Parametrization of Triple Screw Pumps for Aerospace Applications

Allison Putira
PARAMETRIZATION OF TRIPLE SCREW PUMPS FOR AEROSPACE APPLICATIONS

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by

Allison Putira

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by

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A Thesis prepared under the direction of the candidate's committee chairman, Dr. Eric Perrell, Department of Aerospace Engineering, and has been approved by the members of the thesis committee. It was submitted to the School of Graduate Studies and Research and was accepted in partial fulfillment of the requirements for the degree of Master of Science in Aerospace Engineering.

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### SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$A_{CS1}$</td>
<td>Circle Segment Area 1</td>
</tr>
<tr>
<td>$A_{CS2}$</td>
<td>Circle Segment Area 2</td>
</tr>
<tr>
<td>$A_{CASE}$</td>
<td>Pump Case Cross Sectional Area</td>
</tr>
<tr>
<td>$A_{DRIVER}$</td>
<td>Cross Sectional Area of Driver Screw</td>
</tr>
<tr>
<td>$A_{IDLER}$</td>
<td>Cross Sectional Area of Idler Screw</td>
</tr>
<tr>
<td>$A_{PUMP}$</td>
<td>Cross Sectional Area of Fluid Pockets</td>
</tr>
<tr>
<td>$A_{SCREWS}$</td>
<td>Cross Sectional Area of the Screws</td>
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<td>$A_{SURFACE}$</td>
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<td>$A_y$</td>
<td>Epicycloid Area</td>
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<td>$a$</td>
<td>Distance from Driver Screw Axis to Flat Edge of $A_{CS1}$</td>
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<tr>
<td>$b$</td>
<td>Distance from Driver Screw Axis to Flat Edge of $A_{CS2}$</td>
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<tr>
<td>$c$</td>
<td>Perpendicular Distance of Circle Intersection from Horizontal Axis</td>
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<td>Driver Screw Outer Diameter</td>
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<td>$k$</td>
<td>Shape Factor</td>
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<td>$L$</td>
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<td>$N$</td>
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<td>Pump Specific Speed</td>
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<tr>
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<td>$\Delta P$</td>
<td>Pressure Differential</td>
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<td>$\Delta P_{STAGE}$</td>
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</tr>
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<td>$r_2$</td>
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<td>Total Slip</td>
</tr>
<tr>
<td>$s$</td>
<td>Slip per Length</td>
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<tr>
<td>$t$</td>
<td>Tooth Height</td>
</tr>
<tr>
<td>$u$</td>
<td>Fluid Velocity</td>
</tr>
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</table>
\(\Delta X\) Length of Tooth at Outer Radius Perpendicular to Direction of Rotation
\(\Delta x\) Length of Tooth at Outer Radius Perpendicular Screw Angle
\(Y\) Non-Dimensional Inner Diameter of Idler Screw for Constructing Series
\(Z\) Non-Dimensional Tooth Height for Constructing Series
\(A\) \(\frac{1}{\mu} \frac{\partial p}{\partial x}\) When Solving the Slip Equation
\(\alpha\) Angle of Outer Radius of Driver Screw Tooth
\(B\) Integration Constant
\(\beta\) Angle of Inner Radius of Driver Screw Tooth
\(\Gamma\) Integration Constant
\(\gamma\) Epicycloid Angle
\(\delta\) Screw Clearance
\(E\) \(\frac{1}{\mu} \frac{\partial p}{\partial x}\) When Solving the Power Equation
\(\zeta\) Integration Constant
\(\eta\) Integration Constant
\(\eta_{\text{MECHANICAL}}\) Mechanical Efficiency
\(\eta_{\text{TOTAL}}\) Total Efficiency
\(\eta_{\text{VISCOUS}}\) Efficiency Due to Viscous Losses
\(\eta_{\text{VOLUMETRIC}}\) Ratio of Actual to Theoretical Pump Capacity
\(\varphi\) Angle of Rotation of Driver Screw for Determining Epicycloid Curve
\(\varphi_{\text{MAX}}\) Max Angle of Rotation of Driver Screw for Determining Epicycloid Curve
\(\mu\) Dynamic Viscosity
\(\nu\) Coefficient of Friction
\(\xi\) Half of the Angle Subtended by \(A_{CS2}\)
\(\pi\) The Ratio Between a Circles Circumference and Diameter
\(\rho\) Density of Working Fluid
\(q\) Adjustment Factor for Idler Screw Area
\(\sigma\) Screw Angle
\(T\) Torque
\(\tau\) Fluid Shear Stress
\(\theta\) Idler Screw Angle Corresponding to specific \(\delta\) and \(\varphi\) in Global Coordinate System
\(\theta'\) Idler Screw Angle in Local Coordinate System
\(\chi\) Geometric Constant
\(\psi_{\text{MECHANICAL}}\) Empirical Factor for Mechanical Efficiency
\(\psi_{\text{SLIP}}\) Empirical Constant Describing Slip and Pressure
\(\psi_{\text{TORQUE}}\) Empirical Non-Dimensional Distance on which Torque Acts
\(\psi_{\text{VISCOUS}}\) Empirical Constant Describing Viscous Losses
\(\psi\) Half of the Angle Subtended by \(A_{CS1}\)
\(\omega\) Angular Velocity (rad/s)
\(P\) Power
\(P_{\text{MECHANICAL}}\) Power Lost to Mechanical Inefficiencies
\[ P_{\text{SLIP}} \] Power Lost Due to Slip
\[ P_{\text{VISCOUS}} \] Power Lost Due to Viscous Effects
ABSTRACT

Putira, Allison MSAE, Embry-Riddle Aeronautical University, June 2018. Parametrization of Triple Screw Pumps for Aerospace Applications

Triple Screw Pumps, a form of positive displacement pump, offer steady, high-pressure flow. In very small launch vehicles or upper stages turbopumps are not practical. This paper examines the performance of triple screw pumps to determine if they may serve as an alternative to turbopumps for use in small launch vehicles or upper stages. To this end, the geometry of the screws is first determined from the relative sizes of the screws and tooth height. From this — along with the screw angle and screw clearance — the performance parameters of flow rate, output pressure, and efficiency can be computed. These results can be compared against experimental results using a 20mm 5-3-1-4 triple screw pump. The pump was able to produce a flow rate of 37mL/s at 1000rpm and an output pressure of 75.3kPa at 1200rpm — both within expected result range. This document may serve as a starting point for future pump designs and design requirements.
1. Introduction

Triple Screw Pumps, or three screw pumps, are a type of rotary positive displacement pump in which fluid flow through the pumping elements — the screws — is axial. Like all positive displacement pumps, screw pumps do not create pressure but instead simply move fluid from one spot to another; the output pressure of the pump is dependent on the back pressure downstream. The fluid is carried between the threads of the screws and is displaced axially as the screws rotate. These threads create small pockets of fluid and a series of moving seals which form a labyrinth as found in labyrinth seal theory (Karassik, 1976). These pumps have much lower internal velocities than turbopumps and circumferential flow pumps. The screws have a relatively low inertia, which allows these pumps to operate at higher speeds than other rotary or reciprocating pumps. Screw pumps have the highest flow rate to footprint of the positive displacement pumps and forms smooth flow rather than pulsated flow as would be found with a reciprocating pump. These attributes make screw pumps well suited to applications where smooth high-speed flow and minimal churning or agitation is desired.

Screw pumps are currently used in a wide range of applications and markets including "navy, marine, and utilities fuel-oil service; marine cargo; industrial oil burners; lubricating-oil service; chemical processes; petroleum and crude-oil industries; [and] power hydraulics for navy and machine tools" (Karassik, 1976). These pumps can handle a wide variety of viscosities ranging from molasses to gasoline. These pumps are currently manufactured for pressures up to 34.5MPa, and flow rates up to twenty thousand liters per minute (Karassik, 1976). A number of manufacturers produce these pumps including Alfa Laval, IMO Pump, Netzsch, and Applied Pumps.
This thesis examines the possibility of using triple screw pumps as a separate option to turbopumps for space launch vehicles. Although these pumps are produced by a number of manufacturers and give expected performance, little information is available regarding their performance given certain design parameters, as is available for turbopumps. As such, this document will focus on deriving the performance relations that may be used as a starting point for the design of a triple screw pump for use in a space launch vehicle.

<table>
<thead>
<tr>
<th>Impeller type</th>
<th>Radial</th>
<th>Francis</th>
<th>Mixed Flow</th>
<th>Near Axial</th>
<th>Axial</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic shape</td>
<td>Casing</td>
<td>Impeller</td>
<td>Shaft</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Specific speed $N_s$

- SI consistent units: 0.2–0.3, 0.4, 0.6–0.8, 1.0–2.0, Above 2.5
- Efficiency %: 50–80, 60–90, 70–92, 76–88, 75–82

Figure 1.1 Turbopump Types: For some values of specific speed, no option may be feasible (George P. Sutton, 2010)

Turbopumps are limited by their size and pressure output. Pump specific speed is an important dimensionless parameter in turbopump design. The fluid velocity in a given turbopump is proportional to the pump speed, the flow rate is also proportional to the speed, and the pump head is proportional to the square of the pump speed (George P. Sutton, 2010). From this the parameter, pump specific speed may be derived (shown in (Karassik, 1976))

$$N_s = \frac{N\sqrt{Q}}{(p/p)^{3/4}}$$ (1.1)
The specific speed decreases as the flow rates decrease or the pressure increases. The pump speed correlates to the pump diameter. However, there is a limit to how much the pump diameter may be decreased to maintain the pump specific speed. In order for the pump to function at lower pump specific speed the geometry must change to a more centrifugal shape. These shapes operate at a lower efficiency than the more axial pumps. At very low flow rates it can become entirely impractical or impossible to design a useful turbopump.

Beyond this point, pressure fed engines are the only standard option. However, pressure fed systems are quite heavy and tend to have lower performance than pumped stages (George P. Sutton, 2010). One solution to this issue is to use positive displacement pumps, which allow for much lower flow rate while maintaining high output pressures. There have been quite a few cases where positive displacement pumps have been used for small rockets. In 1974, NASA published a document examining the use of a gear style and centrifugal pumps for very low thrust rocket engines (Rocketdyne Division, Rockwell International, 1974). The growth of the small satellite market and small launchers has renewed interest in the use of positive displacement pumps for space launch vehicles such as XCOR's reciprocating pump (XCOR Aerospace, 2018).
Screw Pumps display a number of advantages and disadvantages as compared to reciprocating pumps and other positive displacement pumps. Compared to other positive displacement pumps, triple screw pumps are able to produce high speed flow on a smaller footprint than other pumps. More importantly the flow is smooth, rather than a pulsed flow as in a reciprocating pump. The pressure output is very consistent. The operation of the pump is smooth rather than creating the vibrations associated with a piston pump (Karassik, 1976). The smooth operation is of particular use in launch vehicles, where extra vibration can be quite dangerous. Screw pumps lack the fragile moving rubber seals of piston pumps and as such can operate at much faster speeds (Radolfzell Works). However, screw pumps are subject to galling so care must be taken in the material selection (Karassik, 1976). It is recommended that the driver and idler screws be made of slightly different alloys or bronze to prevent galling. Given the short operational lifetime of a screw pump placed in a launch vehicle, this may not be a concern. Screw pumps, while very effective, are exceptionally difficult to machine and accordingly are quite expensive. Further exacerbating machining difficulty, screw pumps have very close running tolerances (IMO Industries Inc.). This is mitigated somewhat by use of new additive manufacturing techniques which make it much easier to produce the complex shapes of the idler screws and at lower cost.

Though screw pumps can operate in ranges turbopumps cannot, their operational ranges overlap. There are a number of advantages and disadvantages screw pumps display compared to turbopumps. While screw pumps have the same material issues as described previously, the lower operating speeds result in significantly less stress compared to turbopumps. This allows for a broader material selection allowing the use of softer metals or even plastics. Triple screw pumps, unlike turbopumps, do not demonstrate cavitation
and are able to handle entrapped air (though this shouldn’t reasonably occur during operation) and mixed-phase liquid-solid flows provided the solid particles are small enough not to jam the screws (Karassik, 1976). Screw pumps are also able to handle a much wider range of viscosities than turbopumps are, though their performance is sensitive to changes in viscosity (Karassik, 1976). However, turbopumps have a smaller footprint and have a maximum efficiency greater than the triple screw pumps. Very low running clearances could increase the efficiency of the pump. However, this comes with high costs. Screw pumps do not create any suction themselves and require some amount of pressure to push the fluid into the pump (Karassik, 1976). The greatest advantage turbopumps have over triple screw pumps is a long history of use and reliability in space launch vehicles. Despite this, screw pumps might have use for small launch vehicles and upper stages.
2. Geometry

2.1. General Pump Shape

Triple Screw Pumps are made from three intermeshing screws, which create a number of interweaving moving seals pictured in Figure 2.1. The three screws rotate at the same angular velocity, with the two outer screws rotating opposite to the inner screw. The center screw is referred to as the driver screw because it drives the outer screws. The outer screws are called idler screws; they are not driven by an outside motor, but rather the force from the driver screw. The idler screws are formed by epicycloids of the driver screw (Radolfzell Works). As such the driver screw is always in contact with the idler screw. This is further explained in sections 2.2 and 2.3.

![Figure 2.1 Moving Seals and Fluid Pockets of a Triple Screw Pump (DeLaval-IMO) (Karassik, 1976)](image)

There are a number of variations on triple screw pumps including timed and untimed designs and double and single path designs. A timed pump has external gears which drive the idler screws. This reduces galling and friction problems but requires additional external seals which can significantly increase the pump’s cost. A single path design has all the fluid pass through a single passage. A double path design splits the fluid into two paths. This allows for a higher mass flow rate but decreases the maximum output.
pressure. It can also make manufacture more difficult. While these pumps are called triple screw pumps, additional satellite screws may be added provided the satellite screws’ placement are radially symmetrical. In this manner, pumps of four, five, six or more screws are possible. None of these variations significantly affect the analysis and steps in this document; the equations and design procedure may still be applied for any of these variations.

![Figure 2.2 Single Path vs Double Path Pump (IMO Industries Inc.)](image)

### 2.2. Pump Series

Screw pumps appear to be able to be made from almost any series provided general rules are followed: There must be a center core, the tooth height cannot be greater than the outer radius of any of the screws, and the tooth height must be the same on all screws in a pump. The author recommends that the idler screws be the same size. This does not appear necessary for the pump to function, but all of the equations after the Idler Screw Shape hold this assumption. Differently sized idler screws and non-symmetric placing will also misbalance the forces significantly, reducing the maximum possible pressure (Karassik, 1976).

In order to be able to quickly define triple screw pumps, and the apparent lack of
existing classification system and pump series naming system was developed. Pump Series are labeled by a series of three or four numbers: Driver Screw Outer Diameter, Idler Screw Outer Diameter, Tooth Height, and Number of Stages. The number of stages is equal to one half the number of the threads of the screws. The stage length is equal to twice the thread pitch. The first three numbers must always be whole numbers. The last number may be fractional. The number of stages may be dropped as it often does not affect the calculations except for total pressure, torque, and power. Where the number of stages is dropped off, the value is determined assuming the number of stages to be one. The ratio of the Driver Screw Outer Diameter, Idler Screw Outer Diameter, and Tooth Height should be reduced to the lowest common factor. For example, a pump with a 40mm driver screw outer diameter, 24mm idler screw outer diameter, and an 8mm tooth height would be represented as part of the series 5-3-1 and not 40-24-8. Finally, for the special case of a double path triple screw pump the number of stages are written as 2’[Number of stages per side]. For instance, a double path 5-3-1 pump with four stages per side would be labeled as series 5-3-1-2’4.

\[
\text{Driver Screw Outer Diameter} \quad 5 \quad \text{Idler Screw Outer Diameter} \quad 3 \quad \text{Tooth Height} \quad 1 \quad \text{Number of Stages} \quad 4^{1/2}
\]

Figure 2.3 Series Naming System

The 5-3-1 Series is the most common series for commercial triple screw pumps (Radolfzell Works). This specific series is designed such that the outer diameter of the idler screw is equal to the inner diameter of the driver screw (outer diameter minus two tooth heights). Since the screws rotate at the same angular velocity, the relative velocity
between the two surfaces is zero, so that friction and risk of galling is reduced. Following this rule, a series can be created by holding the inner diameter of the idler screw constant while increasing the tooth height. Each step we increase the tooth height adds twice that increase to the idler screw outer diameter and four times that increase in height to the driver screw. The rules of this series may also be satisfied if we increase the inner diameter of the idler screw. This increases the inner and outer diameters of the driver screw by the same amount resulting in a two-dimensional series of screws.

\[
\begin{align*}
[4Z+Y] & \quad [2Z+Y] & \quad [Z] \\
\text{Driver Screw Outer Diameter} & \quad \text{Idler Screw Outer Diameter} & \quad \text{Tooth Height}
\end{align*}
\]

Figure 2.4 Set of Equations for generating two-dimensional series.

Table 2.1 Possible Series Values for Z and Y up to Six. Note that many values repeat and reduce down to values previously listed. 5-3-1 is the most prevalent of this occurring wherever Y and Z are equal.

<table>
<thead>
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<th>Y Value</th>
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<td>5</td>
<td>9-7-1</td>
<td>13-9-2</td>
<td>17-11-3</td>
<td>21-13-4</td>
<td>25-15-5</td>
<td>29-17-6</td>
</tr>
<tr>
<td>6</td>
<td>10-8-1</td>
<td>14-10-2</td>
<td>18-12-3</td>
<td>22-14-4</td>
<td>26-16-5</td>
<td>30-18-6</td>
</tr>
</tbody>
</table>

This document’s graphs focus primarily on a Z value of one and Y values varying between one and six. The variation as Y changes is much greater than the variation as Z
The Shape of the Driver and Idler screws may be determined almost entirely by the Idler and Driver Screw outer diameters and the tooth height. The angle $\gamma$ is called the epicycloid angle and is solely a function of these parameters. The angles $\alpha$ and $\beta$ also describe the screw shape, but are more flexible requiring $\pi = 2\gamma + \alpha + \beta$ to be satisfied.
The pump’s idler screw is formed by an elongated epicycloid or epitrochoid of the driver screw, such that the outer forward tip of the driver screw, point \( A \) shown in Figure 2.9, maintains contact with the idler screw through the rotation until the tip of the driver screw exits the area occupied by the idler screw. Triple screw pumps are designed that all the screws rotate at the same angular velocity.
From these rules, the shape of the idler screw can be derived.

\[ R_1 \cos \varphi + r \cos \theta = R_1 + R_2 - t \quad R_1 \sin \varphi = r \sin \theta \]  

(2.1)

Rearranging these equations to be equal to \( \theta \).

\[ \theta = \sin^{-1}\left(\frac{R_1 \sin \varphi}{r}\right) \quad \theta = \cos^{-1}\left(\frac{R_1 + R_2 - t - R_1 \cos \varphi}{r}\right) \]  

(2.2)

The two equations are combined to yield

\[ \cos^{-1}\left(\frac{R_1 + R_2 - t - R_1 \cos \varphi}{r}\right) = \sin^{-1}\left(\frac{R_1 \sin \varphi}{r}\right) \]  

(2.3)

This equation is rearranged taking the cosine of the equation to yield

\[ \frac{R_1 + R_2 - t - R_1 \cos \varphi}{r} = \cos\left(\sin^{-1}\left(\frac{R_1 \sin \varphi}{r}\right)\right) \]  

(2.4)

Using a trigonometric identity, the equation may be rearranged

\[ \frac{R_1 + R_2 - t - R_1 \cos \varphi}{r} = \sqrt{1 - \left(\frac{R_1 \sin \varphi}{r}\right)^2} \]  

(2.5)

Squaring both sides and multiplying by \( r^2 \) yields

\[ (R_1 + R_2 - t - R_1 \cos \varphi)^2 = r^2 - R_1^2 \sin^2 \varphi \]  

(2.6)

The equation is then rearranged so that the equation is equal to \( r \).
\[ r = \sqrt{(R_1 + R_2 - t - R_1 \cos \varphi)^2 + R_1^2 \sin^2 \varphi} \]  

(2.8)

This equation may be used to give the shape of the epicycloid line on the idler screw. To determine the maximum value of \( \varphi \), the equation is rearranged. First, the above equation is expanded and after some rearranging and use of Pythagorean identities yields

\[ r^2 = \left( R_1 + (R_2 - t) \right)^2 - 2\left( R_1^2 + R_1(R_2 - t) \right) \cos \varphi + R_1^2 \]  

(2.9)

The equation is finally rearranged to be equal to \( \varphi \).

\[ \varphi = \cos^{-1} \left( \frac{(R_1 + R_2 - t)^2 + R_1^2 - r^2}{2R_1^2 + 2R_1(R_2 - t)} \right) \]  

(2.10)

\( \varphi_{\text{MAX}} \) occurs when \( r \) is equal to \( R_2 \) and may be simply substituted into the equation.

\[ \varphi_{\text{MAX}} = \cos^{-1} \left( \frac{(R_1 + (R_2 - t))^2 + R_1^2 - R_2^2}{2R_1^2 + 2R_1(R_2 - t)} \right) \]  

(2.11)

Table 2.2 \( \varphi_{\text{MAX}} \) Angle for Varying Series

<table>
<thead>
<tr>
<th>Series</th>
<th>( \varphi_{\text{MAX}} ) (radians)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-3-1</td>
<td>0.522</td>
</tr>
<tr>
<td>6-4-1</td>
<td>0.505</td>
</tr>
<tr>
<td>7-5-1</td>
<td>0.483</td>
</tr>
<tr>
<td>8-6-1</td>
<td>0.460</td>
</tr>
<tr>
<td>9-7-1</td>
<td>0.440</td>
</tr>
<tr>
<td>10-8-1</td>
<td>0.421</td>
</tr>
</tbody>
</table>

This gives us the maximum angle of \( \varphi \) on the rotation of the driver screw. The driver screw and idler screw turn at the same rate so the \( \gamma \) value is the same on both screws.

The equation derived only gives \( r \) as a function of \( \varphi \) which is on the driver screw, not the
idler screw. At the start of the derivation, an equation was used that relates $\theta$ to $r$ and $\varphi$. $\theta$ was defined on a global axis rather than a local one of the rotating idler screw. To convert between the global coordinate system and the local rotating coordinate system the rotation of the screw, $\varphi$, is simply subtracted from $\theta$ to give $\theta'$. With this, the epicycloid curve may now be determined by gathering the $r$ and $\theta'$ values for varying values of $\varphi$.

$$f(r, \theta') = \begin{cases} r = \sqrt{\left( R_1 + (R_2 - t) \right)^2 - 2\left( R_1^2 + R_1(R_2 - t) \right) \cos \varphi + R_1^2} \\ \theta' = \sin^{-1}\left( \frac{R_1 \sin \varphi}{r} \right) - \varphi \end{cases} \quad 0 \leq \varphi \leq \varphi_{MAX}$$

(2.12)

The driver screw and idler screw turn at the same rate so the $\gamma$ value is the same on both screws. $\gamma$ may be found solving for $\theta'$ at $\varphi_{MAX}$. At $\varphi_{MAX}$, $r = R_2$ that $\gamma$ may be found by

$$\gamma = \sin^{-1}\left( \frac{R_1 \sin \varphi}{R_2} \right) - \varphi$$

(2.13)

Table 2.3 $\gamma$ Angle for Varying Series

<table>
<thead>
<tr>
<th>Series</th>
<th>$\gamma$ (radians)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-3-1</td>
<td>0.459</td>
</tr>
<tr>
<td>6-4-1</td>
<td>0.307</td>
</tr>
<tr>
<td>7-5-1</td>
<td>0.225</td>
</tr>
<tr>
<td>8-6-1</td>
<td>0.174</td>
</tr>
<tr>
<td>9-7-1</td>
<td>0.140</td>
</tr>
<tr>
<td>10-8-1</td>
<td>0.115</td>
</tr>
</tbody>
</table>

Converted into Cartesian coordinates, this set of parametric equations defines an elongated epicycloid or epitrochoid as shown in (Lawrence, 1972).
\[ x = (R_1 - t + R_2) \cos \varphi - R_1 \cos \left( \frac{R_1 - t + R_2}{R_2} \varphi \right) \]

\[ y = (R_1 - t + R_2) \sin \varphi - R_1 \sin \left( \frac{R_1 - t + R_2}{R_2} \varphi \right) \] \tag{2.14}

2.4. Driver Screw Shape

The shape of the driver screw is much simpler than the shape of the idler screw, being a trapezoid when taken in a sectional view. The shape of the driver screw is determined from the parameters set by the idler screw.

![Figure 2.11 Side View of Driver Screw](image)

When taken in a sectional view, the trapezoid shape is immediately clear. The lengths of the regular trapezoid are

\[ l_{R_1} = \frac{\alpha L}{2\pi}, \quad l_{r_1} = \frac{(\alpha + 2\gamma)L}{2\pi} \] \tag{2.15}
The rising and falling edges of the driver screw thread are formed by the legs of the regular trapezoid shown in Figure 2.12. Due to the properties of a helix this shape is maintained that that the legs have a constant slope. Given this the rising and falling edges of the driver screw when viewed as a cross section may be easily calculated.

\[
r = \frac{R_1 - r_1}{\gamma} \varphi + r_1 = \frac{t}{\gamma} \varphi + r_1 = \frac{t}{\gamma} \varphi + R_1 - t
\]  
(2.16)


2.5. Pump Case Cross-Sectional Area

The internal volume of the pump case consists of three intersecting cylinders. The cross-sectional area must be calculated. The easiest way to determine the area of this shape is to break the shape up into its three circular components and subtract the overlapping circular segments. In order to determine the area of these circular segments, the angles $\psi$ and $\xi$ must first be determined. Note that these angles must be in radians.

![Figure 2.15 Diagram of Three Screw Pump Cross Section.](image)

Using the Law of Cosines, the value of $\psi$ may be determined. $\psi$ is equal to the value of $\phi_{\text{MAX}}$. This makes sense as it is the angle that the drive screw no longer interacts with the idler screw.

$$\psi = \cos^{-1}\left(\frac{(R_1^2 + (R_1 + R_2 - t)^2 - R_2^2)}{2R_1(R_1 + R_2 - t)}\right) = \phi_{\text{MAX}} \quad (2.17)$$

With the value of $\psi$ known. The values of $c$ and $a$ may be determined.

$$c = R_1 \sin \psi \quad (2.18)$$
\[ a = R_1 \cos \psi \]  \hspace{1cm} (2.19)

With these parameters the circle segment area may be determined

![Figure 2.16 Diagram to Calculate $A_{CS1}$](image)

The area of the circle segment may be determined by subtracting a triangle from an arc.

\[ A_{CS1} = \frac{2\psi R_1^2}{2} - \frac{2ac}{2} = \psi R_1^2 - ac \]  \hspace{1cm} (2.20)

Unfortunately substituting the values into this equation does not allow for further simplification of the equation.

This process is repeated for $\xi$.

![Figure 2.17 Diagram to Calculate $A_{CS}$](image)
Fortunately, this process is repeated for $\xi$ and $b$.

$$\xi = \sin^{-1}\left(\frac{R_1}{R_2} \sin \psi\right) \quad (2.21)$$

$$b = R_2 \cos \psi \quad (2.22)$$

Using the same process for the previous circle segment the area of this circle segment may be determined as well.

$$A_{CS2} = \xi R_2^2 - bc \quad (2.23)$$

With the area of these circular segments known the cross-sectional area of the pump may be determined.

$$A_{CASE} = \pi R_1^2 + n_{IDLER SCREWS} (\pi R_2^2 - A_{CS} - A_{CS2}) \quad (2.24)$$

### 2.6. Idler Screw Cross Sectional Area

The area of the idler screw will be used in determining the flow rate discussed in Section 3.1. The cross-sectional area of the idler screw is difficult to calculate because the epicycloid curve cannot be explicitly defined. The idler screw is divided up into two sets of arcs. This leaves some area unaccounted for and some area outside of the epicycloid curve. These areas are combined together to be called the epicycloid area (it is the area due to the presence of the epicycloid).
This yields the area as

$$A_{IDLER} = 2(\alpha + 2\gamma)(R_2 - t)^2 + 2\beta R_2^2 + 4A_y$$  \hspace{1cm} (2.25)

$A_y$ is very difficult to calculate analytically and instead is calculated numerically with a trapezoidal Riemann sum.
This yields
\[ A_y = \sum_{i=1}^{n-1} \frac{(r_{n+1} - r_n)(r_n(y - \theta'_n) + r_{n+1}(y - \theta'_{n+1}))}{2} \] \quad (2.26)

Some of the areas are tabulated below. If \( A_y \) is not included in the table, \( A_y \) may be approximated as zero since it is quite small. For a 5-3-1 pump, this would amount to about a 6% error.

<table>
<thead>
<tr>
<th>Series</th>
<th>Area under epicycloid curve: ( A_y/R_2^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-3-1</td>
<td>-0.0265</td>
</tr>
<tr>
<td>6-4-1</td>
<td>-0.0118</td>
</tr>
<tr>
<td>7-5-1</td>
<td>-0.0096</td>
</tr>
<tr>
<td>8-6-1</td>
<td>-0.0098</td>
</tr>
<tr>
<td>9-7-1</td>
<td>-0.0104</td>
</tr>
<tr>
<td>10-8-1</td>
<td>-0.0110</td>
</tr>
</tbody>
</table>

### 2.7. Driver Screw Cross Sectional Area

The area of the driver screw will also be used in determining the flow rate discussed in Section 3.1. The cross-sectional area of the driver screw is easier to calculate.

\[ A_{DRIVER} = \alpha R_1^2 + \beta (R_1 - t)^2 + 2 \int_0^{\gamma} \frac{t}{\gamma} \phi + (R_1 - t) d\phi \] \quad (2.27)

\[ A_{DRIVER} = \alpha R_1^2 + \beta (R_1 - t)^2 + \gamma (2R_1 - t) \] \quad (2.28)
2.8. Pump Case Perimeter

The Perimeter of the Pump Case is another important parameter for describing the slip and power consumption of the pump. The perimeter is split between the idler and driver screws. Referencing Figure 2.12 the individual perimeters are easily determined.

\[
p_{\text{DRIVER}} = 2\pi R_1 - 2n_{\text{IDLER SCREWS}}\psi R_1 \quad (2.29)
\]

\[
p_{\text{IDLER}} = 2\pi R_2 - \xi R_2 \quad (2.30)
\]

\[
p = R_1(2\pi - 2n_{\text{IDLER SCREWS}}\psi) + n_{\text{IDLER SCREWS}}R_2(2\pi - 2\xi) \quad (2.31)
\]
3. Pump Flow Rate, Slip, and Pressure

3.1. Base Flow Rate

A positive displacement pump works by physically moving the fluid from one point to another. The screw pump functions by creating sets of interlocking seals which act as a labyrinth, forming pockets of fluid which move from inlet to outlet. A full rotation of the screw displaces the fluid by the length of one stage or pitch length. From this, the pump flow rate may be determined. The flow rate depends on the size of the fluid pocket, pictured in Figure 2.1. Even though the shape and volume of the pockets are difficult to calculate, the area occupied by the fluid is constant. With this information a basic flow rate can be calculated.

\[ Q = A_{PUMP} L N \]

(3.1)

Figure 3.1 Cross Section of Triple Screw Pump (Adapted from (Radolfzell Works))

Figure 3.2 Cross Section of Experimental Triple Screw Pump
With the cross-sectional area of the screws and the pump case, the cross-sectional area of the fluid pockets may be determined.

\[ A_{PUMP} = A_{CASE} - (A_{DRIVER} + n_{IDLER SCREWS}A_{IDLER}) \]  

(3.2)

These area terms are all functions of outer radius and may be put in terms of \( R_1 \). With this, \( A_{PUMP} \) may be reorganized to be \( A_{PUMP} = kR_1^2 \) where \( k \) is a geometric factor taking into account all of the factors that are not \( R_1 \).

\[ Q = kLR_1^2N \]  

(3.3)

Note that this \( N \) is in Hertz and not rad/s; this is converted to rad/s because it is useful for derivations below. To create consistency with the original flow rate equation presented in the Pump Handbook, \( Q = KD^3N \), the radius is converted to diameter to give

\[ Q = \frac{kLD^2\omega}{8\pi} = \frac{kLD^2N}{4} \]  

(3.4)

If the screw angle is known rather than the length, this will put the flow rate in the same form as the given flow rate equation. This is most easily calculated by treating the shape as a right triangle wrapped around a cylinder, such that when unwrapped the base of the triangle is the length of one stage of the screw and the height of the triangle is the circumference of the screw.

\[ \tan \sigma = \frac{\pi D}{L} \]  

(3.5)
Inserting this equation into the flow rate equation gives our final flow rate equation.

\[
Q = \frac{kD^3\omega}{8\tan\sigma} = \frac{\pi kD^3N}{4\tan\sigma}
\]

(3.6)

This takes the same form for the flow rate equation as is given in the Pump Handbook: \(Q = KD^3N\) (Karassik, 1976).

This flow rate equation is for a single path screw pump. In the case of a double path screw pump the flow rate would simply be multiplied by two.

### 3.2. Slip

If no internal clearances existed, the flow rate would be equal to the flow rate equation given above. It is not practical to make a pump with no clearances and thus, as long as there is a pressure differential some backflow will exist. This backflow is called slip. This slip is primarily affected by pressure, and for practical purposes is not affected by rotational speed (Karassik, 1976). Taking this into account, the flow rate equation becomes

\[
Q_{REAL} = Q - S_{TOTAL} = \frac{kD^3\omega}{8\tan\sigma} - S_{TOTAL}
\]

(3.7)

Due to the epicycloid shape of the idler screw, the driver and idler screw remain in contact along the entire rotation; this forms a thin line where fluid cannot pass through. In an untimed design, the force to drive the idler screw is applied along this line. When operating at pressure, there is substantial force acting on the screws. This drives them together, minimizing the backflow through the internal clearances. Some backflow will still occur, though this is very difficult to calculate and is taken into account as an empirical constant on the slip equation. As the pump runs for an extended period of time, it will wear down. This will increase the slip through the pump. The short operational lifetime of rocket
engines means that this wear is not a concern for this application but should be noted for pumps with a substantial operational lifetime. A timed design will likely have greater slip, as the rotational torque is delivered through external gears and not the meshing screws. In a double path screw pump, the torques due to the pressure are canceled out and will then have more internal slip.

Figure 3.4 Pressure Remains Roughly Constant in Fluid Cavities Dropping Across the Moving Seals (Karassik, 1976)

In order to simplify the problem, several assumptions and approximations are made. The first is that the primary source of slip comes from the external clearances between the screws and the pump case wall. The clearance should be very small relative to length of the channel where the slip is occurring. The flow profile is constant along the length of the channel and does not vary with time. Note this will not be valid for some values of $\alpha$. It is recommended that an $\alpha$ be selected that results in the channel length at least ten times the clearance height. The flow is assumed to be laminar, and the pressure losses will be perpendicular to the screw face at the edge with no vertical component. Finally, the clearance is assumed to be so much smaller than the curvature of the screw, so that it may be treated as flat. This leaves the flow as Combined Poiseuille-Couette Flow (H. Schlichting, 2006).
Figure 3.5 Determining Moving Boundary Condition Along Screw

Figure 3.6 Diagram of Channel Where Slip Occurs (Not to Scale)

\begin{equation}
\frac{1}{\mu} \frac{\partial P}{\partial x} = \frac{\partial^2 u}{\partial y^2}
\end{equation}

Above is the simplification of the Navier-Stokes Equations after the assumptions are applied. $\frac{1}{\mu} \frac{\partial P}{\partial x}$ is taken to be a constant, $A$. Solving the ordinary differential equation yields

\begin{equation}
u = \frac{Ay^2}{2} + By + \Gamma
\end{equation}

Next the boundary conditions are applied and the values of $B$, and $\Gamma$.

\begin{equation}
u(0) = \Gamma = 0, \quad \nu(\delta) = -\omega R \cos \sigma = \frac{A\delta^2}{2} + B\delta
\end{equation}

Since $\frac{1}{\mu} \frac{\partial P}{\partial x}$ was taken to be a constant and there is a definite length for $x$ it may be rewritten as $\frac{1}{\mu} \frac{\Delta P}{\Delta x}$. If $L$ is known instead of $\sigma$ use $\sigma = \tan^{-1} \left( \frac{\pi D}{L} \right)$ to determine $\sigma$. 

\[ u = \frac{1}{2 \mu} \frac{\Delta P \Delta x}{y^2} + \left( -\frac{\omega R \cos \sigma}{\delta} - \frac{1}{2 \mu} \frac{\Delta P}{\Delta x} \right) y \]  
(3.11)

The flow rate may be determined by integrating the velocity function with respect to \( y \) from 0 to \( \delta \).

\[ s = \frac{1}{6 \mu \Delta x} \frac{\Delta P \delta^3}{\delta^3} + \frac{\left( -\frac{\omega R \cos \sigma}{\delta} - \frac{1}{2 \mu} \frac{\Delta P}{\Delta x} \delta \right) \delta^2}{2} \]  
(3.12)

Simplifying this equation becomes

\[ s = \frac{1}{6 \mu \Delta x} \frac{\Delta P \delta^3}{\delta^3} - \frac{\omega R \delta \cos \sigma}{2} \frac{1}{4 \mu \Delta x} \delta^3 = -\frac{1}{12 \mu \Delta x} \delta^3 - \omega R \delta \cos \sigma \]  
(3.13)

Note that the flow in the channel is moving backwards which is expected. The negative sign is already accounted for in equation 3.7 so is removed from the equation. This yields a positive slip value. Note that \( \omega R \cos \sigma \) will always be positive when operating in the forward direction.

\[ s = \frac{1}{12 \mu \Delta x} \frac{\Delta P \delta^3}{\delta^3} + \omega R \delta \cos \sigma \]  
(3.14)

Figure 3.7 Geometric Set Up for Determining \( \Delta x \)

\[ \Delta x_{\text{DRIVER}} = \frac{\alpha}{2\pi} L \sin(\sigma), \quad \Delta x_{\text{IDLER}} = \frac{\beta}{2\pi} L \sin(\sigma) \]  
(3.13)

Note that \( \alpha \) and \( \beta \) must be in radians. Substituting \( L \) out for screw radii yields

\[ \Delta x_{\text{DRIVER}} = \alpha R_1 \cos \sigma, \quad \Delta x_{\text{IDLER}} = \beta R_2 \cos \sigma \]  
(3.14)
To determine the actual slip, the slip/length is multiplied by the perimeter of the pump case. Note that this is the slip only for a single start.

\[ S = \left( \frac{1}{12 \mu} \frac{\Delta P}{\Delta x} \delta^3 + \omega R \delta \cos \sigma \right) (R_1 (2\pi - 4\psi) + n_{\text{IDLER SCREWS}} R_2 (2\pi - 2\xi)) \] (3.16)

Next the corresponding \( \Delta x \)s are substituted in and the second term is distributed. The \( R_1 \) term corresponds to \( \Delta x_{\text{DRIVER}} \) and the \( R_2 \) term corresponds to \( \Delta x_{\text{IDLER}} \).

\[ S = \left( \frac{1}{12 \mu} \frac{\Delta P}{\alpha R_1 \cos \sigma} + \omega R_1 \delta \cos \sigma \right) R_1 (2\pi - 4\psi) \]

\[ + \left( \frac{1}{12 \mu} \frac{\Delta P}{\beta R_2 \cos \sigma} + \omega R_2 \delta \cos \sigma \right) n_{\text{IDLER SCREWS}} R_2 (2\pi - 2\xi) \] (3.17)

Simplifying the equation yields

\[ S = \left( \frac{1}{6 \mu} \frac{\Delta P}{\alpha \cos \sigma} + \omega r_1^2 \delta \cos \sigma \right) (\pi - 2\psi) \]

\[ + n_{\text{IDLER SCREWS}} \left( \frac{1}{6 \mu} \frac{\Delta P}{\beta \cos \sigma} + \omega r_2^2 \delta \cos \sigma \right) (\pi - \xi) \] (3.18)

Note that the pressure component of the slip is not a function of the screw outer radius. Though counter intuitive, this property results from the fact that as the screw outer diameter grows the perimeter increases but the length of the channel also grows in the same proportion.

Finally, an empirical constant, \( \Psi_{\text{SLIP}} \), is applied. This constant accounts for the internal slip, and the slip variations at the internal corners where the screws meet where the assumptions made may not apply.
$$S = \Psi_{SLIP} \left( \frac{1}{6\mu} \frac{\Delta P \delta^3}{\alpha \cos \sigma} + \omega r_1^2 \delta \cos \sigma \right) (\pi - 2\psi)$$

(3.19)

$$+ n_{IDLER SCREWS} \left( \frac{1}{6\mu} \frac{\Delta P \delta^3}{\beta \cos \sigma} + \omega r_2^2 \delta \cos \sigma \right) (\pi - \xi)$$

This is as far as the general slip can be reduced, however in certain operational environments one term will dominate the other. This allows for further simplification. In operation for a space launch vehicle, the pressure will be on the order of megapascals. If we apply this and look at the order of the terms, they may be further simplified by dropping the speed dependent part of the equation.

$$\frac{1}{\sigma(-4)} \sigma(6) * \sigma(-4)^3 + \sigma(2) * \sigma(-1)^2 * \sigma(-4)$$

(3.20)

$$\sigma(-2) + \sigma(-4)$$

(3.21)

This allows the equation to be simplified to

$$S = \Psi_{SLIP} \left( \frac{1}{6\mu} \frac{\Delta P \delta^3}{\alpha \cos \sigma} (\pi - 2\psi) + \frac{n_{IDLER SCREWS}}{6\mu} \frac{\Delta P \delta^3}{\beta \cos \sigma} (\pi - \xi) \right)$$

(3.22)

This simplification must be applied carefully. A particularly viscous fluid, particularly tight clearances, particular values of \(\alpha, \beta, \) or \(\sigma,\) or low pressure could render this simplification invalid or even reverse it so the \(\omega r^2 \delta \sin \sigma\) term becomes dominant.

In order to simplify the equation further the various geometric parameters are condensed into a new parameter, \(\chi.\)

$$\chi = \frac{1}{6\alpha} (\pi - 2\psi) + \frac{n_{IDLER SCREWS}}{6\beta} (\pi - \xi)$$

(3.23)

This simplifies the slip equation to be:

$$S = \left( \frac{\Delta P \delta^3}{\mu \cos \sigma} \Psi_{SLIP} \right) \chi$$

(3.24)
This is the slip for only a single start. To determine the total slip as used in equation 3.7, the slip must be multiplied by the number of starts.

\[ S_{TOTAL} = n_{STARTS}S \]  \hspace{1cm} (3.25)

### 3.3. Volumetric Efficiency

For practical purposes when designing a pump, the slip flow won’t be selected. Instead an acceptable back flow is selected at a specific operating point. Here a new design parameter \( \eta_{VOLUMETRIC} \) may be introduced. \( \eta_{VOLUMETRIC} \) represents the ratio of the actual capacity to the theoretical capacity.

\[ \eta_{VOLUMETRIC} = 1 - \frac{S_{TOTAL}}{Q} \]  \hspace{1cm} (3.26)

This adjusts the mass flow rate equation to

\[ Q_{REAL} = \eta_{VOLUMETRIC}Q \]  \hspace{1cm} (3.27)

Furthermore, since the slip may now be selected as an efficiency might be for a turbine, the slip equation becomes more directive than selective. The viscosity is largely set by the fluid passing through the pump, and the clearances are generally set by the machining capability and budget. Plugging the values of \( S_{TOTAL} \) and \( Q \) into equation 3.23 yields.

\[ \eta_{VOLUMETRIC} = 1 - \frac{8n_{STARTS}\Delta P}{k D^3 \omega \mu \cos \sigma \Psi_{SLIP} \chi} \]  \hspace{1cm} (3.28)

\( \Delta P_{STAGE} \) is simply the \( \Delta P \) from the slip section multiplied by \( n_{STARTS} \). Rearranging the equation to give the pressure drop across each stage yields

\[ \Delta P_{STAGE} = \Delta P n_{STARTS} = \frac{Q \mu \cos \sigma (1 - \eta_{VOLUMETRIC})}{\delta^3 \Psi_{SLIP} \chi} \]  \hspace{1cm} (3.29)

The flow rate without the flow efficiency factor may be substituted in. The
efficiency factor has already been taken into account

\[
\Delta P_{\text{STAGE}} = \frac{k D^3 \omega \mu \cos \sigma (1 - \eta_{\text{VOLUMETRIC}})}{8 \delta^3 \tan \sigma \Psi_{\text{SLIP}} \chi}
\] (3.30)

This equation shows that the \( \Delta P_{\text{STAGE}} \) is most affected by changes to \( D \) and \( \delta \). So long as these two parameters are held constant relative to each other, \( \Delta P_{\text{STAGE}} \) will remain the same. As \( D \) increases, \( \delta \) can remain the same. This means either \( \Delta P_{\text{STAGE}} \) will increase significantly or the efficiency may be greatly improved. Rearranging this equation to give efficiency yields

\[
\eta_{\text{VOLUMETRIC}} = 1 - \frac{8 \Delta P_{\text{STAGE}} \delta^3 \tan \sigma \Psi_{\text{SLIP}} \chi}{k D^3 \omega \mu \cos \sigma}
\] (3.31)

\( \eta_{\text{VOLUMETRIC}} \) should always be a positive real number between zero and one.

### 3.4. Size Effect on Speed

One of the assumptions made is that the flow through the channel is laminar. This requires the Reynolds number be less than 1500 (H. Schlichting, 2006).

\[
Re = \frac{\omega R \rho \delta}{\mu} \sin \sigma \leq 1500
\] (3.32)

This sets an upper limit on how fast the pump can reasonably operate. As the driver screw is larger than the idler screw, the driver screw will be the limiting factor and \( R_1 \) may be used instead of \( R \).

\[
\omega = Re \frac{\mu}{R_1 \rho \delta \sin \sigma}
\] (3.33)

Substituting this into equation 3.6 yields

\[
Q = \frac{k D^2 R \mu}{2 \rho \delta \cos \sigma}
\] (3.34)

The equation may be rearranged to give a minimum outer diameter for a flow rate
3.5. Maximum Pressure

The absolute maximum pressure of the pump (barring structural or seal failure) occurs when $\eta_{\text{VOLUMETRIC}}$ is zero, that the slip is equal to the flow rate.

$$\Delta P_{\text{STAGE MAX}} = \frac{kD^3\omega\mu\cos\sigma}{8\tan\sigma \delta^3 \psi_{\text{SLIP}} \chi} = Q \frac{\mu \cos\sigma}{\delta^3 \psi_{\text{SLIP}} \chi}$$  \hspace{1cm} (3.36)

3.6. Pump Total Pressure

The maximum output pressure is based on the maximum allowable pressure differential across the moving seals. Given this, the number of moving seals can be increased, allowing for higher output pressures. More seals are added simply by extending the pump length.

In this manner, the total pressure may be easily determined.

$$P_{\text{TOTAL}} = \Delta P_{\text{STAGE}} n_{\text{STAGES}}$$  \hspace{1cm} (3.37)

Figure 3.8 Example of Increasing Pressure Capability by Adding Additional Stages (DeLaval IMO) (Karassik, 1976)
4. Power

4.1. Base Power Consumption

As with other pumps, the power consumed is a function of the amount of fluid passing through the pump and the output pressure.

\[ \mathcal{P} = \Delta P_{TOTAL} Q_{REAL} \]  \hspace{1cm} (4.1)

In practice, there are several sources of power loss. These are divided between slip losses, viscous losses, and mechanical losses.

\[ \mathcal{P} = \Delta P_{TOTAL} Q_{REAL} + (\mathcal{P}_{SLIP} + \mathcal{P}_{VISCOUS} + \mathcal{P}_{MECHANICAL}) \]  \hspace{1cm} (4.2)

4.2. Slip Losses

Losses due to slip are relatively easy to calculate. These losses are from fluid that was pressurized but slipped back into the previous chamber. This equation will take the same form as the power equation for a pump.

\[ \mathcal{P}_{SLIP} = 2\Delta P_{STAGE} n_{STAGES} S = 2\Delta P_{TOTAL} S \]  \hspace{1cm} (4.3)

Rearranging this equation to set it up as an efficiency

\[ \frac{\mathcal{P} - \mathcal{P}_{SLIP}}{\mathcal{P}} = \eta_{VOLUMETRIC} \]  \hspace{1cm} (4.4)

\( \eta_{VOLUMETRIC} \) may be plugged directly into the first power equation to give the new power requirement.

\[ \mathcal{P} = \frac{\Delta P_{TOTAL} Q}{\eta_{VOLUMETRIC}} \]  \hspace{1cm} (4.5)
4.3. Viscous Losses

Viscous losses are more difficult to calculate. It is expected that at low speeds viscous losses make up the majority of losses (Karassik, 1976). However, operational ranges for pumps will tend to occur outside of this range. In the operational range, the primary source of viscous losses will be the channels between the screws and the case wall perpendicular to the direction of rotation. The assumptions and simplifications made are similar to that for determining slip: The clearance should be very small relative to length of the channel. The flow profile is constant along the channel and does not vary with time. Note this will not be valid for some values of $\alpha$. It is recommended that an $\alpha$ is selected that results in the channel length being at least ten times the clearance height. The flow is assumed to be laminar and the pressure loses will be anti-parallel to the direction of motion in the channel. Lastly, the clearance is assumed to be much smaller than the curvature of the screw that it may be treated as flat. This leaves the flow as Combined Poiseuille-Couette Flow (H. Schlichting, 2006).

![Figure 4.1 Top Down View of System](image-url)
Figure 4.2 Problem Diagram for Determining Viscous Losses (Not to Scale)

\[ \frac{1}{\mu} \frac{\partial P}{\partial x} = \frac{\partial^2 u}{\partial y^2} \]  

(4.6)

Above is the simplification of the Navier-Stokes Equations after the assumptions are applied. \( \frac{1}{\mu} \frac{\partial P}{\partial x} \) is taken to be a constant, \( E \). Solving the ordinary differential equation yields

\[ u = \frac{Ey^2}{2} + Zy + H \]  

(4.7)

Next the boundary conditions are applied. Since \( \frac{1}{\mu} \frac{\partial P}{\partial x} \) was taken to be a constant and there is a definite length for \( \Delta X \), it may be rewritten as \( \frac{1}{\mu} \frac{\Delta P}{\Delta X} \).

\[ u = \frac{1}{\mu} \frac{\Delta P}{\Delta X} y^2 + \left( -\frac{\omega R}{\delta} - \frac{1}{\mu} \frac{\Delta P}{\Delta X} \right) y + \omega R \]  

(4.8)

The power loss from viscous fluid in this case is equal to the shear stress multiplied by the area of the surface and its speed.

\[ P_{\text{viscous}} = \sum \omega R \tau A \]  

(4.9)

Note this value can never be negative so the absolute value of the equation is taken to avoid sign confusion. \( \tau \) may be found using \( \tau = \mu \frac{du}{dy} \) (H. Schlichting, 2006).

\[ \tau = 2 \frac{\Delta P}{\Delta X} y - \left( \frac{\mu \omega R}{\delta} + \frac{\Delta P}{\Delta X} \right) \]  

(4.10)

Taken at the screw surface and set to an absolute value
\[
\tau(0) = \frac{\mu \omega R}{\delta} + \frac{\Delta P}{\Delta X}
\]

Looking at the order of magnitude of this equation lets us simplify it further. For particularly loose pumps, fast pumps, low pressure, or viscous fluid this may not be valid.

\[
\frac{\sigma(-4) \sigma(2) \sigma(-1)}{\sigma(-4)} - \frac{\sigma(6)}{\sigma(-2)}
\]

\[
\sigma(1) - \sigma(4)
\]

This simplifies the surface shear stress to

\[
\tau = \frac{\Delta P}{\Delta X}
\]

This also confirms the earlier assumption that most of the viscous losses happen in the channels between the screws and case. The surface area is

\[
A_{SURFACE} = n_{STARTS} \Delta X n_{STAGES} (R_1 (2\pi - 2n_{IDLER SCREWS} \psi) + n_{IDLER SCREWS} R_2 (2\pi - 2\xi))
\]

This is obtained by multiplying the length of the outer edge of the screw by the circumference number of stages multiplied by \(n_{STARTS}\) and removing the sections which are not against the perimeter. The equation is not fully solved left in \(\Delta x\) because this will cancel out with the other \(\Delta x\) in the shear stress. An empirical constant \(\Psi_{VISCOUS}\) is added to the equation, however it would be difficult to precisely identify the value of the constant.

Multiplying these all together yields the final power equation

\[
\mathcal{P}_{VISCOUS} = \Psi_{VISCOUS} \omega \Delta P_{TOTAL} n_{STARTS} (R_1^2 (2\pi - 2n_{IDLER SCREWS} \psi) + n_{IDLER SCREWS} R_2^2 (2\pi - 2\xi))
\]

In the same manner as before an efficiency factor may be determined

\[
\frac{\mathcal{P} - \mathcal{P}_{VISCOUS}}{\mathcal{P}} = \eta_{VISCOUS}
\]
In this manner $\eta_{WISCOUS}$ is an efficiency factor and may be plugged directly into the first power equation to give the new power requirement.

\[ P = \frac{\Delta P_{TOTAL} Q_{REAL}}{\eta_{VOLUMETRIC} \eta_{WISCOUS}} \]  

(4.18)

4.4. Mechanical Losses

The mechanical losses include the power needed to overcome the friction of all the parts in the pump including the screws, bearings, gears, external seals etc. It is difficult to calculate the mechanical losses; however, some assumptions may be made that will allow a working estimation for the losses. The friction between the screws should be the largest portion of the mechanical losses at operational speed and pressure. Bearings and external seals are designed to have the minimum friction possible. For untimed designs, the torque is transmitted through the screws themselves and friction cannot practically be minimized. To this end, the torque must first be determined.

\[ T \propto RA \Delta P_{TOTAL} \sin \sigma \]  

(4.19)

Figure 4.3 Diagram of Internal Pressures Creating No Net Torque. Only the total pressure is needed to calculate torque.

The torque is needed to resist the torque created by the pressure drop across the seals. Since the pressure is constant throughout each fluid cavity, the intermediate pressures will cancel each other out and leave no net torque, thus the torque generating pressure differences occurs only on either end of the pump.
These pressures only operate over a half stage, so the area may be rewritten as \( \frac{lt}{2} \) and substituted into the equation. A new term, \( \Psi_{TORQUE} \), is also added since the torque does not act directly at the outer diameter. Due to the nature of the screw shape, it is difficult to estimate where exactly the torque acts and will be more easily determined empirically. This term will then include all the empirical variation in the torque calculation.

\[
T = \sum \frac{\Psi_{TORQUE}RLt}{2} \Delta P_{TOTAL} \sin \sigma \tag{4.20}
\]

The length is substituted out and the radii are substituted into the equation.

\[
T = \Psi_{TORQUE} t \Delta P_{TOTAL} \cos \sigma \left( R_1^2 + n_{IDLER SCREWS}R_2^2 \right) \tag{4.21}
\]

This gives a value for the torque needed to drive the pump. The power loss due to friction is more difficult to determine and will require empirical data. However, an expression for the power may be developed.

\[
P_{MECHANICAL} = \Psi_{MECHANICAL} \omega TV \tag{4.22}
\]

The torque is substituted into the equation. \( \Lambda \) is an empirical factor. \( \nu \) is the coefficient of friction. It will be difficult (if not impossible to) determine the values of \( \Lambda \) and \( \nu \); \( \nu \) will be absorbed into \( \Lambda \) to make it more feasible to find empirical solutions.

\[
P_{MECHANICAL} = \Psi_{MECHANICAL} \Psi_{TORQUE} \omega t \Delta P_{TOTAL} \cos \sigma \left( R_1^2 + n_{IDLER SCREWS}R_2^2 \right) \tag{4.23}
\]

As done before this may be set as an efficiency

\[
\frac{P - P_{MECHANICAL}}{P} = \eta_{MECHANICAL} \tag{4.24}
\]

This may be substituted into the power equation to give the new power requirement

\[
P = \frac{\Delta P_{TOTAL} Q_{REAL}}{\eta_{VOLUMETRIC} \eta_{VISCOUS} \eta_{MECHANICAL}} \tag{4.25}
\]
4.5. Efficiency

The terms to determine the power required unfortunately use a large number of empirical terms which may be difficult or impossible to isolate. The efficiency may be determined by multiplying the three factors determined earlier.

\[ \eta_{TOTAL} = \eta_{VOLUMETRIC} \eta_{VISCOUS} \eta_{MECHANICAL} \] \hspace{1cm} (4.26)

A typical efficiency curve from the pump handbook is shown below. Values between 70% and 80% are typical (Karassik, 1976) though with current technology it is likely substantially higher efficiencies could be achieved.

![Figure 4.4 Typical Pump Efficiency Curve (Karassik, 1976)](image_url)

There are a number of parameters that can be altered to increase the efficiency of the pump. The pressure across the pump is a requirement of the design and cannot reasonably be changed. Decreasing the clearances will have the most significant effect on the volumetric efficiency though this is largely limited by budgets and machining capability. Increasing the outer diameter also has a positive effect on the volumetric efficiency. Both the mechanical and viscous losses appear to be adversely affected by increasing the outer radius, but while they both appear to grow at a rate square of the outer
radius, the volumetric flow rate grows by a cube of the outer radius. The angular velocity
decrease as the size increases reinforcing this effect. Furthermore, increasing the outer
diameter of the pump decreases the angular velocity required to output the needed flow
though this adversely affects the volumetric efficiency and pressure output.
5. Experimental Results

In order to support and verify the analytical findings, an experiment was performed. This experiment aimed to gather data on key metrics of the pump performance, namely the flow rate and the pressure output for a given input angular velocity. The power input is quite difficult to determine and is not included.

The first test pump was a 5-3-1-4 series using a clamshell design style, a gasket between the halves of the clamshell, a 40mm driver screw, and a metal plate on the top held by a silicone gasket which allows for large metal fittings to be attached even though the ports themselves were quite small. The pump was produced from Fused Deposition Modeled ABS with 100% infill. The driving shaft used a long bolt with the head removed. A bronze flange bushing and vacuum grease formed a rudimentary seal around the shaft. The shaft was, in turn, driven by a simple battery powered drill. This pump was successfully able to demonstrate that the shape of screws meshed well together and could transmit fluid. Specific data on flow rates and speeds were not taken as the pump was unable to properly seal and leaked very badly. The pump was unable to achieve a reliably readable pressure. The excessively large seal, shaft running through the seal, and difficulty in creating a gasket and the shaft exit through the high-pressure end of the pump unfortunately made the test pump itself a failure, though the lessons learned were applied to the next test pump.
The second test pump used similar screw design being the same 5-3-1-4 however being substantially smaller at 20mm outer diameter. This pump was produced as a number of parts made from ABS and white Nylon. The pump used a 3mm shaft friction fitted down the driver screw. The pump used six flange bearings, which fit around the shaft and the nubs at the end of the idler screws very well. The case was a bucket style that all the screws are dropped into; this allows the sealed area to be small and entirely on the low-pressure end of the pump. The seal along the shaft was created using a simple long hole which the shaft and the flange bearing on the driver screw fit into. The inlet is divided into two separate tubes to allow the flow to be even into both sides while still allowing the shaft to be centered. The output pipe is threaded directly into the nylon itself. The internal clearances were quite loose, between 0.12mm and 0.24mm. At 1200 rpm, the pump was able to achieve a pressure of about 3ft of water and 16in at around 750 RPM.
The third test pump used many pieces from the second test pump. The primary change was the substitution of the external case for one with tighter tolerances than the second test pump. The screws were unable to be honed to fit well just by turning the screws so they had to be sanded down. While the screws individually fit and slid, they were not able to do so together and would sometimes get stuck. The screws weren't able to fit directly up against each other. This is likely partly due to manufacturing defects and the roughness of the nylon screws. The simplest solution would be to slightly decrease $\alpha$ on the driver screw to allow them to mesh more easily or using some method to reduce surface roughness. Also when these test pumps were designed there was a confusing regarding the value of gamma. It was thought $\varphi_{MAX}$ was equal to $\theta_{MAX}$ both of which would be equal to $\gamma$. The values are very close for the 5-3-1 series so this error was not noticed. However, this difference made it so the screws could not mesh together perfectly and is likely the source of the minor offset.
Two main experiments were performed which define the performance at the two extremes. The first is a flow rate test, which aims to measure the base flow rate of the pump at different rotational speeds. This test used a large reservoir, a drill, a pump, several segments of 1/4 inch plastic tubing, a one-liter flask, and an electric tachometer. The segments of tubing are attached to the pump, the two inlet tubes going into the reservoir, and the outlet tube attached to the flask’s side port. The drill is attached to the shaft of the pump. Finally, some string and a rod are tied to the drill; this acts as a rudimentary throttle. The rod is twisted which tightens the string to push the drill's trigger. The tachometer is used to measure the drill's speed as it must be held at a constant speed. Time measurements are taken every 200mL fill from 400mL to 600mL and from 600mL to 800mL. There is significant possibility for human error both in maintaining consistent rpm and accurately timing the 200mL mark. As such the test is repeated 10 times to average out the error.
The second experiment was similar in setup but measures the maximum output pressure and requires a substantially longer output tube to do this; a tube is run up the side of a building and the height of the column of water is measured at varying RPM.

<table>
<thead>
<tr>
<th>Table 5.1 Third Test Pump Internal Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part</td>
</tr>
<tr>
<td>---------------------</td>
</tr>
<tr>
<td>Idler Screw</td>
</tr>
<tr>
<td>Driver Screw</td>
</tr>
<tr>
<td>Idler Case</td>
</tr>
<tr>
<td>Driver Case</td>
</tr>
</tbody>
</table>

There is some minor variation in the measurement of the screws. Unfortunately, the measurement tools vary in the thickness measured by a few tens of micrometers. While this small variation is well within the margin of error for the parts this small error represents a large variation relative to the size of the channel. With the equipment available, it is not possible to accurately measure the gap though it falls in the range of $80\mu m - 140\mu m$.

The first experiment was a 200mL timed flow rate test. The numbers denote the time needed to pump 200mL of fluid. The 1200rpm test had some battery issues which leads to the greater variance for that value and was often running closer to 1150rpm. The weighted average is determined by removing the four extreme values and averaging the remaining six values.
Table 5.2 Experimental Flow Rate Results for No Pressure Change.

<table>
<thead>
<tr>
<th>Test</th>
<th>RPM</th>
<th>RPM</th>
<th>RPM</th>
<th>RPM</th>
<th>RPM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>400</td>
<td>600</td>
<td>800</td>
<td>1000</td>
<td>1200*</td>
</tr>
<tr>
<td>Test 1</td>
<td>14.11</td>
<td>8.80</td>
<td>12.11</td>
<td>5.55</td>
<td>4.29</td>
</tr>
<tr>
<td>Test 2</td>
<td>13.20</td>
<td>8.14</td>
<td>6.96</td>
<td>5.00</td>
<td>5.57</td>
</tr>
<tr>
<td>Test 3</td>
<td>12.26</td>
<td>9.71</td>
<td>6.79</td>
<td>5.38</td>
<td>4.18</td>
</tr>
<tr>
<td>Test 4</td>
<td>12.76</td>
<td>8.89</td>
<td>7.61</td>
<td>5.57</td>
<td>4.71</td>
</tr>
<tr>
<td>Test 5</td>
<td>12.53</td>
<td>9.27</td>
<td>6.18</td>
<td>4.08</td>
<td>4.84</td>
</tr>
<tr>
<td>Test 6</td>
<td>13.48</td>
<td>9.00</td>
<td>6.25</td>
<td>5.94</td>
<td>6.01</td>
</tr>
<tr>
<td>Test 7</td>
<td>14.97</td>
<td>8.21</td>
<td>6.86</td>
<td>5.32</td>
<td>4.35</td>
</tr>
<tr>
<td>Test 8</td>
<td>13.31</td>
<td>9.18</td>
<td>7.34</td>
<td>4.67</td>
<td>5.61</td>
</tr>
<tr>
<td>Test 9</td>
<td>12.90</td>
<td>8.62</td>
<td>6.06</td>
<td>5.67</td>
<td>4.99</td>
</tr>
<tr>
<td>Test 10</td>
<td>12.98</td>
<td>9.18</td>
<td>6.76</td>
<td>5.94</td>
<td>4.92</td>
</tr>
<tr>
<td>Weighted Average</td>
<td>13.11</td>
<td>8.95</td>
<td>6.82</td>
<td>5.41</td>
<td>5.01</td>
</tr>
</tbody>
</table>

Table 5.3 Volumetric Flow Rates Calculated using Weighted Averages for Time

<table>
<thead>
<tr>
<th>RPM</th>
<th>Experimental Volumetric Flow Rate (L/s)</th>
<th>Expected Volumetric Flow Rate (L/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>0.015</td>
<td>0.021</td>
</tr>
<tr>
<td>600</td>
<td>0.022</td>
<td>0.032</td>
</tr>
<tr>
<td>800</td>
<td>0.029</td>
<td>0.042</td>
</tr>
<tr>
<td>1000</td>
<td>0.037</td>
<td>0.053</td>
</tr>
<tr>
<td>1200*</td>
<td>0.040</td>
<td>0.063</td>
</tr>
</tbody>
</table>
The Experimental Results are lower than the analytical flow rates. For this particular pump $k$ was determined to be 1.58. The slip due to the movement of the screws is not accounted for in the basic flow rate equation. The very small size of this pump results that this slip accounts for about one quarter of the expected flow rate. A significant amount of difference in the expected flow rate is likely due to the experimental setup and the physical design of the pump. The pump has a number of odd shapes near the bottom which are not particularly conducive to flow. In addition, the entrance to the flask was very narrow and the flow passed through a long tube before entering the flask. This would tend to require more pressure to pass through which would lead to proportionally lower flow rates as the speed increased which can be seen in the table below.

Table 5.4 Maximum Output Pressure

<table>
<thead>
<tr>
<th>RPM</th>
<th>Max Height of Water Column (ft)</th>
<th>Experimental Pressure Output (kPa)</th>
<th>Calculated Screw Clearance (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>4.2</td>
<td>12.6</td>
<td>117.6</td>
</tr>
<tr>
<td>600</td>
<td>6.6</td>
<td>19.7</td>
<td>116.0</td>
</tr>
<tr>
<td>800</td>
<td>11.9</td>
<td>35.6</td>
<td>104.8</td>
</tr>
<tr>
<td>1200*</td>
<td>25.2</td>
<td>75.3</td>
<td>93.4</td>
</tr>
</tbody>
</table>

The maximum output pressure was determined by running a tube up the side of a building and measuring the height the fluid rose to at varying RPM. The 1000 RPM value has been omitted from this experiment as it was not able to be read.

Unfortunately, it is difficult to accurately measure the screw clearances. The pump was hand sanded down until it was able to turn smoothly. The individual screws were able to fit and turn in their slots easily before sanding. However, the three screws together were
not able to. It is likely that due to manufacturing defects of the 3D-printed screws they were unable to fully mesh that the idler screws were slightly pushed outward. This effect was likely the cause for the screws requiring substantial additional sanding leading to the variation in the screw sizes. However, the clearances can be estimated using the output pressures shown in Table 5.4. The screw clearances calculated from pressure fall within the range of screw clearances calculated from the dimensions of the screws and case.

There is roughly a 10% variation in the calculation of the screw clearances. In general, these results are from far below the operational range of one of these pumps. The operational range for a 20mm outer diameter pump should be between 6000 to 10000 RPM. At these ranges, various small viscous and mechanical losses likely make up a fairly large portion of the losses like is shown in Figure 4.4. Also, the derivation treated the length of the gap as infinite. At the ends of the gap the flow doesn’t enter perfectly smoothly and likely swirls. This would impede the flow some and would allow for a slightly higher pressure drop across the gap than is listed. It is likely that this effect grows as the speeds inside the pump increase. Lastly, the estimations don’t apply at the intersection of the screws because of converging or diverging fluid. This likely impedes the slip leading to slightly higher pressures in this experiment.
6. Conclusion

Triple Screw pumps may serve as an option to be used instead of turbopumps for small vehicles; however, much more testing must be done to demonstrate the performance and reliability to be comparable to that of current turbopumps. The various empirical factors needed to accurately design one of these pumps need to be determined. This document is able to establish the base physics and relations behind the performance of triple screw pumps. Several things were successfully demonstrated including the creation of the shape for these pumps as well as the linearity of the pumps in their flow rate output depending on the rotational speed and the variation in clearances having a cubic effect on the output pressure. The experimental results were unfortunately primitive. This document may still serve as a good starting point for the design for future screw pumps provided the designer leaves some "wiggle room" in the design itself and has access to better manufacturing techniques.
7. Recommendations

There are a number of ways that the experiments performed could be improved. The experiment setup was very primitive and simple which led to many small errors. The most significant difficulty was the size of the pump itself. The pump performance improves drastically with size. The slip does not depend on the outer diameter of the pump, but the flow rate increases by a cubic factor. Maintaining the same clearance while doubling the pump size increases the output pressure by a factor of eight. The use of 3D Printing made production of parts with suitable tolerances very difficult. Increasing pump size decreases the operational and will make collection of useful data easier. Many more experiments should be performed with a variety of fluids and configurations and a variety of back pressures to form a more complete idea of the pump performance.

There is room to correct and improve the data. The equations derived in this document should apply to most pumps but a number of assumptions were made to reduce the complexity of the math. These errors are accounted for in the various empirical factors, but correcting this will allow the empirical factors to be more consistent between varying pumps. The equations were presented in terms of the driver screw outer diameter and converted the length of the stage to be in terms of the outer diameter which gives the tangent function that shows up in many of equations used. Sigma is assumed to be constant between the two screws though it is not; the length is constant but the angle is different between the two screws. The difference is not particularly significant for reasonable values of sigma but it does create an inaccuracy. Particularly in the case the various parameters were tabulated the difference in sigma between the idler and drive screws should be taken into account.
The data within this document can be further expanded. It seems reasonable that the screws could be reversed such that the idler screws are larger than the driver screws (as a 3-5-1 configuration). This would add some interesting torque problems and the center screw would have to be the epicycloid screw, but this would reduce the number of epicycloid screws that would need to be manufactured which are substantially more difficult to machine than the acme style screws using subtractive machining methods.
REFERENCES


