Dynamical Processes of Gravity Waves Propagation and Dissipation, and Statistical Characteristics of Their Momentum Flux in the Mesosphere and Lower Thermosphere

Bing Cao

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DYNAMICAL PROCESSES OF GRAVITY WAVES PROPAGATION AND DISSIPATION, AND STATISTICAL CHARACTERISTICS OF THEIR MOMENTUM FLUX IN THE MESOSPHERE AND LOWER THERMOSPHERE

by

Bing Cao

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By

Bing Cao

This Dissertation was prepared under the direction of the candidate's Dissertation Committee Chair, Dr. Alan Z. Liu and has been approved by the members of his dissertation committee. It was submitted to the College of Arts and Sciences and was accepted in partial fulfillment of the requirements for the

Degree of

Doctor of Philosophy in Engineering Physics

Dr. Alan Z. Liu, Ph.D
Committee Chair

Dr. Michael P. Hickey, Ph.D
Committee Member

Dr. Shawn M. Milrad, Ph.D
Committee Member

Dr. Jonathan B. Snively, Ph.D
Committee Member

Dr. Terry D. Oswalt, Ph.D
Department Chair, Physical Sciences

Dr. Karen F. Gaines, Ph.D
Dean, College of Arts and Sciences

Dr. Michael P. Hickey, Ph.D
Dean of Research and Graduate Studies

05/05/2017

Date
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Abstract

The mesosphere and lower thermosphere (MLT) (~80–110 km) is dominated by abundant atmospheric waves, of which gravity waves are one of the least understood due to large varieties in wave characteristics as well as potential sources. Gravity waves play an important role in the atmosphere by influencing the thermal balance and helping to drive the global circulation. But due to their sub-grid scale, the effects of gravity waves in General Circulation Models (GCMs) are mostly parameterized. The investigations of gravity waves in this dissertation are from two perspectives: the dynamical processes of gravity wave propagation and dissipation in the MLT region, and the climatology and statistical characteristics of gravity waves as physical basics of gravity wave parameterization. The studies are based on the data acquired from an airglow imager and a sodium lidar, with the assistance of some simulation data from a meso-scale numerical model and GCMs.

To understand the dynamical processes in gravity wave propagation and dissipation, a gravity wave should be resolved as fully as possible. The first topic of this dissertation is motivated by the fact that most observational instruments can only capture part of the gravity waves spectrum, either horizontal or vertical structures. Observations from multiple complementary instruments are used to study gravity waves in 3-D space. There are two cases included in this topic. In case 1, a co-located sodium lidar and an airglow imager were used to depict a comprehensive picture of a wave event at altitude between 95–105 km. Thus, the horizontal and vertical gravity waves structures and their ambient atmosphere states were fully characterized, which suggests that a gravity wave undergoes reflection at two different altitudes and near-critical layer filtering in-between. All the retrieved parameters were then applied to a 2-D numerical model whose outputs help to interpret the observations. In case 2, the lidar system is configured in a 5-direction mode, whose laser beams were pointed to zenith and 30° off-zenith at four cardinal directions. Thus, there is a ~50 km separation at ~90 km altitude between zenith and any off-zenith
directions. Besides the vertical information from traditional lidar measurement profiles, horizontal wavelength and propagation direction are derived from the phase differences among measurements in different directions. With a full set of wave and background parameters, multiple dispersion and polarization relations are examined and the results validate the goodness of different assumptions involved in linear gravity wave theory.

Better knowledge of gravity waves from observational and numerical, as well as theoretical studies directly contribute to the development of physically-based parameterizations. The second topic of this dissertation is about long-term climatology and statistical characteristics of gravity waves observed by an airglow imager. The results provide some insights on how the source spectrum can be specified and tuning factors are constrained in the parameterization. Results from two sites are compared, one is in the middle of the Pacific Ocean, and the other above the Andes Mountains. The difference and similarity provide some clues to the effects of wave sources and background flow on the gravity wave climatology and intermittency in the mesopause region.

Firstly, the long-term climatology of intrinsic wave parameters and propagation direction preferences for high-frequency quasi-monochromatic gravity waves observed by an airglow imager is presented. Wave occurrence and propagation direction are related to convective activities nearby and local background winds. The preferential wave propagation during austral summer is poleward and equatorward during winter. The estimated momentum fluxes show a clear anti-correlation with background winds. Secondly, intermittency of gravity waves near mesopause region is studied. The concept of intermittency is originally from the factors used in wave parameterization schemes to describe the fractional coverage of waves within a large spatial grid and/or temporal period in order to accurately quantify the forcing on the atmosphere by dissipating gravity waves. Intermittency of gravity waves was described by the probability density functions of absolute momentum flux and some diagnostic parameters. An explicit probability function that is a piecewise function of lognormal and power law functions
is obtained from airglow data. The relative importance of abundant waves with smaller amplitudes and rare waves with dramatically large amplitudes were compared. Lastly, the duration of gravity waves in the airglow layer is studied. The observed gravity waves duration in the airglow layer is exponentially distributed. Several mechanisms that could lead to such a distribution are put forward from the perspective of wave breaking due to instabilities and blocking due to evanescent regions. Ducted propagation is also an possible factor.

Through individual cases and statistical studies, this dissertation investigates the dynamical processes and statistical characteristics of gravity waves in the MLT region. The results are expected to provide more insight in both observational and modeling research on gravity waves.
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Chapter 1

Introduction

1.1 Background

Earth’s atmosphere is stably stratified, with density decreasing with altitude. A characteristic of a stably stratified fluid is the ability to support wave motion [Nappo, 2012]. Gravity Waves (GWs) are generated when a fluid parcel is perturbed vertically and gravity/buoyancy act as the restoring forces. Atmospheric gravity waves are ubiquitous in the atmosphere and occur at a variety of spatial and temporal scales. Gravity waves are mostly generated in the lower atmosphere by convection, orography, and front and then propagate upward or downward [Fritts and Alexander, 2003, and references therein]. In the mesopause region, those upward-propagating gravity waves can either dissipate through saturation or break due to critical layers [Lindzen, 1981; Dunkerton and Fritts, 1984; Fritts and Rastogi, 1985; Franke and Robinson, 1999; Vadas et al., 2003], or propagate continuously to higher altitudes and dissipate via viscosity [Liu et al., 2013b; Liu and Vadas, 2013]. The corresponding momentum and energy will be deposited to the mean flow in these cases. This process plays an important role in driving the global scale Brewer-Dobson meridional circulation [Holton et al., 1995; Li et al., 2008; Cohen et al., 2014] and influences the atmospheric thermal balance, leading to a dynamical
1.1. BACKGROUND

rather than radiative equilibrium state in the middle atmosphere [Gierasch et al., 1970; Andrews et al., 1987; Liou, 2002]. The gravity wave momentum transport is directly responsible for the cold summer mesopause [Holton, 1982; Siskind et al., 2012] and reversal of the mesospheric jets, alleviating the cold bias in the Southern Hemisphere winter polar stratosphere and helping to drive the quasi-biennial oscillation (QBO) in the tropical lower stratosphere [Ern et al., 2014] and Semianual Oscillation (SAO) in the upper stratosphere/lower mesosphere [Ern et al., 2015]. At small scales, gravity waves contribute to the instability and turbulence processes in the atmosphere [Fritts, 1984; Fritts et al., 2013]. In the ionosphere, gravity waves contribute to irregularities and traveling ionospheric disturbances [Fritts and Lund, 2011; Liu and Vadas, 2013]. At the meteorological scale, they can initiate and modulate convection and disturb the smooth, balanced state, and lead to instabilities and turbulent mixing. All these processes by gravity waves transfer energy and momentum from wave source regions to other places, and couple the whole atmosphere from the bottom to the top. Therefore, our understanding of gravity wave generation, propagation, and dissipation properties has great implications for both weather and climate applications.

The mesosphere and lower thermosphere (MLT), that is beyond the ceiling altitude of aircrafts and balloons but way below the orbital altitude of spacecrafts, remains one of the least observable and understood regions in the atmosphere. The mesopause at \( \sim 85 \) km altitude is the coldest place in the atmosphere and acts as a transition region between the neutral and ionized atmosphere. Currently, it has only been accessed through rockets and remote sensing techniques with limited temporal or spatial coverage. In this altitude range, the existence of abundant atmospheric waves such as planetary waves, tides and gravity waves make the dynamical processes complicated. Those waves carry energy and momentum from the troposphere and stratosphere, propagate upward and reach large amplitudes due to the extremely low density, and become unstable and dissipate there. This dissipation process deposits momentum and energy into the background atmosphere.
and then drives atmospheric circulations and influences the thermal equilibrium. Those waves act to connect the dynamical processes throughout the whole atmosphere.

In the MLT region, there exist metal and airglow layers. The layer of Na atoms was found globally near 90 km altitude, which is mostly produced by ablation of the cosmic dust that enters the Earth atmosphere from interplanetary space [Plane et al., 2015]. Many airglow emissions resulting from chemiluminescent reactions were also found in the Earth upper atmosphere [Khomich et al., 2008]. Several of these emissions originate within the MLT region (altitude range around 80–100 km) as thin luminous layers with typically thickness of 6–10 km (Full Width at Half Maximum, or FWHM). Historically, the first airglow emissions to be investigated were the visible green OI (557.7 nm) line emission (peak height $\sim$96 km) and the Na line emissions centered at 589.2 nm (peak height $\sim$90 km). But the brightest source of airglow is the hydroxyl (OH) Meinel band emission (peak height $\sim$87 km) which radiates over a broad spectral range (0.7–4.0 $\mu$m) primarily in the near infrared (NIR) region. A lot of studies have revealed that these metal atoms and airglow emissions are very useful tracers to retrieve the atmospheric properties and study the dynamical processes such as the gravity waves and other atmospheric waves such as tides and planetary waves [Hickey and Plane, 1995; Taylor, 1997; Walterscheid et al., 1999; Ejiri et al., 2003; Liu and Swenson, 2003; Suzuki et al., 2007; Li et al., 2009].

### 1.2 Linear Theory of Gravity Waves

Many theoretical studies of gravity waves are based on the linear theory. It commonly used to describe the propagation characteristics of gravity waves. The gravity waves are governed by the Euler equations for a set of fundamental variables $q = (p, \rho, u, v, w)$. In the linear theory, each variable $q$ is expanded into a background state $\bar{q}$ and a small perturbation term $q'$. The background state is generally considered to be steady or slowly
varying and horizontally uniform, but varying in vertical direction. The perturbation \( q' \) is assumed to be much smaller than \( \bar{q} \) and does not affect the background state. The linearization of Euler equations is implemented under different assumptions and different background conditions. Except for waves with very large horizontal scale, the effects of Earth rotation is often ignored. Zhou and Morton [2007] derived the Euler equations for compressible atmosphere with altitude-varying background temperature and wind. Taylor [1931] and Goldstein [1931] derived the 2-D Euler equations with Boussinesq approximation in a continuous shear flow without temperature variations. These specific Euler equations are referred to as Taylor-Goldstein equations [Nappo, 2012]. Fritts and Alexander [2003] derived the Euler equations without wind shear but considered the Earth rotation effects. For more detailed derivations of the linearization of the Euler equations, see Appendix A. All the linearizations finally reach a standard wave equation in vertical direction, represented by

\[
\frac{d^2\phi(z)}{dz^2} + m^2 \phi(z) = 0. \tag{1.1}
\]

Dispersion relation can relate the vertical wavenumber \( m \) to the horizontal wave parameters and background states. For the acoustic-gravity waves in a compressible atmosphere, it is derived from equation (A.6) and the full relation is shown as equation (9) of Zhou and Morton [2007]. For waves with a small intrinsic horizontal phase speed \((c - \bar{u} < 0.5c_s)\), which is valid for most observed gravity waves. The dispersion relation can be simplified as:

\[
m^2 = \frac{N^2}{(c - \bar{u})^2} - k^2 - \frac{1}{4H_s^2} + \frac{1}{c - \bar{u}} \frac{d^2\bar{u}}{dz^2} + \frac{2 - \gamma}{\gamma} \frac{1}{H_s(c - \bar{u})} \frac{d\bar{u}}{dz} - \frac{3}{c_s^2} \left( \frac{d\bar{u}}{dz} \right)^2 + \frac{g}{H_s(c - \bar{u})} \frac{dH_s}{dz} + \frac{1}{2H_s} \frac{d^2H_s}{dz^2} - \frac{3}{4} \left( \frac{1}{H_s} \frac{dH_s}{dz} \right)^2 - \frac{1}{H_s(c - \bar{u})} \frac{d\bar{u}}{dz} \frac{dH_s}{dz}, \tag{1.2}
\]

where \( H_s = R\bar{T}/g \) is the scale height and \( \gamma \) is the ratio of specific heat, and \( c, \bar{u} \) and
1.2. LINEAR THEORY OF GRAVITY WAVES

$c_s$ are observed horizontal phase speed, background wind speed in the direction of wave propagation and speed of sound, respectively. The term $c - \pi$ is the intrinsic horizontal phase speed, denoted as $\hat{c}$. When the atmosphere is incompressible and the background temperature varies slowly within the vertical scale of the wave, we have $c_s \to \infty$ and $dH_s/dz \approx 0$. The dispersion relation (1.2) is reduced to the following form that is derived based on Taylor-Goldstein equation (A.19):

$$m^2 = \frac{N^2}{(c - \bar{u})^2} - k^2 - \frac{1}{4H_s^2} + \frac{1}{(c - \bar{u})} \frac{d^2\bar{u}}{dz^2} - \frac{1}{H_s(c - \bar{u})} \frac{d\bar{u}}{dz}. \quad (1.3)$$

If the wind shear terms are ignored, the dispersion relation (1.3) is reduced to equation (24) in Fritts and Alexander [2003] without the Coriolis term and is also same as the dispersion relation derived by Hines [1960]

$$m^2 = \frac{N^2}{(c - \bar{u})^2} - k^2 - \frac{1}{4H_s^2}. \quad (1.4)$$

If $m^2 > 0$, equation (1.1) will have a wave solution $\phi(z) = Ae^{imz}$ in which the amplitude of $\phi$ varies sinusoidally with altitude with vertical wavelength $\lambda_z = 2\pi/m$. These waves are referred to as internal or propagating waves. If, however, $m^2 < 0$, i.e., $m = im_I$, the wave solution is $\phi(z) = Ae^{-m_1z}$. The wave amplitudes decay exponentially with altitude. These waves are referred to as external or evanescent waves. $m^2$ depends on the intrinsic horizontal phase speed and the background atmosphere. Wave solutions require that $m$ is independent of altitude. Strict independence is not likely since background temperature and wind both vary with altitude. If these variations are relatively slow within a vertical wavelength, the WKB assumption applied.

The dispersion relation is used to diagnose the propagation of gravity waves in the vertical direction. When propagating gravity waves encounter an evanescent region $m^2 < 0$, partial or total reflection can occur. Gravity waves whose propagation is confined between two evanescent layers or between one evanescent layer and the ground
are ducted. When a gravity wave reaches a level at which the wave horizontal phase speed equals the background wind speed in the direction of wave propagation, the wave intrinsic frequency becomes zero and the wave will break and momentum will be deposited to the background flow. This is referred to as critical-layer filtering [Fritts and Alexander, 2003].

Other important relations derived from linearized wave equations are polarization relations that describe the relative phases and amplitudes of various wave quantities. If gravity waves do not undergo dissipation, the complex wave amplitude as defined in equation (A.11) of the relative temperature perturbation $\tilde{T} (= T' / \overline{T})$, zonal wind $\tilde{u}$, meridional wind $\tilde{v}$ and vertical wind $\tilde{w}$ should satisfy the following polarization relations [Fritts and Alexander, 2003; Vadas, 2013; Lu et al., 2015a]:

$$
\frac{\tilde{T}}{\tilde{w}} = \frac{N^2 \left( im + \frac{1}{2H_s} \right) - \frac{\omega^2}{\gamma H_s} (\gamma - 1)}{g \hat{\omega} \left( -m - \frac{i}{2H_s} + \frac{i}{\gamma H_s} \right)}
$$

$$
\frac{\tilde{T}}{\tilde{u}} = \frac{N^2 \left( im + \frac{1}{2H_s} \right) - \frac{\omega^2}{\gamma H_s} (\gamma - 1) \left( \hat{\omega}^2 - f^2 \right) (k_x \hat{\omega} - i f k_y)}{g (N^2 - \hat{\omega}^2) \left( k_x^2 \omega^2 + f^2 k_y^2 \right)}
$$

(1.5)

$$
\frac{\tilde{u}}{\tilde{v}} = \frac{i \hat{\omega} k_x - f k_y}{i \hat{\omega} k_y + f k_x},
$$

where $k_x$ and $k_y$ are zonal and meridional wavenumber ($k^2 = k_x^2 + k_y^2$). The complex ratio of $\tilde{T}/\tilde{w}$, $\tilde{T}/\tilde{u}$ and $\tilde{u}/\tilde{v}$ can be calculated from wave parameters and interpreted as amplitude ratios and relative phase difference of these quantities. On one hand, the missing quantities of observed gravity waves can be estimated through these relations, and on the other hand the discrepancies between observed and theoretical values can be used as an indicator of the wave dissipation. It is possible to estimate higher-order statistical quantities such as gravity wave momentum ($\overline{u'w'}$) and heat ($\overline{w'T'}$) fluxes from these relationships with limited observations [Liu, 2009].

All the physical constants used in the calculations in this dissertation are listed in Table 1.1.
### 1.3. OBSERVATION AND MODELING OF GRAVITY WAVES

Table 1.1: Physical parameters used in this dissertation, specified for neutral atmosphere at 80–110 km altitude.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Names</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_p$</td>
<td>specific heat at constant pressure</td>
<td>1004 JK$^{-1}$kg$^{-1}$</td>
</tr>
<tr>
<td>$c_v$</td>
<td>specific heat at constant volume</td>
<td>717 JK$^{-1}$kg$^{-1}$</td>
</tr>
<tr>
<td>$R$</td>
<td>ideal gas constant for dry atmosphere</td>
<td>287 JK$^{-1}$kg$^{-1}$</td>
</tr>
<tr>
<td>$R_{earth}$</td>
<td>earth radius</td>
<td>6371 km</td>
</tr>
<tr>
<td>$G$</td>
<td>gravitational constant</td>
<td>6.67408×10$^{-11}$ m$^3$kg$^{-1}$s$^{-2}$</td>
</tr>
<tr>
<td>$M$</td>
<td>mass of the earth</td>
<td>5.972×10$^{24}$ kg</td>
</tr>
<tr>
<td>$g = G \cdot M/(R_{earth} + h)^2$</td>
<td>gravitational acceleration</td>
<td></td>
</tr>
<tr>
<td>$H_s = R \cdot \bar{T}/g$</td>
<td>pressure scale height</td>
<td></td>
</tr>
</tbody>
</table>

1.3. Observation and Modeling of Gravity Waves

Currently, the research on gravity waves is mostly divided into two categories, which are separate but closely related. The first is to better understand gravity waves in the atmosphere, including the generation, propagation, and dissipation processes. The second one is to develop physically-based gravity wave parameterizations for the purpose of improving GCMs. Targeted at addressing problems from these two perspectives, scientists have done a lot of work in the last few decades, including theoretical, numerical and observational studies.

Challenges lie in the ability to observe gravity waves and estimate their characteristics and effects, i.e., amplitude, spectrum, linearity, nonlinearity, dispersion and dissipation. Many in-situ and remote sensing techniques have been developed to observe the gravity waves effects on the atmosphere. There are instruments, including, but not limited to, sensors on-board aircraft [Fritts and Nastrom, 1992; Nastrom and Fritts, 1992], high-resolution radiosonde networks [Vincent and Joan Alexander, 2000; Zhang et al., 2012], super-pressure balloons [Vincent et al., 2007; Hertzog et al., 2008] and rockets [Hecht et al., 2004b; Wüst and Bittner, 2008], ground-based radars of various types [Nastrom and Eaton, 2006; Fritts et al., 2010; Liu et al., 2013a], active lidars [Hu et al., 2002; Li
Various data from satellites provide a global view of gravity wave activities and effects, such as from nightglow imagery [Yue et al., 2014; Miller et al., 2015], limb sounding techniques [Jiang et al., 2005; Alexander et al., 2008, 2009] and nadir sounding techniques [Gong et al., 2012; Alexander and Grimsdell, 2013; Hoffmann et al., 2014].

Generally, these observational instruments can only measure part of gravity wave fields. Single-site ground-based techniques like lidar, radar, and limb sounding satellites are limited to providing altitude profiles and can only resolve vertical structures of the wave field. Other techniques like nadir sounding satellites and airglow imager can only retrieve the horizontal structures over certain area. Satellite measurements give valuable global information on atmospheric gravity waves, but typically they have a rather narrow range of observable vertical wavelengths. In some cases, the unobserved horizontal or vertical information, such as wavelength and propagation direction, can be estimated by indirect methods based on the polarization and dispersion relations [Hu et al., 2002; Lu et al., 2015a]. For reliable estimates of wave parameters and characterization of the dissipation process, it is necessary to observe gravity waves as fully as possible in both horizontal and vertical directions, i.e., in 3-D space. Practically, observations from multiple instruments that are complementary in resolving gravity waves are needed.

Among these remote sensing techniques, lidar and airglow imager are used extensively to study gravity waves in the MLT region [Taylor, 1997; Hu et al., 2002; Espy et al., 2006; Li et al., 2007b; Lu et al., 2009; Li et al., 2011; Chen et al., 2013; Fritts et al., 2014; Lu et al., 2015b; Chen et al., 2016]. Lidar measurements provide high resolution vertical profiles at a single location and therefore only resolve gravity wave vertical structures. Airglow imagers capture 2-D images of airglow emissions from thin layers of the atmosphere thus only resolve horizontal information of gravity waves. Such complementary and simultaneous observations from lidar and airglow imager enable the investigation of
small-scale bore/ripple structures and instabilities associated with gravity wave breaking [Hecht et al., 1997; She et al., 2004b; Li et al., 2005; Smith et al., 2005; Cai et al., 2014], estimation of gravity wave momentum flux [Fritts et al., 2014] and gravity wave intrinsic characteristics [Taylor et al., 1995; Suzuki et al., 2013a; Lu et al., 2015a]. Most recently, Bossert et al. [2014] used coordinated sodium lidar and Advanced Mesospheric Temperature Mapper (AMTM) measurements to investigate gravity waves at ALOMAR observatory, Norway. The squared vertical wavenumber $m^2$ was calculated and used as a diagnosis for the altitude range at which gravity waves could freely propagate, become ducted or evanescent. Using similar sodium lidar and Mesospheric Temperature Mapper (MTM) observations at Logan, Utah, Yuan et al. [2016] studied a gravity wave packet with a broad spectrum propagating in the presence of a larger scale wave motion, leading to time and altitude dependent periods and vertical wavelengths. The numerical model of Snively and Pasko [2008] was used to simulate the wave packet, and produced remarkable similarities between the observations and simulation results under relatively idealized conditions.

Complementary to these observational techniques, many high-resolution meso-scale numerical models [Zhang and Yi, 2002; Snively et al., 2007; Yu and Hickey, 2007; Huang et al., 2012; Liu et al., 2013b; Heale et al., 2014a] are also used to investigate gravity wave dynamics by simulating the propagation, interaction and dissipation of gravity waves in given background atmosphere, thus providing a valuable tool to understand the wave processes with nearly continuous 4-D (temporal and spatial) datasets. Carefully-designed numerical modeling experiments and more comprehensive observations have contributed to our understanding of gravity wave characteristics such as scales, periods, phase speeds, possible sources, and their propagation and dissipation processes.
1.4 Gravity Wave Parameterization and Intermittency

The major influences of gravity waves on the middle atmosphere are through their transports of momentum, energy, and constituents. It is now believed that these transports greatly contribute to the large-scale circulation and the thermal and constituent structures of atmosphere, including the seasonal and latitudinal variations of the general circulation. Understanding how momentum and energy are transported and deposited in the atmosphere is dramatically important. The vertical fluxes of momentum and heat by gravity waves are given by

\[
\begin{align*}
\vec{F}_{\text{moment}} &= -\rho \left( u' w', v' w', w' w' \right) \\
F_{\text{heat}} &= -\rho w' T'.
\end{align*}
\]

The gravity wave effects on the atmosphere, i.e., the drag [Fritts and Alexander, 2003] and cooling/heating [Walterscheid, 1981] are defined as the divergence of momentum and heat fluxes:

\[
\begin{align*}
\left( \frac{du}{dt}, \frac{dv}{dt}, \frac{dw}{dt} \right) &= -\frac{1}{\rho} \frac{d}{dz} \left[ \rho \left( u' w', v' w', w' w' \right) \right] \\
\frac{dT}{dt} &= -\frac{1}{\rho} \frac{d}{dz} w' T'.
\end{align*}
\]

In equation (1.7), if the terms on the right such as momentum flux \( \rho u' w' \) are constants, then the gravity waves do not undergo any dissipation or breaking, and the left term \( d\vec{u}/dt \) will be zero. However, if dissipation occurs and momentum flux decreases with altitude, there will be a wave drag acting to slow down or accelerate the background wind, depending on the relative direction between the momentum flux deposited and the mean background wind.

It is a challenging issue of how to represent the effects of gravity waves in GCMs, i.e., the parameterization of gravity waves. The basic idea of gravity wave parameterization
1.4. GRAVITY WAVE PARAMETERIZATION AND INTERMITTENCY

is to include the effects of gravity wave on the mean circulation without actually resolving gravity waves numerically due to the limitations of computational power. Normally, a gravity wave parameterization scheme is built on the linear gravity wave theory in conjunction with a nonlinear mechanism for wave breaking/dissipation, with at least three important components: (1) Source spectrum specified at the source level in the lower atmosphere, (2) Linear propagation upward from source altitude, (3) Nonlinear dissipation mechanism when the waves attain large amplitudes or undergo critical level filtering. Many different parameterization schemes [Lindzen, 1981; Hines, 1997; Alexander and Dunkerton, 1999; Song et al., 2007; Richter et al., 2010] have been proposed and used in GCMs such as the Whole Atmosphere Community Climate Model (WACCM) [Marsh et al., 2013] and the Canadian Middle Atmosphere Model (CMAM) [Beagley et al., 2000]. Currently, these parameterization schemes have many arguments for and against. For the wave sources, at least three different forms of wave sources are included. Orographic-generated gravity waves have zero phase speeds. Convection-generated gravity waves have the full range of phase speed. Inertia-gravity waves are excited near the jet-stream or frontal system by adjustment processes from unbalanced flow. For the wave propagation, horizontal propagation is ignored in the simulation domain. The azimuth direction of wave propagation is limited to be in the wind direction at the source level. This may not be valid for gravity waves in a realistic atmosphere. Also, gravity wave reflection and ducting are not considered in the parameterization. In the dissipation scheme, unconstrained factors are tuned to produce reasonable circulation, temperature structure, and chemical species distribution.

In a gravity wave parameterization, the body force from gravity waves on the background wind is modified from equation (1.7) with an extra factor \( \epsilon \)

\[
\boldsymbol{(X, Y)} = -\frac{\epsilon}{\bar{\rho}} \frac{\partial}{\partial z} \left[ \bar{\rho} (\bar{u}'w' + \bar{v}'w') \right].
\]  

(1.8)
On one hand, a gravity wave needs to be specified with proper amount of momentum flux, so it can break at the correct altitudes. On the other hand, breaking gravity waves have to provide proper magnitude of the forcing, so the model can produce realistic mean winds. This is mostly achieved by tuning the parameter $\epsilon$, which is called the efficiency [Holton, 1982] or intermittency [Alexander and Dunkerton, 1999] factor. Literally, the intermittency factor describes the fraction of time and space of the presence of gravity waves over a long period of time and within a large area. This parameter is tuned to make the average gravity wave forcing more realistic [Alexander and Dunkerton, 1999; Fritts and Alexander, 2003]. It is found that a fairly small value of this factor (~0.1) is needed to produce realistic simulations [Holton, 1982]. Alexander and Dunkerton [1999] define a formula for intermittency for the gravity wave source as the ratio of the average of momentum flux in active-time to its long-term average. The importance of the intermittency parameter, especially in the parameterization of orographic gravity wave drag, is discussed in detail in Alexander et al. [2010].

Lindzen’s scheme [Lindzen, 1981] is the most influential and widely used in current GCMs. When a gravity wave becomes convectively unstable ($|u'| \geq |c-u|$), its amplitude saturates and thus the wave momentum flux decreases, which eventually leads to the acceleration/deceleration of the mean flow. For medium frequency gravity waves ($f \ll \hat{\omega} \ll N$), the dispersion relation can be simplified to

$$\hat{\omega} = N |k| \frac{k}{m}$$

where $m$, $\hat{\omega}$, $N$, and $k$ are the vertical wavenumber, wave intrinsic frequency, buoyancy frequency, and the horizontal wavenumber.

Based on Lindzen’s scheme, Garcia et al. [2007] derived that the saturation momentum flux can be written as

$$\tau^* = \rho \frac{k|U - c|^3}{2N}$$

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1.4. GRAVITY WAVE PARAMETERIZATION AND INTERMITTENCY

where \( \bar{\rho} \) are the atmospheric density, and \( U \) and \( c \) are the background wind and wave phase speed. The convergence of \( \tau^* \) determines the forcing on the background wind

\[
\frac{\partial U}{\partial t} = -\epsilon \frac{1}{\bar{\rho}} \frac{\partial \tau^*}{\partial z} \simeq -\epsilon \frac{k(U - c)^3}{2NH_s}
\]  

(1.11)

In this scheme, fixed gravity wave source spectra, i.e., the momentum flux as a function of wavenumber are specified at the source altitude in each column and then waves propagate upward conservatively. As the simulation goes on, these gravity waves could either propagate continuously to higher altitudes, or dissipate due to larger amplitude or critical levels. The corresponding momentum will be obliterated from the wave field and transferred to the mean flow. The gravity waves can have multiple saturation levels until they reach a critical layer where the waves break and all the momentum flux is deposited to the mean flow. From observations, the momentum deposition by breaking gravity waves is not as easy to measure as the momentum flux itself from freely propagating waves.

The intermittency factors used in the parameterization indirectly reflect the intermittent nature of gravity waves that lies in the fact that gravity waves with different characteristics appear with different probabilities in the atmosphere. Two factors could contribute to the intermittency: One is the wave source because the physical processes that generate gravity waves are random and intermittent. The other is the background atmosphere through which gravity waves propagate [Hertzog et al., 2008; Plougonven et al., 2013; Wright et al., 2013]. Fluctuations in the background wind and temperature cause variations in wave filtering, refraction and dissipation, and contribute to observed wave variabilities at the altitudes above. While the relative contributions from these two factors are difficult to distinguish from observations without comprehensive measurements from the source to the mesopause region. This can be analyzed using the output of GCMs, in which parameterized gravity waves are being dissipated continuously.
throughout model layers.

1.5 Dissertation Outline

This doctoral dissertation focuses on the characteristics of atmospheric gravity waves in the MLT region, based on the data acquired from an airglow imager and a sodium lidar, with the assistance of some simulation data from GCMs and meso-scale numerical models. The dissertation was conducted from two perspectives, one is case studies on the propagation and dissipation of gravity waves. The other one is climatology and statistical characteristics of gravity waves.

This dissertation is organized as follows: Chapter 2 describes the instrumentation and methodology, including lidar and airglow imager, and some important methods we used to process the data. The involved models are also briefly introduced. Chapter 3 presents a case study of using lidar and airglow imager data to investigate the gravity waves in 3-D space. All the retrieved parameters were used in a numerical model to produce a complete picture of the dynamic process. Chapter 4 presents a case study of using data retrieved from a lidar operated in 5-direction mode. Gravity wave parameters are determined fully. Dispersion and polarization relations are investigated in details. Chapter 5 describes the climatology of gravity waves identified airglow images data at Andes Lidar Observatory (ALO), including probability distribution of gravity waves parameters, possible mechanism controlling the propagation direction and relation of gravity wave momentum flux with background winds. Chapter 6 presents the statistical study on the intermittency of gravity waves identified from long-term airglow imager data at Maui and ALO. Intermittency of gravity waves was described by the probability density functions of absolute momentum flux where an explicit probability function was obtained. Chapter 7 presents the statistical study of duration of gravity waves in airglow measurements. Several possible mechanisms are proposed to
1.5. DISSEYATION OUTLINE

explain the specific probability distribution. Chapter 8 summarizes the whole dissertation and suggests some future work related to this dissertation. Chapters 3 and 6 are converted from two published papers of Journal of Geophysical Research-Atmosphere (DOI:10.1002/2015JD023802 and 10.1002/2016JD025173).
Chapter 2

Instrumentation and Methodology

2.1 Instruments and Models

For the MLT region, ground-based remote sensing techniques such as passive and active optical instruments are commonly used to observe atmospheric properties within a local area and over extended period. Meso-scale numerical models and global scale GCMs provide simulations in 4-D (spatial and temporal) and enable the comprehensive investigation of the wave dynamics and their interactions with the background atmosphere. In the following subsections, all the scientific instruments and models involved in the research of this dissertation are introduced.

2.1.1 Lidar

Narrow-band resonance-fluorescence lidars are powerful active remote sensing instruments that measure the fundamental atmospheric quantities including temperature and wind in the mesopause region (80–105 km). By detecting the thermal broadening and Doppler shift of atomic spectral lines of the mesospheric metal atoms such as sodium, the atmospheric temperature and winds can be measured. Figure 2.1 shows the sodium absorption spectra at different temperature and radial winds, the three selected frequen-
Sodium atoms are relatively abundant and have large effective backscattering cross-section for resonance-fluorescence scattering, thus are good tracers for lidar and enable sodium lidar to be deployed globally and contribute a lot in the atmospheric dynamics studies in last two decades [Gardner and Papen, 1995; She et al., 2004a].

![Figure 2.1: Sodium atom absorption spectra at different (left) temperature and (right) radial wind.](image)

The sodium lidar transmits pulsed laser tuned to the sodium D2a line at 589.158 nm into the night sky and the fluorescence scattered photons are collected by telescopes. The temperature and line-of-sight (LOS) winds are derived by determining the shape of the absorption spectrum using a three-frequency technique [She and Yu, 1994; Krueger et al., 2015].

In this dissertation, data acquired from a sodium lidar system operated by the University of Illinois at Urbana-Champaign (UIUC) at several different sites in the last two decades are used, including Starfire Optical Range (SOR) at Albuquerque, NW, Air Force Maui Optical Station (AMOS) at Maui, HI, and Andes Lidar Observatory (ALO) at Cerro Pachón, Chile. The details of the historic and current deployment of the lidar is listed in Table 2.1

The narrow-band sodium lidar technique measures the LOS winds which are along the laser beam. In order to measure complete wind vectors, the laser beam was directed to several off-zenith directions. At SOR (AMOS), the laser beam was coupled with a
2.1. **INSTRUMENTS AND MODELS**

Table 2.1: Basic information of the lidar systems operated at three different sites.

<table>
<thead>
<tr>
<th>Site Name</th>
<th>Site Location</th>
<th>Start Date</th>
<th>End Date</th>
<th>Telescope</th>
<th>Off-Zenith</th>
</tr>
</thead>
<tbody>
<tr>
<td>Starfire Optical Range (SOR)</td>
<td>Albuquerque, NM (35.0°N,106.5°W)</td>
<td>Jun 1998</td>
<td>Nov 2000</td>
<td>Steerable 3.5 m</td>
<td>10°</td>
</tr>
<tr>
<td>Air Force Maui Optical Station (AMOS)</td>
<td>Maui, HI (20.7°N,156.4°W)</td>
<td>Jan 2002</td>
<td>Jun 2007</td>
<td>Steerable 3.67 m</td>
<td>30°</td>
</tr>
<tr>
<td>Andes Lidar Observatory (ALO)</td>
<td>Cerro Pachón, Chile (30.0°S,70.0°W)</td>
<td>Sep 2009</td>
<td>ongoing</td>
<td>Fixed 4×75 cm</td>
<td>20°</td>
</tr>
</tbody>
</table>

A steerable astronomical telescope of 3.5 m (3.67 m) diameter and pointed in five directions: zenith (Z), 10° (30°) off zenith to the north (N), south (S), east (E), and west (W) in the sequence of ZNEZSW. At ALO, four smaller telescopes (75 cm diameter) are equipped, each fixed at one direction, at zenith, 20° off zenith toward east, west and south directions. Figure 2.2 shows a diagram of the lidar operated at 5-direction mode, under 30° (20°, 10°) off-zenith angle, there is about ~50 (32 and 16) km separation distance between any off-zenith and the zenith directions at 90 km altitude.

![Figure 2.2](image)

Figure 2.2: Diagram of the lidar operated in 5-direction detection mode. The laser beam’s off-zenith angle is 30°. A plane wave is shown by the grey scales at 87 km altitude with 200 km horizontal wavelength and wave front is oriented at 60° clockwise from north.

The relation between zonal, meridional and vertical winds (u, v, w) at different
2.1. INSTRUMENTS AND MODELS

directions and line-of-sight (LOS) winds \((V_E, V_W, V_N, V_S)\) are given by

\[
V_E = u_E \sin \theta + w_E \cos \theta \\
V_W = -u_W \sin \theta + w_W \cos \theta \\
V_N = v_N \sin \theta + w_N \cos \theta \\
V_S = -v_S \sin \theta + w_S \cos \theta,
\]

where \(\theta\) is the off-zenith angle. Under the assumption that vertical winds are much smaller than the horizontal winds, the zonal and meridional winds are derived as:

\[
u_E = V_E / \sin \theta \\
u_W = -V_W / \sin \theta \\
v_N = V_N / \sin \theta \\
v_S = -V_S / \sin \theta.
\] (2.2)

Here, \(u_E\) and \(u_W\) (\(v_N\) and \(v_S\)) are zonal (meridional) wind derived from LOS winds at different directions. In order to make complete measurements \((T, u, v)\), we assume homogeneity among measurements of 5 directions with effects of smaller scale waves and/or turbulence ignored.

At ALO, the typical temporal resolution of the measurements is 90 s and spatial resolution is 500 m. At this resolution, the measurement accuracies are \(\sim 1\) K for temperature and and \(\sim 0.5\) ms\(^{-1}\) for vertical winds near peak sodium density altitudes.

2.1.2 Airglow Imager

Airglow refers to the emission of photons in the upper atmosphere via chemiluminescence processes, that mainly result from reaction with species like atomic oxygen, atomic nitrogen, and hydroxyl radicals. In the MLT region, the major types of airglow emission are from hydroxyl (OH) at near-infrared wavelength centered at \(\sim 87\) km, and atmoic
2.1. INSTRUMENTS AND MODELS

Oxygen (OI) at wavelength of 557.7 nm from ~96 km. Variations in airglow emission intensity can be used to infer gravity wave properties [Taylor, 1997; Ejiri et al., 2003; Li et al., 2011, and references therein].

All-sky airglow imagers are equipped with a cooled charge-coupled device (CCD) and a fish-eye lens to collect the emission from all the sky. One or several narrow width bandpass filters are used to distinguish the different emissions from different altitude ranges [Taylor et al., 1995]. The airlgow imager operated at ALO is equipped with two filters to capture OH and OI emission alternately at night during the low moon period throughout the year. The airglow emissions were collected by a 1024 × 1024 CCD array and then binned to a 512 × 512 array to increase signal-to-noise ratio. Figure 2.3 shows the off-zenith distance and the resolution of each pixel with respect to zenith angle of each pixel. When the zenith angle is within ±45°, the airglow images cover an area about 200×200 km² with a resolution better than 1 km/pixel. The integration times for the OH and OI images are 1 min and 1.5 min, respectively.

![Figure 2.3: (Top) Off-zenith distance of each pixel and (bottom) the resolution of each pixel at different zenith angles for OH airglow images. Zenith angles of ±45° are marked by two vertical solid lines.](image-url)
Before airglow images can be used for wave extraction, there are several pre-processing procedures that need to be applied on the raw images. Firstly, all the stars need be removed. Secondly, images need to be unwrapped to remove the spatial distortions due to fish-eye lens and emission intensity variation due to van Rhijn effect. Thirdly, the Milky Way over Cerro Pachón is present and close to zenith most of the time and is much brighter than the airglow emission within the imager observational bandwidth. An additional procedure of removing the Milky Way [Li et al., 2014] is necessary and applied before gravity waves can be identified.

2.1.3 Numerical Models

A nonlinear, fully compressible, two-dimensional numerical model, developed by Snively and Pasko [2008] and updated by Snively et al. [2013], is used to simulate the observed gravity wave processes. The model solves the nonlinear and compressible Euler equations using an adaptation of the Clawpack routines [LeVeque, 2002] for hyperbolic systems of equations. The model solves a Riemann problem at each cell interface by calculating individual characteristic waves and characteristic speeds. These waves are then propagated at each time step and summed up to calculate the flux passing across each cell boundary using a finite volume approach. Dissipation through molecular viscosity and thermal conductivity is solved separately using a time split method. For full details see Heale et al. [2014a] and Snively and Pasko [2008]. The same model has been used extensively to investigate the propagation, dissipation, and interaction of gravity waves in MLT region [Snively et al., 2007; Heale et al., 2014a,b; Heale and Snively, 2015; Yuan et al., 2016].

2.1.4 General Circulation Models (GCMs)

The Whole Atmosphere Community Climate Model (WACCM) is a comprehensive numerical model, spanning the range of altitude from the Earth’s surface to the thermo-
2.1. INSTRUMENTS AND MODELS

sphere. It is developed by coupling the modeling of tropospheric, middle and upper atmosphere using the National Center for Atmospheric Research (NCAR) Community Earth System Model (CESM) as a common numerical framework.

The most recent version of model WACCM4 has 88 vertical levels in pressure coordinate from the surface to $4.5 \times 10^{-6}$ hPa (approximately 150 km) and horizontal resolution of $1.9^\circ$ latitude by $2.5^\circ$ longitude. This resolution is too coarse to resolve the gravity waves, so the effects of gravity waves are parameterized. A recent improvement in the wave source specification in WACCM is to replace the arbitrarily specified wave source spectrum with physically parameterized schemes for the convective source [Beres et al., 2005] and the frontal source [Richter et al., 2010]. For convection, the source spectrum over the forcing region is related to the heating height, heating amplitude and frequency distribution. For frontal systems, a function called ‘frontogenesis’ [Hoskins, 1982; Richter et al., 2010] is used as the indicator of frontal activity. Waves with Gaussian-shaped source spectrum are launched at 500 hPa ($\sim 5.5$ km) level when the ‘frontogenesis’ exceeds certain threshold. For sub-grid scale topography, the source spectrum of orographically-generated gravity waves is specified according the standard deviation of the orography and surface level winds [McFarlane, 1987]. The orographic gravity wave parameterization still uses a tunable parameter to describe the efficiency with which gravity waves are launched, but the non-orographic wave sources uses a source-oriented parameterization. This means the magnitudes of gravity waves are primarily determined by the wave generation mechanisms such as convection or fronts. The intermittency of non-orographic gravity waves already exists in the wave sources.

The implementation of the gravity wave parameterization in WACCM includes the following components: (1) Launch waves with specified source momentum flux phase speed spectra $\tau_0(c_i)$ at a range of phase speed $c_i$ at source level when wave sources are active. (2) Upper atmosphere responses immediately through saturation, critical level filtering or diffusive damping and establish profiles of momentum flux $\tau(c_i, z_j)$ at
relatively complicated mathematical background, and contribute significantly to achieve the main conclusions are described in details. More important methods involved in the data processes and analyses will be discussed within next few chapters.

2.2.1 Type II Chebyshev Filter

Chebyshev filters (Type II) are analog or digital filters with a steeper roll-off, a flat pass-band and some stop-band ripples. They have a good performance in acquiring desired signals whose spectra are too close to the edge, either zero frequency or Nyquist frequency, from raw measurements. These features make it suitable to filter out the gravity waves with a large variety of periods, from raw data such as lidar temperature and wind measurements, whose temporal resolution is limited. The frequency response (gain) of a Type II Chebyshev filter can be described by [Parks and Burrus., 1987]

\[
G_n(w, w_0) = \frac{1}{\sqrt{1 + \frac{1}{\varepsilon^2 T_n^2(\omega_0/\omega)}}}, \quad (2.3)
\]
where $\epsilon$ is the ripple factor, $\omega_0$ is the cutoff frequency and $T_n$ is a Chebyshev polynomial of the $n$th order.

In chapter 3, when a gravity wave packet was extracted from raw lidar and airglow imager data, a Type II Chebyshev bandpass filter was applied in time domain. After repeating tests on the raw data, the filter used in the wave analysis was determined and the frequency response of the filter is shown in Figure 2.4. The cutoff periods (80\% of the amplitude of passband) of the bandpass filter are 17 min and 41 min. The desired signals are preserved without much distortion because of the flat passband, and unwanted signals are suppressed dramatically ($10^{-3}$ or -3 dB) in stop-band.

Figure 2.4: Frequency response of the Type II Chebyshev filter used in wave extraction in (top) linear and (bottom) log scale. Two blue vertical lines mark the periods of 41 min and 17 min.
2.2.2 Linear and Nonlinear Least Square Fitting

Results retrieved from observational data are mostly associated with some randomness, which makes it difficult to understand the mechanisms that may underlie the observations. If an explicitly-specified mathematical model can be derived, the physics underneath could be more obvious and be interpreted more quantitatively, and unobserved results could be predicted from the model. In statistics, least squares fitting is an approach that fits a mathematical or statistical model to data in cases where the idealized value provided by the model for any data point is expressed linearly/nonlinearly in terms of the unknown parameters of the model. Consider a set of $m$ data points, $(x_1, y_1), (x_2, y_2), ..., (x_m, y_m)$, and a model function $y = f(x, \beta)$, which depends on the variable $x = (x_1, x_2, ..., x_m)$ and parameter $\beta = (\beta_1, \beta_2, ..., \beta_n)$, with $m \geq n$. It is desired to find the vector $\beta$ of parameters such that the curve fits best the given data in the least square sense. That occurs when the sum of squares

$$ S = \sum_{i=1}^{m} r_i^2, \quad i = 1, 2, ..., m $$

is minimized, where $r_i = y_i - f(x_i, \beta)$ is the residual/error. The minimum value of $S$ occurs when the gradient is zero. Since the model contains $n$ parameters there are $n$ gradient equations:

$$ \frac{\partial S}{\partial \beta_j} = 2 \sum_i r_i \frac{\partial r_i}{\partial \beta_j} = 0, \quad j = 1, 2, ..., n. $$

In a linear system, the derivatives $\frac{\partial r_i}{\partial \beta_j}$ is only dependent on the variable $x$. So the parameter $\beta$ have a closed-form solution that is unique and can be solved analytically in most cases. But for the nonlinear fitting, the derivatives are functions of both the variable $x$ and parameters $\beta$, so these gradient equations do not have a closed solution. Instead, numerical iterative procedures have to be applied with given initial values for all parameters.
2.2. METHODS

The practical implementation of the non-linear square fitting uses a nonlinear least-square solver \textit{lsqcurvefit} in Matlab Optimization toolbox. The solver tries to find target coefficients \(x\) that solves the problem

\[
\min ||F(x, xdata) - ydata||_2^2 = \min \sum (F(x, xdata_i) - ydata_x)^2,
\]

given input data \(xdata\), and the observed output \(ydata\), where \(xdata\) and \(ydata\) are matrices or vectors, and \(F(x, xdata)\) is a matrix-valued or vector-valued function of the same size as \(ydata\). Optionally, the components of \(x\) can have starting points \(x_0\), which are from tentative estimation, and lower and upper bounds \(lb\), and \(ub\). For more details, see the Matlab documentation.

Least square fitting was used multiple times in this dissertation. In chapter 4, the wave patterns in lidar temperature and wind perturbations are fitted by multiple sinusoidal functions of the same period but with constant phase difference among them, to obtain the wave amplitude and horizontal wavelength. In chapter 6 and 7, the statistical characteristics of gravity wave momentum flux and duration are analyzed and least square fitting is applied on the histograms to obtain the mathematical probability distribution.

2.2.3 Histogram and Probability Density Function

In probability theory, the probability density function (pdf) of a continuous random variable is a function that describes the relative likelihood for this random variable to take on a given value. The probability of the random variable falling within a particular range of values is given by the integral of this variables pdf over that range. The pdf is nonnegative everywhere, and its integral over the entire space is equal to one. For a random variable \(X\) that has probability density function \(f_X(x)\), all the above
mathematical characteristics can be expressed as

\[ f_X(x) \geq 0 \]

\[ Pr[a \leq X \leq b] = \int_a^b f_X(x) \, dx \]  \hspace{1cm} (2.7)

\[ \int_{-\infty}^{+\infty} f_X(x) \, dx = 1. \]

In reality, discrete data are more likely involved. A histogram is a practical approximate of pdf. To construct a histogram, the first step is to ‘bin’ the range of values into a series of intervals, and then count how many values fall into each interval. Normally, the counts are defined as ‘frequency’ (or absolute frequency). Then, the count can be normalized by dividing the total number of values, thus the y-axis of a histogram is called ‘relative frequency’. If the width of the intervals on the x-axis are all unit length, or the relative frequency is normalized to unit interval, then a histogram is an estimate of the probability density function.

In chapter 6, the probability density functions of gravity wave momentum flux are obtained from multi-year airglow imager data and fitted by a piecewise function. In chapter 7, the probability density function of the duration of gravity wave events is also obtained and fitted by an exponential function.

2.2.4 Bootstrapping

A robust analysis of certain results requires necessary estimation of uncertainties. When results are derived from a sample of measurements with certain uncertainties, it is natural to say that the results also have some uncertainties. But it is hard to estimate the uncertainties of the desired results, especially when the uncertainties of the samples are described generally instead of individually. In those cases, we refer to a statistical method to estimate the uncertainties based on bootstrapping.

In statistics, bootstrapping is any test or metric that relies on random sampling with
replacement. Bootstrapping allows assigning measures of accuracy (defined in terms of bias, variance, confidence intervals, prediction error or some other such measure) to sample estimates. Say we have measurements from observation $x_i$, and each measurement has its own uncertainty/error $\delta x_i$ due to measuring error, random error, and other errors. When we count the measurements in a bin to get histogram, there is a probability that a measurement falls into other bins due to the uncertainty/error of this measurement. Therefore, there exists some uncertainties in the counts, and eventually the uncertainties spread to the histogram.

The basic idea of Bootstrapping is that inference about a population from sample data (sample $\rightarrow$ population) can be modeled by re-sampling the sample data and performing inference on (re-sample $\rightarrow$ sample). The method of Bootstrap is roughly based on the law of large numbers, which says, in short, that with enough data the empirical distribution will be a good approximation of the true distribution. Visually it says that the histogram of the data should approximate the density of the true distribution. Suppose we have $n$ data points:

$$x_1, x_2, ..., x_n$$

drawn from a distribution $F$. An empirical bootstrap sample is a re-sample of the same size $n$:

$$x_1^*, x_2^*, ..., x_n^*$$

We could think of the latter as a sample of size $n$ drawn from the empirical distribution $F^*$. For any statistic $v$ computed from the original sample data, we can define a statistic $v^*$ by the same formula but computed instead using the re-sampled data. With this notation we can state the bootstrap principles:

1. $F^* \approx F$.

2. The distribution of $v^*$ approximates the distribution of $v$.

It is a straightforward way to derive estimates of standard errors and confidence
2.2. METHODS

intervals for complex estimators of complex parameters of the distribution, such as percentile points, proportions, odds ratio, and correlation coefficients. Bootstrap is also an appropriate way to control and check the stability of the results. Although for most problems it is impossible to know the true confidence interval, bootstrap is asymptotically more accurate than the standard intervals obtained using sample variance and assumptions of normality.

The procedures of applying Bootstrap method on data are listed as follows:

1. Treat $N$ observed measurements $x_i$ as a sample $X$, randomly select $N$ measurements from the sample with REPLACEMENT and make a new sample $X^*_j$. Please note that in the implementation of the re-sampling, random number generator in Matlab can retrieve $\sim 60\%$ measurements without repeating and the rest $\sim 40\%$ of repeated measurements.

2. Calculate the histogram ($H_j$) of the re-sampled measurements $X^*_j$ with fixed bins and counting method.

3. Repeat procedures 1-2 1000 times and get $H_j(j = 1, 2, 3, ..., 1000)$.

4. In the $k$th bin, sort the $H_j(k)$ in ascending order and find the 5% and 95% percentiles $H_{5\%}(k)$ and $H_{95\%}(k)$. The range between these two percentiles is corresponding to the 90% confidence interval of the histogram in the bin.

2.2.5 Errors and Propagated Errors

The uncertainty/error of raw measurements and derived results can be expressed in a number of ways. The first one is the absolute error $\Delta x$, which is related to measuring processes such as the photon noise of a lidar. Secondly, from statistical perspective, the uncertainty of a quantity is described in terms of the standard deviation $\sigma$ of repeating measurements. The value of a observed quantity and its uncertainty are then expressed
2.2. METHODS

as an interval \([x - \sigma, x + \sigma]\). If the statistical probability distribution of the variable is known or can be assumed, it is possible to derive confidence limits to describe the region within which the true value of the variable may be found. In the non-linear least square fitting, the uncertainties of the fitted parameters are represented by 90% confidence intervals based on parametric estimation. This represents that there is a 90% probability that this interval encompasses the true value of the true parameter.

Mathematically, if a variable \(y\) is the function of multiple experimental measurements \(x = (x_1, x_2, \cdots, x_i, \cdots)\), say \(y = f(x)\). Each measurement \(x_i\) has uncertainty \(\sigma_i\). The uncertainty of \(y\) can be described by propagated uncertainty, which includes the combined effect of the errors of all the measured quantities taking into account the operations in the function. In statistics, propagation of uncertainty is the effect of variables uncertainties on the uncertainty of a function based on them. The following equation shows how the propagated error \(\sigma_y\) is calculated for variable \(y\):

\[
\sigma_y = \sqrt{\left(\frac{\partial y}{\partial x_1}\right)^2 \cdot \sigma_1^2 + \left(\frac{\partial y}{\partial x_2}\right)^2 \cdot \sigma_2^2 + \cdots + \left(\frac{\partial y}{\partial x_i}\right)^2 \cdot \sigma_i^2 + \cdots}.
\] (2.8)

Uncertainties/errors derived from propagation function and Bootstrapping are complementary to each other, when one is incapable of providing uncertainty estimation, the other one could. In the dissertation, uncertainties of majority of results are estimated to show the robustness of the results.
Chapter 3

Observation and Modeling of Gravity Wave Propagation Through Reflection and Critical Layers

3.1 Introduction

The linearized gravity wave theory have long predicted reflection, critical levels and ducting. Several studies have used numerical models to investigate the characteristics of reflection and transmission of gravity wave packets in atmosphere with vertically or horizontally sheared winds and vertically varying temperature, including occurrences of waves trapped between two reflection layers known as ducts [Walterscheid et al., 1999, 2001; Snively et al., 2007; Yu and Hickey, 2007; Snively and Pasko, 2008; Huang et al., 2010; Heale and Snively, 2015]. It was found that waves trapped in ducts can propagate large horizontal distances, depositing their energy and momentum periodically as they leak from the duct [Suzuki et al., 2013b; Heale et al., 2014a]. In addition, it was found that the inclusion of time dependent background winds can lead to a reduction in filtering, as critical levels now become transient [Broutman and Young, 1986; Eckermann,
1997; Sartelet, 2003; Vanderhoff et al., 2008]. Heale and Snively [2015] also found that reflection of a wave can be reduced once the time-dependence of a background wind is considered. Both cases lead to additional upward propagation over time-independent background assumptions.

In this chapter, simultaneous data from collocated sodium lidar and airglow imager is used to depict a gravity wave event in 3-D space. On the night of 16 January 2015, the lidar at ALO observed a persistent wave with a period about 30 min and associated with large vertical wind perturbations. The horizontal and vertical structures of wave packet and its ambient atmosphere states are fully characterized. The observation is unique in that it provides a clear case of both wave reflection and critical level filtering. A numerical simulation is performed with the observed wave parameters using a fully compressible, nonlinear 2-D numerical model, which shows that double reflection leads to a leaky duct and a near-critical level occurs when wave speed approaches background wind speed. The simulation yields a comparison and a confirmation of our interpretation of observations. The chapter is organized as follows: Section 3.2 briefs the basic information of the data used in the study. Section 3.3 presents the observational results from the sodium lidar and airglow imager. Section 3.4 discusses the numerical model setup and simulation results. Finally, the summary and conclusions are presented in Section 3.5.

3.2 Dataset

In this case study, lidar temperature and vertical wind measurements, and OH airglow emission data at ALO are used to analyze a gravity wave event. At ALO, the lidar system is equipped with four telescopes of 75 cm diameter, each fixed at one direction, at zenith, 20° off zenith toward east, west and south directions. The temporal resolution of lidar measurements is 90 sec and spatial resolution is 500 m. The preprocessed airglow
images cover an area about $172 \times 172$ km$^2$ with a resolution better than 1 km/pixel. The integration times for the OH and OI images are 1 min and 1.5 min respectively. Only OH airglow images are used here thus the temporal resolution is 2.5 min. From airglow emission data, we only retrieve gravity wave horizontal information such as wavelength and propagation azimuth angle. Therefore, the amplitude of emission intensity here is simply the normalized intensity without separating the background and wave-induced intensity as other works.

Table 3.1: Basic information of the data used in the studies (Case 1).

<table>
<thead>
<tr>
<th>Case</th>
<th>Site</th>
<th>Instrument</th>
<th>Date</th>
<th>Time</th>
<th>Variables</th>
<th>Resolution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>ALO</td>
<td>Lidar</td>
<td>01/16/2015</td>
<td>02:00-08:30UT</td>
<td>$\rho_{Na}$, T, $w$</td>
<td>500 m/90 sec</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Imager</td>
<td>01/16/2015</td>
<td>00:00-09:00UT</td>
<td>OH</td>
<td>1/2.5 min</td>
</tr>
</tbody>
</table>

3.3 Observational Results

3.3.1 Lidar Measurements

Raw lidar measurements smoothed by a 15-min moving average are shown in Figure 3.1, the perturbation of the vertical wind exceeds $\pm 10$ ms$^{-1}$ at certain altitudes. Distinct wave patterns with a period of about half hour can be identified in the temperature and vertical wind measurements. In order to obtain the dominant periods of the waves, Fourier analysis was applied to the raw lidar data with a 90-sec resolution at all altitudes. Several peaks around $\sim 30$ min were identified in the spectra of temperature and vertical wind. A Chebyshev type II band-pass filter was used to extract the waves from the raw measurements with a 18-min lower 3 dB cutoff period, and a 35-min upper 3 dB cutoff period. The background temperature $\bar{T}$ was obtained using another low-pass filter with a cutoff period of 40 min. Squared buoyancy frequency $N^2$ is calculated as

$$N^2 = \frac{g}{\bar{T}} \left( \frac{\partial \bar{T}}{\partial z} + \frac{g}{c_p} \right), \quad (3.1)$$
where $g$ is the gravity acceleration, $c_p$ is the specific heat at constant pressure. Larger values of $N^2$ indicate more stable atmosphere, while values of negative $N^2$ imply a statically unstable atmosphere. The squared buoyancy frequencies $N^2$ shown in Figure 3.2 reveal that the background atmosphere is mostly stable but layers with relatively smaller values of $N^2$ can be found near 92 km and 98 km.

![Figure 3.1: Smoothed raw lidar (a) temperature, (b) vertical wind and (c) sodium density on the night of 16 January 2015.](image)

The resulting band-pass filtered temperature and vertical winds are shown in Figure 3.3. Perturbations of vertical wind have a lag about 90° with respect to temperature, which matches the polarization relation of gravity waves. Both variables show clear
layered structure. In the temperature perturbations, three layers exist, one is below 92 km, the second centered at 95 km with thickness of \( \sim 2 \) km, and third one above 98 km. In the vertical wind perturbations, only two layers exist and mostly match the temperature perturbation except the perturbations are minimized near 95 km. For the layer below 92 km, the phase of temperature and vertical wind perturbations are almost vertically oriented and the amplitudes of the waves exceed 15 K and 10 ms\(^{-1}\). Smaller values of \( N^2 (\leq 2 \times 10^{-4} \text{ s}^{-2}) \) are also denoted by solid contour lines in Figure 3.3a. Although the atmosphere is found to be mostly stable, there still exist a few relatively unstable layers that would not be favorable for gravity wave propagation. The layered structures of wave patterns are related to the atmospheric stability, with the temperature perturbation maxima corresponding to regions of larger \( N^2 \) (i.e. stable regions). More detailed discussions can be found in later sections about simulation results.

![Figure 3.2: Squared buoyancy frequencies calculated from background temperature. Zero contours are highlighted with thick white lines.](image)

### 3.3.2 Airglow Images

OH airglow images were preprocessed by standard procedures including star removal, coordinate unwrapping and Milky Way removal. Only image pixels within \( \pm 45^\circ \) off-zenith were processed due to their higher resolution (\( \leq 1 \) km/pixel). The preprocessed
3.3. OBSERVATIONAL RESULTS

Figure 3.3: Bandpass-filtered lidar (a) temperature and (b) vertical wind perturbations. The smaller values of $N^2 (0, 1 \times 10^{-4} \text{ s}^{-2}, 2 \times 10^{-4} \text{ s}^{-2})$ are shown by contours in temperature perturbation.

images cover a square area of 172 km in each direction. Firstly, the same band-pass filter that was used in processing the lidar data was applied on the airglow intensity for each image pixel in time domain. Then, a 2-D median filter was used to suppress the noisy and small structures in each image.

In order to demonstrate the temporal evolution of the gravity wave packet in the airglow images, one column and one row of image pixels, that includes the zenith pixel, were extracted from the preprocessed images to make ‘keograms’, i.e. the distance-versus-time plots of airglow intensity. Note that the unit here is normalized airglow emission intensity in percentage. In Figure 3.4, the wave pattern is present and strong from 5:00 UT onwards, matching the lidar measurements near ~87 km in Figure 3.3. A clear phase tilt is found in the north-south direction while not in the east-west direction. This implies that the gravity wave packet propagates mostly southward. Figure 3.5 shows four consecutive airglow images with an interval of 5 min and note that the center area of each image corresponds to the zenith pointing direction of lidar. The wave pattern is distinct in the airglow images and propagate mostly southward. Finally, the horizontal
3.4 Numerical Simulation

3.4.1 Model Setup

The simulation domain is set to be 600 km in the horizontal ($x$-direction) and 170 km in the vertical ($z$-direction), with a resolution of 2-km in horizontal and 0.25-km in vertical. The side and top boundaries are open, and the bottom boundary (ground) is set to be
3.4. NUMERICAL SIMULATION

Figure 3.5: Four consecutive temporally filtered and spatially smoothed airglow images with a 5 min interval. The \( x \) and \( y \) distances are in east-west and north-south directions. The color scale is the same as Figure 3.4.

Table 3.2: Wave parameters identified from lidar and airglow imager.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Date,Time</td>
<td>2015/1/16, 04:30-08:10 UT</td>
</tr>
<tr>
<td>Period</td>
<td>27 (18-35) min</td>
</tr>
<tr>
<td>Horizontal Wavelength</td>
<td>50 km</td>
</tr>
<tr>
<td>Horizontal Wave Speed</td>
<td>30 ms(^{-1})</td>
</tr>
<tr>
<td>Propagation Direction(^a)</td>
<td>190°</td>
</tr>
<tr>
<td>Wave Amplitudes</td>
<td>( \sim )15 K in ( T' ), ( \sim )10 ms(^{-1}) in ( w' )</td>
</tr>
</tbody>
</table>

\(^a\) The direction is measured clockwise from north

closed (reflective). The simulation outputs results every 90 seconds, to be consistent with the lidar measurements, and the simulation runs for \( \sim \)6 hours. The gravity wave packet
3.4. NUMERICAL SIMULATION

is generated by a spectrally-coherent, idealized vertical body forcing applied below the observable altitudes and specified by a Gaussian modulated cosine wave, which has been used frequently in numerical simulation

$$F_z(x, z, t) = A \cdot \exp \left[ - \frac{(x - x_c)^2}{2\sigma_x^2} - \frac{(z - z_c)^2}{2\sigma_z^2} - \frac{(t - t_c)^2}{2\sigma_t^2} \right] \cdot \cos \left[ \omega(t - t_c) - k(x - x_c) \right].$$

(3.2)

The source location is set at $x_c = 200$ km and $z_c = 65$ km (to reduce computational time) and $t_c = 108$ min. The scale of wave is set to $k = 2\pi/50$ km$^{-1}$, $\omega = 2\pi/27$ min$^{-1}$, which are all determined from observations as listed in Table 3.2. The parameters $\sigma_x$, $\sigma_z$ and $\sigma_t$ are 200 km, 65 km and 27 min, respectively. The amplitude $A$ is chosen to be 0.12 ms$^{-1}$, which corresponds to a source amplitude of 0.002 ms$^{-1}$ at tropopause. This amplitude is determined tentatively to match simulation results with those seen in the observations without visible wave breaking, which could diminish the wave amplitudes.

For the numerical simulation, the background condition is very important. The background temperature profile is determined using an average of the lidar temperature between 05:00 UT and 08:00 UT merged with temperature from NRLMSISE-00 [Picone et al., 2002] set to the same location and time. Due to the lidar being operated in zenith mode only, horizontal winds were not available until January 26, 2015. However, the horizontal winds of the following nights show some long-term consistency. Thus, the background horizontal wind used for the simulation is an amalgamation of the HWM-07 winds [Drob et al., 2008] of the same time period, and the averaged lidar winds from January 26 to February 2.

Figure 3.6a–3.6c show the background temperature, horizontal wind projected along the wave propagation direction, and squared buoyancy frequency respectively. Lines in blue show the observations with errors and lines in black show the merged profiles of observations and empirical models, which are used for the simulations. Using equation (1.4), the vertical wavenumber $m$ can be calculated from all the wave and background
3.4. NUMERICAL SIMULATION

Figure 3.6: Background (a) temperature, (b) horizontal wind, (c) calculated squared buoyancy frequency and (d) calculated vertical wavelength. Blue lines with errorbars are observations or calculated directly from observations and black lines are models data merged with observations. Two horizontal dash-dot lines mark the approximate observational altitude range.

parameters. The vertical wavelength is then obtained as \( \lambda_z = 2\pi/m \) for all positive \( m^2 \) and shown in Figure 3.6d. Two reflection layers are found, corresponding to negative \( m^2 \) and denoted by the two broken parts on the curve. One reflection layer is below 85 km and the other is around 103 km. At \( \sim 92 \) km altitude, the wave phase speed approaches the background wind speed, leading to a near-critical layer. At this altitude, the vertical wavelength becomes very small, and the wave phase fronts will be oriented almost horizontally, leading to greater tendency toward large shears and thus instability.

In Figure 3.6, the vertical structures in background temperature and wind indicate the existence of another wave of larger vertical wavelength which creates the reflection and critical layers.

3.4.2 Simulation Results

Due to simplifications in the physics of the simulation, i.e., imposing a time-independent constant background and quasi-monochromatic waves, which may not represent the full spectrum, differences between observations and simulations are expected. However, the
3.4. NUMERICAL SIMULATION

Simulation captures and helps to explain the major features while illustrating the spatial evolution of the wave fields.

As gravity wave packets propagate away from their sources, they will be dispersed, refracted and filtered by the background atmosphere, spreading and depositing the energy and momentum of the packet throughout the atmosphere. Therefore, an instrument at a fixed location relative to the wave source may only capture part of the wave spectrum as a wave passes through the instrument’s field of view. When simulations are compared with lidar observations from a single site, it is important to pick a ‘virtual lidar’ location within the model domain to transform an $x$-$z$ domain into a $t$-$z$ domain. Following the analysis method applied in Yuan et al. [2016], we tested several ‘virtual lidar’ sites and found that results at $x = 340$ km (140 km away from the specified source horizontally) best match the observations, and are thus selected to compare with observations in latter analysis.

![Figure 3.7](image.png)

Figure 3.7: (a) Original and (c) filtered temperature perturbation, and (b) original and (d) filtered vertical wind perturbation at ‘virtual lidar’ site ($x = 340$ km) in simulation domain.

Figure 3.7a and Figure 3.7c (3.7b and 3.7d) depict the original and filtered temperature (vertical wind) perturbations from the ‘virtual lidar’ site. The filtering is done using the same filter as the one used in the lidar and airglow measurements. For easier comparison, the time of numerical simulation is adjusted to match the observation time.
and also called UT time, since the simulation time is arbitrary. A propagating gravity wave packet is generated when the source is active from approximately 4.75 to 5.65 hr (adjusted by adding 3.4 hr from the $t$ in Equation (3.2)). Layered structures are found at similar altitudes to the observations. The amplitudes of temperature and vertical wind perturbations reach maximums of $\pm 10$ K and $\pm 6$ ms$^{-1}$, respectively. The model simulated amplitudes are slightly smaller because only part of spectrum was simulated.

In Figure 3.7a and Figure 3.7c, there is a thin layer right above 90 km with strong and constant negative temperature perturbations and without obvious vertical wind perturbations corresponding to a near-critical layer. It is also in part the result of wave packet dispersion. The three-layered enhancements (87 km, 95 km, and 103 km) in the temperature perturbations correspond to stable regions of relatively large $N^2$. Figure 3.7d shows strong enhancements in vertical wind at $\sim 87$ km and $\sim 101$ km, corresponding to reflection levels as indicated in Figure 3.6d. This is because the wave phase fronts will orientate themselves more vertically as the vertical wavelength increases. Near 95 km above the near-critical layer, the atmosphere is stable and temperature perturbations are strong. However, only weak vertical wind perturbations are found there. This is due to the fact that wave fronts tend to be oriented horizontally, reducing their contribution to the vertical winds. It is also important to note that the reflection appears at 101 km before it does at 87 km, suggesting that the wave reflection at 87 km is, in part, a second reflection from the downward propagating portion of the wave packet that reflected at 101 km.

In order to demonstrate the dynamics of the wave propagation, we select three frames from the $x$-$z$ domain of simulation results at $t = 4.88$ hr, 5.62 hr and 7.38 hr for both temperature and vertical wind. The original frames are shown in Figure 3.8 and the filtered frames are shown in Figure 3.9. At 4.88 hr, a small portion of the wave packet has penetrated the evanescent region near 85 km and continued to propagate upwards. We can see that the main energy center of the wave packet has not yet reached the $x =$
3.4. NUMERICAL SIMULATION

Figure 3.8: Original simulated (a–c) temperature and (d–f) vertical wind perturbations at $t = 4.88, 5.62, 7.38$ hr. Please note that the ‘virtual lidar’ site is at $x = 340$ km and denoted by the vertical dashed line.

Figure 3.9: Filtered simulated (a–c) temperature and (d–f) vertical wind perturbations at $t = 4.88, 5.62, 7.38$ hr.

340 km ‘virtual lidar’ site and is beginning to be reflected at $\sim 85$ km altitude between $x = 100–300$ km.

About 45 minutes later at $t = 5.62$ hr, the wave packet is partially reflected at both 85 km and 101 km, and the energy of the wave is split between the upper and lower reflection levels. The portion of the wave packet at 85 km altitude has still not propagated far
enough horizontally to enter the field of view of the ‘virtual lidar’ and a considerable amount of the wave energy has already been reflected and will not be observable at the ‘virtual lidar’ site. This shows the huge dependance of the relative distance between the instrument site and the wave source, which determines the components of wave spectrum or portion of wave processes that are actually observed.

At $t = 7.38$ hr, we see a clear standing wave type pattern (especially in the filtered cases in Figure 3.9) in the vertical wind at both 85 km and 101 km indicative of strong reflection at both layers and ducting within. While it is not clear from a single frame, the wave at 85 km is in fact the result of reflection from 101 km at earlier times as well as some transmission of the upgoing waves. Also note the horizontal dispersion of the wave packet which now spans $x \sim 100$–500 km. Finally the bottom portion of the wave packet becomes visible at the $x = 340$ km ‘virtual lidar’ site.

### 3.5 Summary and Conclusions

The details of gravity wave reflection and critical level are seldom observed in the atmosphere. Even when they are, the processes are not easy to understand due to incomplete measurements or spatial/temporal coverage of all physical quantities involved. And most numerical simulations are limited in artificially-selected parameters. In this study, the combination of two different datasets reveals a unique and distinct gravity wave packet event that undergoes partial reflection at two altitudes and approaches a near-critical layer in between. We have conducted a detailed and comprehensive investigation of this event. The gravity wave packet was determined with a ground-based period about 18–35 min, a horizontal wavelength $\sim 50$ km and nearly southward propagation direction. The event was also successfully modeled by a mesoscale numerical model, which captures primary features in the observations and provides an opportunity to understand the dynamical processes outside of the limited field of view of the instrumentation.
3.5. SUMMARY AND CONCLUSIONS

The observations show a three-layered structure (larger amplitudes at 90 km, 95 km, 103 km) in the temperature perturbations and a two-layered structure in the vertical winds (peaks at 87 km and 101 km) with amplitudes exceeding 15 K and 10 ms\(^{-1}\) respectively and minima in between these layers. The three-layered structure in temperature corresponds to regions of relatively large \(N^2\) regions (stable regions) and the two-layered structure in vertical wind corresponds to reflection levels which shift the wave to large vertical scales and subsequently large amplitudes. The numerical model predicts the layered structure and approximate amplitude, although only part of the spectra was simulated so amplitudes are slightly underestimated. The model suggests that the wave packet undergoes dual reflection and transmission at \(\sim 85\) km and 101 km altitude and that the portion of the wave seen at later times at lower altitudes is in part the result of reflection and downward propagation of the wave from the upper altitudes. Due to the cancellation effects of gravity waves in the airglow layer [Liu and Swenson, 2003], waves of vertical wavelength around 3 km should be barely visible in the airglow imager. However, the wave reflection enable the waves to be captured by an airglow imager. The model also suggests that the near-critical layer at \(\sim 93\) km altitude leads to enhanced shears and thus instability in the wave field. Notably these features are not clearly apparent when viewing filtered data. The model results reveal the capture of waves within a duct under realistic condition. The dispersion of the wave packet by reflection and near-critical levels was clearly observed, here providing insight into the evolution of gravity wave packet at small scales. The model results highlight that the location of the instrument relative to the source can determine the portion of the wave spectrum and processes that are observed by the instrument.
Chapter 4

Vertical Variation of Gravity Wave Parameters Determined from Five-direction Lidar Measurements

4.1 Introduction

A narrow-band sodium lidar measuring the sodium density, temperature and wind is a powerful tool to study the atmosphere dynamical processes in MLT region. In order to measure the full wind vectors, the laser beam was configured to point to multiple directions, mostly to zenith and off-zenith at several cardinal directions. When we analyze measurements from this type of lidar, the separations of laser beams among different directions are mostly ignored if homogeneity is assumed within the field of view (FOV) of the lidar, i.e., laser beams at different directions. This is true when the effects of smaller scale waves and/or turbulence are small and ignored. Utilizing sodium lidars and airglow imagers collaboratively, a lot of studies have been done on gravity wave characteristics and their effects. In some cases, wave breaking features are analyzed from airglow images and background atmosphere information such as instability is provided.
4.1. INTRODUCTION

by the lidar measurements. [Hecht et al., 1997; She et al., 2004b; Li et al., 2005; Smith et al., 2005; Cai et al., 2014]. In other cases, the horizontal and vertical wave structures are retrieved from airglow images and lidar observations, respectively [Suzuki et al., 2013a; Fritts et al., 2014; Bossert et al., 2014; Lu et al., 2015a; Cao et al., 2016; Yuan et al., 2016].

In WKB approximation, the background horizontal winds $\bar{u}$ and $\bar{v}$ are only functions of altitude. A gravity wave can be assumed as a traveling monochromatic wave and is represented by a sinusoidal function in the form

$$W(x, y, z, t) = A \cdot \exp \left[ i(kx + ly + mz - \omega t + \phi) + \frac{z}{2H_s} \right],$$

(4.1)
of which $x$, $y$ and $z$ are zonal, meridional and vertical coordinates, respectively, and $k$, $l$ and $m$ are zonal, meridional and vertical wavenumber, respectively. $\omega$ and $\phi$ are the observed (Eulerian) angular frequency and phase and $H_s$ is the pressure scale height.

Quasi-monochromatic (QM) gravity waves are frequently observed with airglow imagers, lidars and radars. The quasi-monochromatic gravity waves observed with imagers typically have short horizontal wavelength and high frequency [Walterscheid et al., 1999; Hecht et al., 2001a; Li et al., 2011], while those observed by radars and lidars typically are inertia gravity waves with long horizontal wavelength and low frequency [Hu et al., 2002; Lu et al., 2009; Chen et al., 2013]. In an ideal condition, if a wave defined by equation (4.1) passes the laser beams of a lidar, there will be constant phase differences among the measurements of different directions. These phase differences are more likely to be detected when there is certain match between the temporal resolution of lidar, off-zenith angle, and gravity wave horizontal scale.

In this chapter, we present a case study that a gravity wave was fully resolved only by a single-site lidar system operated in 5-direction mode. From our lidar dataset, phase differences are identified from the data on the night of 14 January 2002 from
Maui/Mesosphere and Lower Thermosphere (Maui/MALT) campaign. The horizontal information of the wave was determined by comparing the notable phase difference among measurements from different directions. And the vertical variations of the wave are also be tracked through the lidar measurements profiles. The chapter is organized as follows: Section 4.2 describes the dataset and analyzing methods. Section 4.3 presents the observational results and diagnostic discussions. Section 4.4 presents a sensitivity study to demonstrate the capability of this method. Finally, the summary and conclusions are presented in Section 4.5.

4.2 Data and Methodology

For the sodium lidar deployed in Maui, Hawaii, the laser beam was coupled with a steerable astronomical telescope (3.67 m diameter) of the Air Force Maui Optical Station (AMOS) at Maui Space Surveillance Site (MSSS). The laser beam was pointed to five directions: zenith ($Z$), 30° off zenith to the north ($N$), south ($S$), east ($E$), and west ($W$), and the return photons are collected by the telescope pointing to the same direction. Figure 2.2 shows the digram of a lidar operated at 5-direction mode. With a 30° off-zenith angle, there is a 50 km separation distance between any off-zenith and the zenith directions at ~90 km altitude. The temperature measurements are available at all 5 directions. Zonal winds are only available at $W$ and $E$, meridional winds are only at $S$ and $N$, and vertical wind is only considered as valid in $Z$ and ignored at rest directions according to equation (2.2). The laser beam was directed to rotate in $ZNEZSW$ sequence with a resolution of 1.7 min. Therefore, the intervals of measurements for zenith and for each off-zenith direction are 5.1 min and 10.2 min, respectively. The spatial resolution is 500 m along the laser beam.

Spectral analysis was applied on the original temperature and wind measurements of different directions, and spectral peaks around the period of 86 min were identified
4.2. DATA AND METHODOLOGY

Table 4.1: Basic information of the data used in the studies (case 2).

<table>
<thead>
<tr>
<th>Case</th>
<th>Site</th>
<th>Instrument</th>
<th>Date</th>
<th>Time</th>
<th>Variables</th>
<th>Resolution</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Maui</td>
<td>Lidar</td>
<td>01/14/2002</td>
<td>06:30-11:00UT</td>
<td>T, u, v, w</td>
<td>500 m/1.7 min</td>
</tr>
</tbody>
</table>

as shown in Figure 4.1. A Chebyshev type II band-pass filter was used to extract the wave from the raw measurements. The frequency response function of the filter is also plotted in Figure 4.1 with cut-off periods are 67-min and 120-min at lower and upper boundaries.

![Figure 4.1: Spectra of temperature perturbations of Z direction. The vertical dashed line indicates a frequency of 0.7 hr\(^{-1}\) and black solid line is the frequency response of the band-pass filter. The unit amplitude of the frequency response is scaled to 2 km altitude.](image)

In the following analysis, the lidar temperature \( T \) at time \( t \) from different directions are referred as \( T^x(t^x)(x = Z, W, E, S, N) \). At each altitude, a non-linear least square fitting is applied on the set of data \( T^x(t^x) \). If a monochromatic plane wave propagates at constant speed and fixed direction in a steady atmosphere, the phase difference over fixed distance is constant. So the fitting functions of four off-zenith directions are related
to Z direction by adding or subtracting a constant phase difference. The objective fitting functions are specified by the following set of equations:

\[
\begin{align*}
    y^Z &= A \cdot \sin(\omega \cdot t^Z + \phi) \\
    y^W &= A \cdot \sin(\omega \cdot t^W + \phi + \psi_x) \\
    y^E &= A \cdot \sin(\omega \cdot t^E + \phi - \psi_x) \\
    y^S &= A \cdot \sin(\omega \cdot t^S + \phi + \psi_y) \\
    y^N &= A \cdot \sin(\omega \cdot t^N + \phi - \psi_y),
\end{align*}
\]

of which \( A \) is the amplitude (temperature perturbation) of the wave, \( \omega = 2\pi/\tau \) is the angular frequency and \( \tau \) (if there is no confusion with temperature, period will be refereed to as \( T \) in other chapters) is the wave period that is determined as 86 min from spectral analysis. For the phases, \( \phi \) is the phase at \( Z \), and \( \psi_x \) and \( \psi_y \) are the phase differences between \( Z \) and \( E \), and \( Z \) and \( N \). The least square fitting is done by minimizing the following residual

\[
\delta T = \sum_Z \left[ A \sin(\omega \cdot t^Z + \phi) - T^Z \right]^2 + \sum_W \left[ A \sin(\omega \cdot t^W + \phi + \psi_x) - T^W \right]^2 + \sum_E \left[ A \sin(\omega \cdot t^E + \phi - \psi_x) - T^E \right]^2 + \sum_S \left[ A \sin(\omega \cdot t^S + \phi + \psi_y) - T^S \right]^2 + \sum_N \left[ A \sin(\omega \cdot t^N + \phi - \psi_y) - T^N \right]^2.
\]

Since the fitting function is non-linear, the desired wave parameters \( (A, \phi, \psi_x, \psi_y) \) do not have a close-form solution and can not be solved analytically. However, numerical iterative procedures are used with given initial guesses for all parameters. Practical implementation of the fitting is done by using optimization toolbox in Matlab [MathWorks, 2016]. Then, the horizontal wavenumbers in zonal and meridional directions.
were derived by
\[ k = \frac{2\pi}{\Delta \cdot \psi_x}, \]
\[ l = \frac{2\pi}{\Delta \cdot \psi_y}. \]  
(4.4)

\( \Delta \) is the spatial separation between zenith and any off-zenith directions at certain altitude. Note that \( \Delta = h \cdot \tan(30^o) \) increases with altitude \( h \). Finally the horizontal wavelength \( \lambda_H \), ground-based phase speed \( c \) and propagation direction \( \theta \) are determined as
\[ \lambda_H = \frac{|k \cdot l|}{\sqrt{k^2 + l^2}}, \]
\[ \theta = \arctan \left( \frac{l}{k} \right) \]  
(4.5)

\[ c = \frac{\lambda_H}{\tau}. \]

Due to the uncertainties of the lidar measurements especially at non-peak sodium density altitudes, the fitted parameters \( (A, \phi, \psi_x, \psi_y) \) have some uncertainties. The uncertainties are demonstrated by the 90% confidence intervals of each parameter from the fitting process. The uncertainties of \( \lambda_H \) and \( \theta \) are derived based on error propagation. It turns out the fitting results are sensitive to the initial guesses due to complexity of the fitting. In the non-linear fitting processes, \( (A, \phi, \psi_x, \psi_y) \) determined from peak sodium density altitudes range (88–92 km) with less uncertainties are used as initial guesses for fitting at other altitudes.

4.3 Results

4.3.1 Fitted Temperature/Wind Perturbation

On the night of January 14, 2002, the sodium lidar was operated continuously in 5-direction mode from around 6:30 to 11:00 UT. The original lidar temperature, horizontal winds measurements are shown in Figure 4.2 and 4.3. The temperature and winds from off-zenith directions are interpolated to same altitude grids as Z direction. The overall
4.3. RESULTS

Figure 4.2: Original lidar temperature measurements at 5 different directions.

Structure in temperature are quite similar among different directions, all with a slow downward phase progression due to tides. More small-scale features are found in the Z direction because of higher temporal resolution. A gravity wave event with a period about 1.5 hr was identified from lidar temperature measurements from 8:30 UT onward near ~90 km altitude.

In order to demonstrate the propagation of quasi-monochromatic gravity waves from the measurements of multiple directions, the temperature and zonal/meridional winds are organized in zonal (W-Z-E) and meridional (S-Z-N) directions. In Figure 4.4 and 4.5, band-pass filtered temperature and zonal/meridional wind perturbations at 91 km are shown in top and fitted ones in bottom panels. Robust phase shifts can be identified in both directions in both temperature and winds. The quasi-monochromatic gravity wave propagate from northeast to southwest and the orientation of wavefront is more
4.3. RESULTS

Figure 4.3: Original lidar wind measurements at 5 different directions. Note it is zonal wind at W and E, meridional wind at S and N, and vertical wind at Z. The colorbar for zonal and meridional winds, and vertical wind are at upper-right and lower-right corner, respectively.

close to north-south since there is larger phase difference in zonal than in meridional direction. Wave amplitudes reach $\sim 8$ K, $\sim 10$ ms$^{-1}$, and $\sim 8$ ms$^{-1}$ in temperature, zonal and meridional winds, respectively. In wind perturbations, horizontal winds were not observable at Z direction but can be retrieved from fitted amplitude and initial phase, and thus added in bottom panels in Figure 4.5 for reference.

When a quasi-monochromatic gravity wave propagates upward, the ground-based period $\omega$ should be constant and horizontal wavelength $\lambda_H$ and propagation direction $\theta$ are also invariable if background atmosphere is treated horizontally homogeneous and steady, within the lidar FOV ($\sim 100 \times 100$ km$^2$) and observation window ($\sim 2$ hr) in this case study. Therefore, the phase differences $\psi_x$ and $\psi_y$ should be invariant with the altitude range where the gravity wave does not dissipate.
4.3. RESULTS

Figure 4.4: (Top) Filtered lidar temperature perturbations at different directions. (Bottom) Fitted temperature perturbations.

Figure 4.5: (Top) Filtered lidar zonal/meridional wind perturbations at different directions. (Bottom) Fitted winds perturbations including winds at Z direction.
4.3. RESULTS

4.3.2 Background and Wave Parameters

Figure 4.6a–4.6c show the background temperature $T$, zonal wind $u$ and meridional wind $v$. They were obtained using a low-pass filter with a cutoff period of 120 min and then averaged over the period of 8:30 to 11:00 UT. Squared buoyancy frequency $N^2$ is calculated based on equation (3.1). Larger values of $N^2$ indicate more statically-stable atmosphere, while values of negative $N^2$ imply an unstable atmosphere. The squared buoyancy frequencies $N^2$ shown in Figure 4.6d reveal that the background atmosphere is mostly stable near 90 and 101 km, with large $N^2$ values about $6–7 \times 10^{-4} \text{ s}^{-2}$.

Figure 4.6: Averaged background (a) temperature, (b) zonal and (c) meridional winds. Calculated (c) squared buoyancy frequency, (d) vertical shear of horizontal wind and (f) Richardson number. Zero and 1/4 are marked by dot-dashed and dashed lines.

Richardson number $Ri$ is commonly used to characterize the dynamical (shear) instability and can be calculated through

$$Ri = \frac{N^2}{(\partial u/\partial z)^2 + (\partial v/\partial z)^2} = \frac{N^2}{S^2}, \quad (4.6)$$

where $S$ is the vertical shear of the horizontal wind, $u_0$ and $v_0$ are the background
4.3. RESULTS

zonal and meridional winds. The atmosphere is typically dynamically unstable when $0 < Ri < 1/4$. Strong horizontal wind shear and negative temperature gradient tends to make the atmosphere dynamically unstable. As shown in Figure 4.6f, $Ri$ approaches $1/4$ in the layers near 87, 95 and 102 km where dynamical instability is likely to occur. And the large wind shears are main factors of these unstable area. The layer near 90 km are relatively stable with large $N^2$ and small wind shear thus a larger Richardson number where it is favorable for wave propagation.

![Figure 4.7: Least-square fitted (a) wave amplitude, (b) initial phase at Z, phase differences at (c) zonal and (d) meridional direction. Thin horizontal lines are 90% confidence intervals.](image)

Figure 4.7a–4.7d show altitude profiles of four fitted parameters ($A$, $\phi$, $\psi_x$, $\psi_y$) with their 90% confidence intervals. Within 88–92 km altitude range, the fitted amplitudes are larger ($\geq 8$ K) and the fitting is more robust with the all the fitted parameters showing less uncertainties. In the fitting, the phases are more sensitive to the raw data than amplitude. The initial phase $\phi$ has large uncertainties but still shows a clear tendency that it increases with altitudes, which implies a downward phase progression. For phase differences $\psi_x$ and $\psi_y$, even with large uncertainties and varying with altitudes, both are relatively constant within the 88–92 km altitude range. Because the least square fitting is done independently at different altitudes, the vertical wavenumber $m$ can be estimated by the gradient of the initial phase $d\phi/dz$.

Figures 4.8a–4.8b show the calculated horizontal wavelength and propagation az-
4.3. RESULTS

Figure 4.8: Calculated (a) horizontal wavelength, (b) propagation azimuth angle and calculated (c) squared vertical wavenumber. Thin horizontal lines without dots are calculated uncertainties. Two vertical solid lines are averaged (a) horizontal wavelength and (b) propagation azimuth angle within 88–92 km altitude range.

Therm institutional angle with their propagated uncertainties. Even with some uncertainties, the horizontal wavelength and propagation azimuth angle shows good consistency within 88–92 km altitude. The mean values within this range are determined as actual wave parameters and listed in Table 4.2. They are used in the calculations for further wave analysis. Above 92 km, there is a clear degradation for both variables, which may indicate that the wave does not propagate to higher altitudes.

Table 4.2: Wave parameters identified from 5-direction lidar (within 88-92 km range).

<table>
<thead>
<tr>
<th>Variables</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Date,Time</td>
<td>2002/01/14, 08:30-11:00 UT</td>
</tr>
<tr>
<td>$\tau$</td>
<td>86 min</td>
</tr>
<tr>
<td>$\lambda_H$</td>
<td>301 km</td>
</tr>
<tr>
<td>$c_H^a$</td>
<td>58 ms$^{-1}$</td>
</tr>
<tr>
<td>$\theta^b$</td>
<td>$-156^\circ$</td>
</tr>
<tr>
<td>$A$</td>
<td>$\sim8$ K in T$'$</td>
</tr>
</tbody>
</table>

$^a$ Observed or ground-based phase speed.
$^b$ The negative direction is measured clockwise from east.
4.3. RESULTS

4.3.3 Wave Diagnosis: Dispersion and Polarization Relations

Since we have a full set of parameters of gravity wave and background atmosphere, it is possible to diagnose the propagation of wave using dispersion and polarization relations. Dispersion relations in equations (1.2), (1.3), and (1.4) are referred as ZQ07, TG02, and FD03 in later analysis. By comparing the differences among different relations, the relative importance of each term in the dispersion equation is evaluated. The squared vertical wavenumbers $m^2$ were calculated and shown in Figure 4.8c. The dispersion relations of ZQ07 and TG02 can well predict the non-evanescent region near 90 km altitude with a thickness of 4 km. This mostly matches the range where wave amplitudes are large as demonstrated in Figure 4.7a. These two dispersion relations show overall similarity but difference exists above 92 km, which is believed due to the large temperature gradient there. In this case, the omission of variation of background temperature and wind produce clear discrepancy in $m^2$ without negative values and lower maximum $m^2$ altitude as predicted by FD03. These inconsistencies in different dispersion relations show that one needs to be careful if certain assumptions and simplifications are properly used.

Table 4.3: Amplitude $(A)$ ratio and phase $(\varphi)$ difference of $\tilde{T}$ and $\tilde{w}$, $\tilde{T}$ and $\tilde{u}$, $\tilde{u}$ and $\tilde{v}$, derived from lidar data and linear wave theory.

<table>
<thead>
<tr>
<th>Variables</th>
<th>$\frac{A(\tilde{T})}{A(\tilde{u})}$</th>
<th>$\varphi(\tilde{T}) - \varphi(\tilde{u})$</th>
<th>$\frac{A(\tilde{T})}{A(\tilde{w})}$</th>
<th>$\varphi(\tilde{T}) - \varphi(\tilde{w})$</th>
<th>$\frac{A(\tilde{u})}{A(\tilde{v})}$</th>
<th>$\varphi(\tilde{u}) - \varphi(\tilde{v})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>% m⁻¹s</td>
<td>Deg.</td>
<td>% m⁻¹s</td>
<td>Deg.</td>
<td>Nan</td>
<td>Deg.</td>
</tr>
<tr>
<td>Data⁵</td>
<td>0.42±0.14</td>
<td>77.92±26.57</td>
<td>5.32±0.47</td>
<td>-48.74±67.11</td>
<td>1.63±0.35</td>
<td>31.18±34.24</td>
</tr>
<tr>
<td>Theory</td>
<td>0.34</td>
<td>99.43</td>
<td>4.97</td>
<td>-84.97</td>
<td>2.22</td>
<td>5.04</td>
</tr>
</tbody>
</table>

⁵ The $\tilde{T}$ here is relative temperature perturbation $T'/\bar{T}$.

b The uncertainties are standard derivations of the quantities within 88–92 km altitude.

Also derived from linear gravity wave theory, polarization relations in equation (1.5) demonstrate the relative amplitudes and phases among non-dissipating gravity wave components of relative temperature, zonal, meridional and vertical winds. The theoretical complex ratio of $\tilde{T}/\tilde{w}$, $\tilde{T}/\tilde{u}$ and $\tilde{u}/\tilde{v}$ can be calculated from all determined wave and
4.4 Sensitivity Study

The success of detecting a gravity wave with this method should not be a coincidence. It is known that different instruments are sensitive to different parts of the spatial and temporal spectra of gravity waves, which is the ‘observation filter’ effects [Alexander, 1998; Gardner and Taylor, 1998; Alexander et al., 2010]. This method based on the phase differences among the measurements from different directions of a lidar also has its own observation filter, that is to say, gravity waves of certain parameters are more favorable to be identified with this method. Here, we present a sensitivity study using Monte-Carlo simulation to verify the goodness of fitted wave parameters and find out the spectral range that this method is capable of resolving a wave.

The configuration of a lidar system is normally fixed, such as the off-zenith angle of lasers and telescopes, the temporal resolution and laser beam rotating squeeze. We
apply a similar configuration as the one used in Maui, with a 30° off-zenith angle, corresponding to a separation $\Delta = 50 \text{ km}$ at $\sim 90 \text{ km}$ altitude and 1-min minimum observation interval, corresponding to a 2-min and 4-min resolution for zenith and off-zenith directions, respectively. Because the derivation of wave parameters in zonal and meridional directions ($k$ and $l$) are independent, only three directions of the laser beam aligned in either zonal or meridional direction is considered. The mathematical functions of a traveling gravity wave are specified by

$$
y^+ = A \sin(\omega \cdot t^+ + \phi + \psi) + \delta
$$

$$
y = A \sin(\omega \cdot t + \phi) + \delta
$$

$$
y^- = A \sin(\omega \cdot t^- + \phi - \psi) + \delta,
$$

of which the definitions of the terms are same as equation (4.2) but an extra term $\delta$ is introduced as error/uncertainty of measurements. The error/uncertainty of lidar measurements is mostly dependent on the signals level or sodium density, we choose a typical of 2 K for temperature measurements. Gaussian distributed random numbers $\delta$ with standard derivation of 2 are added to equation (4.7). The phase difference $\psi$ is related to the wavelength as $2\pi / \lambda \cdot \Delta$. A similar non-linear least square fitting is applied on the data ($y^+, y, y^-$). To qualify the fitting errors, the percentage errors of wave amplitude $A$ and phase difference $\psi$, absolute error of initial phase $\phi$ between fitted results and given values are calculated.

The fitting errors are related to wave amplitude $A$, wavelength $\lambda$ and period $T$. Qualitatively speaking, the amplitude $A$ are expected to be as large as possible, and wavelength $\lambda$ should not be too small ($\leq 2\Delta$) to result in a $2\pi$ ambiguity among different directions nor too large to make the phase difference too little to distinguish. The period $T$ should be smaller than the dataset duration (typically 8–10 hrs) to be determined by spectral analysis, and larger than 5–8 times of the data resolution to resolve the variation of a wave in one wave cycle. For the Monte-Carlo simulation, we carefully choose the
4.4. SENSITIVITY STUDY

Figure 4.9: Percentage errors of (a) wave amplitude $A$, (c) phase difference $\psi$, and absolute error of (b) initial phase $\phi$, between the fitted and real values, with respect to period and wavelength of a gravity wave of amplitude 6 K ($3\delta$).

Wave amplitude $A$ in the range of 2–10 K (1–5 times of $\delta$), wavelength $\lambda$ in the range of 100–500 km (2–10 times of $\Delta$), and period $T$ in the range of 0.5–3 hr. The fitting errors are calculated for artificially-generated waves with specified parameters within the range.

In Figure 4.9, the errors for a wave of amplitude 6 K ($3\delta$) are shown, the percentage errors of wave amplitude, and absolute error of initial phase are less than 5% and independent of wave period and wavelength. The errors decrease clearly when wave amplitude increases (results not shown here). Figure 4.9c shows the percentage error for phase difference $\psi$, the error is clearly proportional to the wavelength, and increases to more than 10% for waves with wavelength longer than 400 km ($8\Delta$). However, period has little influence on the percentage error of phase difference.

Figure 4.10 shows the percentage errors of phase difference $\psi$ with respect to wavelength $\lambda$ for waves of 1.5 hr period and different amplitude $A$. The percentage error increases with wavelength, especially when wave amplitudes are smaller than $3\delta$ with percentage errors larger than 30% for wavelength larger than $6\Delta$. However, when wave amplitudes exceed $3\delta$, the dependence on wavelength is relatively weak, the percentage errors are less than 10%.

This method has its own limitation as any other observational techniques. The simulation shows that wave amplitude and wavelength are the most influential factors and
4.5 Summary and Conclusions

With limited data from a single-site sodium lidar, a medium-scale and medium-frequency gravity wave event on the night of 14 January 2002 at Maui, Hawaii is investigated. The phase difference is the most sensitive variable in the fitting, which is also the most crucial variable to determine wave horizontal information. Here, we provide a preliminary but useful criterion to evaluate the method used in this study. In general, when wave amplitudes exceed 3 times of the uncertainty, the method is adept at detecting most gravity waves of medium-scale and medium-frequency. But when wave amplitudes are smaller, the capability will be reduced a lot for waves with longer wavelength. Note that only one direction is considered in the sensitivity study, the real wavelength of a wave propagating not in cardinal directions is smaller than wavelengths projected to zonal or meridional direction. When the method is extended to waves propagating in any direction, the wavelength of detectable waves is smaller than the results in this sensitivity study.

4.5 Summary and Conclusions

With limited data from a single-site sodium lidar, a medium-scale and medium-frequency gravity wave event on the night of 14 January 2002 at Maui, Hawaii is investigated. The phase difference is the most sensitive variable in the fitting, which is also the most crucial variable to determine wave horizontal information. Here, we provide a preliminary but useful criterion to evaluate the method used in this study. In general, when wave amplitudes exceed 3 times of the uncertainty, the method is adept at detecting most gravity waves of medium-scale and medium-frequency. But when wave amplitudes are smaller, the capability will be reduced a lot for waves with longer wavelength. Note that only one direction is considered in the sensitivity study, the real wavelength of a wave propagating not in cardinal directions is smaller than wavelengths projected to zonal or meridional direction. When the method is extended to waves propagating in any direction, the wavelength of detectable waves is smaller than the results in this sensitivity study.
4.5. SUMMARY AND CONCLUSIONS

differences among measurements from multiple directions provide a unique opportunity to retrieve the horizontal information of the wave. The wave parameters of the wave event is fully determined, with a period of $\sim 86$ min, a horizontal wavelength of 300 km and propagate at $-156^\circ$ azimuth angle. The wave has a observed phase speed of $60$ ms$^{-1}$ and intrinsic phase speed of $40$ ms$^{-1}$. And the background temperature and winds are also determined. The atmosphere is mostly stable under $95$ km and instabilities occur above that as Richardson number approaches $1/4$. The wave packet is propagating upward and gets ducted near $\sim 90$ km altitude, which is indicated by the vertical wavenumber calculated from dispersion relation. With a full set of wave and background parameters, multiple dispersion relations based on different assumptions such as isothermal and windless background, are tested in this study. The comparisons show that the effects of variable background temperature and winds are important in the linear theory, diagnostic analysis based on simplified dispersion relations should be with cautions. Polarization relations are also examined among $\tilde{T}$, $\tilde{u}$, $\tilde{v}$ and $\tilde{w}$. The results of amplitude ratio and phase difference from the observational data and linear theory are mostly consistent but still discrepancies are found. This is expected and may be indicative of possible dissipation of the gravity waves. Maui is in the middle of Pacific Ocean where convection is believed to an important sources of gravity waves. Relations between convection and gravity waves have been observed from lower troposphere to the upper atmosphere [Alexander et al., 1995, 2004; Hoffmann and Alexander, 2010]. In order to analyze the propagation direction preference of gravity waves observed by an airglow imager at the same site, Li et al. [2011] used satellite data to find that the convection activities exist both north and south of Maui (within $10^\circ$ latitude) through the winter time, with more and closer to Maui in the north. So the gravity wave observed by the lidar may originate from some convection from the north.

This study is partly motivated by the fact that most observational instruments can only measure part of gravity waves spectrum, either horizontal or vertical structures.
4.5. SUMMARY AND CONCLUSIONS

In this work, we propose a novel method that uses a single-site lidar configured in multiple-direction observing mode to resolve a gravity wave fully in 3-D space. The sensitivity study demonstrates the capability of this method in detecting medium-scale and -frequency gravity waves when the wave amplitudes exceed certain magnitudes. This could provide some insights to those lidar systems with similar configuration [Hu et al., 2002; Cai et al., 2014; Ban et al., 2015; Liu et al., 2016]. A little different with what is presented here, some of these lidar systems are equipped with 2–4 telescopes that are pointed to several fixed directions depending on research demands. So if their temporal resolution and rotating squeeze are properly configured, more gravity wave cases can be detected and studied with this method. The statistical characteristics of the medium-scale and -frequency gravity waves in the MLT region could benefit the gravity wave parameterization.
Chapter 5

Climatology of Gravity Waves Observed by an Airglow Imager at Cerro Pachón

5.1 Introduction

High-frequency atmospheric gravity waves carry significant amount of momentum. The dissipation and breaking of these waves have large impacts to the circulation through the momentum deposition to the the background flow. Due to the ‘observation filter’ effect, this part of gravity waves spectrum is mostly observed through airglow imaging system [Hecht et al., 2001b; Ejiri et al., 2003; Hecht et al., 2004a; Li et al., 2011]. By studying the wave induced emission intensity perturbations, gravity waves information can be inferred. These gravity waves are revealed with typical horizontal wavelengths of 20–100 km, intrinsic wave periods of 5 to 10 min, and horizontal phase speeds between 30 to 100 ms$^{-1}$ [Taylor, 1997; Ejiri et al., 2003; Li et al., 2011]. The momentum flux estimated from wave induced emission perturbation has an average magnitude of 5–10 m$^2$s$^{-2}$ [Tang et al., 2005a; Li et al., 2011; Tang et al., 2014]. And tidal modulation of gravity wave
momentum flux has been observed through radar data and airglow images. Studies based on airglow observations suggest that the wave propagation in the mid-latitudes often shows an annual variation: polarward in summer and equatorward in winter. Several mechanisms such as critical layer filtering [Taylor et al., 1993], ducted wave propagation [Walterscheid et al., 1999], variations of wave sources location [Nakamura et al., 2003] and Doppler-shifting by the local winds [Li et al., 2011] are proposed to explain the directionality of wave propagation. In general, these mechanisms control the propagation direction cooperatively and are effective at different seasons.

ALO is located at the Andes, whose ridge is aligned generally in the north-south direction and extended over a long distance. Many satellite observations have revealed the existence of gravity waves hotspots over this region in the stratosphere [Hoffmann et al., 2013; Hindley et al., 2015]. Whether these active gravity waves reach higher MLT region before they break remains an unanswered question. By comparing the results with previous deployment of airglow imager at Maui, the similarities as well as differences of wave characteristics, propagation direction, and momentum flux may reflect the generality and specialty of wave sources and background winds at different locations. In this chapter, the multiple-year OH airglow data acquired at ALO is used to study the distribution of the intrinsic gravity wave parameters, dominated propagation direction and possible mechanisms controlling that, and variation of momentum flux and its relation with background wind. Section 5.2 briefs the dataset and methods. Section 5.3 discusses the results. The summary and conclusions are presented in section 5.5.

5.2 Data and Methodology

The all-sky airglow imager from UIUC was installed at ALO since September 19, 2009. The number of OH airlgow images obtained since then are summarized in Table 5.1 for each month. Over more than 6 years, the data acquires are available for all calendar
month, with enough images for robust analysis of seasonal variations of gravity wave climatology.

Table 5.1: The number of OH airglow images for each month at ALO.

<table>
<thead>
<tr>
<th>Year</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>31791</td>
</tr>
<tr>
<td>2010</td>
<td>10453</td>
<td>10879</td>
<td>12639</td>
<td>19083</td>
<td>11529</td>
<td>7630</td>
<td>12600</td>
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<td>7000</td>
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<tr>
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<td>14175</td>
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<td>270</td>
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<td>2679</td>
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<td>2580</td>
<td>2909</td>
<td></td>
<td>3519</td>
<td>2001</td>
<td>1385</td>
<td>194</td>
<td></td>
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<td></td>
<td>14334</td>
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<tr>
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<td>19779</td>
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<td>32956</td>
<td>28430</td>
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<td>16367</td>
<td>24221</td>
<td>316630</td>
</tr>
</tbody>
</table>

High frequency, quasi-monochromatic gravity waves are identified from the images using a series of procedures described in detail in Tang et al. [2002, 2005a,b]. A 2-D spectral method is used to identify gravity waves from airglow images. Three consecutive images \( I_1, I_2, I_3 \) were used to form two consecutive time-differenced (TD) images \( TD_1 = I_2 - I_1, TD_2 = I_3 - I_2 \) for spectral analysis. Wave parameters including wavelength, observed phase speed, propagation direction and relative airglow intensity perturbation \( I_{OH} / \bar{I}_{OH} \) were derived from each set of two TD images. Vertical wavelength is calculated using dispersion relationship (equation (1.4)) with buoyancy frequency in the OH airglow layer derived using temperature from NRLMSISE-00 empirical model [Picone et al., 2002]. The relative airglow intensity perturbation is calculated by dividing the perturbation amplitude \( I_{OH} \) by the average intensity \( \bar{I}_{OH} \) of the star-free and de-trended images after excluding the dark current and background emission, which is assumed to be 30% of total emission intensity [Swenson and Mende, 1994]. The gravity wave momentum flux was calculated based on their intrinsic parameters and the temperature amplitude, converted from \( I_{OH} / \bar{I}_{OH} \) using the airglow model described in Swenson and Gardner [1998] and Liu and Swenson [2003]. The total gravity wave momentum
5.2. DATA AND METHODOLOGY

Flux can be calculated using the following equation:

\[ F_m = \frac{k g^2}{m N^2} \left( \left( \frac{T'}{T} \right)^2 \right) = \frac{k g^2}{m N^2 C_{CF}^2} \left( \left( \frac{T'_{OH}}{T_{OH}} \right)^2 \right) \left( \text{m}^2 \text{s}^{-2} \right), \tag{5.1} \]

of which \( k, m \) are the horizontal and vertical wavenumber, \( N^2 \) is the squared buoyancy frequency and \( C_{CF} \) is the cancellation factor. \( C_{CF} \) is a function dependent on wave intrinsic parameters, especially vertical wavelengths [Liu and Swenson, 2003; Hickey and Yu, 2005]. For each set of images (two TD images or three raw images), there can be zero to multiple gravity waves identified and counted within this 3-min minimal interval. There may be persistent wave events lasting longer than 3 minutes, so a persistent wave event will be counted as several waves in our analysis with slightly varied wave characteristics from different sets of TD images. In this chapter and chapters 6, the analyses are based on waves identified from every set of TD images, while in chapter 7, independent and persistent waves that may last over several sets of TD images are distinguished.

The derivation of intrinsic parameters such as intrinsic phase speed and intrinsic frequency, and calculation of momentum flux require background wind measurements. A meteor radar was deployed nearby to provide continuous hourly-averaged horizontal winds between 80 and 100 km [Franke et al., 2005]. Wind speeds were determined by tracking meteor trail positions and Doppler shifts [Hocking et al., 2001] with the assumption that the horizontal wind field is uniform within a time-height interval and the vertical wind is negligible. The winds around the OH airglow layer are calculated through Gaussian-weighted averaging centered at 87 km. Figure 5.1 shows the horizontal winds measured by the meteor radar near ALO. Clear tidal structure and annual oscillation can be found in both zonal and meridional winds, with a strong eastward wind during austral summer between 00:00 to 06:00 UT time. The meteor radar had some technical issues in mid 2014, thus no more wind data were obtained afterwards. Therefore, for
5.2. DATA AND METHODOLOGY

Gravity waves analysis, we only process the airglow data when the meteor radar wind data were available (2009 to 2014).

Figure 5.1: Monthly mean (top) zonal and (bottom) meridional wind in OH airglow layer from meteor radar.

There are a few things in the data processing that need extra attention when discussing results in the following sections. Firstly, the TD method acts as a high-pass filter and excludes stationary wave features such as mountain waves. The influence of TD method is discussed in details in Section 5.5. Secondly, the Doppler shift correction is applied after the observed (ground-based) wave parameters are obtained instead of shifting the images according to background wind before TD images are obtained as done in Li et al. [2011]. Thirdly, after Nov 2012, some pixels on the imager CCD were broken and a black band about 20 km wide showed up in all airglow images constantly. The bad pixels are cropped which makes the images used for wave extraction smaller than previously used. This brings little difference in extracted wave parameters.
5.3 Results

5.3.1 Wave Characteristics

Figure 5.2: Histograms of gravity wave parameters (from top to bottom, left to right), horizontal wavelength, vertical wavelength, observed phase speed, intrinsic phase speed, period and relative intensity. Small vertical solid lines on top of each bar indicate the 95% confidence interval for each frequency.

Figure 5.2 demonstrates the histograms (frequency) for typical gravity wave parameters, including horizontal wavelength, vertical wavelength, observed phase speed, intrinsic phase speed, intrinsic period and wave amplitude. The bin sizes for them are 2.5 km, 2.5 km, 5 ms\(^{-1}\), 5 ms\(^{-1}\), 1 min and 0.1%, respectively. In order to evaluate the robustness of the histogram, Bootstrapping method is used to estimate the 95% confidence interval for each frequency. In this study, the number of samples are large, the histograms are very reliable as indicated by small statistical uncertainties. The horizontal wavelengths of most waves are less than 100 km with peaks near 20–30 km. The vertical wavelengths are mostly larger than 10 km and with peaks near 15-25 km range. Due to the cancellation effects of intensity perturbations in airglow layer [Liu and Swenson, 2003], waves with vertical wavelength smaller than the thickness of airglow layer will be greatly attenuated in airglow images. These wavelengths are similar to those found in Maui [Li et al.,
and other sites [Taylor et al., 1993; Nakamura et al., 1999; Hecht et al., 2004a; Dou et al., 2010]. As indicated by the calculation of vertical wavelength, most of the waves (78%) identified from airglow images are freely-propagating waves with vertical wavelength larger than 10 km. Note that the calculation of vertical wavelength requires background temperature which is retrieved from empirical model NRLMSISE-00 instead of real-time observations. So the estimation of vertical wavelength is treated as reliable only in climatological perspective. When daily variations are considered, discrepancies are expected.

The observed (ground-based) horizontal phase speeds are peaked near 45–55 ms$^{-1}$, while intrinsic horizontal phase speeds are peaked near 60–70 ms$^{-1}$, which indicates waves mostly propagate against background winds. For the wave intrinsic period, the short-period (high-frequency) waves dominate with period mostly less than 10 min, with peak near 5–6 min. Due to the fact that most gravity waves propagate against the background wind, the waves are Doppler-shifted to higher intrinsic frequency and large vertical wavelength, which makes high-frequency waves more likely to be observed in airglow images. The wave induced emission intensities are less than 2% and peak near 0.5–0.6%.

### 5.3.2 Propagation Direction

The distribution of wave propagation and background wind directions are shown by the histogram in polar coordinate with a 22.5° bin width in azimuth angle. In Figure 5.3, the histograms are organized by month, four rows are summer, fall, winter and spring in Southern Hemisphere. Overall, gravity waves tend to propagate against background wind during later spring to early fall (Nov to Mar). From summer to early fall (Dec to May), the dominant wave propagation direction is mostly southward/polarward. While in winter time (Jun to Aug), the dominate wave propagation direction is northward/equatorward, and opposite-direction relation with background is not that distinct.
In order to explain the preferential propagation direction of waves, potential wave source locations and background wind filtering are considered.

The high-frequency gravity waves tend to propagate upward in a more vertical path. So the waves observed by airglow imager are believe mostly to be generated by convective activities nearby. Tropical Rainfall Measuring Mission (TRMM) satellite data [Tropical Rainfall Measuring Mission (TRMM), 2011] is commonly used to study the tropical and sub-tropical precipitation and associated heat release. The number of pixels detected
by precipitation radar (PR) onboard the satellite is directly related to the strength of convective activities. Level 3 dataset (3A25) that is interpolated into $0.5^\circ \times 0.5^\circ$ grid is used. Area between $90^\circ$W and $40^\circ$W, $10^\circ$S and $37^\circ$S (latitude coverage limit of the TRMM satellite) is selected to quantify the convection around ALO. In Figure 5.11, the monthly mean numbers of convective pixels in each month are shown by colors. The polar histogram of propagation direction of gravity waves is also shown and is centered at ALO. The circle on the map has a radius of 1000 km on earth surface and corresponds to 500 waves for the histogram. The occurrence frequency of gravity waves is quantified as the ratio of the number of identified waves to the number of images. The absolute values may not represent anything physical but indicate the relative likelihood of occurrence of gravity waves in different months. In Figure 5.11, the occurrence frequency and the average number of waves per month are also shown on the map. The occurrence frequencies are high over winter and early summer (Jun to Oct) and low over summer and fall (Jan to May).

On the continent of South America, there are a few notable areas with strong convection on the east side, including Amazon Basin in the tropics and La Palate Basin in the subtropics ($\sim 30^\circ$S). They provide a large amount of moisture and energy for deep convection and precipitation [Insel et al., 2010; Romatschke and Houze, 2010]. ALO is located at the west side of Andes whose typical elevation reaches 4–7 km and thus blocks warm moist air from the east. The convective activities represented by the number of pixels show clear seasonal variations and high correlations with the wave propagation direction. During austral summer (Dec to Feb), the Amazon Basin shows a strong convective activities over a large area. Even the distance is more than 1000 km away from ALO, the waves still have a clear preference of southwestward propagation but with lower occurrence frequencies. From spring to early fall (Sep to Apr), there is also a strong and localized convective source over La Palate Basin to the southeast of ALO. The wave propagation shows a preference of westward or northwestward in some months (Sep,
Nov, Jan, Feb, Mar). And in winter (Jun to Sep), the closest and strongest convective source is in the Pacific Ocean to the southwest of ALO and coast area to the south of ALO, during which the wave propagation is clearly northeastward or northward and the occurrence frequencies are high. Due to the satellite orbit inclination, there is no data retrieved beyond 37°S. Sources of some northward propagation waves such as in Mar and Sep can not be identified from TRMM data. However, they may be generated from convections south of 37°S. There is an important issue regarding the distance between ALO and potential wave source locations. As shown in Figure 5.11, some wave sources are more than 1000 km away from ALO such as Amazon Basin and sources to the east coast of South America. Simulation studies has shown that long-range propagation of gravity waves in MLT region is possible and attributed to the ducted propagation [Hecht et al., 2001a; Snively and Pasko, 2008; Snively et al., 2013; Heale et al., 2014a]. Evidences that can demonstrate direct relation between observed waves at ALO and wave sources are in need.

Critical-layer filtering is an another important mechanism that controls the propagation of gravity waves in the atmosphere. When gravity waves reach a layer where wave observed phase speed equals background wind speed, waves will be absorbed or filtered. Based on equation (A.18), the Doppler-shifted or intrinsic frequency \( \hat{\omega} \) can be related to observed frequency \( \omega \) by

\[
\hat{\omega} = \omega \left(1 - \frac{\bar{u} \cos \phi + \bar{v} \sin \phi}{c}\right),
\]

of which the term \( \bar{u} \cos \phi + \bar{v} \sin \phi \) is the background wind \((\bar{u}, \bar{v})\) projected to wave propagation direction. ‘Blocking diagram’ [Taylor et al., 1993; Medeiros et al., 2003] is introduced to demonstrate the ‘forbidden zone’ of gravity waves, i.e., the range of phase speed \( c \) and propagation azimuth angle \( \phi \) of waves that would be filtered out in certain background wind profiles where \( \hat{\omega} \leq 0 \) is satisfied.
5.3. RESULTS

Figure 5.4: Monthly mean (left) zonal and (right) meridional winds averaged between 00:00 to 06:00 UT at longitude of ALO (71°W), vertical dashed lines indicate the latitude of ALO (30°S).

Currently, there is no complete observations of atmospheric winds from source level to mesopause at ALO. We turn to the model winds from Horizontal Wind Model (HWM07) [Drob et al., 2008], which reasonably reproduces climatological winds. Figure 5.4 shows the latitude-altitude cross section of monthly mean zonal and meridional winds at 70°W in Jan, Apr, Jul and Oct. At ALO, zonal winds are eastward in austral winter in the stratosphere and westward in summer with largest magnitudes exceeding 60 ms⁻¹. Meridional winds magnitudes are much smaller and are mostly polarward but equatorward in summer above 50 km. In Figure 5.5, ‘blocking diagrams’ were plotted for each month using the monthly averaged wind profiles from HWM07 at ALO. They represent the effects of critical layer filtering on gravity waves accumulated in the altitude range from 15 km that is above most convective activities to 87 km that is the peak altitude of OH airglow. The observed phase speed and propagation direction of all waves are also
plotted by scattered dots. The ‘forbidden zones’ of gravity waves predicted by critical layer filtering theory are mostly along west and east directions due to much larger amplitudes of zonal wind component especially in stratosphere. As shown in Figure 5.5, a lot of waves can be found in the ‘forbidden zones’ in some months. But still, areas around small phase speeds showing as hollows in the scattered plots but not matching the ‘forbidden zones’ in some months may indicate the effects of critical layer filtering.

Figure 5.5: Scatter plots of apparent phase speed (0–100 ms\(^{-1}\)) and propagation azimuth angle. Small amount of waves with phase speed large than 100 ms\(^{-1}\) are not included here. Area inside the solid black lines are the ‘forbidden zone’ predicted by critical layer filtering theory.
5.3. RESULTS

Here, critical layers predicted by HWM07 model can not explain the wave propagation direction well. The monthly mean winds retrieved from HWM07 cannot capture the short-period variation of the real winds such as tidal influences, day-to-day variability and any waves that have period longer than gravity waves that are observed by airglow imager. Time-varying background winds make the effects of critical layer filtering reduced because a lot of waves (as much as 70%) have less interaction time with a perceived critical layer and/or some changes in observed phase speed to avoid the critical layer filtering [Heale and Snively, 2015]. This is especially true for the waves observed by airglow imagers that are mostly high-frequency waves, with periods less than 15 min. Figure 5.6 shows the azimuth angle differences between gravity wave propagation and

![Direction Difference](image)

Figure 5.6: The difference between gravity wave propagation and background wind azimuth angles for the waves of different observed phase speed. The percentages on the circles shows the percents of total waves.
background wind directions. Note the wind is simultaneously retrieved from meteor radar with the corresponding waves. For waves with observed phase speed less than 20 m/s, it is prominent that the azimuth angle differences are highly clustered around 180°. This means those waves mostly propagate against the winds, which is an indicator of critical layer filtering of waves propagating along the winds. The concentrated distribution around 180° becomes less notable when observed phase speeds are larger. For waves of phase speed between 20 and 40 m/s, it is still clear most waves propagate toward opposite hemisphere with background wind in polar coordinate. For those faster waves with phase speed larger than 50 m/s, their propagation shows less dependence on background wind and is evenly distributed at all directions. The monthly winds in the OH airglow layer are around 30–40 m/s as shown in Figure 5.1. Winds would be able to filter out waves with observed phase speed similar or smaller than that. Besides the effects of critical layer, waves propagate along the background winds are Doppler-shifted to smaller vertical wavelength thus larger shear may occur to make waves more easily to break down due to instability. However, faster waves more likely penetrate these potential critical layers. Combined with Figure 5.5, results show that the background winds have a large and consistent impact on propagation of gravity waves, especially the slower waves.

5.3.3 Momentum Fluxes

Figure 5.7 shows the monthly mean zonal and meridional gravity wave momentum fluxes ($\langle u'w' \rangle$ and $\langle v'w' \rangle$) with zonal and meridional background winds averaged over 22:00–06:00 UT in the OH airglow layer. Overall, the zonal and meridional momentum fluxes have the magnitudes from several to 10 m²s⁻² with meridional component slight larger than zonal one. Both momentum flux components tend to toward the opposite direction of background winds. Zonal momentum flux is mostly westward and zonal wind is most eastward. There are some intra-seasonal variations in zonal momentum flux and
wind. The opposite directionality between meridional momentum flux and wind is more distinct. Meridional momentum flux shows a clear annual oscillation with northward maximum near austral winter time and southward maximum in summer. Gravity wave momentum fluxes at mesopause altitude are affected by both wave sources in the lower atmosphere and critical layer filtering by the mean flow between the sources and the mesopause [Li et al., 2011]. The change of the momentum flux is related to the variation of the location of primary wave sources, which mostly locate at east and northeast of ALO and south in winter.

Figure 5.7: Monthly mean (top) zonal and (bottom) meridional (left axis, red) momentum flux and (right axis, blue) wind from 2009 to 2014. The zonal and meridional winds are averaged only between 22:00 to 06:00 UT.

5.4 The Effects of Time-Differenced Method

A set of three consecutive images are used to obtain two TD images for spectral analysis. For the airglow imager at ALO, only OH airglow images were captured with a 1 min integration time before 25 Aug 2011. A sample of image sequences is shown in Figure 5.8(A). Two TD images ($TD_1 = I_2 - I_1$, $TD_2 = I_3 - I_2$) are obtained from three
5.4. THE EFFECTS OF TIME-DIFFERENCED METHOD

![Diagram of temporal sequences of OH and OI airglow images.](image)

Figure 5.8: Temporal sequences of OH and OI airglow images. The width of OI images is wider showing the integration time for OI airglow images is 1.5 min while 1 min for OH airglow images. The shadowed OH images are a set of three images selected for TD method.

continuous OH images \((I_1, I_2, I_3)\). In this case, a gravity wave detected from \(TD_1\) and \(TD_2\) is considered to have lasted the duration of three images, i.e., 3 min. Starting on 25 Aug 2011, the imager captures OH and OI airglow images alternately with 1-min and 1.5-min integration time, respectively. The sample of image sequences is shown in Figure 5.8(B). There is one OI airglow image between two closest OH airglow images. In this case, a gravity wave should last at least 6 min to be detected in this set of three OH airglow images. Because of this change, the number of gravity waves identified from OH airglow images obtained at these two different observation modes are not comparable. The minimal durations of gravity waves, i.e., the time of three consecutive OH images, should be taken into consideration when statistics are calculated from the whole dataset.

The TD method was implemented by taking the difference of two consecutive images. This method is equivalent to a high-pass filter and emphasizes the high-frequency gravity waves in wave extraction. The magnitudes of frequency response of TD method can be
5.4. **THE EFFECTS OF TIME-DIFFERENCED METHOD**

described by the following equation:

\[
G(w) = \left| 1 - e^{i\omega} \right|,
\]

of which \( \omega \) is the angular frequency from 0 to \( 2\pi \), which corresponds to the regular frequency from 0 to \( 1/\Delta t \). \( \Delta t \) is the minimum interval between two consecutive airglow images, which could be 1 min or 2.5 min as shown in Figures 5.8(A) and 5.8(B). In Figure 5.9, the frequency response shows that the TD method augments the amplitude of waves with relatively short periods and dramatically suppresses the amplitude of waves with long and extremely short periods, which may make them less likely to be detected. The period ranges that the amplitudes are amplified are different for different time intervals. When \( \Delta t=1 \) min, the period range of strengthened amplitudes is narrow and near the periods of 2–7 min. When \( \Delta t=2.5 \) min, this period range is broader and extends to 15 min. Firstly, the TD method itself may distort the probability distribution of gravity wave parameters shown in Figure 5.2. Secondly, the differences of the minimum interval may also cause some discrepancies in the statistical results.

The discussions above about the frequency response of TD method do not consider the gravity wave amplitudes of different periods. Theoretical [Gardner and Liu, 2014] and...
5.4. THE EFFECTS OF TIME-DIFFERENCED METHOD

Figure 5.10: Wave amplitude of applying TD method on the gravity waves with a theoretical spectrum of $\tilde{\omega}^{-2}$.

Observational [Guo et al., 2017] studies have shown that the power frequency spectrum is approximately proportional to $\tilde{\omega}^{-p}$ for gravity waves of the period range of 5 min to 6 hr, where $p$ is typically around 2. So the gravity wave amplitudes are approximately proportional to wave intrinsic period $\tilde{T}$ ($\tilde{T} = 2\pi/\tilde{\omega}$). The wave amplitudes after applying TD method is calculated by multiplying the gravity wave frequency spectrum calculated from lidar measurements [Guo et al., 2017] by the TD frequency response and is shown in Figure 5.10. From the perspective of wave detection, the TD method attenuates the amplitude of gravity waves of longer period, thus make them less likely to overwhelm the gravity waves of shorter periods.

The estimation above provides some insight on the potential influences of TD method. The TD method should be used with caution. The effects of TD method is largely related to the time interval of images. When results from different sites or observation modes are compared, this difference should be taken into account. For the data acquired at ALO, in order to minimize the discrepancies due to different time intervals, a tread-off is to skip every other image for the time period that only OH airglow images were captured. The selection of a set of three images is shown in Figure 5.8(C) and has a 2-min time interval. In Figures 5.9 and 5.10, the frequency responses of the TD method and theoretical frequency responses for 2-min and 2.5-min time intervals do not show

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5.5 Summary and Conclusions

The long-term dataset from 2009 to 2014, retrieved by an all-sky airglow imager at ALO, is used to investigate the characteristics of high-frequency quasi-monochromatic gravity waves. The typical horizontal wavelengths are around 20–40 km and ground-based horizontal phase speeds are between 40 and 70 ms$^{-1}$. The intrinsic periods of gravity waves cluster approximately 4–10 min. The observed gravity waves tend to propagate against the background wind in most months and also show strong seasonal variations in the propagation direction. The momentum flux estimated from airglow data has a magnitude of several to 10 m$^2$s$^{-2}$ and tends to be toward opposite direction of background winds in airglow layer, especially in meridional direction. These results are consistent with previous studies based on airglow images from other mid-latitude sites such as Fort Collins, CO (20°N) [Tang et al., 2014], Maui, HI (20°N) [Li et al., 2011], Shigaraki, Japan (35°N) [Nakamura et al., 1999] and Urbana, IL (40°N) [Hecht et al., 2001a].

It has been suggested that source locations, background wind filtering and ducting influence the wave propagation together. ALO locates at a place near or within the zone of influence of several remarkable convection sources. During the austral summer, the convection over Amazon Basin is dramatically strong and expands over a vast area. Those waves with southwestward propagation direction could originate from there. Even the stratospheric zonal wind are mostly westward in this season, the wave sources overwhelm the background wind filtering effect in determining the directionality of wave propagation. In winter time, the close convection is over the Pacific Ocean or coast area to the south of ALO, this could mostly explain the northeastward and northward preferential propagation direction. Critical-layer filtering could not explain the propagation
direction preference in many months. Ideally, for the investigation of wave propagation using ‘block diagram’, simultaneous and full wind measurements nearby the imager site are required. However, more comprehensive observations are needed before any conclusion can be drawn on this. The result of this study may not conclude that the anisotropy of propagation direction was almost due to wave filtering by stratospheric winds as Taylor et al. [1993] and Medeiros et al. [2003]. But still, the opposite direction of gravity waves and background wind indicates some filtering effects of critical layer on those slower waves.

The gravity waves observed by the airglow imager at ALO are not generated locally (within 100–200 km) since no convection is identified nearby. As revealed by the TRMM satellite data, the convective activities over South America continent and east Pacific Ocean occurs at certain fixed places in different seasons. The observed waves should propagate from these places either directly or through ducts. Airglow images could not distinguish whether waves are ducted. Further studies could be done from the perspective of potential wave sources. The relationship between the strength of the convective activities and wave occurrence frequency, wave amplitude can be described with some dependence on distance. In other words, it could be possible to quantify the influential area of certain convective activities. This would provide some insight regarding a simplified assumption in the gravity wave parameterization that the horizontal propagation of waves are neglected and higher atmosphere only responses to the waves from the sources underneath.
Figure 5.11: (Color) Convective rain pixels retrieved from TRMM satellite overlapped on the map showing the coastline of South America. The polar histograms show the propagation direction of gravity waves, same as red Figure 5.3. The radius of the circle is $\sim$1000 km on the earth surface and 500 waves for histogram.
Chapter 6

Intermittency of Gravity Wave Momentum Flux in the Mesopause Region

6.1 Introduction

Gravity waves with different characteristics appear with different probabilities in the atmosphere due to the random nature of both gravity wave generation and the variation of the background atmosphere they propagate through. Convection and topography are two important sources of gravity waves. Convective gravity waves show significant temporal variability and are also spatially localized [Alexander et al., 1995; Fritts and Alexander, 2003; Alexander et al., 2004]. Orographic gravity waves are directly related to the flow over the topography of mountain ridges or islands in the ocean. The randomness of the low-level winds thus contributes to the intermittency of gravity waves. As a result, gravity wave dissipation and forcing on the atmosphere are random and often highly intermittent. The extremely large gravity wave forcing associated with breaking of a large amplitude gravity wave can alter the background atmosphere significantly,
6.1. INTRODUCTION

even though these waves may happen rarely and their long-term effect may be small. On the other hand, small amplitude gravity waves are more ubiquitous. Even though their short-term effects are small, they could have a lasting impact on the background atmosphere. In order to understand the comprehensive effects of gravity waves in the atmosphere, both the long-term mean properties of gravity waves and the effects of a minority of large gravity waves need to be studied.

Recently, Hertzog et al. [2012] and Plougonven et al. [2013] investigated the intermittency of gravity wave momentum flux (MF) in the lower stratosphere above Antarctica and the Southern Ocean using balloon data. They found that the probability density functions (pdfs) of gravity wave MF largely follow a lognormal distribution and some deviation from lognormal distribution is found at the larger MF. Monte Carlo simulations were used to show that the lognormal distribution can be explained by the random nature of the background wind variation which affects the critical level filtering. Similar analysis has been done by Wright et al. [2013] on a global scale using HIRDLS satellite data for gravity waves in the 25–65 km altitude range. These works based on different data reveal several important facts about gravity wave intermittency: The intermittency of gravity wave MF in the stratosphere varies with season and altitude; the intermittency over mountain areas is significantly higher than in other areas, represented by a longer tail in pdfs of gravity wave MF; and those rare waves with extremely large MF contribute significantly to the total MF.

In this study, the intermittency of gravity wave MF in the mesopause region, where gravity wave breaking and dissipation are strongest, is investigated for the first time. The analysis is based on a large number of gravity waves identified from multiyear OH airglow measurement (around 87 km altitude) at Maui and ALO. We focus on the statistical characteristics, in particular the pdfs of gravity wave MF at these two sites with distinctively different gravity wave sources. Maui is in the middle of the Pacific with no nearby strong gravity wave sources while ALO is in the Andes Mountains with
strong orographic gravity wave sources. We compare the results of these two sites to examine the differences in intermittency that may be attributable to the differences in wave source and background atmosphere. Based on the pdfs, we also compare the relative importance of gravity waves with large and small MF.

Section 6.2 describes the instrument, dataset and analysis methods used to obtain the pdfs. Section 6.3 presents the pdfs of gravity wave MF at Maui and Cerro Pachón, the intermittency measures based on three diagnostic parameters, comparison of the relative importance of waves with large and small MF, and the seasonal variations of pdfs and intermittency. The significance of these findings are discussed in section 6.4, followed with conclusion in section 6.5.

6.2 Data and Methodology

The data processing procedures are discussed in previous chapter. The total number of identified gravity waves is listed in Table 6.1, together with the number of nights that imager data are available. As revealed in Li et al. [2011], most of the gravity waves identified with this method are high frequency with periods less than 30 min and small scale with horizontal wavelength shorter than 120 km. The results of this study therefore apply to gravity waves in this parameters range only. While lower frequency, larger horizontal-scale gravity waves are not included, they are expected to be less frequent in the mesopause region and carry less MF [Fritts and Vincent, 1987].

While the instrument and algorithm for identifying gravity waves for the two sites are the same, there are two notable differences. First, the Milky Way over Cerro Pachón is present and close to zenith most of the time and is much brighter than airglow emission within the imager observation bandwidth. An additional procedure of removing the Milky Way [Li et al., 2014] is necessary and applied before gravity waves are identified. Second, the exposure time of each image was 2 min at Maui and 1 min at Cerro Pachón.
6.2. DATA AND METHODOLOGY

The longer exposure time in Maui has little influence on the sensitivity to gravity wave detection because most of the gravity wave periods are longer than 5 min. The higher temporal resolution largely contributes to the larger numbers of gravity waves identified at Cerro Pachón than that at Maui as shown in Table 6.1. Better sky conditions at Cerro Pachón are also a contributing factor.

Table 6.1: Statistics of gravity waves at Maui and Cerro Pachón.

<table>
<thead>
<tr>
<th>Site</th>
<th>Start Date</th>
<th>End Date</th>
<th># of Nights</th>
<th># of Wavesa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maui</td>
<td>05/20/2002</td>
<td>06/13/2007</td>
<td>591</td>
<td>12709</td>
</tr>
<tr>
<td>ALO</td>
<td>09/20/2009</td>
<td>11/10/2012</td>
<td>219</td>
<td>60409</td>
</tr>
</tbody>
</table>

a See the definition of the number of waves in the text.

The pdfs of the MF are calculated from the whole data set for each site. For MF values up to 200 m²s⁻², we divide them into small bins and counted the number of waves in each bin, then divided it by the total number of waves, to obtain the occurrence frequency. Since the occurrence frequencies have an extremely large dynamic range, varying 3-4 orders of magnitude from the smallest to the largest MF values, the bin size of MF is chosen to be uniform at a logarithmic scale. As such, the actual bin size is smaller for smaller MF values and increases with the MF value. This approach is similar to that used by Wright et al. [2013] in order to obtain a more reliable estimate of the probability at large MF values. The occurrence frequency in each bin was then normalized by the bin width so the final pdfs were obtained in unit of probability per unit MF, i.e., m⁻²s².

It is important to note that when a wave is identified in a set of two TD images (a group of three consecutive images), it is counted as one wave. If a wave event lasted over multiple sets of TD images, one wave was identified from each set so multiple waves were recorded. Therefore, the number of waves defined in this study is not the number of coherent "wave events", rather it is a quantity proportional to the duration of waves. With this definition, the number of waves within a MF range reflects the
6.3 Results

6.3.1 Probability Density Function

Figure 6.1 shows the pdfs of the absolute MF \( \sqrt{(\langle u'w' \rangle^2 + \langle v'w' \rangle^2)} \) at Maui and Cerro Pachón for the whole data set. The MF derived from the airglow imager data is \( \langle u'w' \rangle \) and \( \langle v'w' \rangle \) in unit of m\(^2\)s\(^{-2}\). For easier comparison with other studies, the corresponding values of \( \rho \sqrt{(\langle u'w' \rangle^2 + \langle v'w' \rangle^2)} \) in unit of pascals are also indicated with a second axis in Figure 6.1, calculated using the mean atmospheric density \( \rho = 5.67 \times 10^{-6} \text{ km}^{-3} \) at \( \sim87 \text{ km} \) altitude according to NRLMSISE-00 model [Picone et al., 2002]. The latter MF is conserved throughout the altitudes when gravity waves do not experience dissipation.

Note that the uniform bin size in logarithm scale corresponds to a bin size of \( \sim10 \text{ m}^2\text{s}^{-2} \) at \( 200 \text{ m}^2\text{s}^{-2} \) and \( \sim0.1 \text{ m}^2\text{s}^{-2} \) at \( 1 \text{ m}^2\text{s}^{-2} \). The pdfs of both sites show some similarity in their shapes with clear peaks near \( \sim1-2 \text{ m}^2\text{s}^{-2} \) and long tails beyond \( \sim10 \text{ m}^2\text{s}^{-2} \). The shape of pdfs shows that there are more gravity waves with smaller MF at Cerro Pachón than at Maui. And there is relatively less probability of large amplitude gravity waves at Cerro Pachón. Close examinations of the pdfs suggest that they follow a log-normal function in the small MF range and a power-law function in large MF range, with the transition near \( \sim16 \text{ m}^2\text{s}^{-2} \) at both sites.

To obtain analytical expressions of the pdfs, we performed a least square fit with the
6.3. RESULTS

Figure 6.1: Histograms of the absolute MF from Maui (red) and Cerro Pachón (blue) in the log-log coordinates. Thick solid curves are pdfs based on the least square fitting of the histograms. The two short vertical lines near 16 m$^2$s$^{-2}$ indicate the transition points between the log-normal distribution on the left and power-law distribution on the right. The dashed lines are the extensions of the log-normal functions, to show the departure from this distribution at large MF. The horizontal axis is absolute MF, with the bottom axis labeled in unit of m$^2$s$^{-2}$ and the top axis labeled in unit of mPa.

The following piecewise function:

\[
y = \begin{cases} 
\frac{1}{\sqrt{2\pi}\sigma x} \exp \left( -\frac{(\ln x - \mu)^2}{2\sigma^2} \right) & \text{if } x \leq x_0, \\
a \left( \frac{x}{x_0} \right)^b & \text{if } x \geq x_0,
\end{cases}
\]

(6.1)

of which $x$ is the absolute MF, $y$ is the corresponding probability density, and $x_0$ is the transition point between the lognormal and power law functions. Continuity between
two functions at $x_0$ is a constraint in the fitting process; i.e.,

$$
\frac{1}{\sqrt{2\pi \sigma x_0}} \exp \left( -\frac{(\ln x_0 - \mu)^2}{2\sigma^2} \right) = a.
$$

(6.2)

The fitting was done in log-log space so the result is not overly weighted by the large probabilities at small MF. In log-log coordinates, the function in (6.1) can be written as

$$
\ln y = \begin{cases} 
- \ln(\sqrt{2\pi \sigma}) - \ln x - \frac{(\ln x - \mu)^2}{2\sigma^2} & \text{if } \ln x \leq \ln x_0, \\
\ln a + b(\ln x - \ln x_0) & \text{if } \ln x \geq \ln x_0,
\end{cases}
$$

(6.3)

so $\ln y$ is a parabolic and a linear function of $\ln x$ in the two regions, respectively. Therefore, instead of fitting the lognormal and power law functions directly, we fit the piecewise function of a parabola and a straight line, with the continuity requirement at the transition point, to obtain values of $x_0$, $\mu$, $\sigma$, $a$ and $b$.

The pdf parameter values from the least squares fit for Maui and Cerro Pachón are listed in the Table 6.2. For the lognormal distribution, $\mu$ is the mean and $\sigma$ is the standard deviation of the normally distributed $\ln x$. The mean value $\mu$ is much larger at Maui, corresponding to $\text{MF} = \exp(1.76) = 5.8 \, \text{m}^2\text{s}^{-2}$, compared with $\text{MF} = \exp(0.75) = 2.1 \, \text{m}^2\text{s}^{-2}$ at Cerro Pachón. This indicates that the MF is on average larger at Maui. The standard deviations $\sigma$ are similar at the two sites. The peak of the pdf is located at $\exp(\mu - \sigma^2)$, which is $0.69 \, \text{m}^2\text{s}^{-2}$ at Cerro Pachón and $1.48 \, \text{m}^2\text{s}^{-2}$ at Maui. The transition points $x_0$ are very close at the two sites, around $16 \, \text{m}^2\text{s}^{-2}$. For the power law distribution, the magnitude of the slope $b$ is slightly larger at Cerro Pachón, indicating a faster decrease of probability of gravity waves with larger MF. The fitted piecewise functions are plotted in Figure 6.1, and they match the probability histograms very well at both sites. Also listed in Table 6.2 are the fractions of waves included in the lognormal and power law regions of pdf functions and their relative contributions to the total MF. It is clear that most of the waves (71.5% at Maui and 94.0% at Cerro Pachón)
are in the log-normal region where MF is relatively small, but waves in the power-law region where MF is large contribute more (81.8% at Maui and 51.4% at Cerro Pachón) to the total MF.

Table 6.2: Fitted parameters of the piecewise functions at Maui and Cerro Pachón, the percentages of waves included in each region and their relative contributions to the total MF.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Lognormal</th>
<th>Transition</th>
<th>Power law</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location</td>
<td>µ</td>
<td>σ</td>
<td>x₀</td>
</tr>
<tr>
<td>Maui</td>
<td>1.76</td>
<td>1.17</td>
<td>71.3%</td>
</tr>
<tr>
<td>Cerro Pachón</td>
<td>0.75</td>
<td>1.06</td>
<td>94.0%</td>
</tr>
</tbody>
</table>

The lognormal distribution in gravity wave MF has also been found in the lower atmosphere with satellite [Alexander and Grimsdell, 2013; Wright et al., 2013] and balloon [Hertzog et al., 2012; Jewtoukoff et al., 2013; Plougonven et al., 2013] measurements. In these studies, the left side of the lognormal distribution, i.e. the decrease of probability toward smaller MF values, were not clearly shown (see Figure 2 in Hertzog et al. [2012], Figure 8 in Alexander and Grimsdell [2013], and Figure 16 in Jewtoukoff et al. [2013]). This makes it difficult to make a proper fit of the lognormal distribution and identify its peak. In Figure 6.1, a more complete picture of the lognormal distribution is shown with the excellent fit of the probability histograms at both sides of the peak. In the large MF region, the broader tail was noted by Hertzog et al. [2012] and Alexander and Grimsdell [2013], and Wright et al. [2013] suggested some linear relation in log-log scale of the pdfs. We are able to confirm that the long tail region indeed follows a power law function as shown by the straight line in Figure 6.1. This power law distribution gives a broader tail (slower decrease with increasing MF) than the log-normal distribution at the large MF region. It indicates that gravity waves with MF larger than \(x₀\) occur more frequently than a lognormal distribution would imply. Hertzog et al. [2012] attributed this broader tail to higher gravity wave intermittency. In the next section, we will study the intermittency in more detail.
6.3. RESULTS

Table 6.3: The mean, median and standard derivation of the MF for all waves, and waves falling in lognormal and power law distribution range.

<table>
<thead>
<tr>
<th>Site</th>
<th>Mean</th>
<th>Median</th>
<th>Std. a</th>
<th>Mean</th>
<th>Median</th>
<th>Std.</th>
<th>Mean</th>
<th>Median</th>
<th>Std.</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>20.33</td>
<td>6.63</td>
<td>34.08</td>
<td>5.20</td>
<td>3.86</td>
<td>4.18</td>
<td>57.89</td>
<td>39.71</td>
<td>44.98</td>
</tr>
<tr>
<td>Lognormal</td>
<td>5.60</td>
<td>2.00</td>
<td>15.00</td>
<td>2.90</td>
<td>1.84</td>
<td>2.95</td>
<td>48.10</td>
<td>30.32</td>
<td>41.33</td>
</tr>
<tr>
<td>Power law</td>
<td>5.60</td>
<td>2.00</td>
<td>15.00</td>
<td>2.90</td>
<td>1.84</td>
<td>2.95</td>
<td>48.10</td>
<td>30.32</td>
<td>41.33</td>
</tr>
</tbody>
</table>

* The std. represents the standard derivation of the MF.

From the perspective of the MF magnitudes, the mean and median values of the MF for all the waves and waves falling in the lognormal and power law distributions are calculated. In Table 6.3, all the median values of MF are smaller than the mean values. Since the probability distributions of MF are highly skewed, the median values could better represent the ‘typical’ magnitude of the MF. The median values of MF are 6.63 m$^2$s$^{-2}$ and 2.00 m$^2$s$^{-2}$ for Maui and Cerro Pachón, respectively. The median MF (3.86 m$^2$s$^{-2}$ and 1.84 m$^2$s$^{-2}$) of the waves falling in the range of lognormal distribution are very close to the overall median MF, compared to the ones for waves in the range of power law distribution. Comparison of the MF in unit of pascals shows that the magnitudes of the gravity wave MF (up to 1 mPa) in MLT region are much smaller than those measured in the lower atmosphere (up to 60 mPa in Hertzog et al. [2012]). This is expected since the saturation and breaking of gravity waves restrict the growth of wave amplitudes when they propagate upward. The gravity waves detected by the airglow imager in the mesopause region, if originated in the lower atmosphere, must have extremely small amplitudes that are not observable there. Alternatively, some of these gravity waves may be generated in the middle atmosphere through secondary wave generation [e.g. Snively and Pasko, 2003; Vadas et al., 2003; Fritts et al., 2009].

6.3.2 Intermittency

Several parameters, such as the Bernoulli proxy and the percentile ratio used in Hertzog et al. [2008], and the Gini coefficient [Plougonven et al., 2013; Wright et al., 2013]
have been proposed to quantify the overall intermittency of the gravity wave MF. The Bernoulli proxy is calculated as

$$\epsilon_1 = \frac{1}{1 + \sigma^*^2 / \mu^*^2},$$

(6.4)

where $\mu^*$ and $\sigma^*$ are the mean and standard deviation of all MF measurements, respectively. Note that $\mu^*$ and $\sigma^*$ are different from $\mu$ and $\sigma$ in (6.1). The Percentile ratio is defined as a ratio of two percentiles (50% and 90%) of MF magnitudes. If a total of $N$ waves are sorted according to their MF magnitude $f_i$ ($1 \leq i \leq N$) so $f_{i-1} < f_i$ for all $i$, the percentile ratio is calculated as

$$\epsilon_2 = \frac{f_{0.5N}}{f_{0.9N}},$$

(6.5)

Hertzog et al. [2008] explained that for these two parameters, values close to 1 indicate continuous occurrence (low intermittency) and values close to 0 indicate large variability or large intermittency.

Following Plougonven et al. [2013], the Gini coefficient is defined as

$$\epsilon_3 = \frac{\sum_{n=1}^{N-1} (n\mu^* - F_n)}{\sum_{n=1}^{N-1} n\mu^*},$$

(6.6)

where $F_n = \sum_{i=1}^{n} f_i$ is the cumulative sum of MF. It is widely used in economics to describe the inequality of wealth. $\epsilon_3 = 0$ corresponds to perfect equality and $\epsilon_3 = 1$ to total inequality. When applied to the intermittency, a large (small) Gini coefficient corresponds to large (small) intermittency, opposite to that represented by $\epsilon_1$ and $\epsilon_2$.

Here we use all three parameters to assess the intermittency of gravity waves from the airglow data. As shown in Table 6.4, the Bernoulli proxy and the percentile ratio both indicate that gravity waves at Maui have larger intermittency, while the Gini coefficient
Table 6.4: Three intermittency parameters at Maui and Cerro Pachón.

<table>
<thead>
<tr>
<th>Location</th>
<th>Percentile Ratioa</th>
<th>Bernoulli Proxy</th>
<th>Gini Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maui</td>
<td>0.114</td>
<td>0.738</td>
<td>0.681</td>
</tr>
<tr>
<td>Cerro Pachón</td>
<td>0.198</td>
<td>0.878</td>
<td>0.694</td>
</tr>
</tbody>
</table>

a The two percentiles are chosen as the 50% and 90%

shows no significant difference. In Plougonven et al. [2013], the average Gini coefficients calculated from balloon observations in the stratosphere (17–19 km altitude) vary from 0.44 over the Southern Ocean to 0.63 over the Antarctic Peninsula. The difference is attributed to the difference between orographic and nonorographic gravity wave sources. The Gini coefficients calculated from the HIRDLS data (25–65 km altitude) are less than 0.4 in all regions with some variations with respect to altitude from the stratosphere to the lower mesosphere [Wright et al., 2013]. Our Gini coefficients at both sites are larger than these lower atmospheric values, indicating higher intermittency in the mesopause region.

The difference in the intermittency measures from the three parameters suggests that there may not be a single best parameter to measure the observed gravity wave intermittency. Bernoulli process treats the gravity wave source as two simple "on" and "off" processes without varying amplitude and, therefore is too simple to reflect the realistic sources with varying amplitudes. The percentile ratio largely depends on two arbitrarily chosen percentiles and cannot adequately represent the whole probability distribution. In addition, as shown in section 6.3.1, the observed pdfs cannot be represented with a single function. To quantify the relationships between the shape of a pdf and the intermittency measures, we performed Monte Carlo simulations with randomly generated MF values that satisfy a given pdf and calculated the three intermittency parameters from these MF values to examine how they vary with the pdf shape. The pdf parameters $\sigma$ and $\mu$ vary in the range of $0.1 \sim 3.0$, which includes our fitted values. The slope $b$ and the transition point $x_0$ are fixed using the fitted values at Maui and Cerro Pachón.
Figures 6.2a–6.2c (Figures 6.2d–6.2f) show the variations of the three intermittency parameters as functions of \( \mu \) (\( \sigma \)), with \( \sigma \) (\( \mu \)) fixed using the values at the two sites. The intermittency parameters calculated from the airglow measurements are indicated by crosses.

Figure 6.2 shows that the percentile ratio and the Gini coefficient are generally consistent, as their values vary with opposite trends (same trends in intermittency) as functions of \( \mu \) as well as \( \sigma \). However, the percentile ratio and the Bernoulli proxy are not always consistent. Comparing the percentile ratio with the Bernoulli proxy as functions of \( \mu \), we can see that they both indicate an increase in intermittency (decrease in parameter values) as \( \mu \) increases up to 1.5. However their trends are opposite when \( \mu > 1.5 \). For variations with \( \sigma \), again they vary in opposite direction for \( \sigma < 1.0 \). Therefore the two intermittency parameters may give opposite change in intermittency with the same change in pdf. These inconsistencies show that one needs to be careful when using these parameters to measure the intermittency.

### 6.3.3 Relative Importance of Large and Small Waves

From the pdfs, it is clear that gravity waves with small MF occur much more frequently than those with large MF. When gravity waves break and deposit momentum to the background atmosphere, their impact is related to the total momentum flux, i.e., the product of MF magnitudes and the durations of the waves. To compare quantitatively the relative contributions of gravity waves with different MF values, we make use of the Lorenz curve [Lorenz, 1905], which, like Gini coefficient, is also used in economics to represent wealth distribution.

We first sorted all detected gravity waves according to their MF magnitudes, and then calculated the cumulative MF as a function of the fraction of waves. This is shown in Figure 6.3a. A point \((x = n/N, y = F_n/F_N)\) on the curve indicates that the bottom \(x\) fraction of waves (ranked according to their MF magnitude) contribute \(y\) fraction to
Figure 6.2: The intermittency parameters as functions of \( \mu \) (a–c) or \( \sigma \) (d–f). In Figures 6.2a–6.2c (Figures 6.2d–6.2f), \( \sigma \) (\( \mu \)) and \( b \) values are fixed using the fitted values in Table 6.4 at the two sites. Crosses in plots mark the fitted pdf’s parameters. The Percentile ratio (Figures 6.2a and 6.2d), Bernoulli proxy (Figures 6.2b and 6.2e), and Gini coefficient (Figures 6.2c and 6.2f) are calculated with Monte-Carlo simulations.

the total MF. When the Lorenz curve is a straight diagonal line, the MF contribution is evenly distributed among all waves. The Gini coefficient is represented as the ratio of the area between this diagonal line and the observed Lorenz curve to the total area below this diagonal line. The closer the curve to the diagonal line, the more uniform the distribution is and thus less intermittency.

A feature to note in Figure 6.3a is that the two Lorenz curves for Cerro Pachón and Maui intersect. The cross point corresponds to 78% percentile of gravity waves at 25% total MF contribution. Because of the crossing of the two curves, the areas between the curves and the diagonal line are similar for the two sites, leading to the close Gini coefficients. This is an evidence that the Gini coefficient is sometimes insufficient to describe the overall MF distribution.

For Maui, the intersection point of the two Lorenz curves corresponds to MF of 23.5 m\(^2\)s\(^{-2}\). This value is about 4 times the mean value in the lognormal region (5.8 m\(^2\)s\(^{-2}\),
6.3. RESULTS

Figure 6.3: (a) Lorenz curves for Cerro Pachón and Maui. The cumulative sum of number of gravity waves \( n \) and MF are normalized to 1. (b) The curve for the cumulative contribution of the waves with MF less than \( m \) to the total MF. The magnitudes of MF is normalized by the maximum of \( M = 200 \text{ m}^2\text{s}^{-2} \).

see section 3.1) and is well beyond the transition point \( x_0 = 16.6 \text{ m}^2\text{s}^{-2} \). Therefore the contribution to the total MF at Maui is mainly from gravity waves in the power-law region with very large MF values, which are highly intermittent. One example of such a large MF wave at Maui was reported by Li et al. [2007a] based on both airglow imager and Na lidar measurements, in which case the MF was about 70 \( \text{m}^2\text{s}^{-2} \). For Cerro Pachón, the intersection point corresponds to MF of 4.9 \( \text{m}^2\text{s}^{-2} \), a little over twice the mean value of 2.1 \( \text{m}^2\text{s}^{-2} \) in the lognormal region and is much less than the transition point \( x_0 = 15.9 \text{ m}^2\text{s}^{-2} \). Therefore at Cerro Pachón a significant portion of the total MF is from gravity waves with small MF.

Figure 6.3(b) provides a different perspective of this distribution. The cumulative contribution to the total MF is plotted against the magnitudes of gravity wave MF instead of number of waves. A point \( (x = m/M, y = F_m/F_M) \) on the plot indicates that all waves with MF less than \( m \) contribute to \( y \) fraction of the total MF. \( M = 200 \text{ m}^2\text{s}^{-2} \) is a chosen maximum MF used to normalize the \( m \) values. The diagonal line corresponds to the case when the pdf is of the form \( f(x) = 1/x \), i.e., the total duration of gravity
waves within a certain MF range is inversely proportional to their MF, so gravity waves with different MF contribute equally to the total MF. The curves of the two sites are both above the diagonal line, indicating that waves with smaller MF contribute relatively more, because they appear more frequently. The curve for Cerro Pachón is higher than that for Maui, indicating that the contribution from waves with small MF is more significant at Cerro Pachón, consistent with the above analysis with Lorenz curves. For the total MF contributed by gravity waves with individual MF within 200 m$^2$s$^{-2}$, 50% is from gravity waves with MF less than 8.5% (32%) of the maximum 200 m$^2$s$^{-2}$ at Cerro Pachón (Maui), and 20% is from gravity waves with MF less than 1.9% (9%) of the maximum at Cerro Pachón (Maui).

The relative contributions can also be quantified by multiplying the gravity wave MF magnitudes with their corresponding probabilities, which yields the relative contributions from gravity waves of different MF magnitudes per unit MF as shown in Figure 6.4. This relative contributions compare the comprehensive effects of gravity waves by considering both the MF magnitude and duration of gravity waves. These two curves show that the relative contributions follow a near normal distribution in the semilog coordinates. Gravity waves with large MF do not contribute a lot to the total MF because they appear so infrequently. The most effective gravity waves have MF values of $\sim2.2$ m$^2$s$^{-2}$ (almost $5.5$ m$^2$s$^{-2}$) at Cerro Pachón (Maui). These values are close to the values of $\exp(\mu)$ discussed in section 3.1.

Finally, a more precise estimate of the total MF should take into account the gravity wave propagation directions. For this purpose, the above analysis was repeated separately for the zonal and meridional MF, $\langle u'w' \rangle$ and $\langle v'w' \rangle$, based on wave propagation directions derived from the airglow image analysis [Tang et al., 2002]. We found that the pdfs of both zonal and meridional MFs (not shown) are very similar to that of the absolute MF at both positive and negative values, with the mean zonal and meridional MFs of $4.39$ and $2.44$ m$^2$s$^{-2}$ at Maui, and $-0.46$ and $0.36$ m$^2$s$^{-2}$ at Cerro Pachón, re-
6.3. RESULTS

Figure 6.4: Relative contribution of waves with different MF values to the total MF.

spectively. These values are comparable to previous studies [Gardner and Liu, 2007; Li et al., 2011] and are very small compared to the MF of those infrequent large waves. Similar to those curves in Figure 6.3b, the curves for net zonal and meridional MFs are shown in Figure 6.5. All the curves are still above the diagonal lines, which confirms that the long-term average of MF is contributed more by waves with smaller MF values, and this disparity is more pronounced at Cerro Pachón than at Maui.

6.3.4 Seasonal Variation

The seasonal variations of the pdfs were obtained by performing the same analysis with data grouped by calendar month from the multiyear data at both sites. The transition points $x_0$ were fixed for all months using the values in Table 6.2 to allow a more consistent comparison. The seasonal variations of $\sigma$ and $\mu$ with their 95% confidence levels are
Figure 6.5: Cumulative contribute to the net zonal (a) and meridional (b) MFs at Cerro Pachón (red) and Maui (blue). A point $(m, p)$ on the curve represents the cumulative contribution to the total MF as a percentage $p$ for waves with MF magnitude less than $m$. The calculation of MF contribution in each direction takes into account the signs and thus represents the net effect.

shown in Figure 6.6. Although there are some uncertainties, noticeable annual and semiannual variations can still be found. $\sigma$ represents the range of variations of MF values for gravity waves in the lognormal region. At Maui, $\sigma$ reaches maximum in April and a secondary maximum in December, and reaches the minimum in October. At Cerro Pachón, $\sigma$ is maximum in January with a secondary maximum in April, and is at minimum in July. The parameter $\mu$ corresponds to the mean MF for gravity waves in the lognormal region. Its seasonal variation are very similar at both sites, with a strong semiannual variations that peaks in winter and summer. This indicates relatively strong gravity wave activities at these seasons. At Cerro Pachón, the maximum value of $\mu$ in winter is much larger than the secondary maximum in summer. This fact matches the results of analyzing the temperature and intensity variances of airglow measurements at El Leoncito (31.8°S, 69.2°W) [Reisin and Scheer, 2004], which is very close to ALO.

The seasonal variations of the slope $b$ of the power law distribution are shown in Figure 6.7. There is a clear annual variation with largest slope in August at Maui and
Figure 6.6: Monthly values of the Lognormal parameters $\sigma$ (a,c) and $\mu$ (b,d) of Maui (a,b) and Cerro Pachón (c,d), with 95% confidence intervals.

November at Cerro Pachón. The minimum occurs in November at Maui and in March at Cerro Pachón, indicating higher intermittency because the smaller slope magnitude indicates a broader tail in the power law region. Also shown in Figure 6.7 are the cumulative probabilities of gravity waves in the power law region. At both sites, they show a clear semiannual variation, with peaks in winter and summer, indicating relatively more gravity waves with large MF during these seasons. This also contributes to the larger total MF-like variations of $\mu$ as shown in Figures 6.6b and 6.6d.

The seasonal variations of the three intermittency parameters are shown in Figure 6.8. Similar to the earlier analysis with the whole data set, the percentile ratio and the Gini coefficient give consistent measures of the intermittency but the Bernoulli proxy
Figure 6.7: Seasonal variations of the Power-law parameters $b$ at (a) Maui and (c) Cerro Pachón, with 95% confidence intervals, and the cumulative probability of the power-law region at (b) Maui and (d) Cerro Pachón.

does not. Both the percentile ratio and the Gini coefficient indicate large intermittency around spring and fall and small intermittency in winter and summer. The Gini coefficients are especially consistent at the two sites in different hemispheres: both have the largest intermittency in the fall, the secondary maximum in the spring, and the smallest intermittency in the summer.

In Hertzog et al. [2008] the long tails of pdfs progressively disappear from late winter to early summer during their campaign. Similar results are also shown in Wright et al. [2013]. This trend corresponds to an increase in the magnitude of $b$, which is also shown in Figure 6.7 from winter to spring at Maui and from summer to winter at Cerro Pachón.
Figure 6.8: Seasonal variations of the intermittency at Maui (red) and Cerro Pachón (blue) as measured by (a) the percentile ratio, (b) the Bernoulli proxy and (c) the Gini coefficient. Large intermittency corresponds to small values in percentile ratio and Bernoulli proxy but large values in Gini coefficient.

Variations of background wind filtering may be the main season for this seasonal changes.

6.4 Discussion

The gravity wave intermittency is mainly influenced by two factors [Hertzog et al., 2008; Plougonven et al., 2013; Wright et al., 2013]. One is the wave source because the physical processes that generate gravity waves are intermittent. The other is the background atmosphere through which gravity waves propagate. Fluctuations in the background wind and temperature cause variations in wave filtering, refraction and dissipation and then influence wave intermittency at the altitudes above. In the lower stratosphere, satellite data [Wright et al., 2013] and balloon data [Plougonven et al., 2013] both show that orographically excited gravity waves always show higher intermittency, as indicated by larger Gini coefficient and broader tails in pdfs of MF. Since the lower stratosphere is close to the gravity wave source, these results imply that the source intermittency is generally higher for orographic gravity waves. The mesopause region is much farther
away from gravity wave sources, and it is expected that the background fluctuations play more important roles in affecting the gravity wave intermittency in this region. The higher intermittency at Maui is likely due to higher variability of the background atmosphere, because Maui is not expected to have a higher number of orographic gravity waves as Cerro Pachón. The consistency in seasonal variation of intermittency at the two sites is another evidence of strong influence of background atmosphere.

When comparing intermittency among different measurements, it is important to note that different instruments are sensitive to different parts of the spatial and temporal spectra of gravity waves. The effects of ‘observation filter’ [Alexander, 1998; Gardner and Taylor, 1998; Alexander et al., 2010] should be carefully considered. As shown in Figure 8a in Alexander et al. [2010], satellite (infrared limb-sounding) and superpressure balloons have different visibilities to the gravity wave spectrum. The airglow images are sensitive to waves with relatively large vertical wavelength (>10 km) and short horizontal wavelength (approximately tens of kilometers) [Li et al., 2011], which covers the part of the spectrum that is ‘invisible’ to satellite limb sounding and balloons. Even though this observation filter may potentially affect the pdfs of detected gravity waves, the remarkable similarities between pdfs in this study and those obtained with different instruments strongly suggest that the lognormal and power law distributions are universal features across the entire gravity wave spectrum.

In the airglow imager data analysis, one consequence of using TD image is that it removes all stationary mountain waves. The mountain waves can be an important source of MF, especially above Cerro Pachón during austral winter. However, only mountain waves generated by steady surface wind propagating in a steady background atmosphere are stationary. Many orographically generated gravity waves are transient or intermittent due to surface wind intermittency or changing background atmosphere and can be detected with our analysis method and are included in the pdfs. Small-amplitude, stationary mountain waves are very difficult to detect, because they are often not steady
6.5. CONCLUSIONS

enough over a long time to be identified. If detectable stationary mountain waves are included, we expect that they may contribute to a little increase of the pdf in the large MF region but will not significantly alter the current results.

We should also point out that the impact of gravity waves on the background atmosphere is related to the MF, but the net forcing is dependent on the momentum deposition from dissipating gravity waves. Nondissipating gravity waves transport momentum but do not impart a net forcing to the background atmosphere. The data from a single layer of airglow emission cannot provide vertical variation of MF; therefore it is inadequate for directly calculating momentum deposition. In addition, if gravity waves are ducted under certain conditions, they can propagate both upward and downward in a layer with a zero net MF. Many observational studies on wave ducting [Isler et al., 1997; Walterscheid et al., 1999; Hecht et al., 2001a; Ejiri et al., 2003; Nielsen et al., 2009] suggest that the ducted gravity waves are highly variable and the percentages of ducted waves vary from a few percent to over 70%. Nevertheless, the mesopause region is where most gravity waves break and deposit their momentum; the statistics of the absolute MF presented here is a good proxy of the overall impact of gravity waves. A more detailed study of gravity wave forcing should take into account wave dissipation and ducting, with additional measurements of airglow from different altitudes or data from other instruments, to resolve the vertical variation of MF. Some numerical models that couple the gravity waves and the response in airglow emissions [Hickey et al., 2010a,b] can provide the MF profiles near airglow layers and determine the gravity wave forcing on the mean flowing.

6.5 Conclusions

We have obtained for the first time the probability density functions (pdfs) of gravity wave MF in the mesopause region at Maui and Cerro Pachón, based on multiyear airglow
image data. The pdfs for gravity waves with smaller MF are found to fit very well with a lognormal distribution. The pdfs in the larger MF region, described as ‘long tail’ in Hertzog et al. [2012], fit very well with a power-law distribution. The transition points between the two different distributions are around $\sim 16 \text{ m}^2\text{s}^{-2}$ at both sites. Because of the large amount of gravity waves, these two distributions are well defined through the fitting process. It enables detailed study of gravity wave intermittency and their relative contributions to the total MF.

The gravity wave intermittency was quantified using three parameters: the Bernoulli proxy, the percentile ratio, and the Gini coefficient. The first two parameters show that gravity waves have higher intermittency at Maui than at Cerro Pachón, while the Gini coefficient shows little difference between the two sites. Monte Carlo simulations were performed to examine the relationships of the pdf with the three intermittency parameters and revealed some inconsistencies. The same change of the lognormal pdf parameters may result in opposite change in intermittency measured with different parameters. This shows the limitations of using these parameters in representing gravity wave intermittency. In general, the percentile ratio and the Gini coefficient give more consistent intermittency measure than the Bernoulli proxy.

Even with these inconsistencies, it is clear that the overall intermittency is much larger at Maui. Mesopause region is farther away from the wave sources. Assuming the source intermittency is larger for orographic gravity waves, as shown in previous studies in the stratosphere, the observed larger intermittency at Maui is indicative of a larger background variability at this site.

We also examined the relative importance of gravity waves in terms of their contribution to the total MF. If measured in terms of the overall time and number of waves, the majority of the total MF is contributed by a small fraction of gravity waves with largest MF. At both sites, during 22% of the time gravity waves with largest MF contribute to 75% of the total MF. However, if measured in terms of MF values, those with small MF
6.5. CONCLUSIONS

contribute relatively more. Gravity waves with MF less than 8.5% (32%) of the maximum MF (200 m²s⁻²) contribute to 50% of the total MF at Cerro Pachón (Maui). In terms of the relative contributions at different MF values, gravity waves with MF around 2.2 m²s⁻² at Cerro Pachón and 5.5 m²s⁻² at Maui are most effective contributors.

Seasonal variations of the pdfs and intermittency are also examined. Clear annual and semiannual variations are found and are remarkably consistent at the two sites. By comparing the Gini coefficient at both sites, it is found that the largest intermittencies are in the fall, with a secondary maximum in the spring. The minimum intermittency occurs in summer and the secondary minimum is in winter. Because of the different characteristics of the gravity wave sources at the two sites, the consistency in seasonal variation is another evidence that the intermittency in the mesopause region is largely determined by the gravity wave propagation conditions associated with the background atmosphere.
Chapter 7

Duration of Gravity Waves in OH Airglow Layer Observed by an Airglow Imager

7.1 Introduction

In the MLT region, there exist multiple airlgow layers such as OH airglow layer that has a typical full-width at half-maximum (FWHM) thickness of \(\sim 7 \text{ km} \) centered at \(\sim 87 \text{ km} \) altitude. Many observations from airglow imaging have shown that quasi-monochromatic, small-scale, high-frequency gravity waves propagate through these layers or break within the layer due to instability. These waves have typical periods of 5-20 min and horizontal wavelength around 20–80 km [Espy et al., 2004; Suzuki et al., 2007; Li et al., 2011]. The breaking of these small-scale gravity waves and occurrence of instabilities is severe and frequent in this altitude range and are mostly indicated by the ‘ripples’, i.e. those ‘wavy’ features with spatial scales about 10 km and time scales similar or shorter than buoyancy period of 5 min, as revealed also by high resolution airglow images. Previously, many complementary and simultaneous observations from lidar and airglow imager enable the
investigation of small-scale bore/ripple structures and instabilities associated with gravity wave breaking [Hecht et al., 1997; She et al., 2004b; Li et al., 2005; Smith et al., 2005; Cai et al., 2014]. Also, many observational and modeling studies reveal the evidences that horizontally long-range propagation of gravity waves airglow layer is related to wave ducting. [Snively et al., 2007, 2010; Suzuki et al., 2013b]. Dispersion relation is a useful measure to diagnose the gravity wave propagation which could be largely influenced by the background atmosphere such as the mean temperature and wind structures. As discussed in Section 1.1, gravity waves can freely propagate at altitude range with positive $m^2$, and get reflected or ducted when encountering region of negative $m^2$, depending on where those evanescent regions are. Ducted waves can travel longer horizontal distances as long as the duct is able to confine it, this enables the effects of gravity waves being exerted on the atmosphere far away from the original source. The evanescent environment of the atmosphere leading to ducted and/or reflected propagation of gravity waves varies as the background conditions. This also determines whether gravity waves could reach airglow layer and be observed.

The duration or lifespan of gravity waves has rarely been studied mostly due to lack of observations that are continuous and long enough. But it has important implications for gravity wave parameterization where it determines how frequently gravity waves exert forcing on the atmosphere. In this chapter, the duration of gravity waves are studied from the statistical perspective. Section 7.2 describes the detailed procedures of gravity wave event identification. Section 7.3 demonstrates the probability density functions and their mathematical expressions for the duration of gravity wave events. Section 7.4 presents a possible mechanism that can explain and lead to the probability distributions. The conclusions and summary are presented in section 7.5.
# 7.2 Wave Events Identification

As stated in chapter 5, there are possibilities that some coherent and persistent ‘wave events’ last longer than the minimum interval for a ‘wave’ that are identified from a set of three consecutive airglow images. In this chapter, the gravity waves are considered from the perspective of complete wave events. Hereafter, the term a ‘wave’ refers to the wave identified from a set of three airglow images and a ‘wave event’ refers to a coherent gravity wave composed of several consecutive ‘waves’. From many individual waves, wave events were distinguished by restricting the parameters of consecutive waves within certain range. Horizontal propagation azimuth angle and wavelength/wavenumber were chosen as the primary criteria because they are most directly retrieved from 2-D airglow images. After some tentative tests, $15^\circ$ and $0.001$ km$^{-1}$ are chosen as threshold values. Meanwhile, the wave period is considered as a secondary criterion.

Wave events identification was implemented in an iterative way. The method started from any wave $w_1$ at $t_1$, then tried to search for a wave in the next time step and also within the threshold propagation azimuth angle and wavenumber range. If such a wave $w_2$ at $t_2(= t_1 + \Delta t)$ was found, the search was restarted from the wave $w_2$ again. Finally, one independent wave event $(w_1, w_2, w_3, ..., w_n)$ was isolated when no more were found and the iteration stopped. The duration of a wave event is calculated as $n \cdot \Delta t$. In some cases, two or more wave events are ‘close’ whose propagation azimuth angle or wavelength are similar. This could make our criterion inapplicable in distinguishing these ‘close’ wave events. Manual intervention was needed to exclude false events and include unidentified events. These procedures were programed and applied on airglow data based on a GUI interface designed using Matlab. After the whole identification procedures on the whole dataset, majority of the waves were identified as part of persistent wave events. The rest of the waves can be treated as wave events with duration of minimum observation interval. Figure 7.1 shows the interface of the GUI, in which coherent wave events are shown as line-connected stars that represent propagation azimuth angle in the left
7.3. **Probability Density Functions of Wave Duration**

The statistics of the identified gravity wave events from two sites are shown in the Table 7.1. More waves are identified in Cerro Pachón than Maui due to the higher temporal resolution data and better sky conditions. Note that the number of wave events only include those with duration longer than minimal observational interval. Based on identified wave events, the duration of gravity wave events are calculated and the their statistics are analyzed. The probability density functions of wave event duration
7.3. PROBABILITY DENSITY FUNCTIONS OF WAVE DURATION

Table 7.1: Statistics of identified wave events at Maui and Cerro Pachón (ALO).

<table>
<thead>
<tr>
<th>Site</th>
<th>Start Date</th>
<th>End Date</th>
<th># of Nights</th>
<th># of Events&lt;sup&gt;a&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maui</td>
<td>05/20/2002</td>
<td>06/13/2007</td>
<td>529</td>
<td>2994</td>
</tr>
<tr>
<td>ALO</td>
<td>09/20/2009</td>
<td>01/30/2014</td>
<td>346</td>
<td>13598</td>
</tr>
</tbody>
</table>

<sup>a</sup> See the definition of wave events in the text.

are obtained by counting the number of wave events in bins of 6 min width and then being normalized by the total number. As shown in Figure 7.2, the probability density functions of both sites follow exponential distribution, i.e. straight lines in semi-log coordinate. The numbers in the parentheses near the tails of the histograms represent the numbers of wave event at the corresponding probabilities. In order to obtain the mathematical probability distribution, a least square fitting is applied on the histograms based on following mathematical formula:

\[
y = \begin{cases} 
\frac{1}{\tau_0} \exp \left( -\frac{x}{\tau_0} \right) & \text{if } x \leq x_0, \\
0 & \text{if } x \geq x_0, 
\end{cases} \quad (7.1)
\]

of which \( \tau_0 \) is the slope of the fitted trendline in semi-log coordinate. The fitting excludes the data points at large duration due to too few samples there. Finally, \( \tau_0 \) is derived as 9.22 (8.28–10.16) min at Maui with 95% confidence interval, and 15.80 (14.34–17.25) min at Cerro Pachón. For exponential distributions in equation ((7.1)), \( \tau_0 \) and \( \tau_0^2 \) are the mean and variance. The average and variance of duration of gravity wave events at Maui are larger than that at Cerro Pachón.
7.4. ONE POSSIBLE MECHANISM: BREAKING OF THE GRAVITY WAVES

Figure 7.2: Probability density functions of the wave event duration at Maui and Cerro Pachón. Thick straight lines are from least square fitting. The numbers in the parenthesis at the tails of histograms are the number of wave events at corresponding probabilities.

7.4 One Possible Mechanism: Breaking of the Gravity Waves

Duration of gravity waves identified from airglow images may be attributed to several factors. In the current image processing algorithm, only an area of $172 \times 172$ km$^2$ is selected for wave extraction and the airglow layer thickness is less than 10 km. Only those waves propagating through this space can be detected by an airglow imager, so the observed duration is equivalent to the time that wave packets penetrate the space vertically and horizontally. If a gravity wave event is simplified as a wave packet with certain scale, thus the duration should be related to the group speed of wave packet. The vertical group speed of the gravity waves can be estimated by [Fritts and Alexander,
2003]

\[ c_{gz} = \frac{-m(\hat{\omega}^2 - f^2)}{\omega \left( k^2 + l^2 + \frac{1}{4H^2} \right)}, \]

(7.2)
of which \( \hat{\omega} \) is the intrinsic wave frequency and \( k, l, \) and \( m \) are zonal, meridional and vertical wavenumber. In Figure 7.3, the distribution of vertical group speed with respect to duration is demonstrated with scattered dots. The vertical group speeds center at 30–40 m s\(^{-1}\) at both sites and there is no clear preference of the vertical group speed for wave events with either short or long duration. Thus, the wave event duration is independent of the propagation of wave packets to a certain extent.

Figure 7.3: Scatter plots of gravity wave vertical group speed with respect to wave event duration at Maui and Cerro Pachón. The contours are wave packet size estimated by multiplying group speed by duration.

Gravity waves were observed to be ducted in the airglow layer and propagated over a large horizontal distance [Simkhada et al., 2009; Suzuki et al., 2013b]. Those waves that are ducted near the airglow layer can present for a longer time in the FOV of an imager. Without enough information, especially the background conditions, we could not diagnose the propagation for each wave event. The scatter plots of wave intrinsic wave period with respect to the duration for all wave event are shown in Figure 7.4. The gravity wave events with longer duration tend to be with shorter intrinsic periods about 4–7 min. From the perspective of wave cycles, these persistent wave events have
7.4. **ONE POSSIBLE MECHANISM: BREAKING OF THE GRAVITY WAVES**

A larger number of wave cycles in time domain. Many numerical simulations confirmed that waves of shorter period are more likely ducted in the airglow layer [Snively et al., 2007, 2010]. In this case, the duration of these gravity waves is largely determined by the lifetime of the ducts that confine the gravity waves in the airglow layer.

![Figure 7.4: Scatter plots of gravity wave intrinsic period with respect to duration of the gravity wave events at Maui and Cerro Pachón.](image)

If the gravity waves break before escaping from the airglow layer, the duration of a gravity wave packet is controlled by two factors, the lifetime of its source and the stability of the atmosphere through which it propagates. If the atmosphere is stable, then a wave will continue to propagate through the region as long as the source still generates the wave. Wave sources like fronts or convective activities, tend to have longer lifetimes, which are typically on the order of a few hours to tens of hours [Hagos et al., 2013]. If the atmosphere is unstable, the wave will break and dissipate its energy as turbulence. With these preconditions, the duration of wave event in a stable atmosphere is controlled by its source, while in an unstable atmosphere, the duration is determined primarily by the occurrence rate of the instabilities.

Due to the cancellation effect, airglow imagers can only detect waves with long vertical wavelengths, which are comparable to or larger than the 10 km thickness of the airglow layer. However, at mesopause heights, the wave spectrum extends over a broad
range. The smallest scale waves have vertical wavelengths of only a few kms and cannot be observed by the imagers. These small-scale waves, in combination with the mean temperature and horizontal wind structure, are responsible for generating unstable regions, which cause the larger vertical wavelength waves to dissipate. The occurrence of these unstable regions is random because the wave field is generally random. The occurrence rate of the unstable regions is related to the mean temperature and wind structure and to the strength of the wave activity, viz. the fluctuation variances of the temperature lapse rate and wind shear [Zhao et al., 2003]. Because the smaller vertical scale waves create the instabilities that cause the larger vertical scale waves to dissipate, the duration of the waves observed by the imagers should not depend on their characteristics such as wavelength, period, propagation direction or even amplitude. All of the larger scale waves should have the same mean duration, which is related to the occurrence rate of the instabilities.

The random number of unstable regions (of sufficient severity and extent to cause a larger vertical wavelength wave to dissipate, viz, the fatal instabilities) observed in a time period of length \( t \) is Poisson distributed. Consider a time interval \((0, t + \Delta t)\) where \( \Delta t \) is so small that at most only one instability occurring in the interval \((0, t + \Delta t)\) is given by

\[
P(n, t + \Delta t) = P(n - 1, t) P(n = 1, \Delta t) + P(n, t) P(n = 0, \Delta t)
= P(n - 1, t) P(n = 1, \Delta t) + P(n, t) [1 - P(n = 1, \Delta t)].
\] (7.3)

We assume the interval \((t, t + \Delta t)\) is so small that the probability of one instability occurring in the interval is proportional to the length of the interval

\[
P(1, \Delta t) \approx \lambda_{in} (t) \Delta t
\]

\[
P(0, \Delta t) \approx 1 - P(1, \Delta t) = 1 - \lambda_{in} (t) \Delta t,
\] (7.4)
where $\lambda_{in}(t)$ is the rate of occurrence of the instabilities, which could vary with time, either long or short scale, and location, but does not depend on the large vertical scale waves being observed. So that equation (7.3) can be written as

$$P(n, t + \Delta t) \approx P(n - 1, t) \lambda_{in}(t) \Delta t + P(n, t) [1 - \lambda_{in}(t) \Delta t].$$

(7.5)

Rearrange terms in equation (7.6) and let $\Delta t$ go to zero, we have

$$\frac{P(n, t + \Delta t) - P(n, t)}{\Delta t} \approx \frac{\partial P(n, t)}{\partial t} = \lambda_{in}(t) [P(n - 1, t) + P(n, t)].$$

(7.6)

The solution of equation (7.6) is Poisson distribution defined as

$$P(n, t) = \left[ \int_{0}^{t} \lambda_{in}(x) \, dx \right]^n \frac{1}{n!} \exp \left[ - \int_{0}^{t} \lambda_{in}(x) \, dx \right].$$

(7.7)

Thus, if we observe a wave in an airglow imager at time $t = 0$, the probability that the lifetime of the wave is less than $\tau$ (cumulative density distribution) is equal to the probability that one or more ‘fatal’ instabilities occur within the interval $(0, \tau)$.

$$P_{\text{life}}(t < \tau) = \sum_{n=1}^{\infty} P(n = 0, \tau) = 1 - \exp \left[ - \int_{0}^{\tau} \lambda_{in}(x) \, dx \right].$$

(7.8)

Then, the probability density function of gravity wave lifetime is simply the derivative of equation (7.8).

$$p_{\text{life}}(\tau) = \frac{dP_{\text{life}}(t < \tau)}{d\tau} = \lambda_{in} \exp \left[ - \int_{0}^{\tau} \lambda_{in}(x) \, dx \right].$$

(7.9)

If the instability occurrence rate $\lambda_{in}(t)$ is constant with respect to time then equation (7.9) reduce to the exponential distribution.

$$p_{\text{life}}(\tau) = \lambda_{in} e^{-\lambda_{in}\tau} = \frac{e^{-\tau/\tau_{GW}}}{\tau_{GW}}.$$  

(7.10)
7.5. DISCUSSIONS

Note that, the mean duration of waves $\tau_{GW}$ is equal to the reciprocal of the ‘fatal’ instability occurrence rate $\tau_{GW} = 1/\lambda_{in}$. The exponential fit to the Cerro Pachón dataset suggests that the mean wave lifetime is 9 min so the instability occurrence rate is $1/9$ min$^{-1}$, while at Maui the mean lifetime is 15 min and the instability occurrence rate is $1/15$ min$^{-1}$.

7.5 Discussions

In this chapter, coherent wave events are isolated from long-term airglow observations from Maui and Cerro Pachón by restricting the consecutive waves parameters to be close under certain threshold. The duration of these wave events is found exponentially distributed and two different sites shows different exponents. Several possible scenarios of gravity waves being observed in airglow images are analyzed. A wave event either propagates through the airglow layer freely, or breaks within the layer due to instability. And the waves are also intermittently blocked by evanescent regions below the airglow layer. Some theoretical analysis is also addressed to explain the probability distribution of wave events duration.

The proposed mechanism treats the wave sources as nearly constant, so waves enter the airglow layer continuously and break before they escape from the layer. So wave duration is related to wave breaking that is controlled by instabilities. Key assumptions leading to the exponentially-distributed probability density functions include instabilities occur randomly and are caused by small vertical scale waves, not those observed by the imager. These assumptions imply: wave duration times are exponentially distributed. Duration times are independent of the observed wave characteristics. Mean duration time is related to background temperature and wind structure and the characteristics of the smaller vertical scale waves which cannot be observed by the imager. Mean duration will vary seasonally and geographically as the wave activity varies depending
on the instability condition.

Those waves that can be resolved by an airglow imager are mostly high-frequency, small-scale gravity waves, which tend to propagate in a more vertical path. It is known that the evanescent area in atmosphere dramatically modify the propagation path, especially the long-range propagation in horizontal direction [Snively et al., 2010; Suzuki et al., 2013b]. The ducting environment for gravity waves has been studied using lidar and airglow imaging data, which varies significantly in time [Bossert et al., 2014]. They show that waves observed in the AMTM may have been intermittently propagating and evanescent at the OH layer at these times due to multiple evanescent regions below 87 km. So the propagation condition for the gravity waves could also be important in determining the wave duration. To be brief, the duration of waves in airglow layer is related to the occurrence of evanescent regions underneath the airglow layer for upward propagation waves. Therefore, with similar assumptions and maths the probability density functions could be derived if the occurrence of evanescent region is also random.

Here, the disappearance of the gravity waves in airglow layer is simply explained by the wave breaking through instabilities or blocking by evanescent regions. Even though the exponentially-distributed probability density function could be mathematically derived, the effects of these factors seem to mix with each other. The whole processes can be analyzed through statistical simulations, in combination with an airglow model that describes the relationship between the observed airglow perturbations and gravity waves [Liu and Swenson, 2003]. With a properly specified wave source below the airglow layer, the duration of wave appearance in the airglow layer can be simulated. By varying the stability of the background atmosphere and a simple wave breaking criteria, the effects of the stability on wave duration can also be simulated. The observed statistical distributions of wave durations will then be used to infer the underlying conditions in the real atmosphere based on the simulation results.
Chapter 8

Summary and Outlook

8.1 Summary

In the MLT region, gravity waves are one of most difficult to be observed comprehensively because they have a variety of spatial and temporal scales, and various potential sources. It is well known that gravity waves indeed help to transfer significant amount of momentum and energy, thus play critical roles in influencing and coupling the atmosphere. A dilemma about gravity waves is that, compared to the large-scale planetary waves and tides that are reasonably understood and well simulated in GCMs, gravity waves are not explicitly represented due to their sub-grid scale. So to develop good physically-based gravity wave parameterization for use in GCMs is essential. And the parameterization is naturally based on a lot analytical, numerical and especially observational studies of gravity wave characteristics. This dissertation aims to explore the fundamental processes how gravity waves propagate and interact with atmosphere and advance the constraints of gravity wave parameterization.

In order to understand the dynamical processes of gravity waves in 3-D space, with less assumptions and simplifications, we use multiple or specially-configured instruments to study gravity waves. Two case studies of gravity waves are presented in this disserta-
8.1. SUMMARY

tion. In the first case, a narrow-band sodium lidar and an all-sky airglow imager reveal a distinct gravity wave event that undergoes partial reflection at two altitudes and approaches a near-critical layer in between. The horizontal wavelength and propagation direction of wave was determined from airlgow images and vertical structures retrieved from lidar temperature and vertical wind. The fully determined wave parameters enable the numerical modeling of the wave event, which suggests that the wave packet undergoes dual reflection and transmission at \( \sim 85 \) km and 101 km altitude where larger vertical winds are observed, and a near-critical layer at \( \sim 93 \) km altitude leading to enhanced shears and thus instability in the wave field. This study demonstrates how a combination of instrumentation and modeling can be used to complement each other to provide a better explanation of gravity wave events. Modeling confirms our interpretation of the observations and provides important insights into gravity wave propagation, reflection and dissipating processes. The wave event is associated with large vertical wind (\( \sim 10 \) ms\(^{-1}\)) and is believed to contribute significantly to the variability of atmosphere. So beyond the characteristics of the gravity waves, it is necessary to study in-depth the effects of gravity waves. It is important to understand how gravity waves deposit momentum and energy when they interact with the background atmosphere through reflection and critical levels. The detailed calculation and analysis of the fluxes of heat and momentum around these levels from observations and simulations are prospective work in the future.

In the second case, a novel method taking advantage of the multiple-direction lidar to resolve the horizontal information of gravity waves is presented. Besides the vertical variation of the wave from traditional lidar measurement profiles, the horizontal wavelength and propagation direction are retrieved from the phase differences among the laser beams pointed to different directions that are about 50 km apart at \( \sim 90 \) km altitude. A gravity wave packet was identified with a horizontal wavelength of about 300 km, a period of 86 min and observed phase speed of 60 ms\(^{-1}\) propagating at \(-156^\circ\) azimuth angle. With a full set of wave and background parameters, multiple dispersion
relations under different assumptions such as isothermal and windless background, are examined in this study. The wave event was confined within a narrow altitude range, believed to be ducted. It turns out variable background temperature and winds counts substantially in the linear theory thus one should carefully evaluate the simplification when apply these dispersion relations. A sensitivity study based on Monte-Carlo simulation provides some guidances about how this method be applied on lidars with similar configuration. There are good chances for medium-scale and medium- to low-frequency gravity waves being detected with this method. Statistical characteristics of the gravity waves in this spectral range, mostly inertia gravity waves, are essential parts in gravity wave parameterizations. With more data obtained, there should be more wave events detected and analyzed with this method.

Generally speaking, case studies can provide details on the dynamical processes of gravity waves such as propagation and dissipation based on limited observations. They can improve our understanding of the gravity waves characteristics, help to validate the linear theory, constrain the gravity wave parameterization. But measurements from a sodium lidar and an airglow imager are mostly confined at 80–110 km altitude range and over a small area. This makes it difficult to track the waves back to the sources, which are mostly located at troposphere and lower stratosphere. The observed gravity waves can either propagate upward from source region nearby, or be ducted or reflected and propagate horizontally over a significant distance. One important topic about the gravity wave parameterization is to specify the gravity waves momentum flux for different wave sources. So a complete gravity waves picture that starts from the troposphere where they are generated to the MLT region where wave dissipation is severe are necessary. Cooperative work from multiple co-located instruments or a network of instruments to study the gravity waves in a comprehensive perspective is urgent. Possible candidates include radiosonde, super-pressure balloon, various types of lidar and airglow imaging, satellites, and other coherent instruments.
In the gravity wave parameterization, a large amount of observations are necessary that can provide reliable estimation of wave parameters such as horizontal wavelengths, phase speed, wave period and vertical wavelengths, and momentum flux. Currently, the parameterization schemes require tuning parameters, intermittency or efficiency factors, to make the proper amount of momentum be deposited at correct altitudes in order to match the model outcome with climatology. As another major topic of this dissertation, long-term observations of gravity waves provide opportunities to investigate the climatology and intermittency of gravity waves near mesopause region. They provide physical basics and references, and are used as validation tools for the parameterizations.

Long-term airglow data at ALO supplements the understanding of high-frequency, small-scale gravity waves in the Southern Hemisphere, especially the continent of South America. The gravity wave characteristics revealed by the airglow imager at ALO show consistency with observations from other mid-latitude sites. But regarding the preferential propagation direction, the ALO shows some uniqueness. With several remarkable convection sources with relatively fixed location in the South American and Pacific Ocean nearby, the observed waves shows high correlation with those potential wave sources. During austral summer, the Amazon Basin acts as an important wave sources while in winter time, wave sources are located at Pacific Ocean and south of ALO. The climatological wind retrieved from the model could not explain the critical layer filtering well. The momentum fluxes deduced from intrinsic wave parameters show anti-correlation with the background wind especially in the meridional direction. Beyond the climatology of the gravity waves, their correlation with convection activities are also important. In the gravity wave parameterization, source specification of gravity waves remains a challenging issue. Convection-generated waves are one of the common ones parameterized in the model. Most observational studies only focus on the wave characteristics and overlooks the relations between the waves and sources, especially the quantitative relation. Neglected horizontal propagation in the parameterization also a topic deserves
8.1. SUMMARY

Further investigation. Waves observed at ALO are usually not generated locally because no convection is found nearby. Convective sources must have their influence zones where generated waves disperse out from the source region. It is beneficial to the wave source specification if wave characteristics such as occurrence frequency and wave amplitude are determined with certain dependence on the distance between the sources and forcing region.

Intermittency of gravity waves are studied for the mesopause region, where wave dissipation is severe and frequent. The idea of intermittency is originally from the factors used in gravity wave parameterization to describe the fractional coverage of waves within a large spatial grid and/or temporal period in order to accurately quantify the forcing on atmosphere of dissipating gravity waves. Intermittency of gravity waves was described by the pdfs of absolute momentum flux where an explicit probability function was obtained. The pdfs for gravity waves with smaller momentum flux are found to fit very well with a lognormal distribution. The pdfs in the larger momentum flux region, described as ‘long tail’, fit very well with a power-law distribution. The transition points between the two different distributions are around $\sim 16 \text{ m}^2\text{s}^{-2}$ at both sites. Because of the large amount of gravity waves, these two distributions are well defined through the fitting process.

It enables detailed study of gravity wave intermittency and their relative contributions to the total momentum flux. The relative importance of abundant waves with smaller amplitudes and rare waves with dramatically large amplitudes were compared. This work provided a new perspective to the gravity waves characteristics in MLT using existing airglow measurements. The statistical properties to be studies are not limited to the traditional mean and standard deviations, rather on the detailed pdfs of various GW properties and how they are affected by other factors. Gravity waves are known to vary significantly at different geographical locations. With many more airglow imager observations around the world, the global distribution of gravity wave intermittency can be derived and used to validate GCMs.
8.2 Outlook

The researches in this dissertation can be expanded in several perspectives. Here are some suggested topics. Firstly, two case studies presented here focus more on the gravity wave characteristics. Since we have complete observations and simulations, the momentum and heat fluxes can be calculated, especially from the simulation results. The wave-induced forcing and heating/cooling can be estimated to quantify the wave-mean flow interaction. It would be very interesting to know how momentum and energy are deposited near the reflection and critical layers. Secondly, the current processing algorithm is based on the monochromatic waves identified from each image. The temporal variations are not directly used to derive the wave information. A better way to extract gravity waves could combine the spatial (2-D) and temporal (1-D) perturbations together and identify the waves as coherent wave events. A wavelet is useful to track the variation of wave periods with respect to time and the motion of the localized wave pattern in space. Thirdly, the high-frequency gravity waves observed by airglow imagers are more likely to be ducted. Ducting is an important factor that should be considered in the explanation of propagation direction preferences and net momentum flux calculation. At ALO, many gravity waves are found to propagate southward, especially during austral summer. There is a long distance (≥1000 km) between ALO and the potential wave sources in the lower atmosphere, represented by heavy convective precipitation. This implies that those waves are likely ducted. More detailed propagation conditions are needed to evaluate the potential ducted propagation. Lastly, the statistical studies of momentum flux intermittency and wave duration provide a new perspective to the understanding of gravity wave characteristics, where a closer linkage between observations and parameterizations in GCMs could be studied. In order to clarify and distinguish the effects of wave sources, background flow, atmosphere instability and other potential factors on these statistical characteristics, studies based on statistical models taking into consideration these different effects are beneficial. The roles of different factors in
8.2. OUTLOOK

determining the shape of probability distribution can be identified through these studies.
Appendix A

Linear Gravity Wave Theories

A.1 Compressible Euler Equation

The potential temperature of a parcel of fluid at pressure $p$ is the temperature that the parcel would acquire if adiabatically brought to a standard reference pressure $p_0$, usually 1000 hPa. It is described by

$$\theta = T \left( \frac{p_0}{p} \right)^{R/c_p} = \frac{p}{\rho R} \left( \frac{p_0}{p} \right)^{R/c_p} = \frac{p}{\rho R} \left( \frac{p_0}{p} \right)^{(\gamma^{-1})/\gamma},$$

(A.1)

where $T$ and $\rho$ are the atmospheric temperature and density, respectively. $\gamma = c_p/c_v$ is the ratio of specific heat. For a compressible atmosphere, the potential temperature is conserved in adiabatic processes

$$\frac{d\theta}{dt} = \frac{p_0^{(\gamma^{-1})/\gamma}}{R} \frac{d}{dt} \left( \rho^{-1} \cdot p^{1/\gamma} \right) = 0
= \frac{p_0^{(\gamma^{-1})/\gamma}}{R} \left[ p^{1/\gamma} \frac{d}{dt} (\rho^{-1}) + \rho^{-1} \frac{d}{dt} (p^{1/\gamma}) \right] = 0$$

(A.2)

$$= \frac{p_0^{(\gamma^{-1})/\gamma}}{R} \left[ -p^{1/\gamma} \rho \frac{d\rho}{dt} + \rho^{-1} \frac{1}{\gamma} p^{1/\gamma-1} \frac{dp}{dt} \right] = 0.$$
A.1. COMPRESSIBLE EULER EQUATION

Then, the relationship between $p$ and $\rho$ is

$$\frac{dp}{dt} = \frac{\gamma p \, dp}{\rho \, dt} = \gamma g H_s \frac{dp}{dt} = \gamma g H_s \frac{d\rho}{dt} = c_s^2 \frac{d\rho}{dt}, \quad (A.3)$$

where $c_s = \sqrt{\gamma g H_s}$ is the speed of sound, $H_s = RT/g$ is the scale height, $g$ is the gravitational acceleration, and $R$ is the ideal gas constant. The material derivative $\frac{d}{dt}$ in Lagrange formulation can be written as

$$\frac{d}{dt} = \frac{\partial}{\partial t} + U \cdot \nabla, \quad (A.4)$$

of which $\frac{\partial}{\partial t}$ is the local derivative in Euler formulation and $U \cdot \nabla$ is convective derivative.

The atmosphere is assumed as inviscid, irrotational (Coriolis force is ignored) and compressible with altitude-varying background temperature and wind. The set of equations of conservation of momentum, thermal energy and mass are:

$$\frac{\partial U}{\partial t} + U \cdot \nabla U = -\frac{1}{\rho} \nabla p + \vec{g}$$

$$\frac{\partial p}{\partial t} + U \cdot \nabla p = c_s^2 \left( \frac{\partial \rho}{\partial t} + U \cdot \nabla \rho \right) \quad (A.5)$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho U) = 0.$$

The set of equations are rewritten in the Cartesian coordinate with positive directions at eastward, northward and upward. $U$ is decomposed into zonal, meridional and vertical
winds \((u, v, w)\).

\[
\begin{align*}
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} \\
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial y} \\
\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial z} - g \\
\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} + v \frac{\partial p}{\partial y} + w \frac{\partial p}{\partial z} &= c_s^2 \left( \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y} + w \frac{\partial \rho}{\partial z} \right) \\
\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} &= 0.
\end{align*}
\] (A.6)

\section{A.2 Incompressible Euler Equation}

The Taylor-Goldstein equation is derived from the 2-D Euler equations with Boussinesq approximation. From equation (A.5), the density perturbations are only considered when they occur in combination with \(g\). A consequence of the separation of the pressure and density changes is that the atmosphere becomes incompressible, and acoustic waves are eliminated. Then the Euler equations become

\[
\begin{align*}
\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{U} &= -\frac{1}{\rho} \nabla p + \mathbf{g} \\
\frac{\partial \rho}{\partial t} + \mathbf{U} \cdot \nabla \rho &= 0
\end{align*}
\] (A.7)

\[
\nabla \cdot \mathbf{U} = 0.
\]

For a 2-D reference plane in the \(x\) and \(z\) directions, the scalar form of Taylor-Goldstein equations is

\[
\begin{align*}
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} \\
\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + w \frac{\partial w}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial z} - g \\
\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} &= 0 \\
\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + w \frac{\partial \rho}{\partial z} &= 0.
\end{align*}
\] (A.8)
A.2. INCOMPRESSIBLE EULER EQUATION

All the terms \( q = (u, w, \rho, p) \) in the equation (A.8) can be written as

\[
q(x, z, t) = \bar{q}(z) + q'(x, z, t),
\]  

(A.9)

where the \( \bar{q}(z) \) is a steady, horizontally uniform background value and only depends on the altitude, and \( q'(x, z, t) \) is a first-order perturbation term. Then equation (A.8) becomes

\[
\begin{align*}
\frac{\partial u'}{\partial t} + \bar{u} \frac{\partial u'}{\partial x} + w' \frac{d\bar{u}}{dz} &= -\frac{1}{\bar{\rho}} \frac{\partial p'}{\partial x}, \\
\frac{\partial w'}{\partial t} + \bar{u} \frac{\partial w'}{\partial x} &= -\frac{1}{\bar{\rho}} \frac{\partial p'}{\partial z} - \frac{\rho'}{\bar{\rho}} g, \\
\frac{\partial u'}{\partial x} + \frac{\partial w'}{\partial z} &= 0, \\
\frac{\partial \rho'}{\partial t} + \bar{u} \frac{\partial \rho'}{\partial x} + w' \frac{d\bar{p}}{dz} &= 0. 
\end{align*}
\]  

(A.10)

Now, we can assume wave-like solutions of the form

\[
q'(x, z, t) = \tilde{q}(z)e^{i(kx - \omega t)},
\]  

(A.11)

of which \( k \) and \( \omega \) are the horizontal wavenumber and frequency. Then, equation (A.10) becomes

\[
\begin{align*}
-iz\bar{u} + i\bar{u}k\bar{u} + w' \frac{d\bar{u}}{dz} &= -\frac{i}{\bar{\rho}} k\bar{\rho}, \\
-iz\bar{w} + i\bar{u}k\bar{w} &= -\frac{1}{\bar{\rho}} \frac{d\bar{p}}{dz} - \frac{\bar{\rho} g}{\bar{\rho}}, \\
iki\bar{u} + \frac{d\bar{w}}{dz} &= 0, \\
-iz\bar{\rho} + i\bar{u}k\bar{\rho} + w' \frac{d\bar{p}}{dz} &= 0. 
\end{align*}
\]  

(A.12)

Note that \( \omega \) is the wave frequency observed in a fixed coordinate, sometimes called observed, ground-based or extrinsic frequency. The intrinsic frequency, \( \hat{\omega} \), is defined as the frequency of a wave relative to the background flow with a speed of \( \bar{u} \),

\[
\hat{\omega} = \omega - \bar{u}k. 
\]  

(A.13)
A.2. INCOMPRESSIBLE EULER EQUATION

\( \hat{\omega} \) is also referred to as Doppler-shifted wave frequency. The Buoyancy frequency is defined as

\[
N^2 = \frac{g}{\hat{\theta}} \frac{\partial \hat{\theta}}{\partial z} = -\frac{g}{\hat{p}} \frac{\partial \hat{p}}{\partial z},
\]  

(A.14)

and the vertical variation of atmosphere density in the isothermal atmosphere is exponentially decreasing with altitude, i.e.,

\[
\hat{p} = \rho_s e^{-z/H_s},
\]  

(A.15)

where \( \rho_s \) is the density at ground level. Solving equation (A.12) for \( \tilde{w} \) gives

\[
\frac{d^2 \tilde{w}}{dz^2} - \frac{1}{H_s} \frac{d \tilde{w}}{dz} + \left[ \frac{k^2 N^2}{\hat{\omega}^2} + \frac{k}{\hat{\omega}} \frac{d^2 \tilde{u}}{dz^2} - \frac{k}{H_s} \frac{d \tilde{u}}{dz} - k^2 \right] \tilde{w} = 0.
\]  

(A.16)

At last, we can simplify this equation by defining a new variable, \( \tilde{q} \), as

\[
\tilde{q} = e^{z/2H_s} \tilde{q},
\]  

(A.17)

and the observed, ground-based or extrinsic horizontal phase speed is

\[
c = \frac{\omega}{\hat{k}} = \frac{\hat{\omega} + \hat{\pi} \hat{k}}{\hat{k}} = \frac{\hat{\omega}}{\hat{k}} + \hat{\pi},
\]  

(A.18)

of which term \( \hat{\omega}/k \) can be defined as intrinsic horizontal phase speed \( \hat{c} \) (\( \hat{c} = c - \hat{\pi} \)). Finally, the Taylor-Goldstein equation takes the form

\[
\frac{d^2 \hat{w}}{dz^2} + \left[ \frac{N^2}{(c - \hat{\pi})^2} + \frac{1}{(c - \hat{\pi})} \frac{d^2 \hat{u}}{dz^2} - \frac{1}{H_s(c - \hat{\pi})} \frac{d \hat{u}}{dz} - \frac{1}{4H_s^2} \right] \hat{w} = 0.
\]  

(A.19)
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