I. Overview

Abstract

A major component of next-generation operational models will include coupling between meteorology and wave models. The achievement of this goal requires improved coupling relationships, physics-based forcing terms in the spectral density transport equation of wave models, and software which facilitates ease of use and is computationally fast in the development of coupled models. Here we investigate a wave-age dependent sea surface roughness for waves for different strength winds.

As the waves in the ocean are unsteady a nonparametric interpretation is needed in order to take into account the growing and decaying waves. We also use a recently derived expression for energy transfer rate to investigate a growing group of waves.

Project background

We are concentrating our efforts on the region around the height were the real part of the complex wave speed is equal to the mean flow velocity. This region called “critical layer” is at the center of Miles’ [1] theory and Lightbrou’s [2] interpretation of growth waves. In this region closed streamlines structures called “cat’s-eye” are developed. The larger these structures are, the more disturbance of the wind flow above the waves occurs. In some previous work, e.g. Drullion & Sajjadi [3], a high-Reynolds number stress model with a constant surface roughness uniformly distributed over a moving water surface was used to show that their size and position are dependent on the wave age and wave steepness, which is in accordance with direct numerical simulations of Sullivan et al [4]. In this study we use the same Reynolds stress model with a wave age dependent surface roughness and an updated growth rate [8] to determine the height of the critical layer and the overall shape and size of the cat’s-eye for a group and a third order Stokes wave.

The analysis of wind over waves is usually carried in a coordinate frame moving at the wave speed (c). Another assumption is related to the unsteadiness of the mean flow over the water surface, where the fluctuation growth is proportional to the rate of the wind. The region of closed streamlines centered at the critical layer is known to be dynamically important for values of the wave age that are not too large.

In our previous work [3][7], we showed numerically that the height of the critical layer and the vertical extent of the cat’s eye structures increase with the wave age and the steepness. Another result was that as a waves grows under the effect of wind, the height of the critical layer and the size of the cat’s-eye structures increase.

Governing equations

We are modelling a turbulent and compressible flow governed by the Navier-Stokes equations

\[ \frac{\partial u}{\partial t} + \nabla \cdot (u \otimes u) = -\nabla p + \frac{1}{Re} \nabla^2 u + g \nabla z + f \]

where:

- \( u \) : Velocity vector
- \( p \) : Pressure
- \( Re \) : Reynolds number
- \( g \) : Gravity
- \( f \): Forcing (e.g., wind stress)

Each quantity is decomposed into a mean and a fluctuating component.

The averaging of the previous equations (using a Fubare average) leads to the Reynolds Averaged Navier-Stokes (RANS) equations.

We used a high Reynolds Stress model closure.

II. Discretization, Boundary conditions, Meshes

Discretisation and solver

- Finite volume method
- Variables located at the center of the cells (collocated)
- Time first order forward
- Bonded 3rd order QUICK (quadratic Interpolation for convective kinematics) (Leonard 1979) scheme for the discretization of the convective fluxes.
- Central difference operator for the pressure and the diffusive fluxes
- The finite volume method and the chosen discretizations lead to penta-diagonal equation in 2D, those systems are solved using a tri-diagonal, matrix algorithm
- Pressure based solver

Wave Profiles

For this study we use third order Stokes waves and groups generated as the superposition of three waves.

Third order Stokes waves profile: \( \omega = \omega_0 + \omega_3 \), \( \omega_0 = \text{amplitude of the first wave}, \omega_3 = \text{amplitude of the third wave} \)

Group profile:

\[ \omega = \omega_0 \left[ \text{cos}(k_1 x) + 4 \text{cos}(k_2 x) + \text{cos}(k_3 x) \right] \]

Where \( \omega_0 \), \( k_1 \), \( k_2 \), and \( k_3 \) are the weights of the second and third waves, \( k_0 \) and \( k_3 \) are defined respectively as: \( k \left( \pi / 2a \right) \) and \( k \left( 3 \pi / 2a \right) \)

Boundary conditions

South: Orbital velocity of the waves (Fig. 1):\n
For example for groups: \( \omega = \omega_0 \left[ \text{cos}(k_1 x) + 4 \text{cos}(k_2 x) + \text{cos}(k_3 x) \right] \)

where \( \omega_0 \) is the group velocity: \( \omega_0 = \frac{8}{\pi} \frac{s}{\mu} \)

North: \( u = 0 \) (velocity boundary condition)

West: periodic boundary conditions

East: Outlet

Mesh

The mesh is generated by the 10 m (U_0) we evaluate:

- The wave induce motion Reynolds stress: \( \tau_{ij} = \mu \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) - \frac{1}{2} \delta_{ij} \left( \frac{\partial v_i}{\partial x_i} \right) \)
- The turbulence stress: \( \tau_{ij} = \rho_0 \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \)
- The frictional stress: \( u_f = \mu \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) - \frac{1}{2} \delta_{ij} \left( \frac{\partial v_i}{\partial x_i} \right) \)

Then the drag coefficient is calculated as: \( C_D = \frac{1}{2} \int_{0}^{L} \frac{\tau_{ij}}{\rho_0 u_f^2} dL \)

Critical Layer for Growing Groups

The growth rate of the wave was calculated as a function of the friction velocity and the wave speed based on the work presented in [6]. For this preliminary result, the growth rate was not recalculated as the mesh was regenerated.

As observed in our previous work [6], as the waves steepen, cat’s-eye structures are formed in each lee of the wave in the group. As the wave grows so do the cat’s-eye structures.

Future work

- Have a growth coefficient dependent on position along the wave.
- Work on changing the profile of the group as the different waves of the group are moving in different horizontal direction.
- Fully couple the calculation of the growth rate into the growing group code.