Feasibility of Circular Orbits for Proximity Operations in Strongly Perturbed Environments around Uniformly Rotating Asteroids

Nicholas Peter Liapis

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FEASIBILITY OF CIRCULAR ORBITS FOR PROXIMITY OPERATIONS
IN STRONGLY PERTURBED ENVIRONMENTS AROUND UNIFORMLY
ROTATING ASTEROIDS

A Thesis

Submitted to the Faculty

of

Embry-Riddle Aeronautical University

by

Nicholas Peter Liapis

In Partial Fulfillment of the

Requirements for the Degree

of

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Daytona Beach, Florida
FEASIBILITY OF CIRCULAR ORBITS FOR PROXIMITY OPERATIONS IN STRONGLY PERTURBED ENVIRONMENTS AROUND UNIFORMLY ROTATING ASTEROIDS

by

Nicholas Peter Liapis

A Thesis prepared under the direction of the candidate’s committee chairman, Dr. Troy Henderson, Department of Aerospace Engineering, and has been approved by the members of the thesis committee. It was submitted to the School of Graduate Studies and Research and was accepted in partial fulfillment of the requirements for the degree of Master of Science in Aerospace Engineering.

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Date

8/13/2019

Date

8/13/19
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SYMBOLS

\( \hat{n}_{f'} \)  Unit vector normal to the other face adjacent to the edge

\( \hat{n}_e \)  Unit vector normal to the edge traveling counterclockwise and the face

\( \hat{n}_e \)  Unit vector normal to the edge traveling counterclockwise about the other adjacent face and normal to the other adjacent face as well.

\( \hat{n}_f \)  Unit vector normal to the face

\( r_1^e, r_2^e \)  The vectors from the spacecraft to the vertices that make up the edge

\( r_1^f, r_2^f, r_3^f \)  The vectors from the spacecraft to the vertices that make up the face

\( r_\ell \)  The vector from the position of the spacecraft to the center of the edge

\( r_f \)  The vector from the position of the spacecraft to the center of the face

\( a_g \)  Gravitational acceleration

\( e_e \)  The length of the edge \(|r_1^e - r_2^e|\)

\( q \)  Position vector of the spacecraft in the asteroid body fixed reference frame

\( \vec{r} \)  Position vector of the spacecraft in the asteroid body centered inertial reference frame

\( \omega \)  Rate of rotation

\( G \)  Universal Gravitational Constant

\( M \)  Mass of the asteroid

\( e \)  Number of edges in the polyhedral model

\( f \)  Number of faces in the polyhedral model
$\nu$ Number of vertices in the polyhedral model

$\sigma$ Bulk Density
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<tr>
<td>JAXA</td>
<td>Japanese Aerospace Exploration Agency</td>
</tr>
<tr>
<td>LIDAR</td>
<td>Light Detection and Ranging</td>
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<tr>
<td>NASA</td>
<td>National Aeronautics and Space Administration</td>
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<td>NAVCAMS</td>
<td>Navigational Camera</td>
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<tr>
<td>NEO</td>
<td>Near Earth Object</td>
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<tr>
<td>PDCO</td>
<td>Planetary Defense Coordination Office</td>
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<tr>
<td>RGB</td>
<td>Red, Green, and Blue</td>
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ABSTRACT

Liapis, Nicholas MSAE, Embry-Riddle Aeronautical University, August 2019. Feasibility of Circular Orbits for Proximity Operations in Strongly Perturbed Environments Around Uniformly Rotating Asteroids.

Asteroids have been mapped and observed since 1801 when an Italian astronomer Guiseppe Piazza discovered Ceres (Serio, Manara, & Sicoli, 2002). Since then, asteroids have been growing in popularity throughout the scientific community because they are thought to hold the information we need to understand how the solar system developed and why life exists on earth, as well as potential precious resources. This research studies different types of orbits that have been performed to date around asteroids and how they can be reworked to require less control effort. Different types of missions that have been sent to asteroids are discussed, as well as the equipment needed for those missions. The use of optimal circular orbits around uniformly rotating asteroids are compared to methods currently used in asteroid science missions. In the process, the dynamics that are used in modeling the system, an optimization method used to map the equilibriums, and how much control effort can be saved by using the equilibrium fields are detailed for smaller asteroids as well as a larger one. Asteroids 216 Kleopatra, 2063 Bacchus, and 101955 Bennu were the focus of this research and significant fuel savings of up to 40% toward the elongated asteroids’ surfaces and 90% toward the spherical asteroids surface.
1. Introduction

There is a growing interest in near Earth asteroids because of the potential answers to impactful events in the history of our solar system as well as the opportunity for collection of precious resources. Through asteroid observation, there have been many asteroids pinpointed as targets for early solar system knowledge, Earth collision avoidance, as well as landing sites for potential resource gathering in the future. This research focuses on the asteroid 216 Kleopatra, which is an asteroid approximately 200 km in length with a strongly perturbed gravitational environment. Kleopatra also has a rotation which allows us to find body fixed equilibrium points around the asteroid where the gravitational force from the asteroid counteracts the centripetal force in the asteroid’s body fixed frame to allow for minimum force required from the spacecraft (Descamps, Marchis, & Berthier, 2011). In this thesis, a method to determine equilibrium points around rotating asteroids using a novel numerical optimization method is developed and then the same method is applied to determine optimal circular orbits. The methods are then translated to asteroids 2063 Bacchus and 101955 Bennu.

1.1 Problem Statement

Sample return missions to asteroids are currently launched from Earth with approximately half of their mass accounted for in the propulsion system. If the space vehicle did not have to expend as much fuel to perform the same mission in close proximity of the asteroid, the launch mass, development cost, and launch cost could be significantly reduced. Low cost alternatives to current asteroid proximity methods are derived and their use is discussed.
1.2 Motivation

With asteroid missions becoming more common, it is now viable to look for better methods for close proximity operations in order to significantly reduce the cost of each mission. If required propulsion can be decreased, it could mean that more samples could be returned to Earth or the same number of samples could be returned much cheaper which means that over time, that money could be redirected into more missions to different asteroids. This work aims to find more efficient ways to hover and orbit the strongly perturbed gravitational environments of asteroids for improved scientific data collection.

1.3 Summary and Contribution of Work

The research performed focuses on the natural dynamics of orbiting an asteroid without taking into account any external potential. The dynamics of an orbit around an asteroid in the body fixed frame are determined and then simplified into hovering with respect to the body fixed reference frame around a uniformly rotating asteroid. This work is then translated into an asteroid centered vehicle orbital frame in order to apply the methods derived to look at the feasibility of circular orbit mission design.

Contributions of Work

- Determine equilibrium point positions around uniformly rotating asteroids;
- Compare minimum force requirements for asteroid proximity operations and orbital maintenance;
- Provide insight into the feasibility of using circular orbit mission designs in highly perturbed gravitational environments around uniformly rotating asteroids.
1.4 Outline of Thesis

Chapter 2 gives background and history on topics that are used in the formulation of the dynamics and the specific systems being studied. It also dives into past missions that have been launched to asteroids in the past. Chapter 3 is the formulation of the dynamics that are used in asteroid missions as well as simplifications for the purposes of this work. Chapter 3 also discusses the determination of the equilibrium points and manifolds around uniformly rotating asteroids. The chapter discusses and analyses results for a simplified asteroid model for conceptual understanding. Chapter 4 discusses the results of where the equilibrium points are located, the optimal places to hover over the specific asteroid, as well as other analyses of the systems. Finally, results are laid out and discussed for the optimal circular orbit around the highly perturbed gravitational fields studied. Then comparisons are made between the proposed mission design and current mission designs. Finally, Chapter 5 discusses important results and future work that can be conducted.
2. Background and History

In this chapter, there will be four main topics of the literature review discussed. First will be key concepts for building up to the work being performed. Second, orbital designs for asteroid observation that will not be analyzed in this research will be noted. Third, specifications on the three asteroids being studied in this analysis will be provided. Finally, three asteroid missions that have been or are currently being performed will be discussed.

2.1 Key Concepts

This section will provide a foundation for understanding Newtonian Gravity and how the potential for a point mass is calculated, Helmholtz’s equation, and Laplace’s equation and Poisson’s equation for gravitational modeling. The section will then review a few common gravitational potential models, as well as the Brillouin sphere.

2.1.1 Newton’s Three Laws of Motion

Law 1: Every object in a state of uniform motion will remain in motion unless acted upon by an external force.

Law 2: Force is equal to the change in momentum and with constant mass, force is equal to the product of the mass times the acceleration.

Law 3: For every action, there is an equal and opposite reaction (Young, & Young, 2007).

2.1.2 Newtonian Gravity

Newtonian gravity is the study of the attraction force between two or more point masses. The concept is illustrated in Figure 2.1.
The gravitational force between the two point masses are equal and opposite. The magnitude of the force is directly proportional to the product of $m_1$ and $m_2$ and inversely proportional to the square of the distance between the two masses ($r$) as shown in Equation (2.1).

$$G = 6.67 \times 10^{-11} \frac{m^3}{kg \cdot s^2}$$

$$F_1 = -F_2 = -\frac{G m_1 m_2 \hat{r}}{\|r\|^2}$$

(2.1)

### 2.1.3 Helmholtz’s Equation

By implementing separation of variables to the wave equation, the Helmholtz equation can be found. The Helmholtz equation, which is a time independent form of the wave equation, is shown in Equation (2.2). The equation is considered Helmholtz’s equation when $k^2 > 0$ (Takahashi, & Scheeres, 2014) and $U$ is the potential of the system.

$$\nabla^2 U + k^2 U = 0$$

(2.2)

*Figure 2.1. Newtonian gravitational attraction of two point masses*
2.1.4 Laplace’s and Poisson’s Equations

Laplace’s equation is a simplified version of the Helmholtz equation where \( k = 0 \). Solutions to Laplace’s equation, shown in Equation (2.3), can be proven to provide an accurate gravitational model for the system outside of the body of the asteroid (Takahashi, & Scheeres, 2014).

\[ \nabla^2 U = 0 \]  

(2.3)

Poisson’s equation, shown in Equation (2.4), must be satisfied in order to have an accurate representation of the gravitational field inside the asteroid.

\[ \nabla^2 U = -4\pi G \rho \]  

(2.4)

These two equations are very important in the development of gravitational models. Currently, the best way to approximate the gravitational field around an object is by using the polyhedral model discussed in section 2.1.10 because it most accurately accounts for the perturbed shape of the gravitational field due to the non-uniformity of the body and because the equation satisfies Laplace’s and Poisson’s equations so the model is accurate all the way down to the surface of the body.

2.1.5 Right Hand Rule

The right-hand rule is a method used for many applications in engineering. In this thesis, the right-hand rule is used to define coordinate systems and operations within. The importance will be to visually understand the dynamics and the directions of forces in the system studied. Figure 2.2 shows the idea of the right-hand rule as it will be applied in this thesis.
2.1.6 Gravitational Potential Introduction

Gravitational potential is the work that would have to be provided to the system per unit mass in order to move an object from a position to the position of the object. The gravitational influence between the bodies will change their paths. This will be very important in the gravitational field calculations. To date, there are a few different methods for determining the dynamic environment that a space vehicle would encounter around any given asteroid. Because of the lack of knowledge about the density of materials that make up the asteroid as well as the exact positioning of those materials under its surface, the gravitational models cannot be accurate without actual data from the asteroid. With this limited knowledge, approximations for the gravitational potential field must be made in order to determine the approximate locations of the minimum control orbits. In the past, approximating the gravitational field with only a few point masses, the mascon approach, or polyhedral modeling have been the most popular for research purposes.

2.1.7 Point Mass Gravitational Potential

By starting with Equation (2.1) and referencing Figure 2.1, the gravitational acceleration can be found by applying Newton’s second law and applying the del

Figure 2.2. Right-hand Rule Visualization (“Right-hand Rule”, n.d.)
operator. Note that for our purposes, we assume there is no change in mass (Muller, & Weiss, 2016).

\[ F = ma \] (2.5)

\[ F_1 = F_2 = m_2 \left( \frac{G m_1 r}{\|r\|^3} \right) \] (2.6)

Note that by selecting \( m_1 \) to be in the acceleration term, we are determining the gravitational potential of the first mass.

\[ a = \frac{G m_1}{\|r\|^3} r = \frac{G m_1 \nabla}{\|r\|} \] (2.7)

Now, the potential for a point mass can be found by applying Equation (2.8)

where \( U \) is the potential. This yields Equation (2.9) which is the potential of a point mass.

\[ a = -\nabla U \] (2.8)

\[ U = -\frac{G m_1}{\|r\|} \] (2.9)

Written in the body frame of the asteroid, replace \( r \) with \( q \) and \( m_1 \) with \( M \) to get Equation (2.10). The sum of the potential from three point masses gives the potential field used in initial approximations for this research. Figure 2.3 shows asteroid Kleopatra approximated as three point masses (Scheeres, 2014).

\[ U = -\frac{GM}{\|q\|} \] (2.10)
2.1.8 Mascon Gravitational Potential

Mascon modeling has the same potential equation as the point mass but has many point masses. Instead of approximating the asteroid as one large mass, mascon is the act of splitting the asteroid into many smaller point masses. This increases the computational expense significantly; however, it allows the density distribution to be much more accurate if that information is known (Scheeres, 2014).

\[
U = \sum_{i=1}^{n} \frac{GM_i}{|q_i|}
\] (2.11)

2.1.9 Constant Density Ellipsoid Gravitational Potential

To develop the constant density ellipsoid gravitational potential model, the semi major axes must be known. Based on the notation given in (Scheeres, 2014), define the semi major axis in the asteroids body fixed x direction as \(\alpha\), the semi major axis in the asteroids body fixed y direction as \(\beta\), and the semi major axis in the asteroids body fixed z direction as \(\gamma\). Also, \(\lambda(r)\) is the maximum real root that will always exist. Given the mass of the asteroid, the gravitational potential can be written as

\[
U(r) = -\frac{3GM}{4} \int_{\lambda(r)}^\infty \frac{\left(\frac{x^2}{\alpha^2 + u} + \frac{y^2}{\beta^2 + u} + \frac{z^2}{\gamma^2 + u} - 1\right)}{\sqrt{(\alpha^2 + u)(\beta^2 + u)(\gamma^2 + u)}} du
\] (2.12)
2.1.10 Polyhedral Gravitational Potential

The homogeneous polyhedron approach to gravitational modeling is a way to model the asteroids gravitational potential all the way down to the surface while also saving computational expense as opposed to a mascon model. It will be explained in more detail as it is the method that will be used throughout the results section of this thesis. The issue with gravitational modeling of asteroids is that the density distribution across the body is unknown; therefore, there will be inaccuracies in the approximation. In order to model the gravitational field of the asteroid, the asteroid’s bulk density is applied uniformly around the entire surface. The homogeneous polyhedron model is created by defining vertices and faces around the entire surface of the asteroid. Figure 2.4 shows the vertices of the asteroid 216 Kleopatra used in the calculations.

*Figure 2.4. Polyhedral vertices that make up asteroid 216 Kleopatra*

Each of the vertices is the corner of several faces connected to it, and in the calculation of the gravitational potential the critical information are the locations of the
faces and edges. Each face is made up of 3 vertices. The number of faces and edges can be determined by Equations (2.13) and (2.14).

\[
f = 2v - 4 
\]

\[
e = 3(v - 2) 
\]

**Figure 2.5.** One face of a polyhedral model made up of edges determined by the vertices

The polyhedral gravitational potential of an asteroid can be stated as Equation (2.15). Note that Equations (2.15) through (2.17) are written in dyadic notation (Scheeres, 2014) and (Park, Werner, & Bhaskaran, 2008).

\[
U(r) = \frac{G\sigma}{2} \left[ \sum_{e \in \text{edges}} r_e \cdot E_e \cdot r_e L_e - \sum_{f \in \text{faces}} r_f \cdot F_f \cdot r_f \omega_f \right] 
\]

\[
\frac{\partial U}{\partial r} = -G\sigma \left[ \sum_{e \in \text{edges}} E_e \cdot r_e L_e - \sum_{f \in \text{faces}} F_f \cdot r_f \omega_f \right] 
\]

\[
\frac{\partial^2 U}{\partial r^2} = G\sigma \left[ \sum_{e \in \text{edges}} E_e L_e - \sum_{f \in \text{faces}} F_f \omega_f \right] 
\]

In order to calculate the gravitational potential and the derivatives of the gravitational potential, the following terms must be computed for every desired position in the gravitational field.

\[
E_e = \hat{n}_f \left( \hat{n}_e \right)^T + \hat{n}_{f'} \left( \hat{n}_e' \right)^T 
\]
\[ F_f = \hat{n}_f \left( \hat{n}_f \right)^T \]  

(2.19)

\[ L_e = \ln \left( \frac{r_1^e + r_2^e + e_e}{r_1^e + r_2^e - e_e} \right) \]  

(2.20)

\[ \omega_f = 2 \arctan \frac{r_1^f \cdot r_2^f \times r_3^f}{r_1^f r_2^f r_3^f + r_1^f (r_2^f \cdot r_3^f) + r_2^f (r_3^f \cdot r_1^f) + r_3^f (r_1^f \cdot r_2^f)} \]  

(2.21)

Based on (Scheeres, 2014), the Laplacian of the potential can be shown to equal 0 when outside the asteroids body and \(4\pi\) inside the asteroid. What this means is that the polyhedral model for asteroids satisfies Laplace’s and Poisson’s equations and the gravitational potential can be estimated all the way down to the asteroids surface. A MATLAB script was developed to calculate the gravitational fields around asteroids 216 Kleopatra, 2063 Bacchus, and 101955 Bennu.

2.1.11 Brillouin Sphere

The Brillouin sphere, also called the circumscribed sphere, is a sphere that contains all mass of the body that originates from the Expansion Center. The gravitational approximation outside of the asteroids Brillouin sphere can be closely approximated with a point mass gravitational potential model. Figure 2.6 shows the circumscribed sphere around asteroid Kleopatra (Takahashi, & Scheeres, 2014).
Though the Brillouin sphere in general can be used as a good approximation of the gravitational potential field, it has its inaccuracies when it comes to highly non-spherical shapes such as asteroid 216 Kleopatra. In order to model the potential field more accurately outside of the Brillouin sphere, the constant density ellipsoidal model can be used as the approximation. The importance of the concept of the Brillouin Sphere is to reduce the computational expense of estimating the gravitational potential.

2.2 Asteroid Orbital Design

Orbit design around asteroids has been researched at length in recent years. By manipulating the dynamics of the system, it is possible to find resonant orbits which can allow a space vehicle to perform proximity operations without risk of colliding with the asteroid. Zero velocity manifolds have been studied around certain equilibrium points in a system as well as Invariant manifolds which can intersect with the asteroid allowing for a natural landing path. These orbits themselves can prove very useful for asteroid
operations in the future but will not be the focus of this research. Instead, this research will focus on the equilibrium point determination itself and transition those same concepts into an optimal circular orbit determination.

2.2.1 Zero Velocity Manifolds

Zero velocity manifolds are positions where the velocity of a spacecraft would be approximately zero with respect to the asteroid body fixed frame. In the study of zero velocity manifolds, Equation (2.22) must be satisfied. $H$ is the Jacobi constant, and $V(r)$ is the effective potential of the system. The specifics for what these are in the dynamics around an asteroid are discussed in chapter 3. By taking a closer look at the equilibrium points in the system, these manifolds develop around the equilibrium points. The linearized system around the equilibrium points are examined to determine the eigenvalues in order to determine the stability of the manifold. This is a different type of manifold than what will be employed in the rest of this thesis; however, it has been studied in length in the past (Jiang, Baoyin, Li, & Li, 2013).

$$V(r) = H$$ (2.22)

2.2.2 Invariant Manifolds

Invariant manifolds is a general term used to describe stable and unstable paths through the dynamical space. In the study of invariant manifolds, the dynamical system is studied to find invariant sets, which are spaces that would naturally transport the spacecraft through without any needed control effort. When the stable sets are discovered, they can help determine the mission plan for a spacecraft because of the lowered control effort. These sets are currently being studied around asteroids in order to find minimum control landing trajectories. These manifolds are also different than what will be discussed in this thesis (Mondelo, Broschart, & Villac, 2010).
2.3 Asteroid Specifications

This section will describe the three asteroids that will be focused on in this research. Asteroids 216 Kleopatra, 2063 Bacchus, and 101955 Bennu will be described and shown, and the asteroids physical parameters will be listed.

2.3.1 216 Kleopatra

Asteroid 216 Kleopatra is an M-type asteroid that has been observed in depth. Due to advancements in optics technology, the asteroid’s shape is well known even though no manmade object has ever traveled close by in order to observe it. The asteroid has been converted into a polyhedral model for gravitational calculations. The reconstructed asteroid is shown in Figure 2.7. The physical parameters that are important to this research are listed in Table 2.1 (Descamps, Marchis, & Berthier, 2011). The model of the asteroid was found on the PDS website (“Shape Models of Asteroids, Comets, and Satellites”, n.d.).

![Figure 2.7. Polyhedral Model of Asteroid 216 Kleopatra](image_url)
Table 2.1

*Kleopatra Physical Parameters*

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimensions (km)</td>
<td>217x94x81</td>
</tr>
<tr>
<td>Mass (kg)</td>
<td>$4.64 \times 10^{18}$</td>
</tr>
<tr>
<td>Bulk Density (g/cm$^3$)</td>
<td>3.6</td>
</tr>
<tr>
<td>Rotation Rate (rad/s)</td>
<td>$3.173 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

### 2.3.2 101955 Bennu

Asteroid 101955 Bennu is a C-type asteroid that has been observed in depth. Bennu is the asteroid that the National Aeronautics and Space Administration (NASA) targeted for its mission OSIRIS-REx discussed in section 2.4.3. The asteroid has been converted into a polyhedral model for gravitational calculations. The reconstructed asteroid is shown in Figure 2.8. The physical parameters that are important to this research are listed in Table 2.2 (Chesley, Farnocchia, Chodas, & Benner, 2014). The model of the asteroid was found on the PDS website (“Shape Models of Asteroids, Comets, and Satellites”, n.d.).
Table 2.2

101955 Bennu Physical Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter ((km))</td>
<td>565</td>
</tr>
<tr>
<td>Mass ((kg))</td>
<td>(7.8 \times 10^{10})</td>
</tr>
<tr>
<td>Bulk Density (\frac{g}{cm^3})</td>
<td>1.26</td>
</tr>
<tr>
<td>Rotation Rate (\frac{rad}{s})</td>
<td>(4.061 \times 10^{-4})</td>
</tr>
</tbody>
</table>

Figure 2.8. Polyhedral Model of Asteroid 101955 Bennu
2.3.3 2063 Bacchus

Asteroid 2063 Bacchus is an S-type asteroid that has been observed in depth. The asteroid has been converted into a polyhedral model for gravitational calculations. The reconstructed asteroid is shown in Figure 2.9. The physical parameters that are important to this research are listed in Table 2.3 (Benner, Hudson, Ostro, & Rosema, 1999). The model of the asteroid was found on the PDS website (“Shape Models of Asteroids, Comets, and Satellites”, n.d.).

![Figure 2.9. Polyhedral Model of Asteroid 2063 Bacchus](image)

Table 2.3

**2063 Bacchus Physical Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter (km)</td>
<td>$1.11 \times 0.53 \times 0.50$</td>
</tr>
<tr>
<td>Bulk Density (g/cm$^3$)</td>
<td>3.3</td>
</tr>
<tr>
<td>Rotation Rate (rad/s)</td>
<td>$1.2 \times 10^{-4}$</td>
</tr>
</tbody>
</table>
2.4 Mission History

The missions focused on are Hayabusa, Hayabusa 2, and OSIRIS-REx. This section goes into a brief overview of the missions and then discusses the instruments used as well as the proximity operations around the asteroids.

2.4.1 Hayabusa

The Hayabusa mission was launched by the Japanese Aerospace Exploration Agency (JAXA) in May, 2003 to asteroid Itokawa and returned a sample of the asteroid to Earth in June 2010. The purpose of the mission was to study the asteroid and the dynamic environment that a spacecraft encounters around irregularly shaped bodies and finally to return a sample to Earth for study. Once at the asteroid, the Hayabusa spacecraft spent time observing Itokawa and finding a suitable landing location. This was not very successful as the spacecraft had difficulties finding the desired landing site. Throughout this observational period, the mission design placed the spacecraft approximately four kilometers away from the surface of the asteroid while always staying on the path between the Earth and the asteroid. This provides three benefits to the mission. First, the vehicle will have the maximum possible time to communicate with Earth. Second, this also means that the solar panels will always have the opportunity to gather power. Last but not least, the surface of the asteroid will be illuminated the majority of the time which allows for better data collection with thermal and RGB sensors. The Hayabusa spacecraft, used LIDAR for ranging purposes with a working range of 50 m to 50 km with a ±1 m accuracy at 50 m range. The spacecraft also had a Laser Range Finder with an operating range of 7 to 100 m. It also had various optical sensors for image collection. Throughout the imaging phase of the mission, the spacecraft
stayed at a distance of 20 km away from the surface of asteroid Itokawa (Kubota, Hashimoto, Kawaguchi, Uo, & Shirakawa, 2006).

2.4.2 Hayabusa 2

The Hayabusa 2 mission was launched by the Japanese Aerospace Exploration Agency (JAXA) in December, 2014 to asteroid Ryugu. It is currently an ongoing mission with the goal of asteroid observation and sample return to Earth. JAXA launched this mission after designing the vehicle and mission in order to account for some of the issues that the Hayabusa spacecraft had in its operation. It is equipped with the same range capability LIDAR and Laser Range Finder as Hayabusa with slightly better accuracy. It was also equipped with optical sensors, Target Markers, and Flash Lamps but those instruments are not as important to the focus of this paper. The mission team selected a home position of 7 km away from the surface of the asteroid and the vehicle has been in proximity at varying distances since June 2018 (Lange, Dietze, Ho, & Kroemer, n.d.).

2.4.3 OSIRIS-REx

OSIRIS-REx is another ongoing asteroid mission launched by the National Aeronautics and Space Administration (NASA) in September, 2016 to asteroid Bennu. The purpose of this mission is to observe the asteroid and return a sample to Earth. The sensors used on this vehicle for relative position sensing are LIDAR and NAVCAMS. The LIDAR used on OSIRIS-REx has a range bias of 20 centimeters with 16,384 individual measurements up to 30 times a second. NAVCAMS are a set of cameras that track stars as well as features on Bennu in order to determine the relative position of the spacecraft. The mission design team plans to place the spacecraft into two different orbits around Bennu which are scheduled to last a total of 100-150 days. These orbits vary in
semimajor axis and inclination with orbital radii of 1.5 km to 2 km. When the spacecraft is not in those orbits, it will be practicing landing maneuvers and be in less efficient proximity operations (Wibben, Williams, McAdams, Antreasian, & Leonard, n.d.).
3. Theory

3.1 Dynamics Introduction

For this research, the dynamic model of a uniformly rotating asteroid was formed. The purpose of this research is to compare the force requirement of hovering over a fixed point in that asteroids body frame as opposed to current methods of sample return missions, and then determine use the same equations to determine the optimal circular orbit at a given radius. The model takes into account the centripetal force from the asteroids rotation with respect to the inertial frame of reference as well as the gravitational forces from the asteroid.

3.2 Lagrangian in the Asteroid Centered Inertial Frame

The Lagrangian for a spacecraft around an asteroid can be calculated by adding the kinetic energy to the potential of the system. Note that the potential is used instead of the potential energy; therefore the terms are added instead of subtracted (Scheeres, 2014).

\[ L = T + U \]

\[ T = \frac{1}{2}v^2 = \frac{1}{2}\dot{r} \cdot \dot{r} \]

\[ U = \text{full potential of the system.} \]

In order to obtain the dynamics of the system from the Lagrangian, the equations of motion can be gathered by following the operation shown in Equation (3.1).

\[ \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) = \frac{\partial L}{\partial q_i} \; ; \; i = 1, 2, \ldots, n \]  

(3.1)

\[ L(q, \dot{q}, t) = \frac{1}{2}\left(\dot{\mathbf{r}} + \mathbf{\omega} \times \mathbf{r}\right) \cdot \left(\dot{\mathbf{r}} + \mathbf{\omega} \times \mathbf{r}\right) + U(C(t) \cdot \mathbf{r}) \]
3.3 Lagrangian in the Asteroid Body Fixed Frame

Following the same notation as used in Orbital Motion in Strongly Perturbed Environments (Scheeres, 2014), the position vector $r$ will be relative to the inertial frame of reference and a position vector $q$ will be expressed in the body fixed frame of the asteroid.

$$r = C(t)q$$  \hspace{1cm} (3.2)

$$\dot{r} = C(t) \left[ \dot{q} + \omega \times q \right]$$  \hspace{1cm} (3.3)

The vector $q$ is the position of the object with respect to the asteroid and $\dot{q}$ is the velocity of the object with respect to the asteroid. Vector $\omega$ corresponds to the rotation of the body with respect to the inertial frame of reference. $C(t)$ is the transformation matrix between the inertial reference frame and the body fixed reference frame. Substituting this into the Lagrangian of the system, the Lagrangian with respect to the body fixed frame of the asteroid can be determined.

$$L(q, \dot{q}, t) = \frac{1}{2} \left( C(t) \left[ \dot{q} + \omega \times q \right] \right) \cdot \left( C(t) \left[ \dot{q} + \omega \times q \right] \right) + U \left( C(t) \cdot q \right)$$  \hspace{1cm} (3.4)

Looking at the first term in Equation (3.4), the transformation matrix $C(t)$ would impact the vectors the same way, the dot product between them would remain the same no matter what $C(t)$ was. Therefore, $C(t)$ can be canceled out in the first term which leaves us with the Lagrangian formulated in the body fixed frame shown in Equation (3.5).

$$L(q, \dot{q}, t) = \frac{1}{2} \left[ \dot{q} + \omega \times q \right] \cdot \left[ \dot{q} + \omega \times q \right] + U \left( C(t) \cdot q \right)$$  \hspace{1cm} (3.5)

Thus, the equation of motion can be found by implementing the operation found
in Equation (3.1). By applying the distributive property to the established Lagrangian, it can be rewritten as:

\[ L = \frac{1}{2} [\dot{q} \cdot \dot{q}] + \dot{q} \cdot [\omega \times q] + \frac{1}{2} [\omega \times q] \cdot [\omega \times q] + U (C(t) \cdot q) \] (3.6)

By applying Equation (3.1), the equation of motion can be found as follows:

\[
\frac{\partial L}{\partial \dot{q}} = \ddot{q} + \omega \times q
\]

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}} \right) = \ddot{q} + \dot{\omega} \times q + \omega \times \dot{q}
\]

\[
\frac{\partial}{\partial q} \left[ \dot{q} \cdot [\omega \times q] \right] = \frac{\partial}{\partial q} \left[ q \cdot [\dot{q} \times \omega] \right] = \dot{q} \times \omega = -\omega \times \dot{q}
\]

\[
\frac{\partial}{\partial q} \left[ \frac{1}{2} [\omega \times q] \cdot [\omega \times q] \right] = \frac{\partial}{\partial q} \left[ \frac{1}{2} [\omega \times q] \right]^2
\]

\[
= [\omega \times q] \cdot \frac{\partial q}{\partial q} [\omega \times q] = \frac{\partial q}{\partial q} \cdot [\omega \times q \times \omega]
\]

\[
= 1 \cdot [\omega \times q \times \omega] = -\omega \times \omega \times q
\]

Therefore,

\[
\frac{\partial L}{\partial q} = -\omega \times \dot{q} - \omega \times \omega \times q
\] (3.7)

Finally, all the terms can be combined to form Equation (3.8).

\[
\therefore \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} = \ddot{q} + \dot{\omega} \times q + 2\omega \times \dot{q} + \omega \times \omega \times q - \frac{\partial U}{\partial q} = 0
\] (3.8)

\[
\ddot{q} + \dot{\omega} \times q + 2\omega \times \dot{q} + \omega \times \omega \times q = \frac{\partial U}{\partial q}
\] (3.9)

\( \frac{\partial u}{\partial q} \) is the derivative of the potential with respect to q which gives the potential force in the
Now, by restricting the problem to uniformly rotating asteroids, the equation of motion simplifies to:

\[ \ddot{q} + 2\omega \times \dot{q} + \omega \times \omega \times q = \frac{\partial U}{\partial q} \]  

(3.10)

In order for a point to be an equilibrium, a vehicle at the point shall not have any acceleration or velocity with respect to the asteroids body centered rotational frame of reference with respect to the vehicle. Therefore, the equation again simplifies to:

\[ \omega \times \omega \times q = \frac{\partial U}{\partial q} \]  

(3.11)

where \( \omega \times \omega \times q \) is the centripetal acceleration due to the rotation of the body fixed frame. For the purpose of this research, the gravitational potential of the system will be calculated using the polyhedral gravitational potential model as explained in section 2.1.10. Therefore, the full equation used to find the equilibrium positions in the system is:

\[ \omega \times \omega \times q = -G\sigma \left[ \sum_{e \in \text{edges}} E_e \cdot q_e L_e - \sum_{f \in \text{faces}} F_f \cdot q_f \omega_f \right] \]  

(3.12)

Equation (3.12) is the main equation that will be used throughout this paper. The equation equates the polyhedral gravitational model to the centripetal acceleration. The regions around the asteroids where this is satisfied are the equilibrium points of the system. After the equilibrium points are found, Equation (3.13) is used along with an optimization algorithm to determine the optimal orbital rate of a spacecraft maintaining a circular orbit around the asteroid. An acceleration is introduced to the equation which represents the acceleration the spacecraft would have to induce in order to satisfy the equation and maintain the circular orbit.
\[ \omega \times \omega \times q + G\sigma \left[ \sum_{e \in \text{edges}} E_e \cdot q_e L_e - \sum_{f \in \text{faces}} F_f \cdot q_f \omega_f \right] + a = 0 \] (3.13)

In Equations (3.12) and (3.13), \( \omega \) is the rate of rotation of the spacecraft’s orbital reference frame with respect to the asteroid body fixed inertial reference frame. \( q \) is the position vector of the spacecraft in the orbital reference frame. \( G \) is the universal gravitational constant. \( \sigma \) is the bulk density of the asteroid. \( E_e, F_f \) are the edge and face dyads respectively. \( \omega_f \) is the signed area of face \( f \) projected onto the unit sphere centered at point \( q \). The symbols \( q_e, q_f \) are the positions of the spacecraft with respect to the edge and the face respectively in the asteroids’ body fixed reference frame.

### 3.4 Equilibrium Determination Introduction

For the purposes of this research, a three dimensional optimization algorithm was used to find the equilibrium points. MATLAB’s built in optimization tool box was utilized to allow the code to converge on the equilibrium points. The objective of this search is to find points in the asteroid’s body fixed frame where the force required to compensate for the natural dynamics of the system is minimized. The dynamics used for this search are the gravitational force from the asteroid as well as the centripetal force from the asteroids’ body fixed frame rotation rate with respect to the inertial frame of reference. Any forces such as gravitational forces from other bodies and solar radiation pressure were not used in this calculation.

### 3.5 Static Optimization

A scalar performance index \( L(u) \) can be defined for the system. In order to find the minimum points in the performance index, the gradient must be found and set equal to zero. When these zeros are found, they could be maxima, minima, or inflection points.
In order to ensure that a minimum is found, the Hessian must be found. If the Hessian is positive definite, and the Gradient is equal to zero, the point is a local minimum. Through MATLABs built in tool box, the code will automatically look for a minimum depending on the input function. The parameters used in the MATLAB code are shown in Table 3.1 (Lewis, Vrabie, & Syrmos, 2012).

Table 3.1

MATLAB Optimization Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Assigned Command</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algorithm</td>
<td>sqp</td>
</tr>
<tr>
<td>MaxIterations</td>
<td>3000</td>
</tr>
<tr>
<td>OptimalityTolerance</td>
<td>$10^{-15}$</td>
</tr>
<tr>
<td>MaxFunctionEvaluations</td>
<td>2000</td>
</tr>
</tbody>
</table>

In order to use this method, a deep understanding of the system being studied is required. The algorithm takes in an initial condition and follows the gradient to the local minimum. Therefore, if the initial conditions tested in the system are never placed in the vicinity of one of the equilibria, the equilibrium will never be found. Figure 3.1 shows an example of the region where the global minimum would be found. If the initial condition is not started in the highlighted circle, the algorithm would converge somewhere else in the field.
If the initial condition is not placed in the region indicated in Figure 3.1, the algorithm will not converge to the local minimum which in this case is the global minimum. A way to work around this is to design a different algorithm such as a genetic algorithm which will test a region and then test another region in the space to check for better values.

### 3.6 Equilibrium Points

To show the process of equilibrium point determination, for this section a simplified mass model of the asteroid is used. The gravitational forces from three point masses as well as a centripetal acceleration from the rotation of the asteroid body fixed frame with respect to the asteroid centered inertial frame are the only accelerations taken into account. In order to model the gravitational acceleration from one of the point masses, the following equation was used:

\[
a_g = \frac{G M}{q^2} \hat{q}
\]  

(3.14)

G is the universal gravitational parameter; M is the mass of the sphere and q is the distance of the satellite with respect to the center of the sphere in the asteroid body fixed coordinate frame. The centripetal acceleration due to the rotation of the asteroid body
fixed frame with respect to the asteroid fixed inertial frame, previously determined in section 3, was used in these calculations as well.

\[ a_c = \omega \times \omega \times q \]  

(3.15)

By applying these equations to the approximated system, an acceleration field around the asteroid is produced as shown in Figure 3.1. The positions and sizes of the point mass approximations are shown in Table 3.2. The equilibrium point’s locations due to the approximations are shown in Table 3.3. The positions, densities and magnitudes were determined in order to mimic the results found in (Jiang, Baoyin, Li, & Li, 2013) for the asteroid Kleopatra.

Table 3.2

<table>
<thead>
<tr>
<th>MATLAB Optimization Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Point Mass</strong></td>
</tr>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>

These results do not accurately represent the potential field around the asteroid Kleopatra and should not be studied further. The purpose of their existence is for the visualization of the existence of the equilibrium points and the convergence of the optimization technique. Polyhedral results on the positioning of the equilibrium points around the bodies studied are stated in chapter 4.
Table 3.3

Simplified Equilibrium Points

<table>
<thead>
<tr>
<th></th>
<th>X Position (km)</th>
<th>Y Position (km)</th>
<th>Z Position (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point 1</td>
<td>141.9473</td>
<td>0</td>
<td>2.2334</td>
</tr>
<tr>
<td>Point 2</td>
<td>-1.2042</td>
<td>0</td>
<td>101.6429</td>
</tr>
<tr>
<td>Point 3</td>
<td>-145.3523</td>
<td>0</td>
<td>5.0125</td>
</tr>
<tr>
<td>Point 4</td>
<td>-3.2979</td>
<td>0</td>
<td>-98.6689</td>
</tr>
</tbody>
</table>


Equilibrium Manifold

In addition to having the Equilibrium points, there exists a field of acceleration around the asteroid where the acceleration of the spacecraft with respect to the asteroid body fixed frame is minimized. In the entire region shown, a 2000 kg spacecraft can hover over the asteroid with minimized force. This is called the equilibrium manifold. On the equilibrium manifold, the gravitational acceleration and the centripetal acceleration

Figure 3.2. Field of acceleration around equilibrium points for the three point mass approximation of asteroid 216 Kleopatra.

3.7 Equilibrium Manifold

In addition to having the Equilibrium points, there exists a field of acceleration around the asteroid where the acceleration of the spacecraft with respect to the asteroid body fixed frame is minimized. In the entire region shown, a 2000 kg spacecraft can hover over the asteroid with minimized force. This is called the equilibrium manifold. On the equilibrium manifold, the gravitational acceleration and the centripetal acceleration
cancel out in such a way that there is a lowered effort needed to maintain position. An important note is that the equilibrium manifold is naturally unstable, and in the absence of a restoring force the spacecraft will drift away from the desired position. An example of the force vectors around the equilibrium manifold is shown in Figure 3.3.

![Figure 3.3. Acceleration direction in Kleopatra’s body fixed reference frame](image)

An important note is that this is only the point mass approximation of the equilibrium band for hovering over the asteroid. Also, if the relative velocity of the spacecraft with respect to the asteroid body fixed frame is not zero, the equations of motion for the system cannot be simplified and the Coriolis and tangential accelerations would have to be taken into account as well.

### 3.8 Computational Burden

Initially, research was performed into understanding how the equilibrium points and the space around them act using the three point mass approximation for the gravitational field. Due to the limited calculations that were performed for each position in the field around the asteroid, these algorithms could run in only a few minutes and would generate a plane of results for the gravitational field around the asteroid as shown in Figures 3.2 and 4.3. When the analysis was expanded into the main focus of the
results, the polyhedral model for the gravitational field around the asteroids was used to ensure accuracy down to the surface of the asteroid as explained in section 2.1.10. Due to the increase in computational expense of using the polyhedral model, the codes took up to a week to produce the same results as Figures 3.2 and 3.3. By implementing MATLAB’s built in parallel computing tool “parfor” into the algorithms, it reduced the run time to approximately 11 hours using 4 cores. In order to generate 3D representations of the gravitational field around the asteroids, using the parallel computing tool and a super computer is recommended. In order to save computational expense, studies can be conducted comparing different gravitational models results outside of the Brillouin sphere as discussed in section 2.1.11.
4. Results

In this chapter, the results from the research performed will be presented and discussed. The results include polyhedral gravitational acceleration mapping of the space around the asteroids, positions of the equilibrium points, force required to maintain the spacecraft’s position in currently used mission design, as well as maintain an optimal circular orbit as a function of the radius. The analysis was performed for a 2000 kg spacecraft throughout this entire section to compare force requirements for the different mission designs.

4.1 Asteroid 216 Kleopatra

This section discusses all results gathered for asteroid Kleopatra.

4.1.1 Kleopatra Gravitational Model

The gravitational field of asteroid Kleopatra was developed with the parameters listed in Table 4.1.

Table 4.1

216 Kleopatra Polyhedral Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertices</td>
<td>2048</td>
</tr>
<tr>
<td>Edges</td>
<td>6138</td>
</tr>
<tr>
<td>Faces</td>
<td>4092</td>
</tr>
<tr>
<td>Bulk Density ( \frac{g}{\text{cm}^3} )</td>
<td>3.6</td>
</tr>
<tr>
<td>Rotation Rate ( \frac{\text{rad}}{s} )</td>
<td>(3.1733 \times 10^{-4})</td>
</tr>
</tbody>
</table>
Using the parameters in Table 4.1, the gravitational field in the asteroid body fixed XY plane was calculated at $z = 0$. In Figure 4.1, the regions inside the asteroid should be ignored as the only space that is important in this analysis is the external gravitational field.

Figure 4.1. Polyhedral gravitational field around 216 Kleopatra. The units for the color bar are $(\frac{km}{s^2})$. Data calculated in intervals of 1 kilometer.

4.1.2 Kleopatra Equilibrium Points

Figure 4.2 shows the field that the spacecraft would encounter if it were stationary in the asteroid body fixed frame of reference. Also, Table 4.2 shows the positions of the equilibrium points of the system based on the polyhedral gravitational model.

Equilibrium points 1 and 2 match closely with the results of analyses from other research papers in the past while points 3 and 4 are different. Due to the complexity of the
gravitational field model, Equilibrium point 4 was moved far off the asteroid body centered \( z = 0 \) plane. This could be caused by the existence of more than 4 equilibrium points in the space around Kleopatra due to the irregular shape.

Figure 4.2. Polyhedral field of acceleration around 216 Kleopatra. The units for the color bar are \( \text{\(\frac{km}{s^2}\)} \). Data calculated in intervals of 1 kilometer.

Table 4.2

216 Kleopatra Equilibrium Points

<table>
<thead>
<tr>
<th>Equilibrium Points</th>
<th>x (km)</th>
<th>y (km)</th>
<th>z (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>142.5091</td>
<td>2.6221</td>
<td>-1.8346</td>
</tr>
<tr>
<td>E2</td>
<td>-141.2245</td>
<td>5.5433</td>
<td>1.8541</td>
</tr>
<tr>
<td>E3</td>
<td>7.6437</td>
<td>-93.3742</td>
<td>.9297</td>
</tr>
<tr>
<td>E4</td>
<td>1.3695</td>
<td>121.0138</td>
<td>22.9255</td>
</tr>
</tbody>
</table>
Looking back at section 3.6, Figure 3.2 is an example of a simplified field of acceleration around the asteroid where all gravitational forces are planar. The purpose was to explain the existence of the equilibrium points by making them visible in one view. In Figure 4.2, the polyhedral model of asteroid 216 Kleopatra, the equilibrium points are not planar, so they are not visible in the figure; however, the relative positions where the field is starting to converge to the equilibrium points are evident in the darker blue regions. The force requirements for a 2000 kg spacecraft to remain at each equilibrium point that was determined through optimization is listed in Table 4.3.

Table 4.3

*Kleopatra Equilibrium Maintenance*

<table>
<thead>
<tr>
<th>Equilibrium Points</th>
<th>Required Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>$8.4202 \times 10^{-6}$</td>
</tr>
<tr>
<td>E2</td>
<td>$2.8657 \times 10^{-5}$</td>
</tr>
<tr>
<td>E3</td>
<td>$2.5956 \times 10^{-5}$</td>
</tr>
<tr>
<td>E4</td>
<td>.6018</td>
</tr>
</tbody>
</table>

The required forces are different due to the tolerances set on the convergence of the optimization algorithm. Equilibrium point 4 is the most difficult to maintain due to a larger difference in the effort requirements as a spacecraft diverges from the point. The reason for the larger value for equilibrium point 4 is because of the step size tolerance that was set. Based on how rapidly the space around the equilibriums differs from equilibriums themselves, the optimality tolerance will be satisfied at different distances away from the true equilibrium. The optimality settings can be found in section 3.5. The equilibrium points of Kleopatra found in modeling in the past are listed in Table 4.4.
force required to maintain those positions based on the gravitational model developed for this paper are listed in Table 4.5 (Jiang, Baoyin, H., Li, & Li, 2013).

Table 4.4

*Kleopatra Equilibrium Points (Jiang, Baoyin, Li, & Li, 2013)*

<table>
<thead>
<tr>
<th>Equilibrium Points</th>
<th>x (km)</th>
<th>y (km)</th>
<th>z (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>142.852</td>
<td>2.45436</td>
<td>1.8008</td>
</tr>
<tr>
<td>E2</td>
<td>-144.684</td>
<td>5.18855</td>
<td>-.282998</td>
</tr>
<tr>
<td>E3</td>
<td>2.21701</td>
<td>-102.102</td>
<td>.279703</td>
</tr>
<tr>
<td>E4</td>
<td>-1.16396</td>
<td>100.738</td>
<td>-.531516</td>
</tr>
</tbody>
</table>

Table 4.5

*Kleopatra Reference Paper Equilibrium Maintenance*

<table>
<thead>
<tr>
<th>Equilibrium Points</th>
<th>Required Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>0.927</td>
</tr>
<tr>
<td>E2</td>
<td>2.997</td>
</tr>
<tr>
<td>E3</td>
<td>4.767</td>
</tr>
<tr>
<td>E4</td>
<td>11.396</td>
</tr>
</tbody>
</table>

Note how the force required to maintain these positions is larger than the force required to maintain the points found in the optimization technique in Table 4.3.

Although the past papers used a polyhedral model for their calculations, the number of vertices used is unclear and the model used for this paper is the most refined model available for Kleopatra on the PDS website.

4.1.3 Non-Orbiting Proximity Operations around Kleopatra

During the Hayabusa missions, the spacecraft maintained the Earth and Sun pointing direction around the asteroids. If that same mission plan was used for a mission
to Kleopatra, the average force required to maintain the position of the spacecraft is shown in Figure 4.3. The red region on the figure is where the orbit would intersect the surface of the asteroid. In order to determine the average force requirement throughout the orbit, the force required to maintain a set distance away from the center of rotation of the asteroid was calculated at an interval of 1 degree throughout the entire field. The reason for determining the average of the gravitational potential around the entire asteroid is because the asteroid is still rotating with respect to the inertial frame of reference while the vehicle is not. Because of the difference in the rotational rate of the two bodies, the spacecraft will experience all 360 degrees longitude around the gravitational field at some point during the mission. The force required to maintain the orbit 114 km away from the center of mass of the asteroid throughout one asteroid revolution is shown in Figure 4.4.

![Figure 4.3. Average force required to maintain distance away from the center of mass of Kleopatra in Earth/Sun pointing direction.](image-url)
In order to compare circular orbits in proximity of the asteroid to the proximity operations used currently, Figure 4.7 shows the average force required to maintain the optimal body fixed orbital frame rotation rate as a function of the radius of the orbit. The red regions on the figures are the orbits that would intersect Kleopatra. All of the data shown were developed using the same optimization scheme as discussed in section 3.5, while varying the rate of rotation of the orbital frame of the spacecraft to minimize the force requirement. Figures 4.5 – 4.7 are all related so if a spacecraft were to maintain a circular orbit at any given radius about the asteroid’s center of mass, the optimal orbit rate, orbital period, as well as average force required is displayed.

*Figure 4.4.* Force required to maintain position 114 km away from the center of mass throughout one revolution of Kleopatra.
**Figure 4.6.** Rotation rates of the optimal circular orbits around Kleopatra as a function of the radius of the orbit.

**Figure 4.5.** Orbital Periods of the optimal circular orbits around Kleopatra as a function of the radius of the orbit.
Figure 4.7. Average force required for the optimal circular orbits around Kleopatra as a function of the radius of the orbit.

Figure 4.6 shows that the optimal orbital period for touching the surface of the asteroid is approximately 5.698 hours. An interesting result is that this orbital rate did not reach the natural rotational rate of the asteroid. This means that, in this circular mission design for any spacecraft with the goal of landing on the asteroid Kleopatra, the circular orbit provides the minimum average force is at a rotation less than the rotation of the asteroid. If the mission of the spacecraft was to maintain an orbit around the asteroid, the equilibrium points would minimize force over time; however, the equilibrium points and the velocities required to maintain them are not in the path of the optimal circular orbit design. Note that the results shown are the optimal orbit results throughout the entire field around the asteroid. Due to the similar rates of rotation of the spacecraft’s orbital frame and the asteroid’s body fixed frame, given limiting factors and time requirements, it is possible to find better rotation rates to minimize fuel for specific segments of the space if
analyzed separately.

The maximum average force can be found by looking at the orbital designs when the spacecraft is right above the surface of the asteroid. A circular orbit about the rotational axis of Kleopatra with a radius of 114 km has an optimal orbital period of 5.698 hours. At this rate, the average force required to maintain the orbit is 15.7 N. Figure 4.7 shows the force required throughout the period of one orbit in the asteroids body fixed frame of reference at the radius of 114 km.

Looking at Figure 4.8, a spacecraft with the goal of landing on Kleopatra by maintaining the optimal circular orbit at the radius of 114 km could achieve the touch down safely with approximately 40.4 N of maximum thrust. Because of the asteroid’s irregular shape and elongation, there is a significant difference between the maximum and minimum thrust to maintain a circular orbit.

![Image](image_url)

*Figure 4.8. Force required to maintain the optimal circular orbit for one orbital period at a radius \( r = 114 \) km around Kleopatra.*
4.1.5 Kleopatra Results Comparison

We can compare the different proximity operations to determine the feasibility of using a circular orbit for specific parts of the mission as opposed to maintaining the Earth/Sun facing direction. Figure 4.9 shows the two results on the same graph and Figure 4.10 shows the percent savings that the spacecraft in the optimal circular orbit on the asteroids $z = 0$ plane could achieve. Note that because of the complexity of the polyhedral model, it is highly likely that the optimal orbit is not on the asteroids body fixed $z = 0$ plane, however, there are still significant cost savings.

![Graph showing percentage savings for using the circular orbit around Kleopatra.](image)

*Figure 4.9. Percentage savings for using the circular orbit around Kleopatra.*
When looking for fuel savings of utilizing a circular orbit around asteroid Kleopatra, it is important to consider that the instruments used in current asteroid missions operate within 50 km of the target. Due to current instrument constraints, compare the force requirements of the two orbital design methods up to a radius of 164 km. At the 164 km mark, the current method of staying on the Earth/Sun facing side of the asteroid would require an average force of 12.4 N. The circular orbit at this radius would cost an average of 7.7 N to maintain. Also, as the spacecraft gets closer to the surface of the asteroid, the circular orbital plan for the spacecraft continues to increase the fuel savings. Inside of 50 km away from the surface of the asteroid, the circular orbit design provides over 30% savings. These results support the conclusion that inserting a spacecraft into a circular orbit can have a significant reduction in the force requirement to perform the mission. Figure 4.11 shows the cost to maintain the two proximity operations.
methods. Due to the tradeoff through the entire space that the optimality algorithm must consider, and the highly irregular shape of asteroid Kleopatra, the optimal circular orbit has a higher force requirement during some parts of the orbit but has an overall savings.

Another benefit in using the optimal circular orbit design around asteroid Kleopatra is the velocity of the spacecraft and the surface of the asteroid during the landing procedure. Reducing the relative velocities could lead to more precise and accurate landing. The difference in the angular rate between the optimal circular orbit at the surface of Kleopatra and the actual rate of rotation of the asteroid is $3.063 \times 10^{-4} \text{ rad s}^{-1}$ where the asteroids rate of rotation is $3.1733 \times 10^{-4} \text{ rad s}^{-1}$. This corresponds to an optimal circular orbit that would have a relative velocity with respect to the surface of $1.26 \text{ m s}^{-1}$ as opposed to the 36.18 m s$^{-1}$ that the currently used Earth/Sun pointing approach would provide.

*Figure 4.11.* Force required to maintain the respective proximity operations as a function of the longitude of asteroid Kleopatra.
Issues with using the circular orbit approach are that it would require more precise knowledge of the relative position of the spacecraft and it would require a more complex algorithm to change the orbital radius while maintaining the circularity of the orbit. As the rotation rate and the radius of the orbit change, the equations of motion in the rotating frame cannot be simplified. The equation of motion for the system when changing the orbit turns back into Equation (3.9). By performing the right-hand rule on the added terms assuming that the orbital radius is decreasing, we can see that the introduced tangential and Coriolis accelerations act in opposite directions based on the change in the optimal circular orbit data as the radius is changed. The direction of the sum of these accelerations assuming a circular orbit cannot be determined without a time relation of the changes. Figure 4.12 is a visualization of the right-hand rule in the rotational frame of the spacecraft with respect to the center of mass of the asteroid. The dotted arrows are the direction of the tangential and Coriolis accelerations to show that they oppose each other.
There are several times throughout the orbit that the natural forces from the system can counteract them in order to lower the orbital radius while in fact decreasing force requirements to maintain the orbit. The positions of the natural descent assistance would vary based on many parameters, but they should be noted. Although these times exist, they may not provide enough of a change each orbit to get to the surface of the asteroid in a reasonable time. Therefore, this introduces a trade off with using the circular orbit technique. The faster the mission calls for the spacecraft to get to the surface of the asteroid, the more force is needed to counteract the tangential and Coriolis accelerations in the rotational frame of reference. This is a tradeoff that will not be studied further in

\[
\begin{align*}
\dot{\omega} \times q & \quad + \quad 2\omega \times \dot{q} \\
\end{align*}
\]
this analysis due to the magnitudes of the tangential and Coriolis accelerations dependence on time.

4.2 Asteroid 101955 Bennu

Asteroid Bennu is the target of the OSIRIS-REx mission. These results provide insight into force requirements for orbiting a small relatively spherical body like Bennu.

4.2.1 Bennu Gravitational Model

The gravitational field of asteroid Bennu was developed with the parameters listed in Table 4.6.

Table 4.6

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertices</td>
<td>1348</td>
</tr>
<tr>
<td>Edges</td>
<td>4038</td>
</tr>
<tr>
<td>Faces</td>
<td>2692</td>
</tr>
<tr>
<td>Bulk Density ( \frac{g}{cm^3} )</td>
<td>1.26</td>
</tr>
<tr>
<td>Rotation Rate ( \frac{rad}{s} )</td>
<td>( 4.061^{-4} )</td>
</tr>
</tbody>
</table>

Using the parameters in Table 4.6, the gravitational field in the asteroid body fixed XY plane was calculated at \( z = 0 \). The results of the calculation are shown in Figure 4.13. Note that the data taken from the PDS website have a few vertex concentration points for the polyhedral gravitational model, one of which can be seen in the figure. The units for the data displayed in the figure are \( \frac{k m}{s^2} \). The regions inside the asteroid should be ignored as the only space that is important in this analysis is the
external gravitational field. The internal forces should be ignored as the spacecraft will not experience them.

Figure 4.13. Polyhedral gravitational field around Bennu. The units for the color bar are \( \frac{\text{km}}{s^2} \). Data calculated in intervals of 5 meters.

4.2.2 Bennu Equilibrium Points

Figure 4.14 shows the field that the spacecraft would encounter if it were stationary in the asteroid body fixed frame of reference. The equilibrium points of Bennu were not found because the relatively spherical shape of the asteroid causes an equilibrium band, also evident in Figure 4.14. Notice that the equilibrium band of Bennu sits on its surface. This means that a spacecraft would have difficulty landing in the region of the band due to the lack of acceleration towards the asteroid. This could make sample collection easier as the samples, no matter the size, would essentially be weightless in this region. A potential danger is that debris could be in a very low orbit.
around the asteroid; therefore, attempting a sample collection in the area could cause damage to the spacecraft.

**Figure 4.14.** Polyhedral field of acceleration around Bennu. The units for the color bar are \( \frac{\text{km}}{\text{s}^2} \). Data calculated in intervals of 5 meters.

### 4.2.3 Non-Orbiting Proximity Operations around Bennu

The average effort required to maintain the position of the spacecraft is shown in Figure 4.15. The calculations for asteroid Bennu are the same as discussed in Section 4.1.3. The red region on the figure is where the orbit would intersect the surface of the asteroid. The force required to maintain the orbit .3 km away from the center of mass of the asteroid throughout one asteroid revolution is shown in Figure 4.16.
Figure 4.15. Average force required to maintain distance away from the center of mass of Bennu in Earth/Sun pointing direction.

Figure 4.16. Force required to maintain position .3 km away from the center of mass throughout one revolution of Bennu.
4.2.4 Optimal Circular Orbit Results around Bennu

In order to compare circular orbits in proximity of the asteroid to the proximity operations used currently, Figure 4.19 shows the average force required to maintain the optimal body fixed orbital frame rotation rate as a function of the radius of the orbit. The red regions on the figures are the orbits that would intersect Bennu. All of the data shown were developed using the same optimization scheme as discussed in section 3.5, while varying the rate of rotation of the orbital frame of the spacecraft to minimize the force requirement. Figures 4.17 – 4.19 are all related so if a spacecraft were to maintain a circular orbit at any given radius about the asteroid’s center of mass, the optimal orbit rate, orbital period, as well as average force required to maintain the optimal orbit are displayed.

*Figure 4.17. Rotation rates of the optimal circular orbits around Bennu as a function of the radius of the orbit*
Figure 4.18. Orbital Periods of the optimal circular orbits around Bennu as a function of the radius of the orbit.

Figure 4.19. Average force required for the optimal circular orbits around Bennu as a function of the radius of the orbit.
Figure 4.18 shows that the optimal orbital period for touching the surface of the asteroid is approximately 5.079 hours. An interesting result is that this orbital rate did not reach the natural rotational rate of the asteroid. This means that in this circular mission design for any spacecraft with the goal to land on the asteroid Bennu, the circular orbit that provides the minimum average force is at a rotation less than the rotation of the asteroid. Note that the results shown are the optimal orbit results throughout the entire field around the asteroid. Due to the similar rates of rotation of the spacecraft’s orbital frame and the asteroid’s body fixed frame, given limiting factors and time requirements, it is possible to find better rotation rates to minimize fuel for specific segments of the space if analyzed separately.

The maximum average force can be found by looking at the orbital designs when the spacecraft is right above the surface of the asteroid. A circular orbit about the rotational axis of Bennu with a radius of .3 km has an optimal orbital period of 5.079 hours. At this rate, the average force required to maintain the orbit is .0088 N. Figure 4.20 shows the force required throughout the period of one orbit in the asteroid’s body fixed frame of reference at the radius of .3 km.
Looking at Figure 4.20, a spacecraft with the goal of landing on Bennu by maintaining the optimal circular orbit at the radius of .3 km could achieve the touch down safely with approximately .02 N of maximum thrust. A few reasons for the force requirement being so low is that the asteroid is relatively small; therefore, the gravitational field around it is weak and the surface in the region is the equilibrium band.

**4.2.5 Bennu Results Comparison**

Looking at the results of using a circular orbit as opposed to not orbiting the asteroid, we can compare them to determine the feasibility of using a circular orbit for the mission. Figure 4.21 shows the two results on the same graph. Figure 4.22 shows the percent savings that the spacecraft in the optimal circular orbit on the asteroid’s \( z = 0 \) plane.
Figure 4.21. Percentage savings for using the circular orbit around Bennu.

Figure 4.22. Average force requirement comparison for the different orbital designs around Bennu.
During the OSIRIS-REx mission to Bennu, the mission plan contained three relatively long-term orbits with radii of approximately 1 km, 1.5 km and 2 km. At a radius between 1.6 and 2.1 km, the orbital period was 61.4 hours. Based on the optimal circular orbit around asteroid Bennu based on the 1348 vertex polyhedral gravitational potential model, the optimal orbital period range for radii between 1.6 and 2.1 km is 70.9 and 106.7 hours. The range is relatively close to the orbital period used for the OSIRIS-REx mission, and the discrepancies could be due to external affects from other bodies, different orbital placement, or the mission did not use the optimal orbit for savings. Orbital data are not available for the other orbits in the mission plan. Figure 4.23 shows the cost to maintain the two proximity operations methods. Due to less of a tradeoff through the entire space that the optimality algorithm must consider, and the more spherical shape of asteroid Bennu as opposed to Kleopatra and Bacchus, the optimal circular orbit always has a lower force requirement than the Earth/Sun pointing proximity operations.

![Graph showing force required to maintain the respective proximity operations as a function of the longitude of asteroid Bennu.](image)

*Figure 4.23.* Force required to maintain the respective proximity operations as a function of the longitude of asteroid Bennu.
Comparing the Earth/Sun facing proximity operations mission approach to the optimal circular orbit data, operations at the 2 km radius have an estimated cost savings of 17% and the 1.5 km radius has an estimated cost savings of 27%. Another advantage is that there would be a reduction in the relative velocity of the spacecraft and the surface of the asteroid during the landing procedure. This could lead to more precise landing locations. What this corresponds to is the optimal circular orbit would have a relative velocity with respect to the surface of $0.019 \, \text{m/s}$ as opposed to the $0.1218 \, \text{m/s}$ that the Hayabusa mission approach of maintaining the spacecraft’s position on the Earth/Sun facing direction would provide.

Another interesting result comes from comparing the percentage savings plots from the three asteroids. Due to the more spherical shape of Bennu, the field of acceleration around it is more radially uniform than Kleopatra or Bacchus. This results in much better savings near the surface and a smaller force requirement distribution over the course of an orbit.

4.3 Asteroid 2063 Bacchus

Asteroid Bacchus is important in this analysis because it is an elongated asteroid like Kleopatra, yet small in size and mass like Bennu.

4.3.1 Bacchus Gravitational Model

The gravitational field of asteroid Bacchus was developed with the parameters listed in Table 4.7.
Using the parameters in Table 4.7, the gravitational field in the asteroid body fixed XY plane was calculated at \( z = 0 \). The results are shown in Figure 4.24. The regions inside the asteroid should be ignored as the only space that is important in this analysis is the external gravitational field.

**Table 4.7**

*Bacchus Polyhedral Parameters*

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertices</td>
<td>2048</td>
</tr>
<tr>
<td>Edges</td>
<td>6138</td>
</tr>
<tr>
<td>Faces</td>
<td>4092</td>
</tr>
<tr>
<td>Bulk Density ( \left( \frac{g}{cm^3} \right) )</td>
<td>3.3</td>
</tr>
<tr>
<td>Rotation Rate ( \left( \frac{rad}{s} \right) )</td>
<td>( 1.2 \times 10^{-4} )</td>
</tr>
</tbody>
</table>

*Figure 4.24.* Polyhedral gravitational field around Bacchus. The units for the color bar are \( \left( \frac{km}{s^2} \right) \). Data calculated in intervals of 15 meters.
4.3.2 Bacchus Equilibrium Points

Figure 4.25 shows the field that the spacecraft would encounter if it were stationary in the asteroid body fixed frame of reference. The units for the color bar are \( \left( \frac{km}{s^2} \right) \). Due to the low rotation rate of the asteroid, the equilibrium points have not fully developed, and they do not provide a significant advantage over the space around them. The natural equilibrium points around asteroid Bacchus are listed in Table 4.8. The force requirements for a 2000 kg spacecraft to remain at each equilibrium point is listed in Table 4.9.

Figure 4.25. Polyhedral field of acceleration around Bacchus. The units for the color bar are \( \left( \frac{km}{s^2} \right) \). Data calculated in intervals of 15 meters.
Table 4.8

*Bacchus Equilibrium Points*

<table>
<thead>
<tr>
<th>Equilibrium Points</th>
<th>Required Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>1.002 x 10^{-5}</td>
</tr>
<tr>
<td>E2</td>
<td>1.180 x 10^{-2}</td>
</tr>
<tr>
<td>E3</td>
<td>2.376 x 10^{-4}</td>
</tr>
<tr>
<td>E4</td>
<td>2.626 x 10^{-8}</td>
</tr>
</tbody>
</table>

Table 4.9

*Bacchus Equilibrium Maintenance*

<table>
<thead>
<tr>
<th>Equilibrium Points</th>
<th>x (km)</th>
<th>y (km)</th>
<th>z (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>1.058</td>
<td>0.758</td>
<td>-0.253</td>
</tr>
<tr>
<td>E2</td>
<td>-1.393</td>
<td>0.225</td>
<td>-0.425</td>
</tr>
<tr>
<td>E3</td>
<td>-0.062</td>
<td>-0.436</td>
<td>0.002</td>
</tr>
<tr>
<td>E4</td>
<td>-0.379</td>
<td>1.031</td>
<td>-0.269</td>
</tr>
</tbody>
</table>

Note how the force required to maintain these positions is larger than the force required to maintain the points found in the optimization technique.

### 4.3.3 Non-Orbiting Proximity Operations around Bacchus

The average effort required to maintain the position of the spacecraft is shown in Figure 4.26. The calculations for asteroid Bennu are the same as discussed in Section 4.1.3. The red region on the figure is where the orbit would intersect the surface of the asteroid. The force required to maintain the orbit .6 km away from the center of mass of the asteroid throughout one asteroid revolution is shown in Figure 4.27.
Figure 4.26. Average force required to maintain distance away from the center of mass of Bacchus in Earth/Sun pointing direction.

Figure 4.27. Force required to maintain position .6 km away from the center of mass throughout one revolution of Bacchus.
4.3.4 Optimal Circular Orbit Results around Bacchus

In order to compare circular orbits in proximity of the asteroid to the proximity operations used currently, Figure 4.30 shows the average force required to maintain the optimal body fixed orbital frame rotation rate as a function of the radius of the orbit. The red regions on the figures are the orbits that would intersect Bacchus. All the data shown was developed using the same optimization scheme as discussed in section 3.5, while varying the rate of rotation of the orbital frame of the spacecraft to minimize the force requirement. Figures 4.28 – 4.30 are all related so if a spacecraft were to maintain a circular orbit at any given radius about the asteroids center of mass, the optimal orbit rate, orbital period, as well as average force required to maintain the optimal orbit are displayed.

![Figure 4.28](image.png)

*Figure 4.28. Rotation rates of the optimal circular orbits around Bacchus as a function of the radius of the orbit.*
Figure 4.29. Orbital Periods of the optimal circular orbits around Bacchus as a function of the radius of the orbit.

Figure 4.30. Average force required for the optimal circular orbits around Bacchus as a function of the radius of the orbit.
Figure 4.29 shows the optimal orbital period for touching the surface of the asteroid. This means that the circular orbit about the rotational axis of Bacchus with a radius of .6 km has an optimal orbital period of 5.272 hours. At this rate, the average force required to maintain the orbit is .104 N. Figure 4.31 shows the force required throughout the period of one orbit at the radius of .6 km. An interesting result is that in this case, the orbital rate exceeded the natural rotational rate of the asteroid. If the mission of the spacecraft was to maintain an orbit around the asteroid, the equilibrium points would minimize force over time, however, the equilibrium points and the velocities required to maintain them are not in the path of the optimal circular orbit design. Note that the results shown are the optimal orbit results throughout the entire field around the asteroid. Due to the similar rates of rotation of the spacecraft’s orbital frame and the asteroid’s body fixed frame, given limiting factors and time requirements, it is possible to find better rotation rates to minimize fuel for specific segments of the space if analyzed separately.

The maximum average force can be found by looking at the orbital designs when the spacecraft is right above the surface of the asteroid. A circular orbit about the rotational axis of Bacchus with a radius of .6 km has an optimal orbital period of 5.272 hours. At this rate, the average force required to maintain the orbit is 0.104 N. Figure 4.31 shows the force required throughout the period of one orbit in the asteroids body fixed frame of reference at the radius of .6 km.
Looking at Figure 4.31, a 2000 kg spacecraft with the goal of landing on Bacchus by maintaining the optimal circular orbit at the radius of .6 km could achieve the touch down safely with approximately .21 N of maximum thrust.

4.3.5 Bacchus Results Comparison

Looking at the results of using a circular orbit as opposed to not orbiting the asteroid, we can compare them to determine the feasibility of using a circular orbit for the mission. Figure 4.32 shows the two results on the same graph. Figure 4.33 shows the percent savings that the spacecraft in the optimal circular orbit on the asteroids $z = 0$ plane.
Figure 4.32. Average force requirement comparison for the different orbital designs around Bacchus.

Figure 4.33. Percentage savings for using the circular orbit around Bacchus.
There is a significant case that can be made for a circular orbital design near the asteroid. An interesting result with asteroid Bacchus is that the difference in the angular rate between the optimal circular orbit at the surface of Bacchus and the actual rate of rotation of the asteroid is $2.11 \times 10^{-4} \text{ rad/s}$, where the asteroid’s rate of rotation is $1.2 \times 10^{-4} \text{ rad/s}$. This means that the asteroid is rotating too slow for there to be a landing accuracy benefit in the optimal circular orbit. The rotation rates correspond to a relative velocity with respect to the surface of $1.16 \text{ m/s}$ when using the circular orbit approach as opposed to the $0.0678 \text{ m/s}$ that maintaining Earth/Sun pointing direction would provide. Although this benefit does not exist in Bacchus’ case, the savings could still be enough to justify the use of the optimal circular orbit for other parts of the mission other than the touch down.

The Coriolis and tangential accelerations that are introduced into the system when changing the orbit in the circularized case would have to be studied further to determine the feasibility. Figure 4.23 shows the cost to maintain the two proximity operations methods. Due to the tradeoff through the entire space that the optimality algorithm must consider, and the elongated shape of asteroid Bacchus, the optimal circular orbit has a larger force requirement at certain points in the orbit than the Earth/Sun pointing proximity operations but still has an overall significant cost savings.
Figure 4.34. Force required to maintain the respective proximity operations as a function of the longitude of asteroid Bacchus.
5. Conclusion

5.1 Contributions

Based on the polyhedral model for the asteroids used in this analysis, the equilibrium points positions in the body fixed reference frames were determined around asteroids 216 Kleopatra, and 2063 Bacchus by varying the position of the spacecraft to minimize the force required to maintain the position. Also, an equilibrium band was shown on the surface of Bennu. The forces required to maintain a 2000 kg spacecraft at the equilibrium points were calculated and discussed. The same optimization algorithm was then used to determine the optimal circular orbits around the asteroids in their $z = 0$ XY planes. In the optimal circular orbit analyses, the rotation rate of the spacecraft’s orbital frame of reference was varied to minimize the force required to maintain the position. A common proximity operation used for asteroid missions is to maintain the spacecraft on the Earth/Sun facing side of the asteroid. The force required to maintain those operations as a function of the distance from the asteroid was also calculated for comparison.

The force requirements for asteroid Kleopatra, there was a maximum percent fuel savings near the surface of 41.7% which slightly dipped before reaching the surface of the asteroid. In addition to the potential savings, the relative velocity of the spacecraft with respect to the surface of the asteroid for utilizing the optimal circular orbit was also significantly reduced as opposed to maintaining the Earth/Sun pointing direction.

For asteroid Bennu, there was a maximum percent fuel savings near the surface of 87.7% at the surface of the asteroid. Also, for asteroid Bennu, the relative velocity of the spacecraft with respect to the surface of the asteroid for utilizing the optimal circular orbit
was also significantly reduced as opposed to maintaining the Earth/Sun pointing direction.

For asteroid Bacchus, there was a maximum percent fuel savings near the surface of 43.3% which slightly dipped before reaching the surface of the asteroid. Unlike the other asteroids, the relative velocity of the spacecraft with respect to the surface of the asteroid for utilizing the optimal circular orbit was increased as opposed to maintaining the Earth/Sun pointing direction. This is due to the low rate of rotation.

The circular orbit approach to mission planning provided significantly less control input requirements around all asteroids. The improvement was maximized around the circular asteroid Bennu with less improvement around non-spherical ones. In addition, the reduction in relative velocity with respect to the surface of asteroids Kleopatra and Bennu is purely a coincidence due to their natural rotation rates. As discussed in Section 4.1.5, a trade off with the control effort required to change the circular orbit and the time it takes to reach the surface of the asteroids will cause a reduction in the fuel savings found in this thesis. For missions that utilize long slow altitude changes with respect to the asteroids, the optimal circular orbits proposed can potentially have a significant fuel savings.

Other factors to consider would be that there would be less communication opportunities for a spacecraft in an orbit in the asteroids rotational plane, the spacecraft would potentially have lower power consumption due to eclipse times from the asteroid, and changing from one optimal circular orbit to another would also cause the relative angular velocity of the spacecraft with respect to the body fixed frame of the asteroid to change as well. Overall, there is a case that can be made for utilizing circular orbit
mission designs for proximity operations around uniformly rotating asteroids.

5.2 Future Work

5.2.1 Introduce External Effects

The analysis performed in this work only considered the gravitational force of the asteroid and centripetal force of the rotational frame of reference of the satellite. This analysis can be expanded to include solar effects as well as how sensitive the results are to other major bodies in the system.

5.2.2 Change in Circular Orbit

In order to continue determining the feasibility of the use of circular orbits in highly perturbed gravitational environments, the tangential and Coriolis forces that are introduced into the system when the orbit is being changed can be studied. The magnitudes of these forces are dependent on the time constraint of the mission due to the rates of change of the orbit and the rate of rotation terms. Additionally, it is possible to develop an analysis into what parts of the orbit the natural force of the system could assist in these changes and look into how long it would take to naturally be pulled towards the surface of the asteroid while still maintaining the optimal circular orbit as a function of the current orbital radius.

5.2.3 Controller Development

In addition to the dynamical analyses that can be performed on the system, a spacecraft can be modeled in the field of the asteroids to start studying the best controllers. Some factors could be not knowing the gravitational environment very well, and state estimation around asteroids.

5.2.4 Equilibrium Manifold Study

The space around the asteroids does not contain only a few points where control is
reduced. A study can be performed into how the natural rotation of the asteroids affects how much space a spacecraft can maintain in the asteroids’ body fixed frames of reference with a maximum control requirement. This could help determine a rotation rate limit based on shape and density in order to narrow down the best targets for asteroid missions in the future.

5.2.5 Compare Gravitational Modeling

The different gravitational models that are currently used for asteroid analyses can be compared to potentially determine a more reliable boundary where a simplified gravitational model of the asteroid is good enough to perform missions. This could help provide insight into how close to the asteroid the spacecraft can get while saving computational expense for the mission.

5.2.6 Image Based Navigation

Image based navigation methods can be studied to determine if there is a consistent correlation between the shape of the asteroid and how perturbed the gravitational field is. Also, this could be taken in another direction and asteroid feature recognition can be used to help the spacecraft determine more accurate relative positioning in order to make the use of circular orbits in highly perturbed gravitational environments more feasible for future missions.

5.2.7 Algorithm Design to Maintain Circularity

In the complex gravitational field of asteroids, if the field is not known exactly, and a spacecraft must rely on sensor data to update the control inputs, the system may be perturbed from its orbital circularity. If this happens without the algorithms knowing, the tangential and Coriolis forces could compound and cause significant and potentially
dangerous changes to the orbital path of the spacecraft. Developing an adaptive algorithm
designed to handle these highly perturbed and relatively unknown gravitational fields
could be another path of study.

5.3 Concluding Remarks

The conclusions found in this thesis pave the way for many additional research
paths as well as providing insight into the feasibility of using circular orbit designs for
proximity operations during asteroid sample return missions. Although more
complexities must be studied, the results look promising for providing a way to plan
asteroid missions so that more weight can be used for other subsystems of the spacecraft
instead of requiring more fuel to perform the same mission.
REFERENCES


Jiang, Y., Baoyin, H., Li, J., & Li, H. (2013). Orbits and manifolds near the equilibrium points around a rotating asteroid. *Astrophysics and Space Science, 349*(1), 83-106


