ is purely real, and the wave is termed 'internal'. The internal wave does allow a phase variation with height and hence also allows a vertical transport of energy. Since the kinetic energy per unit volume is $(1/2) \rho_0(z) [u^2(z) + w^2(z)]$, and $\rho_0(z)$ decreases with increasing height exponentially as $\propto \exp(-z/H)$, so the increasing wave amplitude must vary with increasing height as $\propto \exp(z/2H)$ to conserve energy [Kelley, 1997].

By substitution of (1.16) into (1.15), the result of the dispersion equation is

$$k_z^2 = k_z^2 \left( \frac{N^2}{\omega^2 - 1} \right) - \left( \frac{\omega^2}{\omega^2 - \omega_a^2} \right) \frac{C^2}{2}, \quad (1.17)$$

where the acoustic cut-off frequency is $\omega_a = \gamma g / 2C$, and the isothermal Brunt-Väisälä frequency is $N = (\gamma - 1)^{1/2} g / C$. Also, $N < \omega_a$ is true for an isothermal atmosphere because $\gamma$ is always less than 2. So the wave solutions for $\omega < N$ are internal gravity waves, for $\omega > \omega_a$ they are internal acoustic waves, and for $N \leq \omega \leq \omega_a$ they are evanescent waves (refer to figure 3). The polarization factors are written as [Hines, 1960]

$$P = \gamma \omega^2 \left[ k_z - i(1 - \gamma / 2) g / C^2 \right], \quad (1.18)$$

$$R = \omega^2 k_z + i(\gamma - 1) g k_z^2 - i \gamma g \omega^2 / 2C^2, \quad (1.19)$$

$$X = \omega k_z C^2 \left[ k_z - i(1 - \gamma / 2) g / C^2 \right], \quad (1.20)$$
\[ Z = \omega \left[ \omega^2 - k_x^2 C^2 \right]. \quad (1.21) \]

The energy of the waves is transported at the group velocity which has horizontal and vertical components given by [Hines, 1974a]

\[ p_x \equiv \frac{\partial \omega}{\partial k_x} = \omega k_x \left( \omega^2 - \omega_g^2 \right) / \left( \omega^4 / C^2 - k_x^2 \omega_g^2 \right), \quad (1.22) \]

\[ p_z \equiv \frac{\partial \omega}{\partial k_z} = k_z \omega^3 / \left( \omega^4 / C^2 - k_x^2 \omega_g^2 \right). \quad (1.23) \]

The horizontal component of the group velocity is in the same direction as the horizontal phase velocity \( \omega / k_x \), however the vertical component of the group velocity is in the opposite (same) direction as the vertical phase velocity \( \omega / k_z \) for internal gravity (acoustic) waves [Hines, 1974a]. As the energy propagates upward, the phases of gravity waves move downward.

The energy flux \( (E) \) of the waves is given by [Friedman, 1966]

\[ E = p V, \quad (1.24) \]

where \( p \) and \( V \) are the perturbation pressure and velocity, respectively. One can find that [Yeh and Liu, 1974]

\[ E = E V_g, \quad (1.25) \]

where \( E \) is the wave energy density and \( V_g \) is the group velocity. Thus, the energy is transported at the group velocity.

1.4 Gravity Wave-driven refrigerator

The coupling between the lower and upper atmospheres through gravity waves is now recognized to be of fundamental importance to the dynamics and energetics of the mesopause region [e.g., Hines, 1960; Lindzen, 1981; Fritts, 1984]. The summer polar atmosphere at the mesopause region is actually the coldest place in the Earth's atmosphere (sometimes less than 100K). This amazing fact can be understood only by including the effects of gravity waves.
Before explaining this phenomenon, first we introduce the concept of a Doppler-shifted frequency given by

\[ \Omega = \omega - k \cdot U, \]  

(1.26)

where \( \Omega \) is also known as the intrinsic frequency perceived by locally moving atmosphere, \( \omega \) is the extrinsic frequency observed on the ground, \( k \) is the horizontal wave number, and \( U \) is the mean wind. Gravity waves propagate upward from the dense lower atmosphere into the stratosphere winds, the local fluid perceives it at a Doppler-shifted frequency, lower if the wave travels in the same direction and higher if the two velocities are opposite. If at some height the wind velocity exactly equals the wave velocity such that the wave becomes stationary in the wind frame, the wave merely becomes part of the flow and is absorbed in it. The oppositely directed gravity waves simply go through into the upper levels of the atmosphere. The winds act as a directional filter that results in a net momentum deposition in the upper atmosphere [Kelley, 1997].

Figure 4. Schematic illustrating the allowed and prohibited phase speeds for gravity waves at Wallops Island for winter and summer. Reproduced courtesy of Lindzen, 1981. Note that the summer wind profile prevents stationary waves from entering the middle atmosphere.

9
Why is the summer mesopause so cold?

(1) Without gravity wave forcing

![Diagram showing summer insolation and geostrophic winds](image)

- Geostrophic winds are set up
- Mean horizontal flow increases with height

(2) With gravity wave forcing

![Diagram showing wave breaking, wave reflection and decay, and meridional circulation](image)

Figure 5. The jet stream absorbs waves in one direction but allows them to pass in the opposite. This creates a net momentum source for atmospheric layers above it. Reproduced courtesy of John Cho, Department of Earth, Atmospheric, and Planetary Sciences, Massachusetts Institute of Technology.
As Figure 4 presents, the summer wind profile prevents westward propagating gravity waves from entering the middle atmosphere, but the winter wind profile allows westward propagating gravity waves to enter the middle atmosphere. Eastward (westward) propagating gravity waves will carry eastward (westward) momentum which is eventually deposited into the mean flow in the mesosphere causing eastward (westward) acceleration of the atmosphere. Under the influence of the Coriolis force, a moving particle in the Northern Hemisphere is deflected to the right of the direction of motion, whereas in the Southern Hemisphere it is deflected to the left. So a net momentum source due to the gravity waves reaching the upper atmosphere is compensated by an equatorward shift of the fluid in the summer and a poleward shift in the winter (Figure 5). Such a shift is in turn accompanied by an upwelling in the center of the summer vortex and a downward motion in the winter. The upwelling is a source of adiabatic cooling, whereas the downward flow heats the mesosphere. Thus this meridional circulation creates incredibly cold temperatures in the summer polar mesopause, just like a wave-driven refrigerator [Lindzen, 1981; Kelley, 1997].

Gravity waves that are typically generated in the troposphere by instabilities in the jet stream or convective processes propagate upward into more rarefied regions of the atmosphere with amplitudes that increase with increasing altitude. Eventually, some of these gravity waves become unstable and saturate, depositing their energy and momentum to the surrounding gas. The important role of gravity waves in the transport and redistribution of energy and momentum that reside in the mean flow now are thought to be crucial to the large-scale circulation and structure of the middle atmosphere [Fritts, 1984].

1.5 Cancellation Factor used in the OH nightglow fluctuations

The middle atmosphere cannot be properly understood if we do not consider the complex interactions among radiation, physics, and chemistry [Andrews, et al., 1987]. It is this gravity wave deposition of momentum that is
responsible for the departure from radiative equilibrium in the mesopause region. The direct phenomenon from such a gravity wave effect is that most chemically active species are not in chemical equilibrium at some particular time and location around the mesopause region.

The nightglow intensities of the various mesopause emissions have been observed to oscillate due to internal gravity waves and tides propagating upward through their emission layers. Observations of the large-scale seasonal and latitudinal variations of the nightglow can provide us with details of the large-scale mesopause dynamics. An understanding of these observations requires that the coupling between the dynamics and chemistry relevant to the emission processes be correctly modeled. Many researches have provided a basis for determining the background distributions of minor species, for identifying the wave-driven fluctuations in the nightglow, and for inferring the characteristics of the underlying wave phenomena [Hedin, 1983, 1991; Hecht et al., 1987; Sivjee et al., 1987; Walterscheid et al., 1987; Hickey et al., 1988a, b].

As the development of measurement technology enables the optical experimenters to establish the intrinsic wave parameters (perceived by local atmosphere), the wave energy and momentum fluxes in the upper mesosphere can be better inferred. Vincent [1984] derived the gravity wave energy flux as

\[ F_E = \frac{-\rho_0 \lambda_z^2 g^2}{\lambda_x \tau_{BV} N^2} \left( \frac{T'}{T} \right)^2, \]  

(1.27)

where \( \rho_0 \) is the mass density, \( \tau_{BV} \) and \( N \) are the Brunt-Väisälä period and frequency respectively, \( g \) is the acceleration of gravity, \( T' \) is the temperature fluctuation, \( \bar{T} \) is the undisturbed atmospheric temperature, \( \lambda_z \) is the vertical wavelength, and \( \lambda_x \) is the horizontal wavelength. It follows that the momentum flux derived by Swenson and Liu [1998] is

\[ F_M = \frac{\lambda_x g^2}{\lambda_x N^2} \left( \frac{T'}{T} \right)^2, \]  

(1.28)

Historically a direct measure of wave amplitude from airglow observations appears quite difficult for two reasons. First, the nightglow emissions behave as a
chemical tracer of gravity wave motions because they involve the density fluctuations of minor species (and not fluctuations of the major gas). Second, the nightglow measurements constitute a height integral of the intensity over the entire vertical extent of the emission layer (~10 km) [Hickey et al., 1997].

In order to obtain the wave amplitude from the OH nightglow to calculate atmospheric gravity wave energy & momentum fluxes in the mesosphere, Swenson & Gardner [1998] and Swenson & Liu [1998] were trying to simplify things by making approximations that allow analytical expressions to be used instead of having to rely on complex modeling (like the full-wave model studies [Hickey et al., 1997, 1998]). Swenson and Gardner [1998] defined a so-called Cancellation Factor (CF) for the airglow intensity as

\[
CF = \frac{<I^'>}{<\bar{I}>},
\]

where \( <I^'> \) and \( <\bar{I}> \) (\( <I^'> = \int \Delta Vdz \) and \( <\bar{I}> = \int \bar{V}dz \)) are the integrals of the fluctuating (\( \Delta V \)) and unperturbed (\( \bar{V} \)) volume emission rate over the height of the emission region respectively. Also, \( |T'/\bar{T}|_{z_{OH}} \) is the wave amplitude of the relative temperature fluctuation at the altitude of the OH* emission layer peak (\( Z_{OH} \)).

Then the CF relating the airglow intensity to the wave perturbed atmospheric temperature can be used to calculate the wave fluxes at \( Z_{OH} \) because of the dependence \( |T'/\bar{T}|_{z_{OH}} = ( <I^'> / <\bar{I}> )/CF \), so that the above equations (1.27) and (1.28) become

\[
F_E = -\frac{\rho_0 \lambda_2 g^2}{\lambda x \tau_{by} N^2 CF^2} \left\langle \left( \frac{I'}{I} \right)^2 \right\rangle, \tag{1.30}
\]

\[
F_M = \frac{\lambda_2 g^2}{\lambda x N^2 CF^2} \left\langle \left( \frac{I'}{I} \right)^2 \right\rangle. \tag{1.31}
\]
This thesis is limited to comparing two models, but it also makes a significant improvement in the theory of gravity wave-airglow interactions. From this viewpoint, this study raises questions to further validate the modeling results. Observations of airglow emission can provide some useful information regarding a particular spectrum of gravity waves, however, the limitations on the observational instruments, geophysical locations, and finite research funding make observations of the whole spectrum of gravity waves impractical. On the other hand, improvements in computer hardware resulting in increased memory and efficiency and advancements in newer numerical techniques proposed almost on a daily basis have stimulated modeling simulations in various GW-airglow interaction studies. Unlike observational campaigns, the geophysical conditions and GW spectra in the modeling simulations can be easily varied to obtain perspective results. Thus experimental data along with computational solutions will widen a tremendous application area in the study of middle atmospheric dynamics.
Chapter 2.

Analytical and Numerical modelings for
the OH nightglow responses to various gravity waves

2.1 Gravity Wave-induced fluctuations in the OH nightglow

During the daylight hours molecular oxygen (O₂) is readily dissociated by certain bands of ultraviolet (UV) and extreme ultraviolet (EUV). Atomic oxygen (O) is a reservoir of chemical energy, it may recombine with major gas molecular oxygen to form ozone (O₃), which combines with atomic hydrogen (H) to form the vibrationally excited hydroxyl (OH⁺). OH⁺ may simply radiate the excess energy away in the form of light, or quench with O, O₂, and N₂. OH⁺ and other minor species (e.g., OI 5577) are excellent tracers of the mesopause dynamics and have been used to study the gravity wave-induced perturbations in atmospheric density, temperature, and winds [von Zahn et al., 1987; Bills et al., 1991a, b; Yu et al., 1991; Hecht et al., 1993; Lowe and Turnbull, 1995]. Optical observations [Taylor et al., 1987, 1991a, b, 1995b; Hecht and Walterscheid, 1991; Zhang et al., 1993a; Hecht et al., 1995] and theoretical/numerical models [Hines & Tarasick, 1987; Walterscheid et al., 1987; Hickey, 1988a, b; Schubert & Walterscheid, 1988; Schubert et al., 1991; Tarasick & Shepherd, 1992a, b; Zhang et al., 1993b; Makhlouf et al., 1995; Hickey et al., 1997, 1998; Hickey and Brown, 2002] have been used to investigate gravity wave effects on the emission intensity of airglows.

2.1a Walterscheid et al. [1987] model

Walterscheid et al. [1987] applied Hines' [1960] dynamics to calculate the single level Krassovsky’s ratio \( \eta = \left( \frac{\delta I}{\bar{I}} \right) / \left( \frac{\delta T}{\bar{T}} \right) \) [Krassovsky, 1972], which relates the fluctuation in the intensity (\( \delta I \)) of the nightglow to the fluctuation in
the temperature ($\delta T$) of the emission region (where overbars refer to time-
averaged quantities).

Table 1. The chemical reactions and rate constants for the OH nightglow in
Walterscheid et al. [1987] model.

<table>
<thead>
<tr>
<th>Chemical reaction</th>
<th>Rate constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H + O_3 \rightarrow OH + O_2$</td>
<td>$k_9 = 1.4 \times 10^{-10} \exp(-470/T) \text{cm}^3 / \text{s}$</td>
</tr>
<tr>
<td>$O + HO_2 \rightarrow OH + O_2$</td>
<td>$k_{11} = 4 \times 10^{-11} \text{cm}^3 / \text{s}$</td>
</tr>
<tr>
<td>$O + OH \rightarrow O_2 + H$</td>
<td>$k_7 = 4 \times 10^{-11} \text{cm}^3 / \text{s}$</td>
</tr>
<tr>
<td>$O + O_2 + M \rightarrow O_3 + M$</td>
<td>$k_2 = 1.0 \times 10^{-34} \exp(510/T) \text{cm}^6 / \text{s}$</td>
</tr>
<tr>
<td>$H + O_2 + M \rightarrow HO_2 + M$</td>
<td>$k_{10} = 2.1 \times 10^{-32} \exp(290/T) \text{cm}^6 / \text{s}$</td>
</tr>
</tbody>
</table>

Walterscheid et al. [1987] used the chemical reactions describing the
production and loss of OH in Table 1. Excited hydroxyl (OH) is produced by the
reaction of atomic hydrogen (H) with ozone (O$_3$) and by the combination of
atomic oxygen (O) with perhydroxyl (HO$_2$). The first reaction is generally
acknowledged to be the predominant source of excited OH ($v \leq 9$) [Kaye, 1988].
Loss of excited hydroxyl molecules is due to the reaction of OH with O, effects of
quenching are not considered. The last two reactions for the production of O$_3$
and HO$_2$ close the chemical system. Molecular oxygen (O$_2$) and M (O$_2$ + N$_2$) are
assumed to be part of the background major gas, so that the chemical system
constitutes five equations for the minor constituents OH, H, O$_3$, O, and HO$_2$
[Walterscheid et al. 1987].

The number density or concentration $n$ of any minor species is
determined by the continuity equation

$$\frac{\partial n}{\partial t} = P - L - \nabla \cdot [n \nu + K \nabla (\frac{n}{N})], \quad (2.1)$$
It has been assumed that all species have the same temperature $T$ and velocity $v$ as the background major gas. The terms $P$ and $L$ are rates of volumetric production and loss of minor constituents by the chemical reactions. The quantity $\nabla \cdot (n \nabla)\rho$ provides the dynamical contribution to the rate of change of a minor species number density. The quantity $\nabla \cdot [K \cdot \nabla (n/N)]$ is the eddy diffusion contribution to $\partial n/\partial t$. Here $K$ is the eddy diffusion tensor and $N$ is the major gas number density. The chemical production and loss terms ($P$ and $L$) are generally proportional to the product of a temperature-dependent reaction rate $k(T)$ and the concentrations of reacting species (Table 2). There are five equations, each of the form of (2.1), for the rates of change of $n(\text{OH})$, $n(\text{O}_3)$, $n(\text{H})$, $n(\text{O})$, and $n(\text{HO}_2)$ [Walterscheid et al. 1987].

Table 2. Rates of volumetric production $P$ and loss $L$ of minor constituents by the chemical reactions of Table 1.

<table>
<thead>
<tr>
<th>Species</th>
<th>$P$</th>
<th>$L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>OH</td>
<td>$k_9 n(H)n(O_3)+k_{11} n(O)n(\text{HO}_2)$</td>
<td>$k_7 n(O)n(\text{OH})$</td>
</tr>
<tr>
<td>$O_3$</td>
<td>$k_2 n(O)n(O_2)n(M)$</td>
<td>$k_9 n(H)n(O_3)$</td>
</tr>
<tr>
<td>$H$</td>
<td>$k_7 n(O)n(\text{OH})$</td>
<td>$k_9 n(H)n(O_3)+k_{10} n(H)n(O_2)n(M)$</td>
</tr>
<tr>
<td>$O$</td>
<td>$\ldots\ldots\ldots$</td>
<td>$n(O){k_7 n(OH)+k_{2} n(O_2)n(M)+k_{11} n(\text{HO}_2)}$</td>
</tr>
<tr>
<td>$\text{HO}_2$</td>
<td>$k_{10} n(H)n(O_2)n(M)$</td>
<td>$k_{11} n(O)n(\text{HO}_2)$</td>
</tr>
</tbody>
</table>

Linearization of equation (2.1) yields

$$\frac{\partial n'}{\partial t} = P' - L' - \frac{dn}{dz} \cdot w' - n \nabla \cdot v',$$

where primes denote perturbed quantities, $z$ is altitude, and $w$ is the vertical velocity. The eddy diffusion term has been omitted because near the mesopause, diffusion time scales are typically much longer than gravity wave time scales. The concentration of $O$ changes slowly during the night compared with the fluctuation time scale of interest (gravity wave periods of minutes to
hours), so the slowly varying basic state was considered as steady [Walterscheid et al. 1987].

Assuming that the perturbations are due to plane waves propagating in the x-z plane (x is the meridional direction) one may write

\[
(n', T', \nabla', \ldots) = \left[ \hat{n}'(z), \hat{T}'(z), \hat{\nabla}'(z), \ldots \right] \exp(i(\omega t - k_x x)),
\]

(2.3)

and

\[
\frac{\hat{\rho}'}{\rho} = \frac{n'(M)}{\bar{n}(M)}, \hat{T}', \hat{\nabla}' \propto \exp(1/2H - ik_z)z,
\]

(2.4)

where \( \omega \) is the wave angular frequency, \( k_x \) is the horizontal wave number, \( k_z \) is the vertical wave number, and a circumflex denotes the z-dependent part of the fluctuation. Substitution of (2.3) into (2.2) then yields

\[
i\omega \hat{n}' = \hat{P}' - \hat{L}' - \frac{d\hat{n}}{dz} \hat{w}' - \hat{n} \nabla \cdot \hat{\nabla}'.
\]

(2.5)

The simplified gravity wave equations of Hines [1960] were used by Walterscheid et al. [1987] and also by Hines and Tarasick [1987] to relate \( \nabla \cdot \hat{\nabla}', \hat{w}', \) and \( \hat{n}'(M) \) to \( \hat{T}' \) through complex dynamical factors \( f_1, f_2, \) and \( f_3 \) (given as equations A1, A2, and A3 respectively in Appendix 1 of [Walterscheid et al., 1987]). The internal gravity wave theory included the dynamics of linearized acoustic-gravity waves in an isothermal and motionless atmosphere.

2.1b Hickey [1988] model

Hickey [1988a] added dissipation & Coriolis force to the single level Krassovskky's ratio (\( \eta \)) calculation. Instead of employing the simplified gravity wave equations of Hines [1960], the theory is extended to include the dynamical effects of internal gravity waves propagating in a viscous, thermally conducting and rotating (though windless) isothermal atmosphere. The equations of Hickey and Cole [1987] are employed, neglecting the effects of ion drag. The Coriolis force is included using the shallow atmosphere approximation [Hickey and Cole,
1987]. One of the effects of viscosity and thermal conduction is to dissipate wave energy [Hickey, 1988a].

The basic altitude dependence of wave variables appearing in (2.3) and (2.4) assumed by Walterscheid et al. [1987] is also assumed here as

$$\frac{\hat{\rho}'}{\rho} = \frac{n'(M)}{n(M)} \hat{T}', \hat{\nu}' \propto \exp(1/2H - ik_z z).$$

(2.4)'

Note that $k_z$ is purely real for internal gravity waves when the equations of Hines [1960] apply, but is complex in the studies of Hickey [1988a, b] because Hickey used the more complicatedly quartic dispersion equation provided by Hickey and Cole [1987].

In order to solve equation (2.5) Walterscheid et al. [1987], Hickey [1988a, b] and Hickey et al. [1997, 1998] related all of the forcing terms in (2.5) to the relative temperature fluctuation using complex dynamical factors $f_1$, $f_2$ and $f_3$:

$$\nabla \cdot \nu' = f_1 \frac{\hat{T}'}{T},$$

(2.6)

$$\hat{w}' = f_2 \frac{\hat{T}'}{T},$$

(2.7)

$$\frac{\hat{\rho}'}{\rho} = f_3 \frac{\hat{T}'}{T}.$$ 

(2.8)

The specification of the complex dynamical factors appearing above depends not only on the gravity wave parameters (wave frequency, horizontal wave number, direction of wave energy propagation and properties of the mean state), but also on the particular study. The simplest analytical representations are those given by Walterscheid et al. [1987] and the complexity increases with consideration of additional physical processes as in Hickey [1988a, b] (where $f_1$, $f_2$, and $f_3$ are given as equations (17), (19), and (21) of [Hickey, 1988a], respectively, and also shown in Appendix 1 of this thesis). In the case of the full-wave model [Hickey et al., 1997, 1998] analytical expressions could not be provided and the complex dynamical factors were numerically evaluated.
In Walterscheid et al. [1987] and Hickey [1988a, b] the system of minor species continuity equations (2.5) representing fluctuations in OH, O₃, H, O, and HO₂ is written as

\[
\begin{bmatrix}
\hat{n}'(OH) \\
\hat{n}'(O_3) \\
\hat{n}'(H) \\
\hat{n}'(O) \\
\hat{n}'(HO_2)
\end{bmatrix}
= \begin{bmatrix}
5 \times 5 \\
\text{matrix}
\end{bmatrix}
\begin{bmatrix}
\begin{bmatrix}
T' \\
\frac{T}{T'}
\end{bmatrix}
\end{bmatrix}.
\] (2.9)

The matrices in (2.9) (shown in Appendix 2) are derived from the volumetric production (P) and loss (L) rates of minor constituents in Table 2. A straightforward inversion of the 5 x 5 matrix in (2.9) yields the n' solution vector for the number density perturbations of the minor constituents. Garcia and Solomon [1985] provided tables of the undisturbed minor species density profiles. Walterscheid et al. [1987] and Hickey [1988a, b] assumed that the OH nightglow volume emission rate was directly proportional to the production rate of excited OH because the chemistry of excited OH was not included in their studies. The production rate of excited OH was calculated based on knowledge of the minor species that were included in their models.

2.1c Schubert, Walterscheid & Hickey [1991] model

Hines & Tarasick [1987] and Schubert & Walterscheid [1988] modeled the effects of gravity waves on an emission layer of finite thickness. Schubert et al. [1991] calculated Krassovsky’s ratio \( \langle \eta \rangle \) from an extended, dissipative emission region (where the angle brackets denote an integral over the entire emission region). Krassovsky’s ratio \( \langle \eta \rangle \) for a vertically extended emission region is defined as in Schubert & Walterscheid [1988] and Schubert et al. [1991]:

\[
\langle \eta \rangle = \frac{\langle \delta I \rangle / \langle I \rangle}{\langle \delta T_i \rangle / \langle T_i \rangle}.
\] (2.10)
\langle \eta \rangle \text{ can be considered as a transfer function relating the input } \langle \delta T_i \rangle \text{ to the output } \langle \delta I \rangle . \text{ Reactions between H and O}_3 \text{ and between O and HO}_2 \text{ yield excited OH molecules that subsequently decay and produce the nightglow. Ground-based observations of the OH nightglow record a vertically integrated intensity } \langle I \rangle = \langle \bar{I} \rangle + \langle \delta I \rangle . \text{ The intensity fluctuations } \langle \delta I \rangle \text{ are caused by gravity wave-driven fluctuations in the densities of the involved constituents and temperature. Temperatures inferred from ground-based observations of the OH nightglow are brightness-weighted according to}

\[
\langle T_i \rangle = \int dz T I / \langle I \rangle , \quad (2.11)
\]

where \( z \) is height [Schubert & Walterscheid, 1988; Schubert et al., 1991].

The theory of gravity wave-driven fluctuations in the OH nightglow from an extended source region was generalized to account for Hickey dynamics that included the effects of eddy kinematic viscosity and eddy thermal diffusivity [Schubert et al., 1991]. This generation of the theory was important not only for completing the theory, but also because the effects of eddy kinematic viscosity and eddy thermal diffusivity were expected to be significant for nightglow fluctuations induced by gravity waves with small vertical wavelengths [Hickey, 1988a, b]. When vertical wavelengths of gravity waves are comparable to or smaller than the thickness of the main emission region, the constructive and destructive interferences of OH nightglow signals from vertically separated levels occur. These are exactly the gravity waves whose induced nightglow fluctuations are most affected by cancellation effects associated with an emission layer of finite thickness [Schubert et al., 1991].

2.2 Introduction of the full-wave model combined with the chemical reaction scheme for the OH \((8, 3)\) Meinel emission

Hickey developed a robust, time-independent full-wave model describing the propagation of nonhydrostatic, linear gravity waves in an inhomogeneous atmosphere. The full-wave model gives a solution to the continuity equation, the
momentum equations, the energy equation, and the ideal gas equation and accounts fully for wave reflection. The model includes dissipation due to eddy processes in the lower atmosphere and molecular processes (viscosity, thermal conduction and ion drag) in the upper atmosphere. Height variations of the horizontal winds and mean temperature, as well as the Coriolis force are all included [Hickey et al., 1997; 1998].

The governing equations of wave propagation are given below [Hickey et al., 1997; 1998]:

\[
\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{v} = 0, \tag{2.12}
\]

\[
\rho \frac{D\mathbf{v}}{Dt} + \nabla p - \rho g + 2\rho \Omega \times \mathbf{v} + \nabla \cdot \mathbf{\sigma}_m
\]
\[
+ \mathbf{\nabla} \cdot (\rho \eta_e \nabla \nu) + \rho v_m (\mathbf{v} - \mathbf{v}_i) + \rho K_v \mathbf{v} = 0, \tag{2.13}
\]

\[
\rho \frac{D(c_T T)}{Dt} + \rho \nabla \cdot \mathbf{\sigma}_m : \nabla \mathbf{v} - \nabla \cdot (\lambda_m \nabla T)
\]
\[
- c_T \nabla \cdot \left[ \rho \kappa_e \nabla \theta \right] + \rho v_m (\mathbf{v} - \mathbf{v}_i)^2 + c_T \rho K_N T = 0, \tag{2.14}
\]

\[p = \frac{\rho R^* T}{M}. \tag{2.15}\]

These equations are linearized and used to describe fully compressible, two-dimensional waves. \(\mathbf{v}\) is the velocity with \(x\) (positive southward), \(y\) (positive eastward), \(z\) (positive upward) components \(u, v\) and \(w\), respectively; \(\rho\) is the neutral mass density; \(p\) is atmospheric pressure; \(g\) is the acceleration of gravity; \(\Omega\) is the Earth's angular velocity; \(\mathbf{\sigma}_m\) is the molecular viscous stress tensor; \(\eta_e\) is the eddy momentum diffusivity; \(v_m\) is the neutral-ion collision frequency; \(\mathbf{v}_i\) is the ion velocity; \(c_p\) and \(c_v\) are the specific heats at constant pressure and volume, respectively; \(T\) is temperature; \(\lambda_m\) is the molecular thermal conductivity; \(\kappa_e\) is the eddy thermal diffusivity; \(R^*\) is the universal gas constant; \(M\) is the mean
molecular weight; and \( K_R \) and \( K_N \) are the Rayleigh friction and Newtonian cooling coefficients, respectively [Hickey et al., 1997, 1998].

The operator \( D / D_t = \partial / \partial t + \mathbf{v} \cdot \nabla \) is the substantial derivative, where \( \mathbf{v}(z) \) is the total wind (mean plus perturbation). \( \Theta \) is the potential temperature \( \Theta = T \left( \frac{P_{00}}{P} \right)^{\kappa} \), where \( P_{00} = 1000 \) mbar, \( \kappa = R / c_p \), and \( R \) is the gas constant. The viscous stress tensor is given by [Hickey et al., 2000]

\[
\sigma_y = \mu \left( \frac{\partial \mathbf{v}_y}{\partial x_j} + \frac{\partial \mathbf{v}_j}{\partial x_i} - \frac{2}{3} \delta_y \nabla \cdot \mathbf{v} \right), \tag{2.16}
\]

where \( \mu \) is the viscosity coefficient, \( \delta_y \) is the Kronecker delta function.

The linear wave solutions to these equations are assumed to vary as \( \exp(i(\omega t - k_x - l_y)) \) in time and horizontal coordinates, where \( \omega \) is the wave angular frequency, and \( k \) and \( l \) are the horizontal wave numbers in the \( x \) (meridional) and \( y \) (zonal) directions, respectively. The six linearized equations are reduced to five by eliminating the density perturbation using the linearized ideal gas equation. The remaining five equations are second-order, ordinary differential equations in the vertical coordinate \( z \). This coupled system of equations is solved subject to boundary conditions for the wave variables \( u' \), \( v' \), \( w' \) (the meridional, zonal, and vertical velocity perturbations, respectively), \( T' \), and \( p' \) (the temperature and pressure perturbations, respectively). First, the variables \( u' \), \( v' \), \( w' \), and \( T' \) (collectively \( \Psi' \)) are transformed to new variables (\( \Psi^* \)) through dividing by the square root of the mean atmospheric density, \( \rho^* \) (i.e., \( \Psi^* = \Psi' / \rho^{1/2} \)). The variable \( p' \) is multiplied by this factor (i.e., \( p^* = p' \rho^{1/2} \)). We solve for the transformed variables by expressing vertical derivatives as centered finite differences and then using the tridiagonal algorithm [Bruce et al. 1953] to solve the resulting set of differential equations subject to boundary conditions [Hickey et al., 1997, 1998]. The untransformed lower boundary condition is \( w' = 0 \), and vertical gradients in \( u' \), \( v' \), \( T' \), and \( p' \) are defined based on the equations for an
adiabatic and isothermal atmosphere. At the upper boundary the radiation condition is applied, using the WKB solution described by [Hickey and Cole, 1987]. The upper boundary is chosen to be high enough to ensure that wave reflection from the upper boundary does not influence results at lower altitudes in the model (this was implemented by adjusting the upper boundary height until a WKB wave experiences severe damping within a time scale of one wave period). Newtonian cooling and Rayleigh friction are used to implement a sponge layer. The coefficients $K_N$ and $K_R$ used in the full-wave model are numerically equal and have large values near the sponge layer decreasing to small values far away from the upper boundary. They are numerically equal to the wave angular frequency at the sponge layer, and decrease exponentially away from it. Modeled wave amplitudes become very small at the upper boundary so that an extremely small amount of upward propagating energy is radiated from the model domain where it is presumed to dissipate. A heat source represented as a Gaussian profile of half width 1.5 km and centered near the tropopause is used to provide the wave forcing in the energy equation [Hickey et al., 1997, 1998]. Our results are not dependent on the magnitude of this source because the model is linear so that we later rescale the wave amplitudes to satisfy the values we specify. The final wave variables are obtained by multiplying (for $u'$, $v'$, $w'$, and $T'$) or dividing (for $p'$) the output variables by the square root of the mean atmospheric density $\bar{\rho}$. The finite differential equations in the region between the lower boundary (ground) and the upper boundary (the latter lying between 200 to 500 km) are represented on a grid of ~200,000 points, thus providing a vertical resolution of ~2 m. The model outputs the wave variables $u'$, $v'$, $w'$, $T'$, and $p'$ given the wave angular frequency $\omega$, the horizontal wavelength $\lambda_x$ (equal to $2\pi/\sqrt{k^2 + l^2}$), and the azimuth of propagation $\varphi$ measured from east of geographic north [Hickey et al., 1997, 1998].

The chemical reaction scheme for OH (8, 3) Meinel emission combined in the full-wave model describes the production and loss of OH* as in Table 3.
Table 3. The chemical reactions and rate constants for the OH(8,3) Meinel airglow in the full-wave model

<table>
<thead>
<tr>
<th>Chemical reaction</th>
<th>Rate constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O + OH(\nu = 0) \rightarrow H + O_2$</td>
<td>$k_1 = 4.0 \times 10^{-11} \text{ cm}^3 / \text{s}$</td>
</tr>
<tr>
<td>$H + O_2 + M \rightarrow HO_2 + M$</td>
<td>$k_2 = 2.1 \times 10^{-32} \exp(290 / T) \text{ cm}^6 / \text{s}$</td>
</tr>
<tr>
<td>$O + HO_2 \rightarrow OH(\nu = 0) + O_2$</td>
<td>$k_3 = 4.0 \times 10^{-11} \text{ cm}^3 / \text{s}$</td>
</tr>
<tr>
<td>$O + O + M \rightarrow O_2 + M$</td>
<td>$k_4 = 4.7 \times 10^{-33} (300 / T)^2 \text{ cm}^6 / \text{s}$</td>
</tr>
<tr>
<td>$O + O_2 + M \rightarrow O_3 + M$</td>
<td>$k_5 = 1.0 \times 10^{-34} \exp(510 / T) \text{ cm}^6 / \text{s}$</td>
</tr>
<tr>
<td>$H + O_3 \rightarrow OH^*(\nu = 8) + O_2$</td>
<td>$k_6 = 0.27 \times 1.4 \times 10^{-10} \exp(-470 / T) \text{ cm}^3 / \text{s}$</td>
</tr>
<tr>
<td>$OH^*(\nu = 8) \rightarrow OH(\nu = 3) + h\nu$</td>
<td>$k_7 = 0.569 / \text{s}$</td>
</tr>
<tr>
<td>$OH^*(\nu = 8) + O \rightarrow H + O_2$</td>
<td>$k_8 = 2.5 \times 10^{-10} \text{ cm}^3 / \text{s}$</td>
</tr>
<tr>
<td>$OH^<em>(\nu = 8) + O_2 \rightarrow OH^</em>(\nu - 1) + O_2$</td>
<td>$k_9 = 8.0 \times 10^{-12} \text{ cm}^3 / \text{s}$</td>
</tr>
<tr>
<td>$OH^<em>(\nu = 8) + N_2 \rightarrow OH^</em>(\nu - 1) + N_2$</td>
<td>$k_{10} = 7.0 \times 10^{-13} \text{ cm}^3 / \text{s}$</td>
</tr>
</tbody>
</table>

The excited hydroxyl (OH*) is produced by the reaction of atomic hydrogen with ozone, and lost in several vibrational band emissions and quenching with O, O$_2$, and N$_2$. As mentioned before in the full-wave model, the complex dynamical factors $f_1$, $f_2$, and $f_3$ used in equations (2.6), (2.7), and (2.8) were numerically evaluated instead of provided in analytical expressions.

The procedure for solving (2.5) in the full-wave model is similar to that described before with a few differences. The chemistry used in the full-wave model is more complete and includes the production and loss rates of the excited (radiating) species. Hickey et al. [1997, 1998] simulated gravity wave effects on the OI 5577 airglow using the full-wave model, while Hickey and Walterscheid [1999] also simulated gravity wave effects on the O$_2$ atmospheric airglow. Gravity
wave effects on the OH airglow were simulated using the full-wave model by Hickey [2001], Hecht et al. [2002], and by Huang et al. [2002].

The OH chemistry used in the full-wave model is provided in Table 3. Instead of solving the set of equations (2.5) for five minor species, the full-wave model solves for six minor species (and therefore solves six equations). The additional minor species (OH*(8)) can be specified in the full-wave model because its production and loss (through radiation and quenching) are included in the chemistry.

2.3 The Analytical Model of Swenson and Gardner [1998]

The density response of the neutral atmosphere composed of a minor neutral constituent is governed by the continuity equation

$$\frac{\partial n}{\partial t} + \nabla \cdot (n V) = P - Q,$$

(2.17)

where \( n \) is the number density of the minor constituent, \( V \) is the velocity field, \( P \) is the chemical source term, and \( Q \) is the chemical loss term. It is assumed that diffusion is negligible, only wave-induced dynamics is considered, and the effects of the chemical source and loss terms, \( P \) and \( Q \), are zero. Gardner and Shelton [1985] have shown that the solution to (2.17) has the form

$$n(p, t) = e^{-\chi} n_u(z - \xi),$$

(2.18)

where \( n(p, t) \) is the perturbed density at position vector \( p \) and time \( t \), and \( n_u(z) \) is the steady state density profile in the absence of wave activity. The parameters \( \chi(p, t) \) and \( \xi(p, t) \) are solutions to the partial differential equations

$$\frac{\partial \chi}{\partial t} = \nabla \cdot V - V \cdot \nabla \chi,$$

(2.19)

$$\frac{\partial \xi}{\partial t} = w - V \cdot \nabla \xi,$$

(2.20)

where \( w \) is the vertical velocity,

$$\chi = \ln \left[ 1 + \frac{A}{\gamma - 1} e^{\xi/2H} \cos (\omega t - kx + mz) \right],$$

(2.21)
\[ \xi = \gamma H \ln \left[ 1 + \frac{A}{\gamma - 1} e^{\xi/2H} \cos(\omega t - kx + mz) \right], \]  
(2.22)

and \( \chi = \xi / (\gamma H) \) [Gardner and Shelton, 1985]. \( A \) is wave amplitude, \( \omega \) is wave angular frequency, \( \gamma \) is the ratio of specific heats, \( H \) is the local scale height, \( m \) is the vertical wave number, and \( k \) is the horizontal wave number.

Since the unperturbed atmospheric density is \( \rho_u(z) = \rho_u(0)e^{-z/H} \), the perturbed atmospheric density derived from (2.18) with \( \chi = \xi / (\gamma H) \) is [Swenson and Gardner, 1998]

\[ \rho(z) = \exp\left(\frac{(\gamma - 1)}{\gamma H} \xi\right) \rho_u(z). \]  
(2.23)

Substituting (2.22) into (2.23), the relative atmospheric density perturbation becomes

\[ \frac{\rho(z)}{\rho_u(z)} = \left[ 1 + \frac{A}{\gamma - 1} e^{\xi/2H} \cos(\omega t - kx + mz) \right]^{\gamma-1}. \]  
(2.24)

If the wave amplitude is small enough, (2.24) can be approximated such as

\[ \frac{\rho(z)}{\rho_u(z)} = 1 + e^{\xi/2H} \cos(\omega t - kx + m(z - z_{OH})), \]  
(2.25)

where \( \xi \) is the wave amplitude at the altitude of the OH* emission layer peak \( (Z_{OH}) \), \( 1/\beta \) is the amplitude growth length (= 2H for undamped waves), and \( \omega \) is the intrinsic frequency.

The wave numbers and the wave angular frequency are related by the dispersion relation [Hines, 1960]

\[ m^2 = \frac{(N^2 - \omega^2)}{(\omega^2 - f^2)} k^2, \]  
(2.26)

where \( N \) is the Brunt-Väisälä frequency, \( f = 2\Omega \sin \Phi \) is the inertial frequency, \( \Omega (= 7.292 \times 10^{-5} \text{ rad s}^{-1}) \) is the Earth’s angular velocity, and \( \Phi \) is latitude.

The Meinel band vibrational spectrum of OH* arises from the reactions as [Swenson and Gardner, 1998]
\[ O + O_2 + M \rightarrow O_3 + M, \quad (2.27) \]

\[ k_{27}^{N_2} = 5.70 \times 10^{-34} \left( \frac{T}{300} \right)^{-2.62} \text{ cm}^6 / \text{s}, \]

\[ k_{27}^{O_2} = 5.96 \times 10^{-34} \left( \frac{T}{300} \right)^{-2.37} \text{ cm}^6 / \text{s}, \]

\[ H + O_3 \rightarrow OH^* + O_2, \quad (2.28) \]

\[ k_{28} = 2.6 \times 10^{-11} \text{ cm}^3 / \text{s}. \]

Where \( k_{27}^{N_2} \) and \( k_{27}^{O_2} \) are the rate coefficients of the three-body (O, O_2, M) reaction, and \( k_{28} \) is the rate coefficient of the reaction of atomic hydrogen with ozone which is generally acknowledged to be the predominant source of excited OH \((v \leq 9)\) [Kaye, 1988].

The McDade et al. [1987] model of the OH (8, 3) Meinel band is given as

\[ \frac{L(8)}{A(8,3)} = f(8)[O][O_2]\left\{k_{27}^{N_2}[N_2] + k_{27}^{O_2}[O_2]\right\}/V(8,3), \quad (2.29) \]

where \( V(8,3) \) is the volume emission rate, \( A(8,3) \) is the emission probability, \( L(8) \) is the total atmospheric loss of the \( v = 8 \) vibrational level, and \( f(8) \) is the fraction of the \( v = 8 \) level produced by reaction (2.28). McDade et al. [1987] fit the computed loss profile \( L(8)/A(8,3) \) to the \( O_2 \) profile derived from the MSIS - 83 empirical model [Hedin, 1983] with the result

\[ L(8)/A(8,3) = 260 + 2 \times 10^{-11} \text{ cm}^3 [O_2]. \quad (2.30) \]

Using (2.29) & (2.30), Swenson and Gardner [1998] obtained the following explicit expression for the OH^* volume emission rate:

\[ V(8,3) = \frac{f(8)[O][O_2]\left\{k_{27}^{N_2}[N_2] + k_{27}^{O_2}[O_2]\right\}}{(260 + 2 \times 10^{-11} \text{ cm}^3 [O_2])}. \quad (2.31) \]

For a well mixed atmosphere \([N_2] = 3.54 [O_2]\), Swenson and Gardner [1998] have

\[ \left\{k_{27}^{N_2}[N_2] + k_{27}^{O_2}[O_2]\right\} = K_0(200/T)^{2.62} + 0.267(200/T)^{2.37}[O_2] \]

\[ \approx 1.267K_0(200/T)^{2.5}[O_2], \quad (2.32) \]

where \( K_0 = 5.84 \times 10^{-33} \text{ cm}^6 \text{s}^{-1} \). Substituting (2.32) into (2.31) and using \( f(8) = 0.29 \) give...
\[ V(8,3) = \frac{K_x [O][O_2]^2 (200/T)^{2.5}}{\left(1 + 7.7 \times 10^{-14} \text{cm}^3 [O_2] \right)}, \]  

(2.33)

where \( K_x = 8.25 \times 10^{-36} \text{cm}^6 \text{s}^{-1} \). The \([O_2]_u\) and \(T_u\) profiles are calculated from the MSIS-90 empirical model [Hedin, 1991], and the \([O]_u\) profile is from tables of the undisturbed minor species density profiles [Garcia and Solomon, 1985].

Using the unperturbed \([O]_u\), \([O_2]_u\), and \(T_u\) profiles in (2.33) to calculate the unperturbed \(\text{OH}^*\) volume emission rate \(\bar{V}(8,3)\), then for zenith viewing the mean state emission intensity is given by

\[
<\bar{I}> = \int_{0}^{\infty} \bar{V}(8,3) dz.
\]

(2.34)

Swenson and Gardner [1998] then argued that because the lifetimes of an \(\text{OH}^*\) molecule and ozone are short compared to typical gravity wave periods, the direct redistribution of \(\text{OH}^*\) and \(O_3\) by the waves is negligible. With this assumption, the perturbations in the volume emission rate are solely determined by the perturbed \(O\), \(O_2\), and \(T\) profiles. By differentiating (2.33) with respect to \(O\), \(O_2\), and \(T\), Swenson and Gardner [1998] obtained:

\[
\Delta V(8,3) = \frac{\partial V}{\partial [O]} \Delta [O] + \frac{\partial V}{\partial [O_2]} \Delta [O_2] + \frac{\partial V}{\partial T} \Delta T
\]

\[
\approx \left( \frac{\Delta [O]}{[O]} + \frac{2 + 7.7 \times 10^{-14} \text{cm}^3 [O_2]}{\left(1 + 7.7 \times 10^{-14} \text{cm}^3 [O_2] \right)} \frac{\Delta [O_2]}{[O_2]} - 2.5 \frac{\Delta T}{T} \right) V(8,3)
\]

(2.35)

Since the atmosphere is considered to be well mixed & \(O_2\) is part of the major gas, the \(O_2\) concentration is proportional to the atmospheric density \(\rho\) as

\[
[O_2]/[O_2]_u = \rho/\rho_u.
\]

(2.36)

For long period gravity waves the pressure within a vertically displaced parcel instantaneously adjusts to the environmental pressure, and the pressure fluctuations are negligible, so that

\[
T/T_u = (\rho/\rho_u)^{-1}.
\]

(2.37)

The relative perturbations in the atomic oxygen profile \(([O]/[O]_u)\) are computed by using (2.18) and (2.22) with the \([O]_u\) profile provided by tables of the
undisturbed minor species density profiles [Garcia and Solomon, 1985]. The integral of volume emission rate fluctuations over the entire emission region provides the airglow intensity fluctuation \(< I' >= \int_0^\infty \Delta V(8,3) dz\). The CF was then calculated by Swenson and Gardner [1998] using equation (1.29).

The chemical rate constants used by Swenson and Gardner [1998] are not exactly the same as those used by Hickey [2001] in a full-wave study of gravity waves in the OH nightglow. In order to compare the two models, we repeat the Swenson derivation of (2.31), (2.33), and (2.35) using our rate constant \(k_3 = 1.0 \times 10^{-34} \exp(510/T) \text{ cm}^6/\text{s}\) given in Table 3 for the three-body reaction \((O + O_2 + M \rightarrow O_3 + M)\) to further investigate the method of Swenson and Gardner [1998], but not the rate constant itself.

We use the rate constant \(k_3\) of major gas, and start from (2.31) as

\[
V(8,3) = \frac{f(8)[O][O_2]k_3[M]}{(260 + 2 \times 10^{-11} \text{ cm}^3 [O_2])}, \tag{2.31}'
\]

for a well mixed atmosphere such that \([O_2] = 0.21 \text{ [M]}, \text{ and } f(8) = 0.27\), we obtain

\[
V(8,3) = \frac{K_1'[O][O_2]^2 \exp(510/T)}{(1 + 7.7 \times 10^{-14} \text{ cm}^3 [O_2])}, \tag{2.33}'
\]

where \(K_1' = 4.94 \times 10^{-37} \text{ cm}^6 \text{s}^{-1}\). By differentiating (2.33)' with respect to \(O\), \(O_2\), and \(T\), we obtain:

\[
\Delta V(8,3) = \frac{\partial V}{\partial [O]} \Delta [O] + \frac{\partial V}{\partial [O_2]} \Delta [O_2] + \frac{\partial V}{\partial T} \Delta T \\
\approx \left(\frac{\Delta [O]}{[O]} + \frac{2 + 7.7 \times 10^{-14} \text{ cm}^3 [O_2]}{(1 + 7.7 \times 10^{-14} \text{ cm}^3 [O_2])} \frac{\Delta [O_2]}{[O_2]} - \frac{510}{T^2}\right) V(8,3). \tag{2.35}'
\]

Through replacing (2.31), (2.33), and (2.35) with (2.31)', (2.33)', and (2.35)', respectively, and following the same subsequent steps described before to calculate the CFs from the Swenson method, then we can compare them with the CFs derived from the full-wave model in the later chapter.
Chapter 3.

Comparisons between the Analytical and Numerical results under various atmospheric conditions

3.0 Linear Comparability

The CF for the OH nightglow intensity is a quantity that basically depends on the properties of the atmospheric state (temperature, thermodynamic parameters, concentrations of major constituent N\textsubscript{2}, O\textsubscript{2} and minor constituent O, O\textsubscript{3}, OH, H, HO\textsubscript{2}, and winds condition), chemical reaction rate constants, wave period, and horizontal wavelength. The analytical model of Swenson and Gardner [1998] (hereinafter referred to as SG98) accounted for the nonlinear response of the minor species density to gravity wave perturbations. It is described by equation (2.18),

\[ n(p, t) = e^{-\xi v(z)} n_u (z - \xi), \]

where \( \xi \) is the vertical displacement related to the vertical wind. The relative density perturbation of neutral atmospheric layers is given as equation (2.23),

\[ \rho(z) / \rho_u (z) = \exp\left[ \frac{(\gamma - 1)}{\gamma H} \xi(z) \right]. \]

In Appendix 3, we use the conserved potential temperature \( \theta \) in adiabatic motions with \( D\theta / Dt = 0 \) to calculate \( \xi \) provided \( T'/T \), this approach is consistent with the linearized equation (2.23) under an isothermal and adiabatic atmosphere.

The SG98 model applied for the special case where the mean winds are zero, and for an isothermal and adiabatic atmosphere. The relation between \( \xi \) and \( |T'/T| \) derived from the SG98 model at the altitude of OH\textsuperscript{*} emission layer peak (Z\textsubscript{OH}) is shown in Table 4. The values of \( \xi(Z_{OH}) \) become nonlinear with the increasing amplitude \( |T'/T|_{Z_{OH}} \) larger than 1\%.
Table 4. The relation between $\xi$ and $|T'/T|$ at $Z_{OH}$ (about 87 km)

Let $C_L = \frac{\xi(Z_{OH})}{|T'/T|_{Z_{OH}}}$, where $|T'/T|_{Z_{OH}} = 0.1\%$. So $\frac{C}{C_L} = \frac{\xi(Z_{OH})}{\xi(Z_{OH})/|T'/T|_{Z_{OH}}}$

| $|T'/T|_{Z_{OH}}$   | 0.1% | 1%  | 5%  | 10% |
|---------------------|------|-----|-----|-----|
| $\xi(Z_{OH})$       | 20.454m | 203.631m | 998.479m | 1950.563m |

$C / C_L$ = 1 0.9955 0.9763 0.9536

We further compare the CFs derived from the SG98 model for 0.1%, 1%, 5% and 10% of $|T'/T|_{Z_{OH}}$ to determine the range of wave amplitudes for which the CFs behave linearly (Figure 6). The horizontal wavelength of involved gravity waves is 500 km, and the intrinsic phase velocity ranges from 30 m s$^{-1}$ to 180 m s$^{-1}$. The results consistent with Table 4 show that the CFs behave linearly when the wave amplitude at $Z_{OH}$ is less than or equal to 1%; the CFs behave nonlinearly with wave amplitudes at $Z_{OH}$ increasing from 1% towards 5% and 10%.

In order to evaluate the influences of chemistry and dynamics on the OH nightglow, Walterscheid et al. [1987] defined the chemical time constants in an operational sense. They determined the time scales at which there are differences between their Figure 2 (which included chemical and dynamical effects) and their Figure 9 (which included dynamical effects only). The common method to define these time scales is based on the chemical loss rate of each species, but the nonlinear chemistry causes the system to be highly coupled so that the common method gives misleading results. The chemical time constant

32
for the greatest sensitivity to the \( \text{O}_3 \) scale height is the order of 10 min where dynamical and chemical time scales are comparable, but the chemical time constants for \( \text{H} \) and \( \text{O} \) are even longer than 10 hours when compared to the 10 hours periods of gravity waves [Walterscheid et al., 1987].

By using 0.1\% of \( \left| \frac{T'}{\bar{T}} \right|_{z_{\text{off}}} \) to ensure linearity, in the following sections, for the geophysical location of latitude 39 degree, longitude -106.46 degree, for fixed horizontal wavelengths (\( \lambda_h \)) of 100 km and 500 km, and for 100 different waves with intrinsic velocities ranging from 30 m s\(^{-1}\) to 180 m s\(^{-1}\), we compare the CFs derived from the full-wave model with those derived from the SG98 model under various types of atmospheric conditions.

3.1 Isothermal, Quasi-adiabatic condition

Here the atmosphere is assumed to be isothermal with the fixed temperature (~ 194.7 K) set to the mean value at the altitude of the maximum volume emission rate (VER) for the OH airglow. The mean temperatures of December 15 as a function of altitude derived from the MSIS-90 model are plotted in Figure 2. The unperturbed VER for the OH airglow is plotted as a function of height in Figure 7 where the altitude of the maximum VER is located at ~ 87.2 km. In the quasi-adiabatic condition, the molecular diffusion coefficients in the full-wave model are all significantly reduced to a fraction of their nominal values. In addition, the eddy diffusivity is taken to be a constant equal to a small "background" component (~ 0.1 m\(^2\) s\(^{-1}\)).

The CF differences between the SG98 and full-wave models for a fixed horizontal wavelength of 100 km are shown in Figure 8. The wave intrinsic phase velocities range from 30 m s\(^{-1}\) to 180 m s\(^{-1}\) with corresponding wave periods ranging from ~ 55 minutes to ~ 9 minutes. The vertical wavelength becomes shorter for slower intrinsic phase speed. Consequently the CF decreases with decreasing intrinsic phase speed due to increasing destructive interference between the positive and negative VER fluctuations over altitude [Hines and Tarasick, 1994; Taylor et al., 1995b; Walterscheid et al., 1999]. In order to
assess the relative importance of dynamical and chemical effects, we compare the CF results obtained using all processes together with those obtained using either chemistry or dynamics alone. The CFs derived from the SG98 model using all processes together and the CFs derived from the SG98 model using chemical process only approach each other for the lower frequency, slower waves, but these two curves depart as the waves becoming faster. The neglect of dynamics seems to be unimportant for gravity waves of low frequencies, because the long periods are comparable with the chemical time constants (in the order of 10 min for O₃) with the result that the chemical reactions play a dominant role.

Similarly, for the higher frequencies, faster waves, the CFs derived from the full-wave model including all processes together and the CFs derived from the full-wave model including only dynamical process approach each other. If the periods of faster waves are sufficiently short compared with the chemical time constants then the neglect of chemistry is a tolerable treatment in the high frequency wave region. Also the same trend appears in the full-wave case in the lower frequency region where the CFs derived from the full-wave model including all processes together and the CFs derived from the full-wave model including only chemical process approach each other, but these two curves also diverge as the waves becoming faster.

The CF differences between the SG98 and full-wave models for a fixed horizontal wavelength of 500 km are shown in Figure 9. Again the wave intrinsic phase velocities range from 30 m s⁻¹ to 180 m s⁻¹ with corresponding wave periods ranging from ~ 4 hours 37 minutes to ~ 46 minutes. Figure 9 displays similar tendencies in both models, especially for the much longer period (several hours). In this much slower wave region the CFs derived from the full-wave model including all processes together and the CFs derived from the full-wave model including only chemical process approach each other very closely. It is further confirmed that chemical reactions will play a dominant role when the gravity wave periods are comparable or much longer than the chemical time
constants. During comparing the CFs derived from both models including all processes together, we find the CFs derived from the full-wave model are unexpectedly larger than those derived from the SG98 model displaying in the long period region in Figure 9. It is suggested that the gravity waves with periods of several hours are much more observable due to the inclusion of the more complete chemistry reaction scheme in Table 3.

3.2 Non-isothermal, Quasi-adiabatic condition

In this case the mean temperature profile of December 15 calculated from MSIS-90 model is employed here for a non-isothermal atmosphere (Figure 2). The diffusion coefficients are kept as the same as those used in section 3.1 to maintain quasi-adiabatic condition.

The comparisons of the CFs derived from the SG98 and full-wave models for fixed horizontal wavelengths of 100 km and 500 km are shown in Figure 10 and Figure 11, respectively. In both Figure 10 and Figure 11 the CFs derived from the full-wave model in a non-isothermal atmosphere are obviously greater than those in an isothermal atmosphere described in section 3.1 during the whole range of wave periods. The non-isothermal atmosphere condition could make the gravity waves become much more observable basically. The existence of thermal gradients implies that the Brunt-Väisälä period \( T_B \) is a function of altitude (Figure 3). Short period GWs (~ 10 min periods) will become evanescent near altitudes of ~ 200 km in the thermosphere and be reflected there. These are the faster GWs (~ 160 m s\(^{-1}\) and faster for \( \lambda_h = 100 \) km). Therefore, interference will occur in the airglow region due to the original upward propagating GW and downward propagating reflected GW. The interference depends on vertical wavelength and so depends on phase speed. Figure 10 and Figure 11 show that the CFs derived from the full-wave model in a non-isothermal atmosphere exhibit variations with varying phase speed for the faster gravity waves compared to those in an isothermal atmosphere. Fast GWs propagate upward with appreciable reflection in a non-isothermal atmosphere with vertical wavelengths.
becoming very large in the high frequency region. The variation of temperature with height also influences the temperature-dependent chemical reaction rates that further affect the CFs through the chemical process.

3.3 Isothermal, Non-adiabatic condition

In this case the constant temperature used in section 3.1 and the nominal eddy and molecular diffusion coefficients are employed here. The comparisons of the CFs derived from both the SG98 and full-wave models for a fixed horizontal wavelength of 100 km are shown in Figure 12. Figure 12 shows that under isothermal and non-adiabatic conditions, the CF derived from the SG98 model is a factor of $\sim 0.30$ less than that derived from the full-wave model for a single slow GW of $V_{ph} = 40 \text{ m s}^{-1}$, and the CF derived from the SG98 model is a factor of $\sim 1.47$ greater than that derived from the full-wave model for a single fast GW of $V_{ph} = 170 \text{ m s}^{-1}$. Similar comparisons of the CFs derived from both the SG98 and full-wave models for a fixed horizontal wavelength of 500 km are shown in Figure 13. Figure 13 shows that under isothermal and non-adiabatic conditions, the CF derived from the SG98 model is a factor of $\sim 0.33$ less than that derived from the full-wave model for a single slow GW of $V_{ph} = 40 \text{ m s}^{-1}$, and the CF derived from the SG98 model is a factor of $\sim 2.85$ greater than that derived from the full-wave model for a single fast GW of $V_{ph} = 170 \text{ m s}^{-1}$.

In order to understand the role played by a dissipative atmosphere, we compare the results under a dissipative atmosphere with those under an adiabatic atmosphere for fixed horizontal wavelengths of 100 km (Figure 14) and 500 km (Figure 15), respectively. Although basically a dissipative atmosphere doesn't make any difference to the CFs derived from the full-wave model in the $\lambda_h = 100 \text{ km}$ case (Figure 14), in the $\lambda_h = 500 \text{ km}$ case a non-adiabatic atmosphere makes the CFs derived from the full-wave model slightly larger than those under an adiabatic atmosphere in the low frequency region (Figure 15). Figure 15 shows that under the isothermal condition, the CF derived from the full-wave
model in an adiabatic atmosphere is a factor of ~ 0.81 less than that in a non-adiabatic atmosphere for a single slow GW of $V_{ph} = 30 \text{ m s}^{-1}$. The larger CF corresponds to smaller energy & momentum fluxes due to the relations in equations (1.29) and (1.30). Including eddy viscosity will cause wave energy & momentum to be dissipated for very slow GWs. The results derived from the full-wave model in a non-adiabatic atmosphere (Figure 15) in the high frequency region display variation with varying phase speed (compared to those in an ideal atmosphere) because of reflections occurring for GWs of large vertical wavelength.

3.4 Non-isothermal, Non-adiabatic condition

In this case the mean temperature profile of December 15 calculated from MSIS-90 model (Figure 2) and the nominal eddy and molecular diffusion coefficients are used. Figure 16 displays the comparisons of the CFs derived from both the SG98 and full-wave models for a fixed horizontal wavelength of 100 km under the non-isothermal and non-adiabatic conditions. In Figure 16, the CF derived from the SG98 model is a factor of ~ 0.13 less than that derived from the full-wave model for a single slow GW of $V_{ph} = 40 \text{ m s}^{-1}$, and the CF derived from the SG98 model is a factor of ~ 1.34 greater than that derived from the full-wave model for a single fast GW of $V_{ph} = 170 \text{ m s}^{-1}$. The comparisons of the CFs derived from both the SG98 and full-wave models for a fixed horizontal wavelength of 500 km are shown in Figure 17. Because the results shown in Figure 17 have similar behavior to those shown in Figure 16, they will be discussed no further.

We choose two GWs having intrinsic phase velocities of $50 \text{ m s}^{-1}$ and $110 \text{ m s}^{-1}$, and a horizontal wavelength of 500 km. The amplitude of the temperature perturbation is plotted as a function of height in Figure 18. The results derived from the full-wave model show that the faster GW (larger vertical wavelength) is less dissipated than the slower GW (smaller vertical wavelength). Also Swenson
and colleagues consider undamped GWs in their approach, and so the temperature perturbation amplitudes exhibit identical variations with altitude for the slow and fast GWs in the SG98 model. The different variations of the temperature perturbation amplitudes with height for the slow and fast GWs in the full-wave model not only affect the CFs through dynamical effects, but also influence the temperature-dependent chemical reaction rates that further affects the CFs.

3.5 Non-isothermal, Non-adiabatic condition with Tidal winds

In this section the effect of background winds on the cancellation factor derived from the full-wave model is studied using an empirical model of the mean winds [Hedin et al., 1996]. Mean winds depend on position (altitude, latitude and longitude), season, local time, and also vary with the level of solar and geomagnetic activities at thermospheric altitudes. Consideration of a large number of different wind profiles in our simulations is beyond the scope of this thesis. Instead, in order to demonstrate differences between the cancellation factors calculated with the two models when winds are included only in the full-wave model, we choose to consider a single wind profile. Hickey and Brown [2002] demonstrated that mean winds can have a significant influence on the observation of some gravity waves propagating within airglow emission layers. Accordingly, we choose a wind profile such that some of the eastward propagating gravity waves considered will encounter critical levels that have an effect on the calculated airglow emission fluctuations.

Simulations are performed for December 15 condition using the mean state temperature (Figure 2) and nominal eddy diffusion coefficients discussed earlier. The mean meridional and zonal winds are shown as a function of height in Figure 19. The zonal wind is larger than the meridional wind throughout the OH airglow region and so our wave simulations are based on wave propagation in the zonal direction (the direction of maximum wind effect).
We have found that when mean winds are included in the full-wave model simulations the variation of the cancellation factor with extrinsic phase velocity ($V_{ph}$) is not as first expected for phase velocities less than about 56 m s$^{-1}$. Before presenting the calculated cancellation factors we first present detailed results for two gravity waves, which allows us to understand and explain this unexpected behavior. These two gravity waves have a horizontal wavelength ($\lambda_h$) of 100 km and extrinsic phase velocities of 50 m s$^{-1}$ and 70 m s$^{-1}$, respectively.

The intrinsic phase velocity for each wave is plotted as a function of altitude in Figure 20. The slower gravity wave ($V_{ph} = 50$ m s$^{-1}$) experiences critical levels at $\sim$ 43.3 km and $\sim$ 73.6 km altitude (where its intrinsic phase velocity is zero), whereas the faster gravity wave ($V_{ph} = 70$ m s$^{-1}$) does not experience a critical level. The temperature perturbation amplitude calculated using the full-wave model (where the perturbation source locates at 60 km) is shown as a function of altitude for the two gravity waves in Figure 21. The amplitude of the faster gravity wave ($V_{ph} = 70$ m s$^{-1}$) varies smoothly as a function of height and gradually increases from the altitude shown about 78 km up to about 113 km, after which its amplitude decreases with increasing altitude as a consequence of molecular dissipation. The amplitude of the slower gravity wave ($V_{ph} = 50$ m s$^{-1}$) decreases rapidly with increasing altitude at the altitudes shown in the figure. This behavior is consistent with severe damping due to eddy diffusion associated with the small intrinsic phase velocities and small vertical wavelengths in the vicinity of the critical level near 73 km altitude. Above about 80 km altitude where the intrinsic phase velocity of the wave has increased again dissipation is small and the wave amplitude increases smoothly with increasing altitude in a manner similar to that of the faster wave. In our model we set all wave temperature perturbation amplitudes equal to each other ($\sim 0.2$ K) at the altitude of maximum volume emission rate (VER) for the OH airglow (that is, at $\sim$ 87 km altitude in Figure 7). The result of this is that for the slower wave ($V_{ph} = 50$ m s$^{-1}$) the temperature perturbation amplitude is extremely large at the lower altitudes shown in the figure as a consequence of the critical level at $\sim$ 73 km altitude.
The real part of the VER perturbation ($\Delta V$) is shown for the two gravity waves as a function of altitude in Figure 22. For the fast gravity wave ($V_{ph} = 70$ m s$^{-1}$, dashed-dotted curve) $\Delta V$ is approximately zero at the lowest altitudes shown in the figure and grows slowly with increasing altitude while fluctuating between positive and negative values. Although the slow gravity wave ($V_{ph} = 50$ m s$^{-1}$, solid curve) exhibits similar behavior to this at most altitudes shown, at the lowest altitudes shown $\Delta V$ is extremely large. In order to calculate the brightness fluctuations, $\Delta V$ is integrated over altitude (the imaginary part of the VER fluctuations are similarly integrated, but for the sake of brevity we only consider the real part of $\Delta V$). When $\Delta V$ is integrated for the fast wave there is much cancellation between the positive and negative contributions to the integral with the result that the brightness fluctuation is relatively small. However, for the slow wave the large values of $\Delta V$ near the lower boundary survive these cancellation effects with the result that the brightness fluctuation is large. Because the cancellation factor is proportional to the brightness fluctuation divided by the temperature perturbation at the altitude of maximum undisturbed OH VER, and because the latter has a constant value of ~ 0.2 K for all waves considered, the cancellation factor essentially depends only on the brightness fluctuation in our full-wave simulations. Therefore, because the slow wave encounters a critical level near about 73 km altitude its associated brightness fluctuations are extremely large (for a temperature perturbation amplitude of ~ 0.2 K at the altitude of maximum undisturbed OH VER). This means that the cancellation factor is significantly greater for those gravity waves experiencing critical levels at altitudes below the OH airglow region. Our results give a numerical cancellation factor value of ~ 12.6 for the slow wave versus a value of ~ 5.6 for the fast wave.

It should be noted that because gravity waves experiencing critical levels at altitudes below the OH airglow region will be strongly dissipated, their energy and momentum fluxes will be substantially reduced at OH airglow region altitudes. For reasonable lower atmospheric sources this means that, in fact, these waves would have very small and probably undetectable amplitudes at
airglow altitudes. Therefore, it is unlikely that our simulated cancellation factors that are influenced by critical levels could ever be verified by observation.

The cancellation factor calculated using the SG98 model, and using the full-wave model (both with and without winds included) is shown as a function of extrinsic phase velocity for the wave with $\lambda_h = 100$ km in Figure 23. The cancellation factor derived from the SG98 model (solid curve) increases smoothly from a value of $\sim 0.1$ for $V_{ph} \approx 35$ m s$^{-1}$ to a value of $\sim 5.8$ for $V_{ph} = 180$ m s$^{-1}$. The cancellation factor derived from the full-wave model calculated without winds (dotted curve) increases from a value of $\sim 1.1$ for $V_{ph} = 30$ m s$^{-1}$ to a value of $\sim 3.4$ for $V_{ph} = 180$ m s$^{-1}$. Differences between these full-wave model cancellation factors and the SG98 values are greatest for the slower waves. The full-wave model cancellation factor calculated including winds (dashed-dotted curve) varies from very large value of $\sim 7000$ for $V_{ph} = 30$ m s$^{-1}$ to a value of $\sim 4$ for $V_{ph} = 180$ m s$^{-1}$. Most of the values of cancellation factor for the slow waves calculated with the full-wave model including winds are not shown on this figure because they are extremely large as a result of critical level encountering at altitudes below the OH airglow layer (as discussed previously). In addition, such large values of cancellation factor are unlikely to ever be observed (as previously discussed).

The cancellation factor calculated using the SG98 model, and using the full-wave model (both with and without winds included) is shown as a function of extrinsic phase velocity for the wave with $\lambda_h = 500$ km in Figure 24. There is no further discussion here since the results shown in Figure 24 have similar behavior to those shown in Figure 23.
Figure 6. Linear & nonlinear CF comparisons in the SG98 model

\[ \lambda_h = 500 \text{km} \]

Intrinsic Phase Velocity (m/s)

Cancellation Factor

- --- \( T' \) 0.1\% at Z_OH
- - - - \( T' \) 1\% at Z_OH
- --- - \( T' \) 5\% at Z_OH
- --- --- \( T' \) 10\% at Z_OH
Figure 7. The Background Volume Emission Rate

Altitude (km)

Background VER (# m^{-3} s^{-1})
Figure 8. Isothermal and quasi-adiabatic atmosphere

\[ \lambda_h = 100 \text{km} \]
Figure 9. Isothermal and quasi-adiabatic atmosphere

$\lambda_h = 500\text{km}$
Figure 10. Non-isothermal and quasi-adiabatic atmosphere

$\lambda_h = 100\text{km}$

- SG98 --- chemical and dynamical processes
- SG98 --- chemical process only (hairline)
- Full-Wave --- chemical and dynamical processes
- Full-Wave --- chemical process only
- Full-Wave --- dynamical process only
- Full-Wave --- complete processes in isothermal atmosphere

Intrinsic Phase Velocity (m/s)

Cancellation Factor

$10^{-1}$ $10^1$
Figure 11. Non-isothermal and quasi-adiabatic atmosphere

$\lambda_h = 500$ km

<table>
<thead>
<tr>
<th>Line Style</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid</td>
<td>SG98 --- chemical and dynamical processes</td>
</tr>
<tr>
<td>Dashed</td>
<td>Full-Wave --- chemical and dynamical processes</td>
</tr>
<tr>
<td>Dashed-Dot</td>
<td>Full-Wave --- chemical process only</td>
</tr>
<tr>
<td>Dotted</td>
<td>Full-Wave --- dynamical process only</td>
</tr>
<tr>
<td>Dashed-Dotted</td>
<td>Full-Wave --- complete processes in isothermal atmosphere</td>
</tr>
</tbody>
</table>
Figure 12. Isothermal and non-adiabatic atmosphere

$\lambda_h = 100\text{km}$

Cancellation Factor

$10^0$ $10^1$

Intrinsic Phase Velocity (m/s)

30 50 70 90 110 130 150 170

SG98 --- chemical and dynamical processes
SG98 --- chemical process only (hairline)
Full-Wave --- chemical and dynamical processes
Full-Wave --- chemical process only
Full-Wave --- dynamical process only
Figure 13. Isothermal and non-adiabatic atmosphere

\[ \lambda_h = 500 \text{ km} \]
Figure 14. Isothermal atmosphere

$\lambda_h = 100 \text{km}$

Cancellation Factor

Intrinsic Phase Velocity (m/s)
Figure 15. Isothermal atmosphere

\[ \lambda_h = 500 \text{km} \]

Graph showing the relationship between Cancellation Factor and Intrinsic Phase Velocity (m/s). The graph includes three types of atmosphere:
- **SG98** — adiabatic atmosphere
- **Full-Wave** — non-adiabatic atmosphere
- **Full-Wave** — adiabatic atmosphere

The x-axis represents the Intrinsic Phase Velocity (m/s), ranging from 30 to 170. The y-axis represents the Cancellation Factor, ranging from $10^{-1}$ to $10^1$. The graph illustrates how the Cancellation Factor changes with varying Intrinsic Phase Velocity for each type of atmosphere.
Figure 16. Non-isothermal and non-adiabatic atmosphere

\[ \lambda_h = 100 \text{km} \]

- SG98 — chemical and dynamical processes
- SG98 — chemical process only (hairline)
- Full-Wave — chemical and dynamical processes
- Full-Wave — chemical process only
- Full-Wave — dynamical process only

Intrinsic Phase Velocity (m/s)

Cancellation Factor

30 50 70 90 110 130 150 170

10^{-1} 10^{0} 10^{1}
Figure 17. Non-isothermal and non-adiabatic atmosphere

\[ \lambda_h = 500 \text{km} \]

Cancellation Factor

Intrinsic Phase Velocity (m/s)
Figure 18. Non-isothermal and non-adiabatic atmosphere

\[ \lambda_h = 500 \text{km} \]
Figure 19. Meridional and zonal mean winds profile of Dec 15
Figure 20. Intrinsic velocity profile

Extrinsic Vph = 50 m/s
Extrinsic Vph = 70 m/s
Figure 21. Non-isothermal, dissipative atmosphere with tidal winds of Dec 15

\[ \lambda_h = 100 \text{km} \]
Figure 22. The Volume Emission Rate Perturbation

\[ \lambda_h = 100 \text{km} \]

- Real (\( \Delta V \)) (\# m\(^{-3}\) s\(^{-1}\))
- Altitude (km)

Extrinsic Vph = 50 m/s
Extrinsic Vph = 70 m/s
Figure 23. Non-isothermal and dissipative atmosphere of Dec 15

\( \lambda_h = 100 \text{km} \)

- **SG98** — no winds
- **Full-Wave** — azimuth eastward with winds
- **Full-Wave** — no winds

Extrinsic Phase Velocity (m/s)

Cancellation Factor

Extrinsic Phase Velocity (m/s)
Figure 24. Non-isothermal and dissipative atmosphere of Dec 15

\( \lambda_h = 500 \text{km} \)

- SG98 --- no winds
- Full-Wave --- azimuth eastward with winds
- Full-Wave --- no winds

Extrinsic Phase Velocity (m/s)
Conclusions and scope for future work

The SG98 model demonstrates the significance of cancellation effect in the induced perturbation of OH emission intensity for GW of short vertical wavelength. Swenson and colleagues employed the relatively simple reactions of OH chemistry (compared with Table 3), and applied the analytical model to an isothermal and adiabatic atmosphere where the mean winds are zero. It is fairly possible for the SG98 model to be extended to handle the nonlinear issues of the OH nightglow responses when GW amplitudes increase.

The magnitudes and phases of the OH brightness fluctuations depend on the disturbing GW amplitudes, periods, horizontal wavelengths and the steady state of atmosphere. Within the context of linear GW theory, in order to quantitatively interpret the GW-induced variations in the OH nightglow intensity, we introduce the full-wave model combined with the chemical reactions scheme for OH (8, 3) Meinel emission (Table 3) to provide a more accurate and also more realistic approach to study the GW-Airglow interactions.

Including additional minor species (OH*(8)) specified in the chemical scheme through radiation and quenching influences on the CFs derived from the full-wave model in the long GW period region where the chemical processes dominate. For GWs with periods much longer than the prime chemical time constants (for example, several hours period), the CFs derived from the full-wave model in this region are not expected as those derived from the SG98 model. For a single slow GW with period of ~ 3 hours 28 minutes (corresponding to \( V_{ph} = 40 \) m s\(^{-1}\) and \( \lambda_h = 500 \) km in Figure 17), a factor of ~ 0.15 difference in the CFs between the SG98 and full-wave models could make up to a factor of ~ 46.28 difference in the wave fluxes, because the wave fluxes are proportional to the inverse-square of CF.

Due to the real atmosphere we apply to is a non-isothermal and dissipative atmosphere, the seeming odd large valves of CF in the long period
(e.g., several hours) region can then be better interpreted. Since the lower frequency GWs are more dissipated than the higher frequency GWs, when the slower GWs rise into the rare atmosphere, the effects of viscosity and thermal conduction are provided more time to damp them away than those faster GWs. The energy of the slower GWs may be severely dissipated even before they reach the altitude of the OH* emission layer peak. Those larger values of CF result from more dissipation to the long period GWs (the energy flux depends on the inverse-square of the CF).

The extremely large values of CF occurring in the slow GW region when we further include the tidal winds are strong evidences of the GW-critical layer interaction. The critical layer occurs when the Doppler-shifted frequency is zero where the GW’s horizontal phase velocity becomes equal to the mean flow velocity. The interaction of a GW with the mean flow near the critical layer results in a severe GW attenuation with much of its energy and momentum being absorbed by the mean flow. The horizontal winds vary in a complicated way with local time and seasons. The zonal wind profile (since GWs are traveling in eastward direction) will determine certain GWs encountering the critical layer under the altitude of OH nightglow layer peak. Then most of their energy and momentum have been blocked under the altitude of OH nightglow layer peak, that’s the reason for the CFs derived from those certain GWs turning out to be extremely large values.

There are still much further works to do to find out the behavior of the GW-critical layer interaction, this instability results from the strong coupling between GWs and background winds. Actually the mean winds will simultaneously vary with the GWs exchanging their momentum and energy to the background. As GWs propagate upward, the vertical momentum flux divergence plays a role to decelerate the mean flow in the mesosphere [Swenson and Liu, 1998]. So a new time-dependent and/or nonlinear GW model being developed afterward will provide better chance to explain the wind-wave interactions. Moreover, this initial study of the monochromatic GWs implies that a completely spectral calculation of those CFs in the OH nightglow be worth to be further investigated.
We have compared one model (the Hickey numerical full-wave model) to another model (the Swenson analytical model), but the verification of the models can only be made by comparing model results with measurements of GWs. The comparisons between models in this thesis involve GWs with fixed horizontal wavelengths of 100 km and 500 km, periods ranging from ~9 minutes to ~4 hours 37 minutes, and vertical wavelengths ranging from ~10 km to ~67 km. So those measurements for verification must be complete enough to allow a thorough comparison with the models. It will require airglow imaging measurements of GWs, mean wind measurements (to provide knowledge of the intrinsic GW periods), and a measurement of the GW amplitude in the major gas. GW amplitude (and its height variation) could be obtained using a Na lidar or suitable radar (e.g., Median Frequency radar). Gardner and Taylor [1998] examined the observational limits for lidar, radar, and airglow imager measurements of middle atmosphere gravity waves. As a consequence of the constructive and destructive interferences of OH nightglow signals from vertically separated levels, the range of the GWs seen by OH imagers are associated with GWs having vertical wavelengths comparable to or greater than the thickness of the main OH emission region (~10 km). According to Gardner and Taylor [1998], airglow imagers observe the long vertical wavelength, short-period waves, while the lidars and radars observe the short vertical wavelength GWs. Although fortunately lidars, radars, and imagers are often most sensitive to GWs in largely different regions of the spectrum, their combined coverage excludes the long vertical wavelength, long-period waves. Gardner and Taylor [1998] showed that GWs with periods longer than about 5 hours, vertical wavelengths exceeding 15 – 20 km, and horizontal wavelengths exceeding ~1000 km were not sampled. So the measurements required for verifying the results presented in this thesis need to be co-located and made in a campaign style, covering a long time period under different geophysical conditions & covering a wide range of wave parameters (e.g., vertical wavelength). Even then, there are still some results of this modeling study that can never be completely verified because of their unobservability (e.g., the critical level effect described in section 3.5 of Chapter
3). But once we obtain the observed airglow brightness fluctuation and the wave amplitude observed directly from lidar or radar observations, we can obtain the observational CF. The CF that the numerical model or analytical model generates should be validated by comparing it to the experimental CF. Once its validity has been established, the modeling can be used for various prediction purposes, within the limits imposed by the assumptions on which it was based.

Recently Hickey and Brown [2002] derived the wave amplitude by normalizing the model-derived airglow fluctuation amplitude to that observed from the ground during the ALOHA-93 campaign. The model [Hickey and Brown, 2002] provided momentum and energy fluxes as a function of height as well as the flux divergences, from which the mean state forcing was evaluated. Also, the importance of critical level on airglow fluctuations was emphasized.
Appendix 1.

The complex dynamical factors $f_1, f_2,$ and $f_3$ connect $\nabla \cdot \mathbf{v}', \mathbf{w}',$ and $\mathbf{n}'(M)$ respectively to $\hat{T}'$ of a linearized acoustic–gravity wave propagating in a viscous, thermally conducting and rotating (though windless) isothermal atmosphere [Hickey, 1988a]:

$$f_1 = i\omega \left\{ vR - \frac{1}{(\gamma - 1)} \right\}, \quad (A1.1)$$

$$f_2 = \frac{(\omega / k_x) \left\{ x_1 + x_3 \left\{ vR - (\gamma - 1)^{-1} \right\} \right\}}{(x_2 - i\alpha x_3)}, \quad (A1.2)$$

$$f_3 = \frac{\left\{ i\alpha x_1 + x_2 \left\{ vR - (\gamma - 1)^{-1} \right\} \right\}}{(i\alpha x_3 - x_2)}. \quad (A1.3)$$

Where

$$x_1 = (i\alpha - \kappa)(\phi - c^2\phi^{-1}) + 3i\alpha \eta',$$

$$x_2 = -\eta'(\kappa - 3i\alpha)(\kappa + 2i\alpha) + \left\{ \eta'(4R - 1) - \beta' \right\}(\phi - c^2\phi^{-1} + \eta'),$$

$$x_3 = \kappa(\phi - c^2\phi^{-1}) - 2i\alpha \eta'.$$

Also,

$$\phi = 3\eta' R - \beta' \quad R = \kappa^2 - i\alpha \kappa + 1$$

$$\kappa = \frac{k_x + (i/2H)}{k_x} \quad \alpha = \frac{1}{k_x H}$$

$$\beta' = \frac{\omega^2}{gHk_x \bar{\nu}} \quad \eta' = \frac{i \omega \mu}{3 \bar{\rho}}$$

$$\nu = \frac{i \lambda \bar{T}k_x}{\omega \bar{\rho}} \quad c = \frac{f \omega}{gHk_x^2}$$

$$f = 2\Omega \sin \theta$$
Here $\bar{p}$ is the unperturbed pressure, $\bar{T}$ is the unperturbed temperature, $\omega$ is the wave angular frequency, $k_x$ is the horizontal wave number, $k_z$ is the complex vertical wave number, $g$ is the acceleration of gravity, $H$ is the pressure scale height, $\mu$ is the coefficient of eddy viscosity, $\lambda$ is the coefficient of eddy thermal conduction, $f$ is the Coriolis parameter, $\Omega$ is the Earth’s angular velocity, and $\theta$ is the latitude. The complex vertical wave number $k_z$ can be obtained from the quartic dispersion equation of Hickey and Cole [1987].

Equations (A1.1) – (A1.3) are derivable after some algebra from the nondimensional equations of momentum, continuity, and energy (first using the ideal gas equation to eliminate the pressure) [Hickey, 1988a]

\[
(\phi + \eta') \hat{u}_1 - ic\hat{v}_1 + \eta'(\kappa - 3i\alpha)\hat{w} + \frac{\hat{p}}{\hat{\rho}} + \frac{\hat{T}}{\hat{T}} = 0
\]  
\[\hat{v}_1 = -ic\phi^{-1} \hat{u}_1 \]  
\[\eta'(\kappa + 2i\alpha) \hat{u}_1 + [\eta'(4R - 1) - \beta']\hat{w}_1 + \kappa \frac{\hat{p}}{\hat{\rho}} + (\kappa - i\alpha) \frac{\hat{T}}{\hat{T}} = 0 \]  
\[\hat{u}_1 + (\kappa - i\alpha)\hat{w}_1 = \frac{\hat{p}}{\hat{\rho}} \]  
\[\hat{u}_1 + \kappa \hat{w}_1 = \left(\frac{1}{(\gamma - 1) - \nu R}\right) \frac{\hat{T}}{\hat{T}} \]

where $\rho$ is mass density, $u$ is the meridional velocity, $v$ is the zonal velocity, $w$ is the vertical velocity, and the velocity components have been nondimensionalized by multiplication of $k_z/\omega$, e.g., $\hat{u}_1 = k_z \hat{u}/\omega$. 

66
Appendix 2.

The 5 X 5 matrix on the left side of equation (2.9) is [Walterscheid et al. 1987]

\[
\begin{pmatrix}
  i\omega + k_7 \tilde{n}(O) & -k_9 \tilde{n}(H) & -k_9 \tilde{n}(O_3) \\
  0 & i\omega + k_9 \tilde{n}(H) & k_9 \tilde{n}(O_3) \\
  -k_7 \tilde{n}(O) & k_9 \tilde{n}(H) & i\omega + k_9 \tilde{n}(O_3) + k_{10} \tilde{n}(O_2) \tilde{n}(M) \\
  k_7 \tilde{n}(O) & 0 & 0 \\
  0 & 0 & -k_{10} \tilde{n}(O_2) \tilde{n}(M)
\end{pmatrix}
\]

\[
\begin{align*}
-k_{11} \tilde{n}(HO_2) + k_7 \tilde{n}(OH) & -k_{11} \tilde{n}(O) \\
-k_2 \tilde{n}(O_2) \tilde{n}(M) & 0 \\
-k_7 \tilde{n}(OH) & 0 \\
i\omega + k_7 \tilde{n}(OH) + k_2 \tilde{n}(O_2) \tilde{n}(M) + k_{11} \tilde{n}(HO_2) & \tilde{k}_{11} \tilde{n}(O) \\
\tilde{k}_{11} \tilde{n}(HO_2) & i\omega + \tilde{k}_{11} \tilde{n}(O)
\end{align*}
\]

The 5 X 1 matrix on the right side of equation (2.9) is [Walterscheid et al. 1987]

\[
\begin{pmatrix}
  \tilde{n}(H) \tilde{n}(O_3) k_9 \left(\frac{470}{T}\right) - f_2 \frac{d}{dz} \tilde{n}(OH) - f_1 \tilde{n}(OH) \\
  k_2 \tilde{n}(O) \tilde{n}(M) \left\{ -n(O_2) \left[ f_3 - \left(\frac{510}{T}\right) \right] + \beta f_3 \tilde{n}(M) \right\} - k_9 \left(\frac{470}{T}\right) \tilde{n}(H) \tilde{n}(O_3) - f_2 \frac{d}{dz} \tilde{n}(O_3) - f_1 \tilde{n}(O_3) \\
  -k_9 \tilde{n}(H) \tilde{n}(O_3) \left(\frac{470}{T}\right) - k_{10} \tilde{n}(H) \tilde{n}(M) \left\{ \tilde{n}(O_2) \left[ f_3 - \left(\frac{290}{T}\right) \right] + \beta f_3 \tilde{n}(M) \right\} - f_2 \frac{d}{dz} \tilde{n}(H) - f_1 \tilde{n}(H) \\
  -k_2 \tilde{n}(O) \tilde{n}(M) \left\{ \tilde{n}(O_2) \left[ f_3 - \left(\frac{510}{T}\right) \right] + \beta f_3 \tilde{n}(M) \right\} - f_2 \frac{d}{dz} \tilde{n}(O) - f_1 \tilde{n}(O) \\
  k_{10} \tilde{n}(H) \tilde{n}(M) \left\{ \tilde{n}(O_2) \left[ f_3 - \left(\frac{290}{T}\right) \right] + \beta f_3 \tilde{n}(M) \right\} - f_2 \frac{d}{dz} \tilde{n}(HO_2) - f_1 \tilde{n}(HO_2)
\end{pmatrix}
\]

Where $T$ is temperature in Kelvins, and $\beta$ is the constant mixing ratio for the molecular oxygen (O2) with respect to major gas (M), i.e. $\tilde{n}(O_2) = \beta \tilde{n}(M)$ and $\tilde{n}'(O_2) = \beta \tilde{n}'(M)$.
Appendix 3.

Here we try to calculate $\xi$ provided by $T'/\bar{T}$. We start from the potential temperature $\theta$ is a quasi-conserved quantity for adiabatic motion.

$$\frac{D\theta}{Dt} = \frac{\partial \theta'}{\partial t} + w \frac{d\theta}{dz} = 0,$$

where we assume $\theta = \bar{\theta} + \theta'$ and $\theta' \propto \exp(i\omega t)$.

Then $i\omega \theta' + w' \frac{d\bar{\theta}}{dz} = 0$, but $\theta = T\left(\frac{p_0}{p}\right)^{\kappa}$, where $p_0 = 1000$ mbar is a standard pressure, $\kappa = R/c_P = \frac{\gamma - 1}{\gamma}$.

So

$$\frac{d\ln \bar{\theta}}{dz} = \frac{d\ln \bar{T}}{dz} - \kappa \frac{d\ln p}{dz},$$

$$\Rightarrow \frac{1}{\theta} \frac{d\bar{\theta}}{dz} = \frac{1}{T} \frac{d\bar{T}}{dz} - \kappa \left( -\frac{1}{H} \right).$$

$$\Rightarrow \frac{d\bar{\theta}}{dz} = \frac{\bar{T}}{T} \frac{d\bar{T}}{dz} + \kappa \frac{\bar{T}}{H}.$$

We assume $w' \propto \exp(i\omega t)$, so $\xi = \int w'dt = \frac{w'}{i\omega}$. Then

$$i\omega \theta' + \bar{\theta} w' \left\{ \frac{1}{T} \frac{d\bar{T}}{dz} + \kappa \frac{1}{H} \right\} = 0,$$

$$\Rightarrow i\omega \frac{\theta'}{\bar{\theta}} + w' \left\{ \frac{1}{T} \frac{d\bar{T}}{dz} + \kappa \frac{1}{H} \right\} = 0,$$

$$\Rightarrow i\omega \frac{\theta'}{\bar{\theta}} + i\omega \xi \left\{ \frac{1}{T} \frac{d\bar{T}}{dz} + \kappa \frac{1}{H} \right\} = 0,$$

$$\Rightarrow \xi \left\{ \frac{1}{T} \frac{d\bar{T}}{dz} + \kappa \frac{1}{H} \right\} = -\frac{\theta'}{\bar{\theta}} = -\left( \frac{T'}{T} - \kappa \frac{p'}{p} \right).$$
If we further assume that the pressure of the air parcel instantaneously adjusts to the environmental pressure during the displacement, i.e. \( \frac{p'}{p} = 0 \), and also under an isothermal atmosphere, i.e. \( \frac{dT}{dz} = 0 \), we get 
\[ \xi = \frac{H}{\kappa} \left( -\frac{T'}{T} \right). \]
For linear perturbation, \( \frac{\rho'}{\rho} \approx -\frac{T'}{T} \), so 
\[ \xi \approx \frac{H}{\kappa} \frac{\rho'}{\rho} = \frac{\gamma H}{\gamma - 1} \frac{\rho'}{\rho}. \]

From equation (2.23), 
\[ \rho / \rho_u = \exp \left[ \frac{(\gamma - 1)}{\gamma H} \xi \right], \]
then for linear perturbation 
\[ \xi = \frac{\gamma H}{\gamma - 1} \ln \left( \frac{\rho}{\rho_u} \right) = \frac{\gamma H}{\gamma - 1} \ln \left( 1 + \frac{\rho'}{\rho} \right) \approx \frac{\gamma H}{\gamma - 1} \frac{\rho'}{\rho}. \]

The approach shown here is consistent with the linearized equation (2.23) under an isothermal and adiabatic atmosphere.
References


Taylor, M. J., M. B. Bishop, and V. Taylor, All-sky measurements of short period waves imaged in the OI (557.7 nm), Na (589.2 nm) and near infrared OH and O\(_2\) (0,1) nightglow emissions during the ALOHA – 93 campaign, *Geophys. Res. Lett.*, 22, 2833 – 2836, 1995b.


