Experimental and Computational Analysis of a 3D Printed Wing Structure

Aryslan Malik

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EXPERIMENTAL AND COMPUTATIONAL ANALYSIS
OF A 3D PRINTED WING STRUCTURE

A Thesis
Submitted to the Faculty
of
Embry-Riddle Aeronautical University
by
Aryslan Malik

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Requirements for the Degree
of
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EXPERIMENTAL AND COMPUTATIONAL ANALYSIS

OF A 3D PRINTED WING STRUCTURE

by

Aryslan Malik

A Thesis prepared under the direction of the candidate’s committee chairman, Dr. Claudia Moreno, Department of Aerospace Engineering, and has been approved by the members of the thesis committee. It was submitted to the School of Graduate Studies and Research and was accepted in partial fulfillment of the requirements for the degree of Master of Science in Aerospace Engineering.

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4/24/2019
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SYMBOLS

\( m_b \)  
Total body mass

\( l_b \)  
Mass inertia

\( V_b \)  
Velocity

\( \Omega_b \)  
Angular velocity

\( g_E \)  
Gravitational vector

\( T_{bE} \)  
Transformation matrix

\( \phi_b^T \)  
Rigid body modal matrix about c.g.

\( \tilde{M}_f \)  
Generalized modal mass matrix

\( \tilde{K}_f \)  
Generalized modal stiffness matrix

\( \tilde{\Sigma}_f \)  
Generalized damping matrix

\( \eta_f \)  
Vector of elastic modal displacements

\( \Phi_f^T \)  
Flexible modal matrix

\( p^c \)  
Vector of aerodynamic forces and moments

\( \phi \)  
Velocity potential

\( V_\infty \)  
Free stream velocity

\( v_b \)  
Velocity generated by vortex

\( n_i \)  
Normal vector to the panel \( i \)

\((u, v, w)_{ij}\)  
Velocities induced by vortex \( j \) on collocation point \( i \)

\( \Gamma_j \)  
Circulation

\( \Delta L_i \)  
Lift of the panel \( i \)

\( b_i \)  
Bound vortex length

\( \rho \)  
Air density

\( w_{ij} \)  
Induced normalwash at \( i^{th} \) panel

\( c_j \)  
Chord length of the \( j^{th} \) panel

\( K \)  
Kernel function

\((x_i, y_i)\)  
Coordinates of the \( i^{th} \) collocation point

\((\xi_j, \sigma_j)\)  
Coordinates along the doublet line of the \( j^{th} \) panel

\( \omega \)  
Frequency at which the lifting surface is oscillating

\( V \)  
Free stream velocity

\( \Delta p_j \)  
Pressure difference across the doublet at the \( j^{th} \) panel

\( k \)  
Reduced frequency

\( \bar{q} \)  
Free stream dynamic pressure

\( S \)  
Diagonal matrix of panel areas

\( F_{aero} \)  
Aerodynamic force distribution

\( D \)  
Normalwash matrix

\( h_i \)  
Heave displacement of \( i^{th} \) panel

\( \theta_i \)  
Pitch displacement of \( i^{th} \) Panel

\( M \)  
Mach number

\( C_L \)  
Lift coefficient
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>3D</td>
<td>Three Dimensional</td>
</tr>
<tr>
<td>AAW</td>
<td>Active Aeroelastic Wing</td>
</tr>
<tr>
<td>ABS</td>
<td>Acrylonitrile Butadiene Styrene</td>
</tr>
<tr>
<td>AIC</td>
<td>Aerodynamic Influence Coefficients</td>
</tr>
<tr>
<td>AR</td>
<td>Aspect Ratio</td>
</tr>
<tr>
<td>CAD</td>
<td>Computer Aided Drawing</td>
</tr>
<tr>
<td>CFD</td>
<td>Computational Fluid Dynamics</td>
</tr>
<tr>
<td>cm</td>
<td>Centimeter</td>
</tr>
<tr>
<td>CSD</td>
<td>Computational Structural Dynamics</td>
</tr>
<tr>
<td>DIC</td>
<td>Digital Image Correlation</td>
</tr>
<tr>
<td>DLM</td>
<td>Doublet Lattice Method</td>
</tr>
<tr>
<td>DoF</td>
<td>Degree(s) of Freedom</td>
</tr>
<tr>
<td>ERAU</td>
<td>Embry-Riddle Aeronautical University</td>
</tr>
<tr>
<td>FE</td>
<td>Finite Element</td>
</tr>
<tr>
<td>FEA</td>
<td>Finite Element Analysis</td>
</tr>
<tr>
<td>FEM</td>
<td>Finite Element Method, Finite Element Model</td>
</tr>
<tr>
<td>ft</td>
<td>Feet</td>
</tr>
<tr>
<td>g</td>
<td>Gram</td>
</tr>
<tr>
<td>GPa</td>
<td>Gigapascal</td>
</tr>
<tr>
<td>GVT</td>
<td>Ground Vibration Test</td>
</tr>
<tr>
<td>Hz</td>
<td>Hertz</td>
</tr>
<tr>
<td>in²</td>
<td>Square inch</td>
</tr>
<tr>
<td>kg</td>
<td>Kilogram(s)</td>
</tr>
<tr>
<td>lbs</td>
<td>Pound(s)</td>
</tr>
<tr>
<td>m</td>
<td>Meter(s)</td>
</tr>
<tr>
<td>m/s</td>
<td>Meter(s) per second</td>
</tr>
<tr>
<td>m²</td>
<td>Square meter</td>
</tr>
<tr>
<td>mm</td>
<td>Millimeter</td>
</tr>
<tr>
<td>MPa</td>
<td>Megapascal</td>
</tr>
<tr>
<td>NASA</td>
<td>National Aeronautics and Space Administration</td>
</tr>
<tr>
<td>SAnD</td>
<td>Structural Analysis and Design</td>
</tr>
<tr>
<td>SISO</td>
<td>Single input, single output</td>
</tr>
<tr>
<td>SWBT</td>
<td>Symmetric Wing Bending Torsion</td>
</tr>
<tr>
<td>UAV</td>
<td>Unmanned Aerial Vehicle</td>
</tr>
<tr>
<td>VLM</td>
<td>Vortex Lattice Method</td>
</tr>
</tbody>
</table>
ABSTRACT

Malik, Aryslan MSAE, Embry-Riddle Aeronautical University, March 2019. Experimental and Computational Aeroelastic Analysis of a 3D Printed Wing Structure.

Correct prediction of aeroelastic response is a crucial part in designing flutter or divergence free aircrafts within a designated flight envelope. The aeroelastic analysis includes specifically tailoring the design in order to prevent flutter (passive control) or eliminate it by applying input on control surfaces (active control). High-fidelity models such as coupled Computational Fluid Dynamics (CFD) - Computational Structural Dynamics (CSD) can obtain full structural and aerodynamic behavior of a deformable aircraft. However, these models are so large that pose a significant challenge from the control systems design perspective. Thus, the development of an aeroelastic modeling software that can be used for further control design is the main motivation of this thesis. In addition, an aeroelastic analysis of a topologically optimized wing geometry will serve as a validation tool of the software. Initially, a 3D printed prototype of the wing is validated against static deformation tests as well as dynamic Ground Vibration Tests (GVT). The developed model is compared against the commercial software Nastran/Patran. Further plans include experimental aerodynamic test of 3D printed wing in the new Embry-Riddle Aeronautical University’s (ERAU) wind tunnel to validate the proposed model.
1. Introduction

Aeroelasticity is a phenomenon that requires a thorough analysis of the combination of multiple forces such as aerodynamic, elastic and inertia forces. Contemporary airframes are becoming more flexible which in turn makes the aeroelastic analysis a crucial part of aircraft design (Livne, 2017). Aeroelasticity, generally, can be divided into static aeroelasticity and dynamic aeroelasticity. Static aeroelasticity includes major phenomena such as divergence and aileron reversal. Dynamic aeroelasticity includes the flutter phenomenon (Chinmaya & Venkatasubramani, 2009).

To be more specific an example of undesired aeroelastic phenomena is described. An aerodynamic surface (e.g., wing, canard or tail) experiences aerodynamic force normal to the airstream that increases with the square of speed and the angle of incidence which is the angle between the corresponding aerodynamic surface and the air flow (Chinmaya & Venkatasubramani, 2009). This aerodynamic force which is generally called lift will usually twist the lifting surface with its leading edge up about its elastic axis because the center of pressure is located in front of the elastic axis. This twist of the lifting surface increases the angle of incidence experienced by the corresponding aerodynamic surface which in turn increases the aerodynamic force that increases the twist further and so on until the system reaches an equilibrium condition. Undesired phenomena such as divergence occurs when the given lifting surface is deformed such that the applied aerodynamic load is increased or when the aerodynamic load is moved so as to increase the twisting effect on the structure which deflects the structure further until it fails.

When the aileron is commanded to deflect downward, the upward lift force is
created so that the aircraft can perform, for instance, a rolling maneuver. This additional lift force also creates a pitch-down moment about the elastic axis, since the deflected control surface is located behind the elastic axis. This pitch-down moment causes elastic rotation of the lifting surface section in a nose-down direction inducing down lift. The aileron is considered to be reversed when the induced down lift force exceeds the commanded up lift force created by the aileron. Extensive research was focused on the study of the dynamic phenomena that resulted in development of sophisticated analytical and computational techniques that would ensure that the design is free of flutter or any other undesired aeroelastic phenomena (Livne, 2017). One example of such research works could be NASA Armstrong’s Active Aeroelastic Wing (AAW) demonstrated in Figure 1.1 which used active control of the leading edge flaps (NASA Armstrong Fact Sheets - Active Aeroelastic Wing, 2018). In essence, these control surfaces were deployed to eliminate the aileron reversal by compensating the twisting effect of the wing.

![Figure 1.1 Nasa Armstrong's Active Aeroelastic Wing (2004).](image)

Having established an understanding of static aeroelastic phenomena an example of
dynamic aeroelastic instability is outlined. Flutter is a dynamic aeroelastic phenomena that is produced by combination of elastic, inertial and unsteady aerodynamic forces that eventually causes airframe’s vibration. Generally, flutter is more complex problem because it involves the vibration of the structure. In order to understand flutter phenomenon imagine rectangular unswept wing fixed rigidly on its root chord and mounted in a wind tunnel. If a wing in this configuration is disturbed without any airflow in the wind tunnel, the perturbation is damped by the structural damping of the wing structure. When the airflow speed in the wind tunnel is gradually increased the damping rate, at first, increases as well. However, as the airspeed is increased further a point is reached where the damping decreases rapidly. Critical flutter speed characterizes the condition at which constant steady amplitude is maintained by the interaction of the airflow and the structure. At airspeeds higher than the critical a small vibration in the structure could possibly trigger oscillations with increasing amplitude leading to failure of the structure. The first recorded flutter incident dates back to 1915 and involved Hadley Page’s bomber Bi-plane shown in Figure 1.2 with ‘violent oscillations’ of the tail flutter problem (Chinmaya & Venkatasubramani, 2009). Follow-up inspection showed that the reason for flutter was that the fuselage’s torsional mode coupled with independently actuated anti-symmetrical elevators’ mode (Lanchester, 1916).
For several decades the flutter phenomenon was the primary focus of the research in the field of aeroelasticity (Kehoe, 1995), and it is also the primary focus of this thesis. There are different models for aeroelastic analysis and currently computational fluid dynamics and computational structural dynamics coupling is at highest modeling fidelity level. These models can provide accurate representation of the dynamic as well as aerodynamic behavior of a deformable aircraft (Livne, 2017). However, there are certain limitations to this model which makes it less practical. Firstly, it is so large that it takes considerable amount of time to simulate, so in the design environment where myriad number of simulations are required the usage of this model is still impractical. Secondly, because of the same reason the math models become too involved which poses challenges to a control system designer. Moreover, the interdisciplinary nature of the phenomenon leads to simplifications and several assumptions during formulation of the aeroelastic model therefore requiring experimental validations. Thus, the main objectives of this thesis are:

1) Develop a custom software for coupling of aerodynamic and structural wing forces.

2) Design an aeroelastic wing experiment for the new ERAU wind tunnel.
General equations related to the development of a custom aeroelastic modeling software are shown as follows:

\[
\begin{bmatrix}
    m_b (V_b + \Omega_b \times V_b - T_{bE} g_E)
    \\
    I_b \dot{\Omega}_b + \Omega_b \times (I_b \Omega_b)
\end{bmatrix} = \Phi_b^T P^c \tag{1.1}
\]

\[
\bar{M}_f \ddot{\eta}_f + \bar{\Sigma}_f \dot{\eta}_f + \bar{R}_f \eta_f = \Phi_f^T P^c \tag{1.2}
\]

where,

1. \( m_b \): total body mass
2. \( I_b \): mass inertia
3. \( V_b \): velocity
4. \( \Omega_b \): angular velocity
5. \( g_E \): gravitational vector
6. \( T_{bE} \): transformation matrix
7. \( \Phi_b^T \): rigid body modal matrix about c.g.
8. \( \bar{M}_f \): generalized modal mass matrix
9. \( \bar{R}_f \): generalized modal stiffness matrix
10. \( \bar{\Sigma}_f \): generalized damping matrix
11. \( \eta_f \): vector of elastic modal displacements
12. \( \Phi_f^T \): flexible modal matrix
13. \( P^c \): vector of aerodynamic forces and moments

The structure of the aeroelastic modeling software consists of several integral parts. First is aerodynamic finite element method which is an aerodynamic analysis tool and like structural analysis it is based upon finite element approach. Since dynamic aeroelastic phenomenon is analyzed, unsteady aerodynamic forces should be considered.
These forces are generated when the flow is disturbed by the moving structure. The unsteady aerodynamics analysis allows computation of a matrix that correlates the forces acting on the lifting surface due to the displacement of the lifting surface’s structure. The elements in this matrix are complex, which account for phase lags between the movement of the structure and the forces. This matrix depends on reduced frequency and Mach number and is computed by Doublet Lattice Method (DLM). Second part of the model is the structural model, which is a structural analysis method that utilizes finite element method. The structural grid is independent of the aerodynamic grid. Since, structural grid points usually do not occupy same spatial coordinates with elements of an aerodynamic grid and degrees of freedom of the structural grid may also differ from the aerodynamic grid, an aero-structure coupling is required which is usually based upon method of splines. This interpolation is a crucial feature because it allows the choice of structural and aerodynamic elements to be based upon independent considerations (Rodden & Johnson, 1994). Both custom Matlab Aeroelastic Code and Nastran/Patran are based on this type structure of aeroelastic modeling software. Prior experimental data and Nastran’s Aeroelastic Module which is used on a par with its pre/post processor Patran serve as a source of verification and comparison for the custom code that is developed throughout the thesis work (Matlab Aeroelastic Code).

As the second objective is to design an aeroelastic wing experiment in the new ERAU wind tunnel, the setup is first analyzed via simulations in Nastran. For most of the simulations and experiments with the goal of validation, a specific wing geometry was chosen. This wing geometry corresponds to the Northrop Grumman’s F-5 fighter wing. The Computer Aided Drawing (CAD) model of the wing was provided by the Embry-
Riddle Aeronautical University’s (ERAU) Structural Analysis and Design (SAnD) Lab.
The internal structure of this wing is topologically optimized with a specific geometry
and configuration of spars and ribs. General data regarding the geometry of the full-scale
wing is summarized in Table 1.1.

Table 1.1

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sweep-back of quarter chord</td>
<td>24°</td>
</tr>
<tr>
<td>Leading edge sweep</td>
<td>32°</td>
</tr>
<tr>
<td>Aspect ratio</td>
<td>~4</td>
</tr>
<tr>
<td>Taper ratio</td>
<td>0.2</td>
</tr>
<tr>
<td>Wing area</td>
<td>17.28m²</td>
</tr>
<tr>
<td>Wingspan</td>
<td>8.13m</td>
</tr>
</tbody>
</table>

Simulations in Nastran are verified and validated as well. The structural FEM is
verified with static experiment and GVT, for this purpose a smaller scale wing is 3D
printed from an ABS plastic. Static experiment is carried out by applying a load on a 3D
printed wing prototype and Digital Image Correlation (DIC) is chosen as data acquisition
system. After experimental validation of simulations a flutter analysis is carried out in
Nastran/Patran environment. The future work would include the validation of the Matlab
Aeroelastic Code with a wind tunnel test that would require the manufacturing,
specifically 3D printing a large (2ft) wing, and the design of the test setup rig to mount
accelerometers on the wing’s surface inside wind tunnel test section.
The structure of the thesis is as follows: Chapter 2 describes aerodynamic modeling approach that includes both steady aerodynamics (VLM) and unsteady aerodynamics (DLM). The chapter also includes the validation of both steady and unsteady aerodynamic models. Chapter 3 describes the structural model (FEM) developed for the Matlab Aeroelastic Code. Chapter 4 covers the interpolation between aerodynamic and structural grids and intermediate spline grid. Chapter 5 describes flutter analysis in Nastran/Patran, static and GVT experiments with 3D printed wing and preliminary experimental setup in wind tunnel. Chapter 6 provides concluding remarks and future direction for the work.
2. Aerodynamic Modeling

Aerodynamic modeling is crucial part of aeroelastic analysis and modeling. Derivation of aerodynamic equations starts from more general fluid dynamics equations. These more general equations are usually simplified to represent the most important physical aspects of aerodynamic flow (Shames, 1982; Anderson, 1984; Katz & Plotkin, 1991). Example of such simplifications can be attributed to Lifting Line Theory developed by Ludwig Prandtl in 1920’s that was, at that time, one of the first aerodynamic models predicting aerodynamic forces acting on a finite lifting surface (Shames, 1982; Anderson, 1984). Prandtl Lifting Line Theory assumes that the aerodynamic flow is irrotational, inviscid and incompressible. Such flow is also known as potential flow (Shames, 1982; Anderson, 1984). Mathematical implications of such assumptions are represented in the following equations:

From the continuity equation, in an incompressible and inviscid flow:

$$\nabla \cdot \mathbf{v} = 0$$  \hspace{1cm} (2.1)

As the flow is irrotational a velocity potential can be introduced:

$$\mathbf{v} = \nabla \phi$$  \hspace{1cm} (2.2)

Thus, as a result velocity potential satisfies Laplace’s equation:

$$\nabla^2 \phi = 0$$  \hspace{1cm} (2.3)

Thenceforward, over the last century modeling of aerodynamic flows significantly developed so as to envisage different flow conditions and characteristics that range from subsonic potential flow to a more involved supersonic, viscous, compressible flow that is described by different underlying assumptions applied to Navier-Stokes equations (Shames, 1982; Anderson, 1984).
Time varying nature of unsteady flow is crucial because it affects the mathematical modeling of unsteady aerodynamics used in aeroelastic analysis. When a lifting surface experiences aerodynamic load it is deformed which changes its aerodynamic shape, and in turn alters the flow characteristic around it. This process is not instantaneous, thus unsteady aerodynamic modeling is of primary importance to capture the forces acting on a body in a time varying flow. There are various methods solving unsteady aerodynamics for the purpose of aeroelastic analysis ranging from the highest fidelity level CFD-CSD coupled solvers, which discretize Navier-Stokes equations, to the lowest fidelity level potential flow solvers based on the strip theory and 2-D infinite wing assumption (Livne, 2017).

High fidelity CFD-CSD coupled solvers are capable of capturing accurate structural and dynamic behavior of the lifting surface, yet computationally expensive and involved which renders them as impractical as far as the control system design perspective is concerned (Livne, 2017). Contrarily, aerodynamic models based on strip theory are relatively simple and computationally inexpensive compared to high fidelity models. However, strip theory may lack accuracy required for the aeroelastic analysis (Livne, 2017). Potential flow based panel methods can be considered as middle tier fidelity aerodynamic models which satisfy the requirements of aeroelastic analysis and at the same time not as computationally demanding as high fidelity models while still retaining reasonable accuracy of lifting characteristics of finite wings, and readily applicable to control system design (Katz & Plotkin, 1991). Panel methods such as Vortex Lattice Method (VLM) and Doublet Lattice Method (DLM) are used for the purpose of aerodynamic modeling where the former is used for steady part of the solution and the
latter is essentially unsteady oscillatory extension of the VLM.

### 2.1 Vortex Lattice Method

The VLM is a potential flow solver that utilizes planar surfaces (panels) to represent lifting surfaces (e.g. wing, canard, tail etc.) (Moran, 1991). According to Kutta-Joukowski theorem the vortices represent lift and are placed on the quarter chord of each panel. Boundary condition of zero normal velocity on the panel surface is satisfied on so-called collocation (control) point of each panel that is located at 3/4 of the chord (Katz & Plotkin, 1991). Described positioning of vortices and collocation points is not a theoretical law but rather a rule of thumb that works well for this method and also it is known as “1/4 – 3/4 rule” (Mason, 1998). *Figure 2.1* demonstrates the placement of the bound vortices and control points.

![Figure 2.1 Vortex and control points layout (Mason, 1998)](image)

Generally, the procedure for the VLM panel method is as follows:
Firstly, the lifting surface is discretized into lattice of quadrilateral panels. Generally a collection of panels describing lifting surface is referred to as aerodynamic grid. Secondly, the zero normal flow condition is satisfied on every panel’s collocation (control) point. This is defined by the addition of already known free stream velocity $V_\infty$ and unknown velocities $v_b$ generated by vortices:

$$(V_\infty + v_b) \cdot n = 0$$

Equation 2.4 is satisfied at every collocation point, and combined with Biot-Savart Law that leads to the following compact formulation of influence coefficients (more detailed derivation is presented in (Katz & Plotkin, 1991)):

$$a_{ij} = (u, v, w)_{ij} \cdot n_i$$ (2.5)

where, $n_i$ is the normal vector to the panel $i$ and $(u, v, w)_{ij}$ represent velocities induced by vortex $j$ on collocation point $i$. Combining above Equation 2.5 leads to the following linear system of equations:

$$
\begin{bmatrix}
 a_{11} & a_{12} & \cdots & a_{1N} \\
 a_{21} & a_{22} & \cdots & a_{2N} \\
 \vdots & \vdots & \ddots & \vdots \\
 a_{N1} & a_{N2} & \cdots & a_{NN}
\end{bmatrix}
\begin{bmatrix}
 \Gamma_1 \\
 \Gamma_2 \\
 \vdots \\
 \Gamma_N
\end{bmatrix}
=
\begin{bmatrix}
 -V_\infty \cdot n_1 \\
 -V_\infty \cdot n_2 \\
 \vdots \\
 -V_\infty \cdot n_N
\end{bmatrix}
$$ (2.6)

where $\Gamma_j$ are unknown circulations. After circulations are calculated the lift can be obtained as follows:

$$\Delta L_i = \rho V_\infty \times \Gamma_i \Delta b_i$$ (2.7)

where $\Delta L_i$ is the lift of the panel $i$ and $b_i$ is the bound vortex length. In order to calculate the total lift of the lifting surface all panels’ lift contributions are summed.
2.1.1. VLM Validation

A specific wing geometry was analyzed in order to validate the VLM code that is based on a code developed by University of Minnesota UAV lab (Kotikalpudi, 2017). This specific geometry was chosen because there are experimental results available for this geometry that can be used to validate against. The wing under consideration has an aspect ratio of 3, quarter chord sweep angle of 45, taper ratio of 0.5 and zero dihedral angle. Panels are distributed in such a way that there are 6 panels in chordwise direction and 8 panels in spanwise direction resulting in 48 panels for half of the wing, and if symmetry is considered – 96. The wing geometry generated in the Matlab Aeroelastic Code’s VLM is shown in Figure 2.2 and wing geometry generated in Tornado VLM is presented in Figure 2.3.

![Geometry of the wing](image)

*Figure 2.2* Wing geometry in Matlab Aeroelastic Code’s VLM
Figure 2.3 Wing geometry in Tornado VLM

In order to validate the Vortex Lattice Method (VLM) the results of the Matlab Aeroelastic code were compared against experimental results and numerical results obtained by Albano-Rodden VLM (Albano & Rodden, 1969) as well as Tornado VLM developed by Department of Aeronautics at the KTH Royal Institute of Technology (Melin, 2000). Flight conditions are as follows: the angle of attack is 4 degrees and the Mach number is 0.8. Particularly, the pressure coefficients at the root chord, shown in Figure 2.4, and tip chord, demonstrated in Figure 2.5, of a tapered and swept wing at an incidence in a steady flow were compared.
The results indicate that the developed VLM overestimate the pressure at the leading edge and underestimate it further down the chord line in the case of root chord, which can be observed in Figure 2.4. As far as tip chord is concerned (Figure 2.5) it can be inferred that the developed VLM vice-versa underestimate the pressure at the leading edge and slightly overestimate it starting from the half chord. Moreover, it is crucial to note that the results obtained by VLM almost identically match the results obtained using Tornado VLM.

### 2.2 Doublet Lattice Method
Essentially, DLM can be considered as a panel method relying on potential flow assumption with unsteady oscillatory extension of the steady VLM discussed in section 2.1 (Albano & Rodden, 1969). The VLM is extended to account for oscillatory doublets of constant strength to the bound vortex along the quarter-chord of each box. Similarly to VLM, the points located at 3/4 of any given panel’s chord are normalwash calculation points, also known as collocation points (Katz & Plotkin, 1991; Albano & Rodden, 1969). Normalwash is the normal flow to the panel’s surface normalized with respect to the freestream velocity (Katz & Plotkin, 1991; Albano & Rodden, 1969). Both doublet lines and freestream flow induce normalwash. The DLM utilizes the normalwash distribution to calculate the pressure distribution across panels that describe the lifting surface.

The relation between normalwash distribution and pressure distribution is produced in an involved process of derivation which includes several simplifying approximations and is shown in detail in (Albano & Rodden, 1969; Rodden, Taylor & McIntosh, 1998). Thus, a concise overview pertinent to the utilization of the DLM to aeroelasticity is presented henceforth.

As it was mentioned previously the main objective of the DLM is to relate the normalwash distribution due to free stream generated by doublet lines to a corresponding pressure distribution acting on the lifting surface undergoing oscillatory motion with a specific frequency. In order to accomplish this goal as a first step the lifting surface is discretized to a lattice similar to the aerodynamic grid used in VLM. However, in this case a doublet line is placed on the quarter chord of each panel.

The second step is to construct the downwash matrix $D$ that is obtained by
computing the normal washes induced by each doublet line at all collocation points on the lifting surface. The normal wash produced at the $i^{th}$ collocation point due to the doublet line on $j^{th}$ panel in terms of pressure distribution is given as a following integral:

$$w_{ij} = \frac{c_j}{8\pi} \int_{-\frac{L}{2}}^{\frac{L}{2}} K(x_i, y_i, \xi_j(l), \sigma_j(l), \omega, V) \Delta p_j dl$$  \hspace{0.5cm} (2.8)$$

where,

1. $w_{ij}$: induced normal wash at $i^{th}$ panel
2. $c_j$: chord length of the $j^{th}$ panel
3. $K$: Kernel function relating the normal wash produced by an infinitesimal acceleration doublet to the pressure difference across it
4. $(x_i, y_i)$: coordinates of the $i^{th}$ collocation point at which normal wash is computed
5. $(\xi_j, \sigma_j)$: coordinates along the doublet line of the $j^{th}$ panel
6. $\omega$: frequency at which the lifting surface is oscillating
7. $V$: free stream velocity
8. $\Delta p_j$: pressure difference across the doublet at the $j^{th}$ panel

Total normal wash at the $i^{th}$ panel can be calculated by summing contributions of every panel as follows:

$$w_{ij} w_i = \sum_{j=1}^{N} D_{ij} \Delta p_j$$  \hspace{0.5cm} (2.9)$$

$$D_{ij} = \frac{c_j}{8\pi} \int_{-\frac{L}{2}}^{\frac{L}{2}} K(x_i, y_i, \xi_j(l), \sigma_j(l), \omega, V)$$  \hspace{0.5cm} (2.10)$$

where $N$ is the total number of panels describing the lifting surface. The resulting
normalwash \( w_i \) is a harmonic function because the pressure distribution \( \Delta p_j \) is harmonic function as well. The Equations 2.9-2.10 can be re-written in matrix form as follows:

\[
\vec{w} = D \vec{p}
\]  

(2.11)

where \( \vec{w} \) denotes a vector of \( N \times 1 \) dimension containing all total induced normalwash at each panel produced by doublet lines at each panel, \( D \) represents the normalwash matrix of \( N \times N \) dimension, and \( \vec{p} \) is the pressure difference vector of \( N \times 1 \) dimension that contains pressure difference across each panel. It is worth to mention that the \( D_{ij} \) depends only on the geometry of the lifting surface and known flow condition. Thus, the \( D \) matrix can be obtained by integrating the kernel function \( K \) along each doublet line. The detailed process of integration can be found in (Albano & Rodden, 1969; Watkins, Woolston & Cunningham, 1959). The crucial aspect of the formulation is that the equations given in (Albano & Rodden, 1969; Watkins, Woolston & Cunningham, 1959) allow the usage of non-dimensional parameter called reduced frequency that combines free stream velocity, oscillating frequency and given reference chord presented as follows:

\[
k = \frac{\omega \bar{c}}{2V}
\]  

(2.12)

Thus, for a given geometry of the lifting surface the \( D \) matrix becomes a function of solely reduced frequency. As shown previously, the \( D \) matrix maps the pressure difference across the panels produced by respective doublet lines to the induced downwash at each panel’s collocation point. Since the pressure vector is unknown, a further step is taken by inverting the \( D \) matrix. The inverted \( D \) matrix is also known as Aerodynamic Influence Coefficients (AIC) matrix (Kotikalpudi, 2017). The equations solving for the pressure difference vector are given as:
The next step is to apply zero net normal flow boundary condition which is a physical constraint stating that there cannot be any flow passing perpendicularly to the discretized panels (Kotikalpudi, 2017). Ideally this condition should be satisfied across the entire surfaces of all panels, however, in practice it is satisfied on the collocation points (Kotikalpudi, 2017). In the model the boundary condition is satisfied by relating the induced normalwash vector \( \bar{w} \) produced by the doublet lines to the normalwash distribution vector \( \bar{w}_\infty \) due to the free stream as follows:

\[
\bar{w} + \bar{w}_\infty = 0
\]

Thus, the pressure difference can be found using the normalwash distribution vector due to the free stream:

\[
\bar{p} = -[AIC(k)]\bar{w}_\infty
\]

Using the Equation 2.16 the pressure distribution of an oscillating lifting surface can be found from the free stream normalwash distribution. \( \bar{w}_\infty \) vector is calculated from the given flow condition and the motion of the lifting surface. For small angles the normalwash vector \( \bar{w}_\infty \) is identical to the angle of attack. Vectors \( \bar{p} \) and \( \bar{w}_\infty \) are both harmonic functions in oscillating frequency \( \omega \).

It can be observed that the vector \( \bar{w}_\infty \) contains the normalwash of each individual panel. This implies that if an elastic deformation of a given lifting surface can be approximated by the motion of the discretized panels, the corresponding normalwash vector \( \bar{w}_\infty \) can be computed (Kotikalpudi, 2017). Consequently, Equation 2.16 can be readily used to obtain the pressure distribution across panels corresponding the elastic
deformation of the lifting surface (Kotikalpudi, 2017). The aerodynamic force distribution can be calculated as follows:

\[ F_{aero}(k) = \bar{q}S\bar{p} \]  \hspace{1cm} (2.17)

\[ F_{aero}(k) = \bar{q}S[AIC(k)]\bar{w} \]  \hspace{1cm} (2.18)

where \( \bar{q} \) is the free stream dynamic pressure and \( S \) is a diagonal matrix of panel areas (Kotikalpudi, 2017). The obtained aerodynamic force acts at the midpoint of the doublet line at each panel.

The DLM is generally more involved than the VLM, and thus involves more assumptions and approximations. The implications of such approximations is that at zero oscillating frequency the DLM result is not as accurate as the result obtained by VLM (Katz & Plotkin, 1991; Albano & Rodden, 1969; Rodden, Taylor & McIntosh, 1998). Therefore, for the same aerodynamic grid in order to improve the accuracy of the DLM result it is suggested to superimpose unsteady solution of the DLM to the steady solution of the VLM as given in (Rodden, Taylor & McIntosh, 1998).

In order to incorporate steady solution of the VLM into the unsteady part of the DLM solution it is required to obtain the incremental downwash matrix that represents solely unsteady effects. Incremental downwash matrix is found by obtaining the downwash matrix twice using the DLM, first, it is calculated at the given frequency and subsequently it is computed at zero frequency. This is demonstrated in the following Equation 2.19:

\[ D_{unsteady}(k) = D_\omega(k) - D_0 \]  \hspace{1cm} (2.19)

where \( D_{unsteady} \) is the incremental downwash matrix that represents only the unsteady part of the DLM solution, \( D_\omega \) is the downwash matrix computed at the given frequency \( \omega \)
and $D_0$ is the downwash matrix calculated at zero frequency (Kotikalpudi, 2017). Note that the downwash matrix at zero frequency $D_0$ is not a function of reduced frequency $k$ since by setting the oscillating frequency to zero the reduced frequency becomes zero as well.

Total solution is obtained by combining unsteady part of the DLM solution which is the incremental downwash matrix $D_{unsteady}$ and steady part is provided by the VLM solution, in an essence the VLM solution compensates the subtracted $D_0$ and bolsters the accuracy of the result. The steady part is obtained as shown in section 2.1. It is important to mention that downwash matrices are compatible only if the same aerodynamic grid is used for both the computation of incremental downwash matrix using the DLM and for obtaining steady state downwash matrix using the VLM. Total downwash matrix is obtained as follows:

$$D_{total}(k) = D_{unsteady}(k) + D_{VLM}$$  \hspace{1cm} (2.20)

where $D_{total}(k)$ is the total downwash matrix that consists of both steady state downwash matrix $D_{VLM}$ computed using the VLM and unsteady incremental downwash matrix $D_{unsteady}$. The improved AIC matrix is given as follows:

$$[AIC(k)] = D_{total}^{-1}(k)$$  \hspace{1cm} (2.21)

As it was mentioned earlier, in order to use the DLM in aeroelastic analysis it is necessary to express the free stream normalwash vector $\bar{w}_\infty$ so that it accounts for the elastic deformation of a lifting surface (Kotikalpudi, 2017). This topic is discussed in detail in the following DLM validation section.

\textbf{2.2.1. DLM Validation}

The DLM is verified with experimental data and previous DLM codes. The DLM
code used in this thesis is based on a work of University of Minnesota UAV Lab (Kotikalpudi, 2017).

Firstly, a lift distribution on a rectangular wing with aspect ratio of 3 oscillating in bending mode is used as a source of validation. Experimental measurements of lift distribution are provided by (Lessing, Troutman & Menees, 1960). Dimensional sketch of the model is given in Figure 2.6.

Figure 2.6 Sketch of experimental model (Lessing, Troutman & Menees, 1960) Corresponding geometry generated in the DLM code is given in Figure 2.7
The wing is oscillating in a bending mode that can be described by Equation 2.22:

\[ \vec{h} \approx 0.1804|\frac{y}{s}| + 1.702\left(\frac{y}{s}\right)^2 - 1.136\left(\frac{y}{s}\right)^3 + 0.253\left(\frac{y}{s}\right)^4 \]  \hspace{1cm} \text{(2.22)}

where, \( \vec{h} \) is the heave (vertical displacement) of the lifting surface, \( y \) is the span-wise coordinate, \( s \) is the span of the lifting surface, in this case it is the span of the wing. In the case of the DLM code \( y \) takes discrete values since the lifting surface (wing) is discretized to 8 panels chord-wise and 8 panels span-wise. For a given specific case \( y \) becomes a vector and takes the span-wise coordinates of the downwash points. Thus, \( \vec{h} \) becomes a vector as well allowing the usage of the DLM techniques described in section 2.2.

The deformation vector \( \vec{h} \) that describes the bending of the wing cannot be directly used in aeroelastic analysis, as it was mentioned before the elastic deformation of the lifting surface given by the vector \( \vec{h} \) should be first expressed in the form the free stream
normalwash vector $\bar{w}_\infty$ so that Equation 2.23 can be used to obtain the pressure distribution:

$$\bar{p} = -[AIC(k)]\bar{w}_\infty$$  \hspace{1cm} (2.23)

The next step is, thus, to calculate the normalwash on panels due to their corresponding heave and/or pitch motion. Pitch motion is considered in the model to make it more general. Special matrices called differentiation matrices $D_1$ and $D_2$ are constructed in order to relate the motion of the panels to their corresponding normalwashes (Kier & Looye, 2009). The differentiation matrix $D_1$ maps the panels’ displacement to the downwash at the collocation point, and $D_2$ maps the panels’ velocity to the downwash at the same point. Equations 2.24-2.25 represent the relation of motion and corresponding normalwash:

$$u_{aero}^i = \begin{bmatrix} \theta_i \\ h_i \end{bmatrix}$$  \hspace{1cm} (2.24)

$$w_i = D_{1i}u_{aero}^i + D_{2i}\dot{u}_{aero}^i$$  \hspace{1cm} (2.25)

where, $u_{aero}^i$ is the aerodynamic degrees of freedom (DoF) of, $\theta_i$ is the pitch displacement, $h_i$ is the heave displacement, $w_i$ is the downwash at collocation point, $D_{1i}$ is the displacement differentiation matrix, $D_{2i}$ is the velocity differentiation matrix, $\dot{u}_{aero}^i$ is the velocity of the $i^{th}$ panel’s DoF. It should be noted that the heave displacement does not produce any downwash at the collocation point; however the pitch displacement produces the equivalent amount of downwash for small angles because the rotation of the panel about its pitch axis results in perpendicular flow at the collocation point (Kotikalpudi, 2017). Also, both heave velocity $\dot{h}_i$ and pitch rate $\dot{\theta}_i$ induce downwash at the collocation point given by $-\dot{h}_i/V$ and $\dot{\theta}_i c_i/4V$ respectively, where $c_i$ is the chord
length of $i^{th}$ panel (Kier & Looye, 2009). The normalwash is, thus, formulated as follows:

$$D_{1i} = [1 \ 0]$$  \hspace{1cm} (2.26)

$$D_{2i} = \frac{2}{c} \left[ \frac{c_i}{4} - 1 \right]$$  \hspace{1cm} (2.27)

$$w_i = D_{1i}[\theta_i \ h_i]^T + D_{2i}[\dot{\theta}_i \ \dot{h}_i]^T \frac{\bar{c}}{2V}$$  \hspace{1cm} (2.28)

The differentiation matrix $D_{2i}$ is normalized with respect to the reference chord $\bar{c}$ so that $\bar{c}/2V$ factor is isolated which later is combined with oscillating frequency $\omega$ in order to be expressed as reduced frequency $k$. Both $\theta_i$ and $h_i$ are harmonic functions of oscillating frequency $\omega$, thus $\dot{\theta}_i$ and $\dot{h}_i$ can be expressed as follows:

$$\theta_i = \theta_0 e^{i\omega t}$$  \hspace{1cm} (2.29)

$$h_i = h_0 e^{i\omega t}$$  \hspace{1cm} (2.30)

$$\dot{\theta}_i = i\omega \theta_i$$  \hspace{1cm} (2.31)

$$\dot{h}_i = i\omega h_i$$  \hspace{1cm} (2.32)

The normalwash can be rewritten as:

$$w_i = (D_{1i} + ikD_{2i})[\theta_i \ h_i]^T$$  \hspace{1cm} (2.33)

The differentiation matrices $D_{1i}$ and $D_{2i}$ are computed for all panels and combined in block-diagonal manner in the DLM code so that the total differentiation matrices $D_1$ and $D_2$ are obtained.

In the case of the validation, the wing is oscillating in bending mode and thus its $i^{th}$ panel has heave $h_i$ and zero pitch $\theta_i$. Thus, the normalwash on $i^{th}$ panel can be expressed as follows:
\begin{align*}
    w_i^{val} &= (ikD_{2i})[0 \quad h_i]^T \quad (2.34) \\
    D_{2i} &= \frac{2}{c} \left[ \frac{c_i}{4} \quad -1 \right] \quad (2.35) \\
    w_i^{val} &= ik\frac{2}{c} \left[ \frac{c_i}{4} \quad -1 \right][0 \quad h_i]^T \quad (2.36) \\
    w_i^{val} &= -ik\frac{2}{c} h_i \quad (2.37)
\end{align*}

It can be noticed that the differentiation matrix \( D_{1i} \) is absent since it is only related to pitch \( \theta \), which is zero in the case of a lifting surface oscillating in bending mode. The pressure acting on the panels can be computed using Equation 2.28:

\[ \bar{p} = -[AIC(k)]w^{val} \quad (2.38) \]

The result of pressure distribution on the root chord is given in Figure 2.8 and the pressure distribution on the tip chord is shown in Figure 2.9. Mach number \( M \) is 0.24 and the reduced frequency \( k \) is set to 0.47 to match the experimental flight condition.

\textit{Figure 2.8 Root chord pressure distribution, bending mode}

It can be observed from Figure 2.8 that the DLM slightly overestimate the
imaginary and real values of the pressure distribution.

*Figure 2.9* Tip chord pressure distribution, bending mode

*Figure 2.9* demonstrates that the code is in good agreement with the experimental results for the case of tip chord pressure distribution.

The custom DLM code is also verified with other DLM codes, namely N5KQ and N5KA where former code uses more accurate quartic approximation in the kernel numerator and the latter utilizes parabolic approximation (Rodden, Taylor, McIntosh & Baker, 1999). In this case, authors studied the effect of panel’s aspect ratio (AR) on rectangular wings that are pitching about their mid-chord at $M = 0.8$. The AR of the wing is 2 and it is divided into 10 equal span-wise strips, after that the number of chord-wise panels is varied from 5 to 100 that, in turn, varies the ARs of panels from 0.5 to 10.0. Moreover, the reduced frequencies are varied as well from $k = 0.1$ to 2.0. It should be noted that the normalwash vector was calculated differently in this case because the panels describing lifting surface are pitching about their respective mid-chords. The results for lift coefficient $C_L$ are presented below in Table 2.1 where both imaginary and real values are given.
Table 2.1

Comparison of lift coefficients for pitching wing with AR = 2.0, 10 panels span-wise, varying number of chord-wise panels and $M = 0.8$

<table>
<thead>
<tr>
<th>$k$</th>
<th>Chord-wise panels</th>
<th>N5KQ Real</th>
<th>N5KQ Imaginary</th>
<th>N5KA Real</th>
<th>N5KA Imaginary</th>
<th>Custom DLM Real</th>
<th>Custom DLM Imaginary</th>
</tr>
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<tr>
<td>0.1</td>
<td>5</td>
<td>2.968</td>
<td>0.3626</td>
<td>2.968</td>
<td>0.3626</td>
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<td>2.975</td>
<td>0.3653</td>
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<td>100</td>
<td>4.953</td>
<td>1.200</td>
<td>4.932</td>
<td>1.300</td>
<td>4.8974</td>
<td>2.0569</td>
</tr>
<tr>
<td>2.0</td>
<td>5</td>
<td>4.652</td>
<td>2.380</td>
<td>4.652</td>
<td>2.380</td>
<td>2.7568</td>
<td>4.1813</td>
</tr>
<tr>
<td>2.0</td>
<td>10</td>
<td>5.396</td>
<td>1.814</td>
<td>5.461</td>
<td>1.729</td>
<td>5.2472</td>
<td>2.0019</td>
</tr>
<tr>
<td>2.0</td>
<td>20</td>
<td>5.720</td>
<td>1.393</td>
<td>5.681</td>
<td>1.449</td>
<td>5.7793</td>
<td>1.1570</td>
</tr>
<tr>
<td>2.0</td>
<td>50</td>
<td>5.840</td>
<td>1.194</td>
<td>5.730</td>
<td>1.378</td>
<td>5.8561</td>
<td>0.9452</td>
</tr>
<tr>
<td>$k$</td>
<td>Chord-wise panels</td>
<td>N5KQ</td>
<td>N5KA</td>
<td>Custom DLM</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-----</td>
<td>-------------------</td>
<td>------</td>
<td>------</td>
<td>------------</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Real</td>
<td>Imaginary</td>
<td>Real</td>
<td>Imaginary</td>
<td>Real</td>
<td>Imaginary</td>
</tr>
<tr>
<td>2.0</td>
<td>100</td>
<td>5.854</td>
<td>1.170</td>
<td>5.735</td>
<td>1.371</td>
<td>5.8636</td>
<td>0.9216</td>
</tr>
</tbody>
</table>

*Figure 2.10 through Figure 2.17 show the real and imaginary $C_L$ of custom DLM code, which is shown in blue line, compared with the real and imaginary $C_L$ of N5KQ (shown in red) and N5KA (shown in cyan) as the number of chord-wise panels is increased from 5 to 100.*

*Figure 2.10 Comparison of $C_L$ (real) for rectangular AR = 2 wing, $M = 0.8$, reduced frequency $k = 0.1$*
Figure 2.11 Comparison of $C_L$ (imaginary) for rectangular AR = 2 wing, $M = 0.8$, reduced frequency $k = 0.1$

Figure 2.12 Comparison of $C_L$ (real) for rectangular AR = 2 wing, $M = 0.8$, reduced frequency $k = 0.5$
Figure 2.13 Comparison of $C_L$ (imaginary) for rectangular AR = 2 wing, $M = 0.8$, reduced frequency $k = 0.5$

Figure 2.14 Comparison of $C_L$ (real) for rectangular AR = 2 wing, $M = 0.8$, reduced frequency $k = 1.0$
Figure 2.15 Comparison of $C_L$ (imaginary) for rectangular AR = 2 wing, $M = 0.8$, reduced frequency $k = 1.0$.

Figure 2.16 Comparison of $C_L$ (real) for rectangular AR = 2 wing, $M = 0.8$, reduced frequency $k = 2.0$. 
Figure 2.17 Comparison of $C_L$ (imaginary) for rectangular AR = 2 wing, $M = 0.8$, reduced frequency $k = 2.0$

The results indicate that the custom DLM code is in good agreement with other developed codes, and its imaginary and real parts seem to converge for the same number of chord-wise panels, however, it should be noted that for high reduced frequencies starting from $k = 0.5$ the imaginary part is slightly off as it can be seen in Figure 2.13, Figure 2.15 and Figure 2.17. Apart from that, the real part of the $C_L$ almost matches the values of other DLM codes through the whole range of reduced frequencies as shown in Figure 2.10, Figure 2.12, Figure 2.14 and Figure 2.16.
3. Structural Modeling

In order to describe the structure of the wing a Finite Element Method (FEM) is utilized. Since the aerodynamic model allows the panels to pitch and heave, triangular plate bending element is chosen so that the structural grid describing the lifting surface can translate and rotate as well to account for heave and pitch motion introduced by the aerodynamic modeling.

The FEM code developed for the Matlab Aeroelastic Code is based on several models presented in (Singiresu, 2017). Since triangular bending element is considered, there are 3 nodes per element, and each node has 3 DoF which sums to 9 DoF per element. The DoF of triangular bending element are shown in Figure 3.1.

![Figure 3.1](image)

*Figure 3.1 Nodal DoF of a triangular plate in bending (Singiresu, 2017)*

In Figure 3.1 a single element with 3 nodes is presented. Each node has 3 DoF:
$q_1$, which is the vertical (transverse) translation in \( z \), \( q_2 \), which is the slope (rotation) in \( x \) and \( q_3 \), which is the slope (rotation) in \( y \). The magnitude of the translation of the first node is given by \( w(x_1, y_1) \), similarly the rotation in \( x \) axis is given by \( \frac{\partial w}{\partial y}(x_1, y_1) \) and the rotation in \( y \) is shown as \(-\frac{\partial w}{\partial x}(x_1, y_1)\). The thickness of the plate is denoted by \( t \).

Since, there are 9 displacement DoF per element, the assumed polynomial for the displacement function \( w(x, y) \) should also contain nine constant terms (Singiresu, 2017). The chosen displacement model is the nonconforming element (T-9) (Tocher, 1962) and is given as:

\[
w(x, y) = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 x^2 + \alpha_5 xy + \alpha_6 y^2 + \alpha_7 x^3 + \alpha_8 (x^2 y + xy^2) + \alpha_9 y^3
\]

\[
w(x, y) = [\eta] \bar{\alpha}
\]

\[
[\eta] = [1 \ x \ y \ x^2 \ xy \ y^2 \ x^3 \ (x^2 y + xy^2) \ y^3]
\]

\[
\bar{\alpha} = \begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\vdots \\
\alpha_9
\end{bmatrix}
\]

The constants \( \alpha_1, \alpha_2, \ldots, \alpha_9 \) from the vector \( \bar{\alpha} \) are determined from the nodal conditions:

\[
w(x, y) = q_1, \ \frac{\partial w}{\partial y}(x, y) = q_2, \ -\frac{\partial w}{\partial x}(x, y) = q_3 \ \text{at} \ (x_1, y_1) = (0, 0)
\]

\[
w(x, y) = q_4, \ \frac{\partial w}{\partial y}(x, y) = q_5, \ -\frac{\partial w}{\partial x}(x, y) = q_6 \ \text{at} \ (x_2, y_2) = (0, y_2)
\]

\[
w(x, y) = q_7, \ \frac{\partial w}{\partial y}(x, y) = q_8, \ -\frac{\partial w}{\partial x}(x, y) = q_9 \ \text{at} \ (x_3, y_3)
\]

Note that the local coordinates are chosen in such a way that the origin is placed at node 1, thus \((x_1, y_1) = (0, 0)\), the local \( y \) axis is the line connecting the nodes 1 and 2,
and the local \( x \) axis is pointing towards the node 3 which is demonstrated in Figure 3.1.

Single element’s DoF can be put in matrix form as:

\[
\begin{bmatrix}
q_1 \\
q_2 \\
\vdots \\
q_9
\end{bmatrix} = \begin{bmatrix}
\eta_1 \\
\eta_2 \\
\eta_3 \\
\eta_4 \\
\eta_5 \\
\eta_6 \\
\eta_7 \\
\eta_8 \\
\eta_9
\end{bmatrix} = [\eta][\bar{\alpha}]
\]  
(3.8)

where,

\[
[\eta] = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & y_2 & 0 & 0 & y_2^2 & 0 & 0 & y_2^3 \\
0 & 0 & 1 & 0 & 0 & 2y_2 & 0 & 0 & 3y_2^2 \\
0 & -1 & 0 & 0 & -y_2 & 0 & 0 & -y_2^2 & 0 \\
1 & x_3 & y_3 & x_3^2 & x_3y_3 & y_3^2 & x_3^3 & (x_3^2y_3 + x_3y_3^2) & y_3^3 \\
0 & 0 & 1 & 0 & x_3 & 2y_3 & 0 & (2x_3y_3 + x_3^2) & 3y_3^2 \\
0 & -1 & 0 & -2x_3 & -y_3 & 0 & -3x_3 & (-y_3^2 + 2x_3y_3) & 0
\end{bmatrix}
\]  
(3.9)

Any point on the element experiences transverse \( w \) (in \( z \)-axis) and in-plane \( u \) (in \( x \)-axis) and \( v \) (in \( y \)-axis) displacements. Thus, the strain-displacement relations can be expressed as:

\[
\varepsilon_{xx} = \frac{\partial u}{\partial x} = -z \frac{\partial^2 w}{\partial x^2}
\]  
(3.10)

\[
\varepsilon_{yy} = \frac{\partial v}{\partial y} = -z \frac{\partial^2 w}{\partial y^2}
\]  
(3.11)

\[
\varepsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = -2z \frac{\partial^2 w}{\partial x \partial y}
\]  
(3.12)

The strains can be expressed in matrix form as:

\[
\varepsilon = [\bar{B}]\bar{\alpha} = [B]\bar{q}
\]  
(3.13)

where,
\[
\begin{bmatrix}
[B]
\end{bmatrix} = -z \begin{bmatrix}
0 & 0 & 2 & 0 & 0 & 6x & 2y & 0 \\
0 & 0 & 0 & 0 & 2 & 0 & 2x & 6y \\
0 & 0 & 0 & 2 & 0 & 0 & 4(x + y) & 0
\end{bmatrix}
\]  
(3.14)

\[
[B] = [\bar{B}][\bar{\eta}]^{-1}
\]  
(3.15)

The element stiffness matrix in local coordinates can be expressed as:

\[
[k^e] = \iiint_{V^e} [B]^T [D] [B] dV
\]  
(3.16)

where, \( V^e \) is the volume of the element and \([D]\) is the flexural rigidity matrix given by:

\[
[D] = \frac{E}{(1 - \nu^2)} \begin{bmatrix}
1 & \nu & 0 \\
\nu & 1 & 0 \\
0 & 0 & \frac{1 - \nu}{2}
\end{bmatrix}
\]  
(3.17)

The transverse displacement is expressed as:

\[
w(x, y) = ([\eta][\bar{\eta}]^{-1})\bar{q}
\]  
(3.18)

The in-plane displacements are expressed as:

\[
u = -z \cdot \frac{\partial w}{\partial x}
\]  
(3.19)

\[
v = -z \cdot \frac{\partial w}{\partial y}
\]  
(3.20)

Combined translational displacements can be shown as:

\[
\begin{bmatrix}
u(x, y) \\
w(x, y)
\end{bmatrix} = \begin{bmatrix}
-z \cdot \frac{\partial [\eta]}{\partial x} \\
-z \cdot \frac{\partial [\eta]}{\partial y}
\end{bmatrix} [\bar{\eta}]^{-1} \bar{q} = [N_1][\bar{\eta}]^{-1} \bar{q} = [N]\bar{q}
\]  
(3.21)

\[
[N_1] = \begin{bmatrix}
0 & -z & 0 & -2xz & -yz & 0 & -3x^2z & -z(y^2 + 2xy) & 0 \\
0 & 0 & -z & 0 & -xz & -2yz & 0 & -z(2xy + x^2) & -3y^2z \\
1 & x & y & x^2 & xy & y^2 & x^3 & (x^2y + xy^2) & y^3
\end{bmatrix}
\]  
(3.22)

The consistent mass matrix can be evaluated as:
\[
[m^e] = \iiint_{V^e} \rho [N]^T [N] dV
\]

(3.23)

\[
= \iiint_{V^e} \rho ([\tilde{\eta}]^{-1})^T [N_1]^T [N_1] [\tilde{\eta}]^{-1} dV
\]

(3.24)

3.1 FEM Validation

The developed FEM is validated with the results provided by (Clough, 1965). The setup is shown in Figure 3.2.

Figure 3.2 Square plate schematic

The square plate is simply supported at three corners and subjected to a vertical load at the fourth corner as shown in Figure 3.2. The plate is square with a side of 8 inches and its thickness \( t \) is 1 inch, the elastic modulus \( E \) is 10,000 lbs/in\(^2\) and Poisson’s ratio is 0.3. Transverse load is applied on Point B and its magnitude is 5 lbs.

Displacements at Point A (center) and Point B are recorded.

The FEA was also carried out in Nastran so that the results of the FEM code can be verified with the output of Nastran. The analysis was performed with different mesh sizes with increasing numbers of elements per side from \( N_e = 4 \) to \( N_e = 24 \).

Table 3.1 demonstrates the displacements of point A and point B shown in Figure 3.2. The “Experiment” column in Table 3.1 depicts the displacements of point A and B...
obtained experimentally by (Clough, 1965). It is important to mention that structural
grids generated in Nastran are identical to the ones generated in the Matlab FEM code.
The “Nastran” and “Matlab” columns in Table 3.1 show the displacements of nodes
corresponding to the central point A and corner point B shown in Figure 3.2 generated
using Nastran code and Matlab code respectively.

Table 3.1
Displacements of point A and B

<table>
<thead>
<tr>
<th>Mesh Size</th>
<th>Point</th>
<th>Nastran</th>
<th>Matlab</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>N = 4</td>
<td>A</td>
<td>0.06912453</td>
<td>0.0672</td>
<td>0.0624</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0.2764981</td>
<td>0.2690</td>
<td>0.24960</td>
</tr>
<tr>
<td>N = 8</td>
<td>A</td>
<td>0.07034770</td>
<td>0.0646</td>
<td>0.0624</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0.2813908</td>
<td>0.2583</td>
<td>0.24960</td>
</tr>
<tr>
<td>N = 12</td>
<td>A</td>
<td>0.07165316</td>
<td>0.0638</td>
<td>0.0624</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0.2866126</td>
<td>0.2551</td>
<td>0.24960</td>
</tr>
<tr>
<td>N = 16</td>
<td>A</td>
<td>0.07259741</td>
<td>0.0634</td>
<td>0.0624</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0.2903896</td>
<td>0.2536</td>
<td>0.24960</td>
</tr>
<tr>
<td>N = 20</td>
<td>A</td>
<td>0.07326044</td>
<td>0.0632</td>
<td>0.0624</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0.2930418</td>
<td>0.2528</td>
<td>0.24960</td>
</tr>
<tr>
<td>N = 24</td>
<td>A</td>
<td>0.07373901</td>
<td>0.0631</td>
<td>0.0624</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0.2949560</td>
<td>0.2522</td>
<td>0.24960</td>
</tr>
</tbody>
</table>
Figure 3.3 and Figure 3.4 show the data in Table 3.1 as the mesh size is varied from $N_e = 4$ to $N_e = 24$.

It can be observed from Figure 3.3 and Figure 3.4 that the Matlab code’s result is converging very close to the experimental value similarly to the Nastran’s output which is converging to a value slightly closer to the experimental value for the same number of elements per side.
4. Grid Interpolation

As it was discussed in section 2.2, the DLM provides the aerodynamic force distribution on an aerodynamic grid describing a lifting surface at a given frequency and for a given normalwash distribution. However, in order to obtain the normalwash distribution corresponding to a lifting surface’s elastic deformation the aerodynamic model should interact with the structural model. Moreover, the effect of aerodynamic forces on the structural grids should be computed as well.

*Figure 4.1* Example of 2x2 aerodynamic grid

*Figure 4.1* demonstrates an aerodynamic grid with collocation points (shown as blue circles) and doublet lines (shown as dashed blue lines) that has 4 panels in total. Panel boundaries are represented as solid blue lines. The flow direction is also shown in *Figure 4.1*. The next step is to superimpose FEM on the aerodynamic grid which is
demonstrated in Figure 4.2. It should be noted that the selected FEM has triangular elements, and in this demonstration every aerodynamic grid’s element has corresponding 4 triangular bending elements (boundaries of FEM are shown in black dashed lines) which can be seen in Figure 4.2. Black dots represent structural grid’s nodes.

Figure 4.2 FEM superimposed on 2x2 aerodynamic grid

FEM grid superimposed on aerodynamic grid can be observed in Figure 4.2. The FEM elements are triangular and their boundaries are shown in black dashed lines, the nodes of the structural grid are presented as black solid circles. As it was mentioned back in Chapter 3, the structural elements are triangular bending elements which have 3 DoF per node, which means that there are 9 DoF per structural element. The aerodynamic
grid, however, has 2 DoF per panel (element): pitch about centerline parallel to the y-axis and heave. Since the FEM was validated in section 3.1 the next step is to interconnect the aerodynamic grid with structural grid so that structural DoF are projected to the aerodynamic grid and the normalwash vector can be computed for a given elastic deformation. This is achieved by using grid interpolation technique that can relay information from FE grid to the aerodynamic grid and vice-versa. Generally, the elastic deformation provided by the FE grid is relayed to the aerodynamic grid so that the normalwash vector is computed. In the other direction, the aerodynamics forces are transmitted to the structural grid using the same grid interpolation. Considering linear interpolation, this can be described by the following transformation matrices:

\[ u_{aero} = T_{as}u_{struc} \]  \hspace{1cm} (4.1)

\[ F_{struc} = T_{sa}F_{aero} \]  \hspace{1cm} (4.2)

where, \( u_{aero} \) is aerodynamic panels’ DoF, \( u_{struc} \) is the structural nodes’ DoF, \( F_{struc} \) is the force applied on the structural grid, \( F_{aero} \) is the force acting on the aerodynamic grid, \( T_{as} \) is the transformation matrix that gives aerodynamic panels’ DoF displacements or forces given structural grid’s displacement or forces, \( T_{as} \) is the opposite transformation matrix (Kotikalpudi, 2017). It is important to note that the \( F_{aero} \) is a vector that consists of lift and pitching moments of each panel about their midpoints, and it can be expressed as:

\[ F_{aero}^i = \begin{bmatrix} F_i \\ M_i \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix} F_{aero}^i = T_{sf}^i F_{aero}^i \]  \hspace{1cm} (4.3)

where, \( F_{aero}^i \) vector consisting of force acting on \( i^{th} \) panel’s midpoint and moment about its midpoint, whereas \( F_{aero}^i \) is the force acting on \( i^{th} \) panel’ quarter-chord (Kotikalpudi,
The transformation matrix $T_F$ can be constructed in diagonal manner to account for all panels.

Also, the transformation matrices are transpose of one another:

$$T_{as} = T_{sa}^T$$  \hspace{1cm} (4.4)

This is true because the interpolation of aerodynamic forces on to the structural grid requires structural equivalence, which implies that the load vectors $F_{struc}$ and $F_{aero}$ deform the structure identically (Rodden 1959; Rodden & Johnson, 1994). From structural equivalence, it can be shown that:

$$\delta u_{struc}^T F_{struc} = \delta u_{aero}^T F_{aero}$$  \hspace{1cm} (4.5)

and,

$$\delta u_{aero} = T_{as} \delta u_{struc}$$  \hspace{1cm} (4.6)

$$\delta u_{aero}^T = \delta u_{struc}^T T_{as}^T$$  \hspace{1cm} (4.7)

thus,

$$\delta u_{struc}^T [F_{struc} - T_{as}^T F_{aero}] = 0$$  \hspace{1cm} (4.8)

$$F_{struc} = T_{as}^T F_{aero}$$  \hspace{1cm} (4.9)

It can be seen that the $T_{as}^T$ actually equals to $T_{sa}$ as shown. Thus, it is required to find transformation in one direction only (Kotikalpudi, 2017).

In order to obtain such transformation surface spline theory for thin surfaces is utilized. The surface splines used in this thesis are based on the work of (Harder & Desmarais, 1972). This method is a mathematical tool that interpolates between grids using infinite thin plate deformation equations. However, this is a two-step process since firstly the structural deformations have to be represented as deformations on infinite thin plate and secondly using the surface spline method these deformations are interpolated to
match with aerodynamic panels’ DoF. Similar technique is used in Nastran which is discussed in detail in (Rodden & Johnson, 1994).

An infinite thin plate has only 1 DoF since it can only deform in the direction normal to its surface (Kotikalpu, 2017). Thus, it is required to represent the deformation of structural grid that has 3 DoF per node purely as 1 DoF heave deformations. This is done by constructing spline grid. Figure 4.3 demonstrates the spline grid superimposed on FE grid and aerodynamic grid. The nodes of spline grid are shown as red solid circles, and its boundaries that match with FE grid’s boundaries are shown in red dashed lines and solid red line show additional nodes that are added to each structural grid’s node.

![Spline grid](image)

*Figure 4.3 Spline grid*
The DoF of Node$_1$ shown in Figure 4.3 can be represented as heave motion of spline nodes attached to the structural grid’s Node$_1$:

\[
\begin{bmatrix}
h_{sp_1} \\
h_{sp_2} \\
h_{sp_3}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & l_1 \\
1 & 0 & 0 \\
1 & 0 & -l_2
\end{bmatrix}
\begin{bmatrix}
h_{Node_1} \\
\theta_{Node_1} \\
\beta_{Node_1}
\end{bmatrix}
\] (4.10)

where $h_{sp_i}$ is the heave displacement of spline grid’s $i^{th}$ node, $l_1$ is the distance between $sp_1$ grid and $sp_2$ grid, $l_2$ is the distance between $sp_2$ grid and $sp_3$ grid, $h_{Node_1}$ is the heave displacement (z-axis translation) of Node$_1$, $\theta_{Node_1}$ is the bending (rotation about x-axis) of Node$_1$, $\beta_{Node_1}$ is the twist (rotation in y-axis) of Node$_1$. The total transformation matrix between structural grid and spline grid is formed by obtaining spline grid’s DoF for each node of structural grid, so that it can be expressed as:

\[u_{spline} = T_{spline}u_{struc}\] (4.11)

where $u_{spline}$ contains the deformations of spline grid which consists of purely heave motion. After the deformation of spline grid is obtained the deformations at the locations of aerodynamic panel midpoints can be found using the infinite surface spline theory (Kotikalpudi, 2017). This interpolation between spline grid and aerodynamic grid can be expressed as $T_{ips}$, so that the interpolation between structural grid and aerodynamic grid can be expressed as:

\[T_{as} = [T_{ips}][T_{spline}]\] (4.12)

The detailed derivation of $T_{ips}$ matrix is shown in (Rodden & Johnson, 1994; Harder & Desmarais, 1972). It is important to note that this approach is general in a way that it allows having independent structural and aerodynamic grids.
5. Analysis of the F-5 wing

The F-5 wing was analyzed in Nastran environment and the analysis included modal, flutter and static. Moreover, a static experiment and GVT was conducted to validate the FE model of the wing. The flutter analysis was performed in Nastran/Patran environment. Patran was used as a graphical user interface to setup the analysis by specifying the geometry, mesh, properties of material, loads, boundary conditions and flight condition. Since, the flutter analysis is of interest aeroleastic module built-in within Nastran was used.

As it was mentioned in introductory section 1, a specific wing geometry was chosen for aeroelastic analysis which is Northrop Grumman’s F-5 fighter wing. The FE model of the wing was provided by the Embry-Riddle Aeronautical University’s (ERAU) Structural Analysis and Design (SAnD) Lab (Tamijani et al., 2018; Locatelli et al., 2013). General characteristics of the wing are listed in Table 1.1. The rendering of the wing geometry is presented in Figure 5.1.

![Rendering of the F-5 wing](image_url)

*Figure 5.1 Rendering of the F-5 wing*

The internal structure of the wing is topologically optimized which is demonstrated
in Figure 5.2 and Figure 5.3.

Figure 5.2 Wing internal structure (rib-spar geometry)

Figure 5.3 Wing box with hidden top surface

5.1 Static experiment of 3D printed wing prototype

Prior to flutter analysis, a static experiment was carried out in “Structures” lab at ERAU to validate the FE model made in Nastran. A small version of the wing was 3D printed in ERAU 3D printing workshop using “Makerbot Replicator 2X” 3D printer
Important issue that was considered prior to the experiment was the mechanical properties of the printing material. Since the wing has complex internal structure (Figure 5.2, Figure 5.3) and it was planned to analyze the displacement of the wing structure under static load, it was required that the printing material and the final structure had uniform modulus of elasticity. In other words, it was desired that the printing material had the same modulus of elasticity throughout the whole structure. Another issue was that there are enclosed spaces between spars, ribs, top and bottom surface as shown in Figure 5.3, so that if support material was used it would have been trapped within the enclosures changing the properties of the final printed structure. Furthermore, because of the same reason it would have been challenging to remove the support material from those enclosures once the manufacture is complete. Thus, it was decided that the structure of the wing is manufactured vertically and without support material. Dimensions of the manufactured wing was limited by the dimensions of “Makerbot Replicator 2X” 3D printer which is 24.6 cm x 16.3 cm x 15.5 cm (MakerBot Industries, 2018). As the wing was manufactured vertically, and, geometrically, the half-span of the wing is the largest dimension of the model it was decided to use the maximum allowable vertical print dimension of the “Makerbot Replicator 2X” 3D printer that resulted in the actual span-wise dimension of the wing to be 153 mm as shown in Figure 5.4.
Figure 5.4 Top view with dimensions of the 3D printed wing

The material was chosen from the list of offered materials provided by ERAU 3D printing workshop. The wing was printed from “HATCHBOX” Acrylonitrile Butadiene Styrene (ABS) 3D printer filament with elastic modulus of $E = 1.8 \text{ GPa}$. For the static analysis the wing was fixed along the root chord as can be seen in Figure 5.5 and the load was applied at the center of the tip chord. The same boundary conditions were equally applied to the FE model shown in Figure 5.6.

Figure 5.5 3D printed F-5 wing fixture
Figure 5.6 FE model’s boundary conditions

Loads were applied on the 3D printed wing so that the wing itself was not damaged. This was achieved by adhering double-coated foam squares on the lower surface of the wing and gluing “L” shaped aluminum extrusion to the foam square using cyanoacrylate based glue. The load was varied by adding weights to a plastic bag that was mounted on the “L” extrusion as shown in Figure 5.5.

The displacement of the tip chord trailing edge point was recorded for varying loads for both the experiment and simulation in Nastran and the data is presented in Table 5.1.

Table 5.1

<table>
<thead>
<tr>
<th>Load (g)</th>
<th>Displacement (mm)</th>
<th>Experiment</th>
<th>Nastran</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>2</td>
<td>1.52</td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>3.5</td>
<td>2.28</td>
<td></td>
</tr>
<tr>
<td>400</td>
<td>4.2</td>
<td>3.05</td>
<td></td>
</tr>
<tr>
<td>900</td>
<td>6.8</td>
<td>6.848</td>
<td></td>
</tr>
<tr>
<td>Load (g)</td>
<td>Displacement (mm)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>---------</td>
<td>-------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Experiment</td>
<td>Nastran</td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td>7.7</td>
<td>7.61</td>
<td></td>
</tr>
<tr>
<td>1300</td>
<td>9</td>
<td>9.93</td>
<td></td>
</tr>
<tr>
<td>1500</td>
<td>10.5</td>
<td>11.45</td>
<td></td>
</tr>
<tr>
<td>1800</td>
<td>13.2</td>
<td>13.738</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>15.5</td>
<td>15.22</td>
<td></td>
</tr>
</tbody>
</table>

*Figure 5.7* Displacement of tip chord trailing edge point as a function of load

Experimental results of the static experiment and the results of Nastran are in good agreement as demonstrated in *Figure 5.7*, thus, bolstering the confidence in further aeroelastic analysis within the Nastran’s aeroelastic module.
5.2 Static test of 3D printed wing using DIC

A different static experiment utilizing DIC as the data acquisition tool was performed. The specimen in this case was the same 3D printed wing that was used for a static experiment described in section 5.2. However, this time DIC was used to obtain the map of z-axis displacements of the upper surface of the 3D printed wing.

The VIC-3D system based on the principle of Digital Image Correlation was used to measure the displacement. The VIC-3D system requires an applied random speckle pattern on a specimen and a calibration procedure since two cameras are used to capture 3-Dimensional measurements of displacements.

Since the 3D printed wing was printed from black ABS plastic a white spray paint was applied on the top surface of the wing as it is shown in Figure 5.8. The cameras were positioned vertically pointing downwards as it is demonstrated in Figure 5.8 and at a proper distance so that cameras’ resolution is fully utilized.
Figure 5.8 Static test, DIC setup with (a) a speckle pattern and (b) position of cameras

The load was applied and varied similarly as in section 5.2 by means of adding weights to a plastic bag that was mounted on the “L” extrusion as shown in Figure 5.5.

Table 5.2

Static tests performed with VIC-3D

<table>
<thead>
<tr>
<th>Test Number</th>
<th>Load (g)</th>
<th>VIC-3D</th>
<th>Nastran</th>
<th>Absolute Error (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>0.525</td>
<td>0.76</td>
<td>0.235</td>
</tr>
<tr>
<td>3</td>
<td>150</td>
<td>0.83</td>
<td>1.14</td>
<td>0.31</td>
</tr>
<tr>
<td>4</td>
<td>200</td>
<td>1.105</td>
<td>1.52</td>
<td>0.415</td>
</tr>
<tr>
<td>5</td>
<td>250</td>
<td>1.38</td>
<td>1.9</td>
<td>0.52</td>
</tr>
<tr>
<td>Test Number</td>
<td>Load (g)</td>
<td>VIC-3D</td>
<td>Nastran</td>
<td>Absolute Error (mm)</td>
</tr>
<tr>
<td>-------------</td>
<td>----------</td>
<td>--------</td>
<td>---------</td>
<td>---------------------</td>
</tr>
<tr>
<td>6</td>
<td>300</td>
<td>1.68</td>
<td>2.28</td>
<td>0.6</td>
</tr>
<tr>
<td>7</td>
<td>350</td>
<td>1.97</td>
<td>2.66</td>
<td>0.69</td>
</tr>
<tr>
<td>8</td>
<td>400</td>
<td>2.25</td>
<td>3.04</td>
<td>0.79</td>
</tr>
<tr>
<td>9</td>
<td>450</td>
<td>2.54</td>
<td>3.42</td>
<td>0.88</td>
</tr>
<tr>
<td>10</td>
<td>500</td>
<td>2.82</td>
<td>3.8</td>
<td>0.98</td>
</tr>
<tr>
<td>11</td>
<td>550</td>
<td>3.12</td>
<td>4.18</td>
<td>1.06</td>
</tr>
<tr>
<td>12</td>
<td>600</td>
<td>3.4</td>
<td>4.56</td>
<td>1.16</td>
</tr>
<tr>
<td>13</td>
<td>650</td>
<td>3.74</td>
<td>4.94</td>
<td>1.2</td>
</tr>
<tr>
<td>14</td>
<td>700</td>
<td>4</td>
<td>5.32</td>
<td>1.32</td>
</tr>
<tr>
<td>15</td>
<td>750</td>
<td>4.3</td>
<td>5.7</td>
<td>1.4</td>
</tr>
<tr>
<td>16</td>
<td>800</td>
<td>4.58</td>
<td>6.08</td>
<td>1.5</td>
</tr>
<tr>
<td>17</td>
<td>850</td>
<td>4.86</td>
<td>6.46</td>
<td>1.6</td>
</tr>
<tr>
<td>18</td>
<td>900</td>
<td>5.25</td>
<td>6.84</td>
<td>1.59</td>
</tr>
<tr>
<td>19</td>
<td>950</td>
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<td>1.72</td>
</tr>
<tr>
<td>20</td>
<td>1000</td>
<td>5.8</td>
<td>7.6</td>
<td>1.8</td>
</tr>
</tbody>
</table>

As it is shown in Table 5.2 several tests were carried out with loads varying from 100 to 1000 grams and for all test cases the Nastran simulation result showed slightly larger value of deformation. It can be noticed that the first test has a load of zero grams and the reason why first test’s load is zero grams is that the consequent tests with nonzero
loads are compared to the first no-load case that acts as a reference so that z-axis displacements are obtained rather than z-axis position map of the top surface. The first no-load case is demonstrated in Figure 5.9. It can be observed that the whole surface is colored in green which demonstrates that this test is the reference no-load case with zero z-axis displacements.

*Figure 5.9 DIC test 1, no-load*

The results obtained using VIC-3D were compared to Nastran simulation and are in Figure 5.10 through Figure 5.13 below for several load cases.
Figure 5.10 250-gram load case with (a) VIC-3D results and (b) Nastran results
Figure 5.11 500-gram load case with (a) VIC-3D results and (b) Nastran results
Figure 5.12 750-gram load case with (a) VIC-3D results and (b) Nastran results
Figure 5.13 1000-gram load case with (a) VIC-3D results and (b) Nastran results

The comparison between FEM and experiment indicates that the Nastran slightly overestimates the z-axis displacement for every load case as it is demonstrated in a summarized data plot given in Figure 5.14. Nevertheless, the displacement patterns look
similar with some differences in the area close to the fixed root chord, which could be attributed to the fact that Nastran uses linear approximation for the displacements.

![Graph showing maximum z-axis displacement as a function of load](image)

*Figure 5.14 Maximum z-axis displacement as a function of load*

The discrepancy between the slopes of FEM and experiment shown in *Figure 5.14* is attributed to the fact that in Nastran simulations a certain value of elastic modulus was used (1.8 GPa). However, the elastic modulus of the 3D printed part can be affected by the manufacturing conditions.

### 5.3 Ground Vibration Test of the 3D printed wing

In addition to the static test a GVT test was carried out with the 3D printed wing. The test was conducted using dynamic shaker Modal Exciter 2060E which can apply forces up to 267 N at frequencies between 1-6000 Hz.

For the purpose of this test a modified wing with a flange demonstrated in *Figure 5.15* was 3D printed so that a more accurate fixed root boundary condition can be achieved.
The wing was fixed horizontally via clamps as it is demonstrated in Figure 5.16. The shaker was connected to the structure from below with a stinger. A PCB 208C01 force sensor was also mounted between the stinger and the wing to measure the excitation force as shown in Figure 5.17. The force sensor is capable of measuring forces of ±45 N within a frequency range of 0.01-36000 Hz with a sensitivity of 112.41 mV/N.
In order to measure the acceleration signals four PCB 352A24 miniature lightweight accelerometers were used. Utilized accelerometers are capable of measuring...
accelerations of $\pm 490 \text{ m/s}^2$ within a frequency range of 1-8000 Hz and with 10.2 mV/(m/s$^2$) sensitivity. The data from accelerometers and force sensors was sampled at 2048 Hz frequency. The accelerometers were placed on the top surface of the wing structure and were advanced span-wise towards root chord for each test case which is demonstrated below in Figure 5.18.

In total 8 test cases were performed which resulted in 32 accelerometer output signals. As far as the input is concerned: the structure was excited by sine sweep wave input on dynamic shaker from 1 to 300 Hz in 600 seconds. This frequency range was chosen because the Nastran simulation showed that the 1st torsional mode has a natural frequency of 289.3 Hz.

An example of time domain response is shown in Figure 5.19. The data shows the accelerometer output signal and the input excitation force signal. The time response data for all 8 cases was transformed to frequency response data by utilizing empirical transfer function estimation and then it was analyzed in Matlab to obtain the natural frequencies and mode shapes within 0 to 300 Hz frequency range. Figure 5.20 shows the single input-single output (SISO) frequency responses of the first test case and it can be observed that there are several peaks of magnitude, some of which correspond to natural frequencies of the structure.
Figure 5.19 Time domain response corresponding to case 1

Figure 5.20 Frequency response from input force to acceleration response for case 1

After 32 individual SISO systems’ frequency responses were obtained, the System Identification Toolbox from Matlab was used to identify the natural frequencies and estimate the model that would reproduce the dynamic behavior of the 3D printed wing. A state-space model with 32 states was found to be suitable in order to accurately
approximate the experimental results. The estimated model’s states are all stable as shown in Figure 5.21. The worst and the best fits between the experimental data and the estimated model is presented in Figure 5.22.

**Figure 5.21** Hankel Singular Values for 32 order model

The analysis of the estimated model identified 4 modal frequencies within a range of frequencies from 1 to 300 Hz. The stability of the estimated model’s modal parameters was checked as the order of the underlying model was varied, which is demonstrated in Figure 5.23. The inspection of the plot suggests that there are 4 modal frequencies which have values of approximately 65, 95, 165 and 285 Hz. However, in order to identify the wing’s 1st bending and 1st torsion modes the mode shapes of the corresponding modal frequencies were visualized and compared to the Nastran modal simulation results.
Figure 5.22 The worst (top) and the best (bottom) fit of the 32 order state-space model

Figure 5.23 Stabilization diagram

Figure 5.24 and Figure 5.25 provide a comparison between experimentally obtained natural frequencies and mode shapes and simulated ones obtained in Nastran. The Nastran simulation showed that the 1st bending mode’s frequency is 95.99 Hz which is very close to the experimental 94.31 Hz frequency shown in Figure 5.24. The 1st torsion mode’s frequency obtained in Nastran is 273 Hz which is close to the experimentally
identified frequency of 283.3 Hz as it is shown in Figure 5.25. The mode shape of the 1\textsuperscript{st} bending is very close to the experimentally obtained mode shape, however the 1\textsuperscript{st} torsion mode shape obtained in Nastran has some discrepancies when compared to the experimentally obtained torsional mode which can be observed in Figure 5.25. This difference can be explained by the fact that Nastran modal analysis utilizes linearized model whereas experimentally obtained mode shape possesses non-linear dynamics that makes it look like a combination of torsional mode and residual minor effect of bending mode.

![Figure 5.24](image.png)

*Figure 5.24* 3D printed wing’s experimental (top) and simulated (bottom) 1\textsuperscript{st} bending mode
Moreover, it can be noticed that the experimental values of frequencies are slightly smaller than the values obtained from simulation. This discrepancy can be attributed to the fact that the damping is not considered in Nastran simulation whereas the structural damping slightly reduces the natural frequency for the case of experiment. Similarly, the viscous damping might marginally contribute to this difference. Nevertheless, the experiment proved that the Nastran simulation accurately approximates the dynamic response of a 3D printed wing structure.

Since only the wing is planned to be tested, the flutter mechanism was chosen to be symmetric wing 1st bending/1st torsion flutter (SWBT) as it is experimentally simplest.
mode to investigate and furthermore requires a simple fixture at the root (Pankonien, Reich, Lindsley & Smyers, 2017).

Prior to the manufacturing of the wing for the wind tunnel experiment, a feasibility study was performed in Patran/Nastran simulation environment to investigate whether the chosen flutter mode is attainable given dimensions and material. Thenceforth, the following section describes the future experimental setup for flutter analysis and is followed by the flutter analysis within the Nastran.

5.4 Experimental setup of the wing in ERAU wind tunnel

In order to experimentally test topologically optimized 3D printed internal structure of the wing, the new Embry-Riddle Aeronautical University’s (ERAU) wind tunnel located at “MicaPlex” innovation complex was selected as to leverage its fairly large test section, which is 4 feet high, 6 feet wide and 12 feet long, and its nominal achievable flowspeed with mean turbulence intensity of less than 0.5%, which is 350 feet per second (0.3 Mach) (Langer, 2018). The depiction of ERAU wind tunnel test section is shown in Figure 5.26.
ERAU wind tunnel’s range of flowspeed allows for a more unrestrained 3D printed wing designs because it expands the choice of dimensions and material to be used by providing a wider range for the to-be-tested wing’s stiffness that would inflict required flutter mechanism for experimental analysis.

There are several ways of placing the wing in the wind tunnel for aerodynamic tests. Placing the wing vertically attached to the ground or to the top wall (ceiling) of the test section could be considered as one of configurations (Ballman et al., 2011; Matsuzaki, Ueda, Miyazawa & Matsushita, 1989). Another configuration of placing the wing is to attach it to the sidewall of the wind tunnel horizontally (Scott, Coulson, Castelluccio & Heeg, 2011; Tang & Dowell, 2001) or vertically (Pankonien, Reich, Lindsley & Smyers, 2017). Further variation of this method is to attach the wing horizontally to a splitter plate (Ricketts & Doggett, 1980; Heeg, Wieseman & Chwalowski, 2016; Huang, Zhao & Hu, 2016). After thorough consideration it was decided to install wing model inside ERAU wind tunnel vertically mounted on a splitter
plate as demonstrated in Figure 5.27. The main reasons for this decision are the ease of manufacturing of the splitter plate, simple fixture using existing screw-threads on a turntable (no modification of ERAU wind tunnel section will be required) and ability to change angle of attack for each run. Wing mount that is connected to the splitter plate is perforated so as to allow positioning the wing at desired angle of attack with a step of 2 degrees.

*Figure 5.27 Visualization of ERAU wind tunnel section with wing model a) Rendered b) Isometric sectioned and c) Isometric side views.*

The dimensions of the wing for the experimental analysis in wind tunnel is limited by the height of the test section, which is 4 feet. The dimensions of the wing that fits the
5.5 Modal and aeroelastic analysis in Nastran/Patran

Prior to flutter analysis a modal analysis was performed in Nastran with a scaled wing made from polypropylene material called “Durus” $E = 1.1 \, \text{GPa}$ produced by “Stratasys” company (Stratasys, 2018). The dimensions of the scaled wing are shown in Figure 5.28.

![Figure 5.28 Top view of the wing model with dimensions](image)

The results of the analysis showed that the first natural frequency is 18.66 Hz and the corresponding mode was 1st bending which is demonstrated in Figure 5.29. The second natural frequency yield a value of 57.19 Hz with the corresponding mode of 1st torsion which is shown in Figure 5.30. 2nd bending and 2nd torsion frequencies are at 71.79 and 122.5 Hz respectively. 1st torsional mode’s frequency was found to be lower than the 2nd bending mode, resulting in a favorable symmetric wing bending torsion flutter mechanism. Symmetric Wing Bending Torsion (SWBT) flutter mechanism is experimentally the simplest flutter mode to investigate since it does not include a rigid...
body degree of freedom (Pankonien, Reich, Lindsley & Smyers, 2017), easing the mounting implementation via fixed root which is crucial for the future wind tunnel test.

![Figure 5.29 1st bending at 18.66 Hz](image1)

**Figure 5.29** 1st bending at 18.66 Hz

![Figure 5.30 1st torsion at 57.19 Hz](image2)

**Figure 5.30** 1st torsion at 57.19 Hz

Preliminary flutter analysis was carried out in Nastran’s aeroelastic module. A case study was conducted in which 2 wing dimensions (2ft and 3ft) were tested. The results are $v - g$ and $v - f$ plots that demonstrate the velocity versus damping and velocity versus frequency curves, where $g$ represents the structural damping of the vibration. The velocity at which the curve on the $v - g$ plot passes the x-axis so that the value of $g = 0$ is called the flutter speed. It is possible to determine the frequencies of the modes at flutter speed by picking the value of the velocity at which the curve passes the x-axis in **Figure 5.31** and finding values of frequencies corresponding to that velocity in **Figure 5.32**.
Figure 5.31 $v - g$ plot of F-5 wing, “Durus” material, $E = 1.1 \, GPa$, 2 feet

Figure 5.32 $v - f$ plot of F-5 wing, “Durus” material, $E = 1.1 \, GPa$, 2 feet
Figure 5.33 $v - g$ plot of F-5 wing, “Durus” material, $E = 1.1 \, GPa$, 3 feet

For the case of 2 feet wing model the flutter speed was calculated to be 195 m/s as shown in Figure 5.31 and Figure 5.32. If the model size is increased to 3 feet it can be seen that the corresponding flutter speed is decreased. This is demonstrated in Figure 5.33 and
Figure 5.34 where flutter speed was found to be 160 m/s. The data relevant to the flutter analysis is tabulated in Table 5.3.

Table 5.3

*Case study design of flutter model via stiffness and dimension control*

<table>
<thead>
<tr>
<th>Case</th>
<th>Material Characteristic Dimension</th>
<th>Natural Frequencies (Hz)</th>
<th>Predicted Flutter Speed (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1st Bending</td>
<td>1st Torsion</td>
</tr>
<tr>
<td>Durus (E=1.1GPa) -</td>
<td>2 feet root chord</td>
<td>18.7</td>
<td>57.2</td>
</tr>
<tr>
<td>Homogeneous</td>
<td>3 feet root chord</td>
<td>13.3</td>
<td>40.4</td>
</tr>
</tbody>
</table>

As it can be seen from Table 5.3 the flutter speed is significantly higher than the ERAU wind tunnel’s flow speed limit of 100 m/s. If a lower flutter speed is desired, elastomeric breaks can be incorporated in the wing model to reduce its torsional stiffness which in case would lower the torsional frequency thus lowering the flutter speed (Pankonien, Reich, Lindsley & Smyers, 2017).

Another case study was conducted to see how the flutter speed changes as the thickness of the wing model’s walls are varied from 1.5 mm to 5 mm. The results are demonstrated in Figure 5.35, Figure 5.36 and Figure 5.37. For the case of 1.5 mm wall thickness the flutter speed was obtained to be 112 m/s as shown in Figure 5.35. As the thickness is increased to 2.5 mm the flutter speed increases as well and reaches 139 m/s as demonstrated in Figure 5.36. Finally, Figure 5.37 demonstrates a wing configuration with a wall thickness of 5 mm which has the flutter speed of 195 m/s. This simulation demonstrates that even by decreasing wing walls’ thickness to 1.5 mm the flutter is still not reached given ERAU wind tunnel’s limitation of 100 m/s.
Figure 5.35 $v - g$ plot of F-5 wing, "Durus" material, $E = 1.1 \text{ GPa}$, 2 feet, 1.5 mm

Figure 5.36 $v - g$ plot of F-5 wing, "Durus" material, $E = 1.1 \text{ GPa}$, 2 feet, 2.5 mm
Figure 5.37 $v - g$ plot of F-5 wing, "Durus" material, $E = 1.1 \text{ GPa}$, 2 feet, 5.0 mm
6. Concluding remarks and future work

In the course of this work a significant part of aeroelastic modelling software was developed. Steady aerodynamics was modeled using vortex lattice method (VLM) and the unsteady aerodynamics was modeled using doublet lattice method (DLM). Triangular bending elements were used as a basis for a finite element method (FEM) in this work. The interpolation required to relay the results obtained from the DLM into the structural modal space (FEM) was developed as well. Moreover, a static, modal and aeroelastic analyses of F-5 wing were performed in the framework of Nastran’s solver. Consequently, a prototype 3D printed F-5 wing was manufactured and a static and ground vibrational test (GVT) analyses for the purpose of validation were carried out.

The static experiment performed using 3D printed wing prototype and digital image correlation (DIC) technique showed that the finite element (FE) model is in good agreement with the experiment. Furthermore, a ground vibration test (GVT) showed that the computational modal analysis performed in Nastran is in good agreement with the experimental results. The results of the flutter analysis performed in Nastran showed that the 2 feet wing made from “Durus” 3D printing material is not going to flutter within the limit of ERAU wind tunnel’s maximal flowspeed of 100 m/s. Additional flutter analysis in Nastran was carried out to see if the reduction in wing walls’ thickness leads to an attainable wing bending torsion flutter. The results of this analysis showed that even reducing thickness to 1.5 mm does not result in wing fluttering below 100 m/s.

The analyses and experiments performed in this work allow for several new avenues of additional research work as well as experiments to be explored. Experimental flutter analysis results can be obtained by testing the larger 2 feet wing inside the ERAU
wind tunnel as explained in Chapter 5. Results of this analysis might be used as a source of validation for the developed Matlab Aeroelastic Code.
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