Integrating Physics Informed Neural Networks with the Finite Element Method for Solving Inverse Problems

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Introduction

This research introduces a novel framework called Finite Element Physics-Informed Neural Networks (FE-PINNs) for solving complex problems in engineering. Building upon the strengths of traditional Physics-Informed Neural Networks (PINNs) and the Finite Element (FE) method, FE-PINN offers an efficient and accurate approach for solving challenging inverse problems in civil engineering. PINNs [1] use neural networks to approximate physical systems while enforcing conformance with the systems' governing equations as a soft constraint during the optimization process, which allows system parameters to be updated alongside the weights of the PINN. FE-PINN extends this approach by using PINNs to solve the system of equations resulting from applying the FE method to potentially complicated real-world systems, while updating unknown system parameters simultaneously with neural network weights. The architecture closely resembles traditional PINNs but exhibits advantages such as faster convergence, reduced data requirements, and simplified loss functions. The effectiveness of FE-PINN is demonstrated through a 2D linear elastic full waveform inversion problem, where it not only accurately estimates elastic modulus values with less than 0.01% error, but also provides an efficient surrogate model which can be used for forecasting. The success of FE-PINN in this simplified problem provides grounds for optimism that it can be applied to more intricate systems – a direction the authors intend to explore in future research endeavors.

FE-PINN Workflow

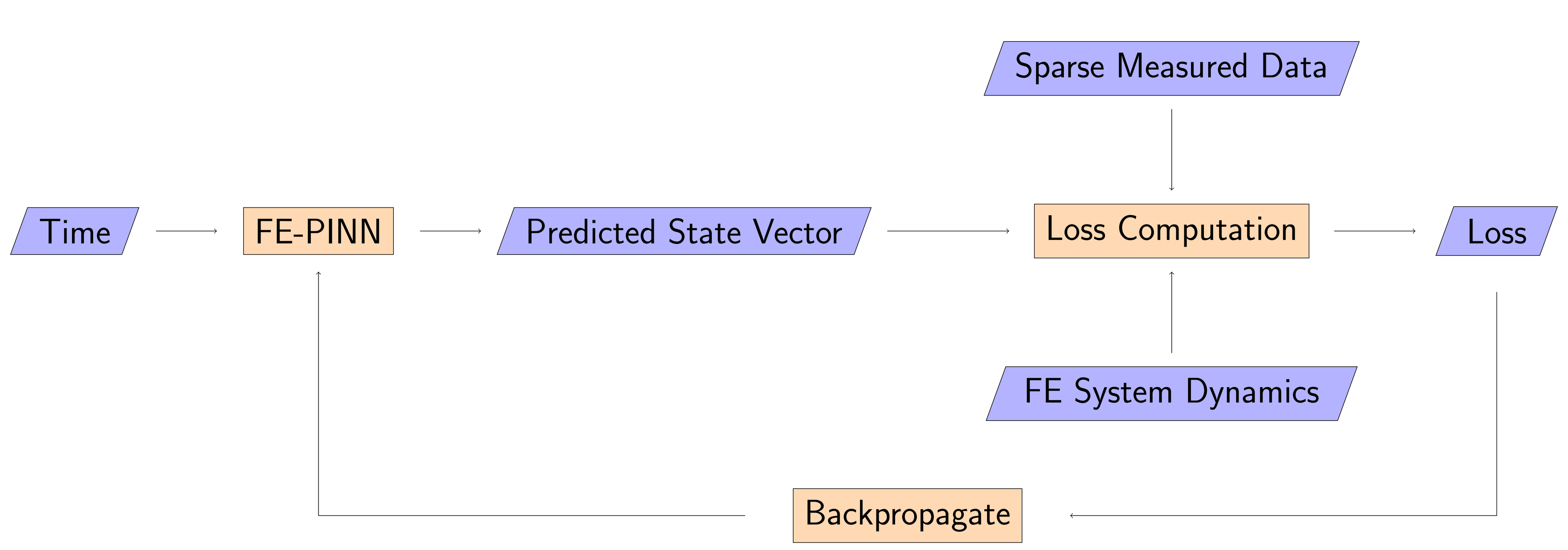

Figure: A graphical representation of the FE-PINN training process.

- Given time t, FE-PINN predicts the displacement vector $u(t)$
- Autodifferentiation is used to obtain $\dot{u}(t)$ and $\ddot{u}(t)$
- Data loss: mean-squared error of misfit between predicted and measured displacements
- Physics loss: mean of squared residual of dynamic equation
- Total loss is a linear combination of data and physics loss
- Total loss minimized using ADAM optimizer, updating neural network weights along with physical parameters

Numerical Experiment

Finite Element Model

$$f(t) = -1000\, e^{-(t-1.45)^2}$$

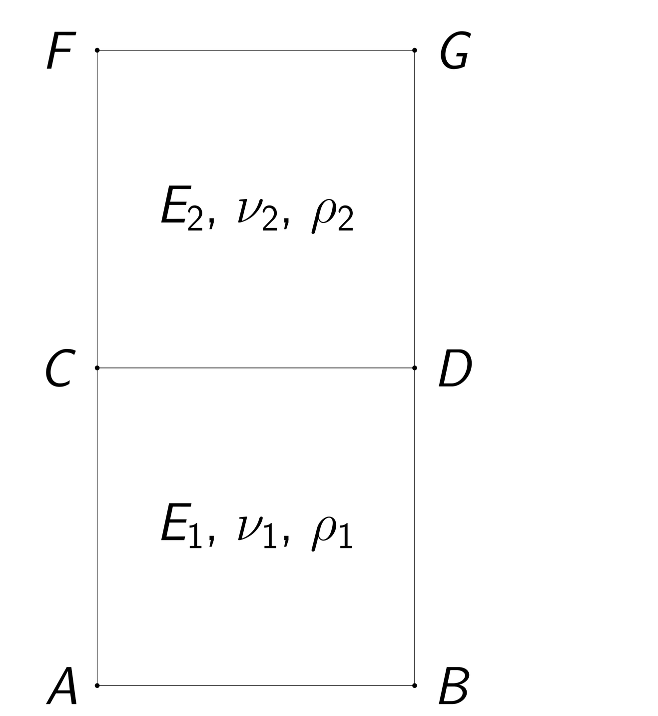

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Figure: Simulated soil column. Nodes A and B are fixed. Displacement history of nodes F and G used to train FE-PINN. Rayleigh damping is assumed.

Parameter Values

- $\nu_1 = \nu_2 = 0.3$
- $\rho_1 = \rho_2 = 2000 \text{ kg} \cdot \text{m}^{-3}$
- $E_1 = 69.2307$ MPa
- $E_2 = 23.0769$ MPa
- $a_0 = 0.149824$
- $a_1 = 0.000435366$

Dynamics

The dynamics of this system are given by the equation

$$\mathbf{M}\ddot{u}(t) + \mathbf{C}\dot{u}(t) + \mathbf{K}u(t) = f(t) \tag{1}$$

Loss Function

The loss function is formulated as follows.

$$\mathcal{L} = \mathcal{L}_D + \frac{1}{NM}\sum_{i=1}^{N}\sum_{j=1}^{M}\left(\mathbf{M}\ddot{u}(t_j) + \mathbf{C}\dot{u}(t_j) + \mathbf{K}u(t_j) - f(t_j)\right)_i^2 \tag{2}$$

where $\mathcal{L}_D$ denotes data loss. At each optimization step, the values of E_1 and E_2 are updated along with the parameters of the neural network.

Results

Parameter	True Value	FE-PINN Estimate	Percent Accuracy
E_1	69.2307 MPa	69.2302 MPa	99.999%
E_2	23.0769 MPa	23.0764 MPa	99.998%

Table: Comparison of FE-PINN's estimates vs. ground-truth values.

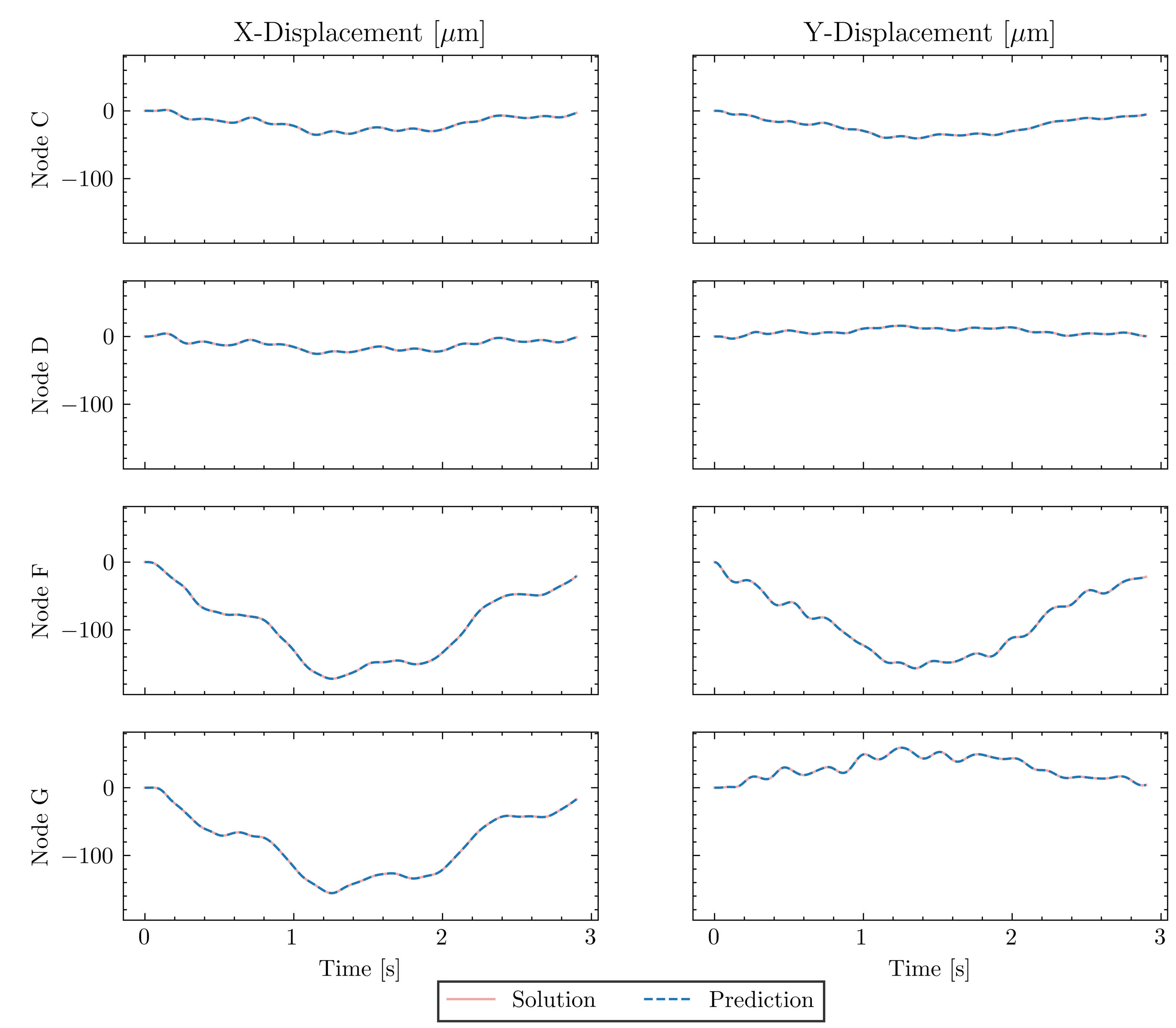

Figure: True solution data (obtained with a FE simulation) compared to FE-PINN prediction. Due to FE-PINN's high accuracy, the curves are nearly identical.

Future Goals

Due to the strong performance of FE-PINN on this simplified problem, the authors are motivated to attempt the following.

- Apply FE-PINN to data from a real, 3D geophysical experiment
- Evaluate the performance of FE-PINN against traditional PINNs and FE model-updating schemes.

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