Control Driven Scaling Effects of Motor and Rotors for Urban Air Mobility Design

Agustin A. Giovagnoli

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CONTROL DRIVEN SCALING EFFECTS OF MOTOR
AND ROTORS FOR URBAN AIR MOBILITY DESIGN

A Thesis Submitted to the Faculty
of Embry-Riddle Aeronautical University
by
Agustin A. Giovagnoli

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of
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CONTROL DRIVEN SCALING EFFECTS OF MOTOR
AND ROTORS FOR URBAN AIR MOBILITY DESIGN

by

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A Thesis prepared under the direction of the candidate’s committee chairman, Dr. Richard Anderson, Department of Aerospace Engineering, and has been approved by the members of the thesis committee. It was submitted to the School of Graduate Studies and Research and was accepted in partial fulfillment of the requirements for the degree of Master of Science in Aerospace Engineering.

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DEDICATION

To the memory of my beloved grandfather Eduardo L. Raviola, who always supported me and taught me to keep my head up.
AKNOWLEDGEMENTS

Firstly, I wish to thank my parents, Mariela and Jorge, and family who have supported me in all aspects since my first steps in my aviation career. All my accomplishments would have not been possible without them by my side. I want to thank my committee chair, Dr. Anderson, for the opportunity to work at the Eagle Flight Research Center, allowing me to combine my engineering knowledge and pilot experience. This lead me to support very interesting projects like this thesis, which helped me become more proficient in my field and to learn even more about aviation. In addition, I would also like to thank Dr. Steven Daniel and my friend Sergio Bacca for sharing their knowledge and guide me through some stages of my research. Also, I am very grateful with Dr. J. Gordon Leishman for guiding and introduce me to rotorcraft aerodynamics.

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### SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega$</td>
<td>Rotor rotational speed</td>
</tr>
<tr>
<td>$U$</td>
<td>Resultant flow velocity</td>
</tr>
<tr>
<td>$U_T$</td>
<td>Tangential velocity component</td>
</tr>
<tr>
<td>$U_R$</td>
<td>Radial velocity component</td>
</tr>
<tr>
<td>$U_p$</td>
<td>Perpendicular flow velocity component</td>
</tr>
<tr>
<td>$\theta_b$</td>
<td>Collective pitch angle</td>
</tr>
<tr>
<td>$\phi_\lambda$</td>
<td>Inflow angle of attack</td>
</tr>
<tr>
<td>$J$</td>
<td>Inertia matrix</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass</td>
</tr>
<tr>
<td>$R_{CG}$</td>
<td>Aircraft CG location</td>
</tr>
<tr>
<td>$r_n$</td>
<td>Motor position vector with respect to CG, n=1,2,...,8</td>
</tr>
<tr>
<td>$\dot{\omega}$</td>
<td>Body angular accelerations vector</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Body angular rates vector</td>
</tr>
<tr>
<td>$M_x, M_y, M_z$</td>
<td>Rolling moment, pitching moment, yawing moment</td>
</tr>
<tr>
<td>$\dot{p}, \dot{q}, \dot{r}$</td>
<td>Roll, pitch, yaw accelerations</td>
</tr>
<tr>
<td>$p, q, r$</td>
<td>Roll rate, pitch rate, yaw rate</td>
</tr>
<tr>
<td>$\ddot{\phi}, \ddot{\theta}, \ddot{\psi}$</td>
<td>Euler accelerations</td>
</tr>
<tr>
<td>$\dot{\phi}, \dot{\theta}, \dot{\psi}$</td>
<td>Euler rates</td>
</tr>
<tr>
<td>$\phi, \theta, \psi$</td>
<td>Euler angles</td>
</tr>
<tr>
<td>$\ddot{x}, \ddot{y}, \ddot{z}$</td>
<td>Earth frame accelerations</td>
</tr>
<tr>
<td>$\dot{x}, \dot{y}, \dot{z}$</td>
<td>Earth frame velocities</td>
</tr>
<tr>
<td>$x, y, z$</td>
<td>Earth frame position</td>
</tr>
<tr>
<td>$F_x, F_y, F_z$</td>
<td>Net force in each axis</td>
</tr>
<tr>
<td>$DCM_E^B$</td>
<td>Body to Earth Direction Cosine Matrix</td>
</tr>
<tr>
<td>$x_{ref}, y_{ref}, z_{ref}$</td>
<td>Position commands</td>
</tr>
<tr>
<td>$\dot{x}<em>{ref}, \dot{y}</em>{ref}, \dot{z}_{ref}$</td>
<td>Velocity commands</td>
</tr>
<tr>
<td>$\dot{x}<em>{p</em>{ref}}, \dot{y}<em>{p</em>{ref}}, F_{z_{p_{ref}}}$</td>
<td>Pilot control commands</td>
</tr>
<tr>
<td>$r_{p_{ref}}$</td>
<td>Pilot control heading command</td>
</tr>
</tbody>
</table>
\[ \phi_{ref}, \theta_{ref}, \psi_{ref} \] Euler angles commands

\[ p_{ref}, q_{ref}, r_{ref} \] Angular rates commands

\[ M_{x_{ref}}, M_{y_{ref}}, M_{z_{ref}} \] Stabilizing and control moments commands

\[ \theta_{cmd_n} \] Collective pitch control command for the \( n^{th} \) motor with \( n = 1, 2, \ldots, 8 \)

\[ K_{P_x}, K_{I_x}, K_{P_y}, K_{I_y}, K_{P_z}, K_{I_z} \] Angular rates pseudo controller gains

\[ K_{P\phi}, K_{P\theta}, K_{P\psi} \] Euler angles pseudo controller gains

\[ K_{P_\dot{x}}, K_{P_\dot{y}}, K_{P_\dot{z}} \] Velocities pseudo controller gains

\[ K_{P_x}, K_{P_y}, K_{P_z} \] Position pseudo controller gains
ABSTRACT

Through this thesis research the problems of controllability and propulsion associated with scaling-up consumer drones to vehicles that may carry significantly larger payloads, including passengers will be analyzed and tested. Controllability is mainly compromised due to the increasing response time of a larger rpm controlled rotor. This requires a more powerful motor, which translates into heavier and larger devices compromising the thrust-to-weight ratio. Collective pitch control at constant rpm is proposed as a first approach to mitigate the controllability problem, and it is tested in a MATLAB Simulink environment. This solution, linked to a Non-linear Dynamic Inversion controller, is simulated as part of the Personal Aerial Vehicle Embry-Riddle aircraft, which serves as test bed. The simulation includes the electric motor, rotor and aircraft mathematical models, which are developed in this research.

Included in this thesis are motor sizing and weigh analyses as well as a thrust-to-weight ratio study, which allows to identify the scaling-up effects in consumer drones’ propulsion plant. This portion of the thesis is closely linked to the behavior displayed in the simulation, which leads to conclude that collective pitch control at constant rpm can mitigate controllability drawbacks. However, due to the size and weight of electric motors increasing very rapidly, it is demonstrated that, while it is possible to obtain an optimal solution where controllability and thrust-to-weight ratio are in balance, scaling-up consumer drones is a highly complex and limited task.
1. INTRODUCTION

1.1 Motivation and Background

The desire for more efficient and environmentally friendly vehicles has become a focal point for several companies around the world. BMW, Chevrolet, Ford and Tesla, to name a few, have introduced hybrid and fully electric cars on the market. While emission problems are partially solved, the issues related to safety and traffic have not been addressed by these vehicles. According to the U.S. Bureau of Transportation Statistics (USBTS), as shown in Figure 1.1, ground transportation is still responsible for the highest number of U.S. fatalities in the last few years.

![Average Fatalities Comparison Chart](image)

**Figure 1.1: 2014 - 2016 Average Fatalities Comparison Chart.**

Also, the USBTS reports 268,799,083 registered vehicles in 2016, with the number increasing since 2010 as shown in Figure 1.2, this partly explains why drivers find themselves in traffic jams especially during rush hours.
It is important to point out that the data displayed in Figure 1.2 corresponds to the U.S. only. In a document released by UBER (Holden and Goel, 2016), it is referenced that people in San Francisco spend over 200 hours commuting between work and home in a year. In Mumbai, India the average commuting time is over 90 minutes. Similarly, over a one year period, residents in Sydney and Los Angeles spend an equivalent of seven weeks commuting, with two of those weeks spent in gridlock. All of this wasted time commuting translates into lost productivity, excess fuel costs and less time with loved ones.

As a result of these drawbacks, the concept of a large network of small aircraft that allow short and long distance commuting is being evaluated by the aerospace industry. There are several companies that started to research and develop concept aircraft that can satisfy the efficiency and environmental demand. Some of these companies are small start-ups like Kitty Hawk, which is developing the CORA and Flyer aircraft, while large companies like AIRBUS are working on the Vahana. These concept aircraft are shown in Figure 1.3 already in-flight with great potential for the future market.
The new mobility idea is expected to present several advantages to including low cost, flexibility, noise reduction, and autonomous technology, which will increase safety and reliability. Currently, helicopters, like the Airbus EC155 Dauphin shown in Figure 1.4, are the closest widely used aircraft to be related to this concept. Also, V-STOL aircraft are currently in the market but they are not available for civilian applications; except for the AugustaWestland AW609 (Figure 1.4), which is under the certification process.

Both helicopters and V-STOL have several disadvantages including noise, low efficiency, pollution, elevated fuel consumption and they are very expensive to reach massive production. The new generation of V-STOL promises to overcome these drawbacks by applying new technologies like Distributed Electric Propulsion, streamline designs, high-efficiency wings, flight controls, and stability strategies.

The other side of the picture is based not on the vehicle itself but on the required infrastructure. This presents a challenge given that not all cities and areas are ready to
receive “flying cars” traffic. The main reason behind it is the construction of takeoff and landing areas, which is difficult to achieve in high density or busy cities like New York. However, applying the knowledge and experience based on helicopters operations there are some strategies that can deal with this problem. For example, limiting the number of vehicles operating in an area at a given time. Also, the so call “vertiports” would be areas allowing multiple takeoff and landing operations, providing support and services, while “vertistops” would be individual landing pads devoted to drop-off and pick-up only resembling the helipad on top of skyscrapers (Holden and Goel, 2016).

In aviation, takeoff and landing are two of the most critical phases of flight, hence, vertiports and particularly vertistops will demand that these aircraft have a highly efficient flight control and stability system. A trained pilot will be required to operate them but it must be assisted by a pilot augmentation system to meet FAA safety standards. Given that these vehicles hover before departure and touch down, their behavior at this point is that of a helicopter, which is naturally unstable and highly dependent on stabilizing pilot inputs. This is due to pitch/roll coupling, which is known as pendulum instability (Padfield, 2008). Most of the current designs, like Vahana and CORA, resemble small drones in quad or octo-configurations contributing to a more stable design but the need of control authority is imperative.

There are several types of pilot augmentation systems which are also associated with autonomous flight. These stability and control systems are varied but the most common one is feedback linearization, which combined with the adequate controller in a cascade control strategy can provide good performance. In this research study, this approach is used in combination with a Non-linear Dynamic Inversion controller to stabilize and control PAVER during the hovering phase of flight, which is discussed in detail in section 2.5. The PAVER design shown in Figure 1.5 has gone through iterations before reaching its current state. It can be seen that the design trend among these future aircraft is similar, and they stick to propellers as a means to generate thrust, however, such setup brings challenges and limitations.
Since the drones era started to emerge and become popular, the idea of just scaling a drone to carry passengers or goods has been on the sight of companies and individuals. For example, a man in the UK has combined fifty four drones motors-rotors to make a vehicle that he can fly himself on (Men, 2015). One of the main problems associated with scaling a motor-propeller setup in drones like quadcopters or octocopters is the response time to thrust demand. Given that this is controlled by rpm variation, it depends on the power plant to generate enough torque to produce rapid thrust change. This issue can be mitigated by applying collective pitch control at constant rpm. A portion of this research will address the changes in required torque, motor weight and motor sizing based on a blade increasing dimension.

1.2 Problem Statement

Aviation is transitioning to hybrid and electric power plants, enabling a reduction in exhaust and engine noise pollution. This, coupled with increased ground traffic congestion, has motivated the idea of a quiet aerial vehicle suitable for use in urban transportation settings. A vehicle of this type would facilitate rapid commuting within short distances negating surface traffic congestion. The proposed ultra-low noise rotor solution requires high torque at low rpm which is achievable with electric motors coupled...
with advanced controls. These requirements suggest the design of a multicopter aircraft with tilting capability for Urban Air Mobility (UAM) similar to scaled up consumer drones.

In recent years, there has been a proliferation of these small scale consumer drones in this configuration. The primary focus of the research presented here is the scalability of consumer drone technology to full scale human rated Urban Air Mobility vehicles. There are two areas of primary concern: the scalability of electric energy storage for propulsion and the controllability scaling using fixed pitch rotors. The primary research problem of this work is the viability of scaling up consumer drone propulsion systems with fixed rotors and solutions to the case where there are scaling limitations.

In the face of scaling limitation, alternative hardware configurations and control law algorithms will be explored to allow for scaling of consumer drone configurations to UAM sized aircraft. The inherent coupling of propulsion, controls and weight make this a unique problem that requires investigation of all three in the design of a viable vehicle. At a top level, this research seeks a solution for propulsion pods of UAM vehicles under real constrains of control, thrust production and weight.
2. LITERATURE REVIEW

2.1 Electric Motor Modeling

2.1.1 Motor Equations

To derive a mathematical model for the motor, consider the following electric motor schematic known as the equivalent circuit Younkin (2001).

\[ V_A - i_A R_m - L_m \frac{di_A}{dt} - K_B \dot{\theta}_m = 0 \] (2.1)

Solving for the current time derivative in equation 2.1 leaves

\[ \frac{di_A}{dt} = \frac{1}{L_m} (V_A - i_A R_m - K_B \dot{\theta}_m) \] (2.2)
Given that the motor is driving the propulsion assembly, it will have a load or counter-acting torque, which must be included in the motor characterization. This will not only increase the overall fidelity of the simulation, but it will also allow to test how is the motor reacting to the different torque variations as a result of the change in collective pitch. In addition, the link between the rotor and the motor is a transmission that alleviates the load on the driving plant helping to cope with high torque demand. This topic is detailed in subsection 3.1.2.

The next step is to obtain the motor torque equation and generate a relationship with the electric equations. Recall Newton’s second law as applied to a rotating object, where \( T = J \alpha \). Figure 2.2 shows the rotating portion of the motor assembly, which, unlike most common motors, the rotating shaft is attached to the armature. This kind of motors are called Outer Rotor Permanent Magnet DC Motor.

![Figure 2.2: Motor Armature - Render by Daniel Posada.](image-url)

Considering the two torques acting in the system, the mechanical equation can be expressed as:

\[
T_{\text{net}} = J_t \ddot{\theta}_m
\]  
(2.3)

Next, expanding equation 2.3 and solving for the angular acceleration yields,
where,
\[ \ddot{\theta}_m = \text{Motor angular acceleration} \]
\[ T_m = \text{Motor torque} \]
\[ T_R = \text{Rotor torque} \]
\[ J_m = \text{Inertia of the armature} \]
\[ J_R = \text{Inertia of the rotor} \]

For simplicity, the relationship between the electrical and mechanical equations was done using a State-space representation. Also, for this research study, the simulation does not have a ground reaction model, which leads to the following issue: the motors take approximately twelve seconds to go from 0 to operational rpm. This results in a simulation error given that the aircraft would display a free fall behavior until it achieves the required rpm to control and recover. This issue can be easily addressed by the State-space model, which, unlike transfer functions, it allows to set an initial condition for the motor states. Hence, the simulation starts with the motor rpm at its operational speed.

Once the states are defined and linearization is performed the system can be expressed as follows:

\[
\begin{bmatrix}
\dot{X}_1 \\
\dot{X}_2
\end{bmatrix} = \begin{bmatrix}
0 & \frac{K_T}{J} \\
-\frac{K_n}{L_m} & -\frac{R_m}{L_m}
\end{bmatrix} \begin{bmatrix}
X_1 \\
X_2
\end{bmatrix} + \begin{bmatrix}
-\frac{1}{J} & 0 \\
0 & \frac{1}{L_m}
\end{bmatrix} \begin{bmatrix}
T_R \\
V_A
\end{bmatrix}
\]

\[
[Y] = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix} \begin{bmatrix}
X_1 \\
X_2
\end{bmatrix}
\]

Note that the inputs to the State-space are the rotor torque demand, \( T_R \), and the applied voltage, \( V_A \), while the outputs are the angular velocity of the motor and the current.
2.2 Rotor Modeling

Rotorcraft are quite unique when it comes to lifting force generation. These machines produce the required thrust by spinning the blades, which are linked to the driving shaft by the rotor head as in Figure 2.3. This last component is one of the most critical ones given that it is not only the point where the blades are attached but it also houses the swash plate, which is the controls actuator. Note that there are three rotor functions that the pilot can control: vertical lifting force, the horizontal force for forward or rearward flight and forces and moments for attitude and position control (Leishman, 2006).

Figure 2.3: Rotorhead MBB BK 117 Helicopter - Redback Aviation.

One of the rotorcraft particularities is that they are quite efficient in hovering given that they do not require to spin the blades at high speed to produce enough thrust. For example, for a light helicopter like the Schweizer 300 displayed in Figure 2.4, the rotor speed is maintained at 440rpm to 460rpm (Professional-Helicopter-Services, 2016).
This requires long blades, which translates into large inertia, hence, due to the load on the shaft it is not possible to produce a rapid change in rpm to control the vehicle. The key is in the variation of collective pitch, which by keeping the rpm constant it allows to accelerate a large mass of air. Figure 2.5 clarifies this concept.

While this study focuses on utilizing only collective pitch at constant rpm to control the aircraft, it is important to point out that most rotorcraft also use cyclic control, which is embedded with collective pitch control in the swash plate. This control allows the pilot
to produce forward, rearward or lateral movement by tilting the rotor disk as shown in Figure 2.6.

![Figure 2.6: Cyclic control translation - FAA-H-8083-21A.]

### 2.2.1 Basics of Rotor Aerodynamics

Consider a rotor blade subdivided spanwise in small sections, the lifting force generated by each element along the blade corresponds to its local angle of attack (AoA) and dynamic pressure. In hover flight, the flow velocity variation along the blade is symmetric with respect to the rotation axis (azimuthal symmetry). At this location, the velocity is zero and increases radially and linearly to its maximum at the tip, where aerodynamic vortices occur (Leishman, 2006). This velocity distribution along with other parameters like azimuth angle $\psi$ can be seen in Figure 2.7. The azimuth angle defines the position of the blade along the circumference of the disk being $\psi = 0$ at the downstream position.

Note that Figure 2.7 assumes hover flight only, while in translational flight azimuthal symmetry is no longer valid given that a portion of the disk would be generating higher lift.
2.2.2 Hovering Flight Aerodynamics

Hovering flight occurs when the rotorcraft is flying stationary on a point. Since there is no displacement the flow is either upward or downward. However, strong vortices at the tip still exist and they have a very complex structure to study and produce a mathematical model. This issue is addressed by the Rankine-Froude momentum theory, which allows to analyze the basic performance and develop a model to obtain thrust and power. While this is a first-order prediction, it is also a strong baseline for further analysis of rotor aerodynamics (Leishman, 2006).

2.2.3 Conservation Laws Applied to a Hovering Rotor

In physics, there are three elements that are conserved: mass, momentum, and energy. To study these elements in the aerodynamics field, it is common to define a control volume enclosing the area to be analyzed along with a set of assumptions. In this case, the control volume is defined as shown in Figure 2.8, and it encloses the rotor and its wake.
The assumptions applied to this analysis include a one-dimensional flow, meaning that there are no changes in the fluid properties across parallel planes to the rotor disk but along the vertical position, parallel to the axis of rotation. A quasi-steady flow implying that the properties at a point do not change with time and, lastly, incompressible and ideal, where there is no viscous shear within the fluid.

In Figure 2.8 there are 4 sections. The first one located above the disk plane is labeled as 0, where the flow velocity, $v_0$, is zero. The next section is 1, located right above the disk area A, which is followed by section 2 located right below it. The last section known as the vena contracta (Leishman, 2006) is located far downstream, and it is labeled with $\infty$. Note that the velocity $v_i$ is associated with the acceleration of the mass of air at the rotor-disk plane, which is increased to velocity $\infty$ at the vena contracta.

### 2.2.4 Blade Element Theory (BET)

The BET was designed by three scientists: William Froude -1878, David W. Taylor -1893 and Stefan Drzewiecki between 1892 and 1920 (Revolvy, 2008). However, along the years, there were several contributions and improvements by other scientists like Glauert, Reissner, and Theodorsen to name a few (Leishman, 2006).
This theory is widely known to serve as a base to analyze rotor aerodynamics focusing on radial and azimuthal loading of the rotor disk. The blade is subdivided into small sections and it is assumed that each portion, \( dy \), acts as a quasi-2D airfoil to produce forces and moments, whose resultant is obtained by integrating those forces along the blade and one revolution. To account for a 3D case, BET adds tip losses in the analysis, as well as, empirical factors.

One of the downsides in this analysis is attributed to the modeling of the induced velocity, which is the one occurring at the rotor disk. One way to address this issue is expanding the BET to BEMT (Blade Element Momentum Theory), which allows to perform a deeper analysis and to interrelate more parameters that are at play. While this extended version of the rotor analysis is far more detailed, in this research, the BET equations will based on the assumption of a uniform induced velocity distribution at the rotor plane.

Figure 2.9: Blade sectioning and velocities acting on the blade element (Leishman, 2006).

Figure 2.9 shows the resultant flow velocity \( U \) acting on the blade element of width \( dy \). The radial and tangential components of the resultant velocity are identified by \( U_R \) and \( U_T \). However, based on the independence principle, which states that above a certain Mach number, several aerodynamic properties become asymptotically independent of the free stream (Li et al., 2012), the component \( U_R \) can be ignored. Note that this case applies only to hovering flight because in forward flight the radial component affects the drag on the blade (Leishman, 2006).

The resultant velocity also has a third component as seen in Figure 2.10, which is
labeled as $U_p$. This velocity acts perpendicular to the rotor disk and it is composed by the induced velocity when in stationary flight, and the flow velocity during a climb.

The angle $\phi_\lambda$ is known as the inflow angle of attack, which modifies the relative flow velocity vector, in turn changing the AoA. This causes the lift vector to change given that by definition it acts perpendicular to the relative wind. Recall that lift is inevitably associated with induced drag, which is one of the elements demanding power from the rotor shaft.

![Diagram of velocities and forces acting on the blade element](image)

Figure 2.10: Velocities and forces acting on the blade element (Leishman, 2006).

From figures 2.9 and 2.10 it is possible to derive the expression for the three main elements (thrust, torque, and power) for a particular portion of the blade. Recall that thrust is the perpendicular force to the disk, then $F_z$ is analogous to thrust. Also, while only one portion of the blade is being analyzed, its effects have to be multiplied by the number of blades, $N_b$. Similarly, torque is related to the parallel force $F_x$ acting perpendicular to the blade span at a distance $y$ from the shaft. Lastly, power is directly related to torque by the rotational speed $\Omega$.

In rotorcraft aerodynamics is common to use non-dimensional analysis, specially because it is effective when it comes to compare different rotors or helicopters. Also, it is helpful for developing the required relationship between collective pitch and thrust.

In this branch of aviation, there is a convention to turn parameters into non-dimensional form. This is based on dividing length values by $R$ and velocities by
\( V_{tip} = \Omega R \). In addition, the coefficients for thrust, torque, and power are given by

\[
dC_T = \frac{dT}{\rho AV_{tip}^2} \quad (2.6)
\]

\[
dC_Q = \frac{dQ}{\rho AV_{tip}^2 R} \quad (2.7)
\]

\[
dC_P = \frac{dP}{\rho AV_{tip}^2} \quad (2.8)
\]

Replacing \( V_{tip} = \Omega R \) in the equations above it becomes apparent that \( P = \Omega Q \), which means that \( dC_Q \equiv dC_P \).

The thrust and power coefficients corresponding to a particular element along the blade are given by equations 2.9 and 2.10 respectively Leishman (2006).

\[
dC_T = \frac{1}{2} \sigma C_l r^2 dr \quad (2.9)
\]

\[
dC_Q \equiv dC_P = \frac{1}{2} \sigma C_l r^2 \lambda + \frac{1}{2} \sigma C_d r^3 dr \quad (2.10)
\]

Equations 2.9 and 2.10 present two new parameters. One of them is \( \sigma = \frac{N_b c}{\pi R} \) known as the rotor solidity, and \( r = \frac{\gamma}{R} \) (non-dimensional span). Note that \( r \) is the parameter over which the span wise integration of the forces will occur, being \( r = 0 \) at the root and \( r = 1 \) at the tip of the blade.

The net coefficient of thrust is given by

\[
C_T = \frac{1}{2} \sigma \int_{r=0}^{r=1} C_l r^2 dr \quad (2.11)
\]

\[
C_T = \frac{1}{2} \sigma C_{l_a} \left[ \frac{\theta_b}{3} - \frac{1}{2} \sqrt{\frac{C_T}{2}} \right] \quad (2.12)
\]

To obtain the net power coefficient it is required to consider profile drag. Note that
equation 2.10 is composed by $C_l$ and $C_d$, where $C_l$ is part of the induced drag and $C_d$ is a product of viscous effects, hence, associated with profile drag.

$$C_P = \int_{r=0}^{r=1} dC_T \lambda dr + \int_{r=0}^{r=1} \frac{1}{2} \sigma C_{d0} r^3 dr$$ (2.13)

The left-hand side of equation 2.13 is the ideal power, while the right-hand side is the induced power. Since uniform inflow was assumed for this analysis, the drag coefficient is constant along the blade, then this parameter becomes $C_{d0}$ (zero-lift drag coefficient), which is an airfoil constant.

The solution to the integral of the two power components under this assumptions yields

$$C_P = \frac{C_l^{3/2}}{\sqrt{2}} + \frac{1}{8} \sigma C_{d0}$$ (2.14)

For further details on the derivation of the thrust, power and torque equations refer to chapter 3 of the book “Principles of Helicopter Aerodynamics” by J. Gordon Leishman

2.3 Scaling Motor and Rotor

2.3.1 Rotational Inertia

The rotational inertia or moment of inertia is a measure of the opposing effect that a body has against a change in its rotational speed, which occurs as a result of a turning force (torque). The moment of inertia is based on mass distribution of the body with respect to an axis of rotation, which can be internal or external (Britannica, 2018).

A general expression defining a body’s moment of inertia is shown by equation 2.15, where $R$ is the distance to the rotation axis and $dm$ is the infinitesimal particle of mass. This equation assumes rotation about an axis located at the center of mass.

$$I = \int_{B} R^2 dm$$ (2.15)

For a helicopter case, equation 2.15 is applied to the blade, where the shape is that of
an airfoil extruded a distance $R$ from the axis of rotation. Blades can be simple presenting a rectangular planform or more complex involving span wise twist and taper. These factors along with the type of airfoil dictates the shape of the blade, which defines its mass distribution. Given that in helicopters the blades rotate about an axis located at the root, equation 2.15 has to be solved applying the parallel axis theorem shown in equation 2.16

$$I = I_{cm} + md^2$$ \hspace{1cm} (2.16)

### 2.3.2 Scaling Issues Associated with Rotors

When scaling up a rotor in general, there are several variables to take into consideration and that can adversely affect the design. These variables include tip Mach number, aerodynamics moments, structure bending and stress, and weight related loads (Jamieson and Branney, 2012). In addition, scaling up a rotor translates into a larger mass, which based on the previous discussion produces a larger inertia, hence, the torque required to spin the rotor is increased as well.

The motor driving the rotor has to produce enough power to keep it rotating at a certain speed. As the size of the rotor increases, the power required to maintain the same rotational speed rises. This is the main point where scaling a motor-rotor assembly becomes very complex for thrust vector controlled aircraft like common drones. If the rotor is scaled up to a point where the motor is unable to produce the power required, the rpm will decrease and produce a drop in thrust. Also, the latency of the rpm variation is significantly increased. Therefore, the aircraft will no longer be controllable given the there is no inherent stability in this kind of vehicles.

Assuming that the motor is redesign to have a fast response and to cope with the required power and torque demand, it will happen at a cost of larger size and weight, which in aircraft design has to be minimized.

The use of blade collective pitch control at constant rpm offers a solution to mitigate the scaling problem. When collective pitch angle is increased then torque demand is also
increased, but rpm are kept constant by means of a motor controller or governor. While thrust has a dynamic response associated to the aerodynamics of the blade pitch angle (Leishman, 2006), the thrust change is faster than the rpm control case.

2.4 Aircraft Modeling

All moving objects can be represented by a mathematical model regardless of its shape and mass. This model is usually defined by a set of non-linear differential equations. A typical 6 degree of freedom (6DOF) aircraft model is governed by a set of twelve ordinary differential equations (ODE), which include the rotational subsystem, translational subsystem, Euler Angle-Kinematic, and navigation equations. This chapter outlines the derivation of the 12 ODEs for PAVER, which will be used to generate the vehicle simulation for testing.

2.4.1 Reference Systems

The development of a mathematical model of a moving particle initially requires to define the reference frames on which the equations will be based. In aviation there are two commonly used reference frames, one called the inertial or Earth reference system whose axes are North, East, Down (NED) and it is fixed on the Earth surface. The second system is the body reference frame, which is defined by the x, y and z-axis and it is located at the aircraft center of mass.

The direction of the axes in the inertial frame is implied by the name whereas N points to the Earth North, E points to the Earth East and D points to the center of the Earth. In the case of the body reference frame, the direction of the axes is based on the assumption made to derive the equations. Aircraft are known to have three body axes: longitudinal, lateral and vertical, which are represented by x, y, z respectively as depicted in Figure 2.11.
2.4.2 Reference Systems Relation

In the mathematical modeling of an aircraft, there are parameters that are measured in the Earth reference frame, and others must be measured in the body frame. Therefore, to link those parameters, both systems are interrelated by a series of orthogonal frame rotations. These are classified in two different types, namely successive and non-successive leading to multiple possible rotations (Sidi, 2000).

In aviation, it is common to find the successive 3-2-1, which corresponds to the rotations about yaw, pitch and roll axis respectively as represented by Figure 2.12. The position of the center of mass with respect to the inertial frame is defined by the vector

\[ r = [x \ y \ z]^T. \]

Figure 2.11: Reference frames.

Figure 2.12: Yaw, pitch and roll ration frames.

From each rotation, it is possible to obtain its correspondent rotation matrix.
Therefore, following the 3-2-1 combination and based on the diagrams from Figure 2.12
the rotation equations are (Yechout et al., 2003):

\[
\begin{bmatrix}
  x_1 \\
  y_1 \\
  z_1
\end{bmatrix} =
\begin{bmatrix}
  \cos \psi & \sin \psi & 0 \\
  -\sin \psi & \cos \psi & 0 \\
  0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  \hat{i}_E \\
  \hat{j}_E \\
  \hat{k}_E
\end{bmatrix} =
R_{\psi}
\begin{bmatrix}
  \hat{i}_E \\
  \hat{j}_E \\
  \hat{k}_E
\end{bmatrix}
\] (2.17)

\[
\begin{bmatrix}
  x_2 \\
  y_2 \\
  z_2
\end{bmatrix} =
\begin{bmatrix}
  \cos \theta & 0 & -\sin \theta \\
  0 & 1 & 0 \\
  \sin \theta & 0 & \cos \theta
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  y_1 \\
  z_1
\end{bmatrix} =
R_{\theta}
\begin{bmatrix}
  x_1 \\
  y_1 \\
  z_1
\end{bmatrix}
\] (2.18)

\[
\begin{bmatrix}
  \hat{i}_B \\
  \hat{j}_B \\
  \hat{k}_B
\end{bmatrix} =
\begin{bmatrix}
  1 & 0 & 0 \\
  0 & \cos \phi & \sin \phi \\
  0 & -\sin \phi & \cos \phi
\end{bmatrix}
\begin{bmatrix}
  x_2 \\
  y_2 \\
  z_2
\end{bmatrix} =
R_{\phi}
\begin{bmatrix}
  x_2 \\
  y_2 \\
  z_2
\end{bmatrix}
\] (2.19)

To obtain the Body reference frame coordinates from a vector fixed in the Earth
reference frame, it is not required to go through the rotations one by one. Instead, the
Direction Cosine Matrix (DCM) can be applied, which is obtained by multiplying the
rotation matrices from equations 2.17, 2.18 and 2.19 as follows:

\[
\begin{bmatrix}
  \hat{i}_B \\
  \hat{j}_B \\
  \hat{k}_B
\end{bmatrix} =
R_{\psi}R_{\theta}R_{\phi}
\begin{bmatrix}
  \hat{i}_E \\
  \hat{j}_E \\
  \hat{k}_E
\end{bmatrix}
\] (2.20)

Conversely, given that the DCM matrix is orthogonal, the post-multiplication of the
rotation matrices transposed would yield the Earth coordinates of a vector located in the in
the Body reference frame.

\[
\begin{bmatrix}
  \hat{i}_E \\
  \hat{j}_E \\
  \hat{k}_E
\end{bmatrix} = (R_{\psi})^T (R_{\theta})^T (R_{\phi})^T
\begin{bmatrix}
  \hat{i}_B \\
  \hat{j}_B \\
  \hat{k}_B
\end{bmatrix}
\] (2.21)
As a result, the DCM matrix for coordinate transformation from Earth to Body reference frame is given by equation 2.22.

\[
DCM_E^B = \begin{bmatrix}
\cos \theta \cos \psi & \sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi & \cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi \\
\cos \theta \sin \psi & \sin \phi \sin \theta \sin \psi + \cos \phi \cos \psi & \cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi \\
-\sin \theta & \sin \phi \cos \theta & \cos \phi \cos \theta 
\end{bmatrix}
\] (2.22)

2.4.3 Kinematics

The attitude rates of the vehicle are given by the Euler angular rates. However, it is necessary to define those parameters in the Body reference frame given that they are required states to develop the rotational subsystem. The rates in the Body frame are represented by \( p, q, r \), and they are calculated as follows (Perez Rocha, 2016):

\[
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix} = R_\phi R_\theta R_\psi \begin{bmatrix}
0 \\
0 \\
\psi
\end{bmatrix} + R_\phi R_\theta \begin{bmatrix}
0 \\
\dot{\theta} \\
0
\end{bmatrix} + R_\phi \begin{bmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix}
\] (2.23)

Solving equation 2.23, the body angular rates are given by:

\[
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix} = \begin{bmatrix}
1 & 0 & -\sin \theta \\
0 & \cos \phi & \sin \phi \cos \theta \\
0 & -\sin \phi & \cos \phi \cos \theta
\end{bmatrix} \begin{bmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix}
\] (2.24)

Similarly, to perform a kinematic transformation from Body to Earth, equation 2.24 can be solved to obtain the Euler rates.

\[
\begin{bmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix} = \frac{1}{\cos \theta} \begin{bmatrix}
\cos \theta & \sin \phi \sin \theta & \cos \phi \sin \theta \\
0 & \cos \phi & -\sin \phi \\
0 & \sin \phi & \cos \phi
\end{bmatrix} \begin{bmatrix}
p \\
q \\
r
\end{bmatrix}
\] (2.25)
2.4.4 Rotational Subsystem

From Newton’s second law of motion, it is known that the angular momentum dynamics are given by the total sum of moments applied to a rigid body, as represented by equation 2.26. Here, \( \mathbf{M}(t) = \left[ \sum M_x \sum M_y \sum M_z \right]^T \) are the total moments acting around the \( x,y,z - \text{axis} \) respectively, and \( \mathbf{H} = \mathbf{r} \times (m \mathbf{V}) \) is the angular momentum of the aircraft (Yechout et al., 2003).

\[
\frac{d \mathbf{H}}{dt}_E = \mathbf{M}(t) \tag{2.26}
\]

Given that these equations are derived under the assumption of a rigid body, the angular momentum expression can be defined as:

\[
\mathbf{H} = I \mathbf{\omega} \tag{2.27}
\]

Where \( I \) is the inertia tensor and \( \mathbf{\omega} = [\omega_x \omega_y \omega_z]^T \) is the angular velocity with respect to the Earth frame. The inertia tensor is defined by a 3x3 matrix containing the moment of inertia about each rotation axis and the product of inertia as shown in equation 2.28. Note that in this thesis the inertia matrix is assumed to be time invariant. The external moments and angular momentum can be calculated with respect to an arbitrary point, but choosing the center of mass of the object as the reference point allows to simplify the equations.

\[
J = \begin{bmatrix}
J_{xx} & J_{xy} & J_{xz} \\
J_{xy} & J_{yy} & J_{yz} \\
J_{xz} & J_{yz} & J_{zz}
\end{bmatrix} \tag{2.28}
\]

To obtain an expression for the angular momentum with respect to the inertial frame, it is possible to find the derivative of \( \left( \frac{d \mathbf{H}}{dt}_E \right) \) by means of the Transport Theorem (Sidi, 2000). Consider a vector \( \mathbf{B} = (b_1 \mathbf{\hat{i}}_b + b_2 \mathbf{\hat{j}}_b + b_3 \mathbf{\hat{k}}_b) \) fixed in the Body frame, which is moving with respect to the Earth frame. Then the derivative of this vector is given by

\[
\left( \frac{d \mathbf{B}}{dt}_E \right) = \left( \frac{d \mathbf{B}}{dt}_B \right) + \mathbf{\omega}_B \times \mathbf{B}.
\]

Applying this theorem to equation 2.26, the angular
momentum with respect to the inertial frame results in the following expression.

\[
\left( \frac{d\vec{H}}{dt} \right)_E = \left( \frac{d\vec{H}}{dt} \right)_B + \omega \times \vec{H}
\]  

(2.29)

Taking the derivative of equation 2.27 and replacing terms in equation 2.29, it yields,

\[
\left( \frac{d\vec{H}}{dt} \right)_E = J \frac{d\omega}{dt} + \omega \times (J \omega)
\]  

(2.30)

Finally, the equation for rotational dynamics is obtained by replacing equation 2.30 in equation 2.26 and solving for the angular acceleration vector \( \dot{\omega} \).

\[
\dot{\omega} = J^{-1} \{ -\omega \times (J \omega) + \vec{M}(t) \}
\]  

(2.31)

### 2.4.5 Translational Subsystem

The translational movement of the aircraft can also be modeled by applying Newton’s second law of motion, which states that the sum of forces applied to the center of mass is directly proportional to the mass of the object and the acceleration in the same direction the force is applied. Recall that Newton’s second law requires a transformation to relate the inertial and body axes system (Yechout et al., 2003). Applying the transport theorem, in this case, the equation yields,

\[
\begin{bmatrix}
\sum F_x \\
\sum F_y \\
\sum F_z
\end{bmatrix} = m \begin{bmatrix}
a_x \\
a_y \\
a_z
\end{bmatrix} = m \left( \frac{d\vec{V}}{dt} \right)_E = m \left[ \left( \frac{d\vec{V}}{dt} \right)_B + \omega \times \vec{V}_B \right]
\]  

(2.32)

Given that in this research study the vehicle has eight motors in the vertical axis, and assuming no tilting for the hovering phase, the only force acting on the body is the motors thrust \( F_Z \). Then equation 2.32 can be rewritten as follows:
\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{z}
\end{bmatrix} = DCM^E_B \begin{bmatrix}
\dot{u} \\
\dot{v} \\
\dot{w}
\end{bmatrix}
\] (2.33)

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{z}
\end{bmatrix} = \frac{1}{m} \begin{bmatrix}
0 & c\theta c\psi & s\phi s\theta c\psi - c\phi s\psi & c\phi s\theta c\psi + s\phi s\psi & 0 \\
0 & c\theta s\psi & s\phi s\theta s\psi + c\phi c\psi & c\phi s\theta s\psi - s\phi c\psi & 0 \\
mg & -s\theta & s\phi c\theta & c\phi c\theta & -F_z
\end{bmatrix}
\] (2.34)

Finally, the translational equations for this vehicle are obtained by solving equation 2.34.

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{z}
\end{bmatrix} = \begin{bmatrix}
0 & \cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi \\
0 & \cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi \\
g & \cos \phi \cos \theta
\end{bmatrix}
\] (2.35)

2.5 Control Approach

2.5.1 Feedback Linearization

Physical models are created to represent the dynamics of a moving object or vehicle. These models are non-linear by nature and require to be linearized to be controlled by means of classical or linear control techniques. Feedback linearization has been developed in the 1980's and it has been successfully implemented to solve a variety of control problems including airplanes, helicopters, robots and biomedical devices to name a few (Slotine and Li, 1991)(Krstic et al., 1995).

This method can be understood as a technique to obtain an equivalent representation of a simple system from its original complex form. The process involves the algebraic transformation of the non-linear system into a full or partial linear system. Unlike the linear approximation technique, in this case, the new system is obtained by exact state transformation and feedback.
2.5.2 Non-linear Dynamic Inversion

There are different types of feedback linearization techniques, one of them is the non-linear dynamic inversion (NLDI) which given certain conditions allows the transformation of a non-linear system into a linear one. This control architecture has been proven to be a robust solution for a limited type of non-linearities and unmodeled or incorrectly modeled uncertainties. Also, this control technique serves as a solid baseline control law for the in-flight testing of advanced controllers (Miller, 2011).

It has to be noted, that the NLDI control approach has limitations that do not allow its application or make the controller unreliable. One of these conditionals is that the NLDI application requires the differential equations of the model to be invertible, then it is important to understand that this controller is highly dependent on the accuracy of the model. The problem associated with a good characterization of the system relies on the implementation stage.

In a simulation environment, the controller acts in a perfect or ideal scenario, where several factors like unmodeled effects or physical differences are not considered when tuning the controller. As result, when the NLDI is tested in the real vehicle, the controller does not warrant a stable behavior. One common but simple example is the inertia matrix, which plays an important role in the rotational equations of motion. This matrix may be different between the simulation and implementation stage due to different factors. These include but are not limited to a person sitting on the vehicle, a drastic change in the vehicle shape or unmodeled heavy equipment.

2.5.3 NLDI Derivation

Consider the general representation of a MIMO non-linear system as given by equation 2.36

\[
\dot{x} = f(x) + g(x)u(t)
\]  

(2.36)

Here, \( \dot{x} \) represents the state vector while \( f(x) \) and \( g(x) \) are non-linear vector functions, and \( u(t) \) is the system inputs vector. Assuming that \( f(x) \) and \( g(x) \) are invertible
it is possible to develop the feedback linearization and obtain the linear representation of
the system.

Solving equation 2.36 for the input $u(t)$ it yields

$$u(t) = g(x)^{-1}[v(x) - f(x)]$$ (2.37)

Note that the vector function $g(x)$ must be invertible $\forall t$ to find the exact state
transformation and apply the desired control law. Replacing equation 2.37 into equation
2.36 the new linear dynamics are:

$$\dot{x} = v(x)$$ (2.38)

where $v(x)$ is the virtual controller yielding the closed loop linear. The virtual
controller can be a PID or any other state-feedback linear control the designer deems
appropriate for the application. The PID controller is one of the simplest ones and in
combination with a NLDI was proved to work successfully in quadcopters and
octocopters (Garcia Herrera, 2017). Therefore, this controller was selected for this study
and applied at each inversion during the control development stage.

The NLDI basic control architecture is shown in Figure 2.13. Note that $X_{ref}$ represent
the desired states, which along with the feedback signal generates the error to be
processed by the pseudo controller (PID) and produce the desire dynamic response. At
this point is when the dynamic inversion takes place and generates the inputs to the plant.

Figure 2.13: Generic NLDI schematic.
3. METHODOLOGY

3.1 Motor Parameter Calculation

3.1.1 Hardware

The eight motors powering PAVER are Kontronik Pyro 750-50 XL, which are rated at 4.5kW. Each of them has 14 magnetic poles, delta-winding configuration, and three phases.

Figure 3.1: Kontronik Pyro 750-50 XL.

The power source is a pair of Pulse Ultra lithium polymer battery per motor connected in series. These are 22.2V batteries, which have a 5500mAh capacity at a discharge rate of 65C.

Figure 3.2: Pulse Ultra Battery.

The modeling of the electric motor was made obtaining the required parameters experimentally by means of a motor test bed, where one motor was set up to run as a generator linked to the driving motor. The latter was controlled by the ESC and had the two batteries in series as the energy source.
3.1.2 Transmission

The current rotor configuration at a peak thrust of 60 pounds demands approximately 13lbs-ft of torque. Therefore, to reduce the load on the motor, it is used a transmission composed by a gear train and a standard synchronous belt as shown in Figure 3.4.

The overall reduction factor provided by this transmission was calculated experimentally and theoretically. The expression governing the input and output rotational velocity is based on the number of teeth of the gears, hence, given by the following expression:

$$\omega_R = \frac{N_1}{N_2} \frac{N_3}{N_4} \omega_m$$  \hspace{1cm} (3.1)

Here, $N_1$ is the gear attached to the shaft of the motor, while $N_2$ is the gear driven by the belt. $N_3$ is linked to $N_2$ by a shaft and drives $N_4$ which is the rotor blade assembly gear.
3.1.3 Electrical Parameters

As stated in the hardware discussion, the characterization of the motor was achieved by means of the motor test bed shown in Figure 3.3. The motor was run at different rpm, which were recorded by the manual tachometer, while the generator was connected to an oscilloscope to obtain the correspondent back EMF values.

3.1.4 Mechanical Parameters

At this point, the missing parameters to complete the motor characterization are the mechanical constants \( J_m \) and \( J_R \). The first one corresponds to the inertia of the armature which was generated from the render displayed in Figure 2.2. The inertia of the rotor was calculated by approximation applying equation 3.2, which is based on the thin rod approximation for the blade’s inertia.

\[
J_R = \rho \pi r^2 R \left( \frac{R^2}{3} + h^2 + hR \right)
\]  

(3.2)

Figure 3.5: Simplified Rotor Blade Diagram.

Note that the expression \( \rho \pi r^2 R \) in equation 3.2 is the mass of the rod, which is based on aerospace grade carbon fiber. This particular rotor being used for PAVER is composed by two blades, hence, the inertia \( J_R \) must be doubled for this calculation.

The list of the mechanical parameters and the resulting inertia are listed in Table 3.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density, ( \rho ) [lb/ft(^3)]</td>
<td>86.4</td>
</tr>
<tr>
<td>Rod radius, ( r ) [ft]</td>
<td>0.014583</td>
</tr>
<tr>
<td>Single blade span, ( R_t ) [ft]</td>
<td>2.083</td>
</tr>
<tr>
<td>Hub, ( h ) [ft]</td>
<td>0.5</td>
</tr>
<tr>
<td>Single blade inertia, ( J_R ) [lb-ft(^2)]</td>
<td>0.607</td>
</tr>
<tr>
<td>Total rotor inertia, ( J_T ) [lb-ft(^2)]</td>
<td>1.214</td>
</tr>
</tbody>
</table>
3.1.5 Torque Constant vs. Back EMF Constant

Referring back to the motor state space presented by equation 2.5, it can be seen that the parameter $K_T$ known as the torque constant has not been defined yet. The reason behind is that when this constant is expressed in metric units $K_T = K_B$, which was obtained from Figure 4.2. This equivalency between the two constants is derived and demonstrated in section 4.2, Fundamental Concepts, of the book “Brushless Permanent Magnet Motor Design 2nd Edition” by Dr. Duane Hanselman.

3.2 Motor Dimensions and Rotor Inertia Variation

In section 2.2 it was stated that rotorcraft produce thrust by accelerating a large mass of air changing collective pitch while keeping constant rpm. This is due to the large load imparted by the big blades on the shaft. As the blade aspect ratio (AR) increases so does its inertia, which translates in higher torque and power demand. In this section, it is analyzed how motor size and weight changes with the increasing size of the rotor blade. While the method used in this analysis is only an approximation under a set of assumptions, it gives a clear idea of how the numbers behave in such a situation.

3.2.1 Motor Size Approximation

The first step in this analysis is to derive the blade’s inertia, which is not only composed by the blade itself, but it also has the hub portion where the blade attaches to the rotor center. However, two configurations were considered, the first one assuming a full blade while, the second one assumes a hub portion, which is more realistic. The main difference lies on the moment arm from the axis of rotation to where blade’s mass starts.

When it comes to calculate the inertia of an object there are three main parameters: mass, reference axis (or axis of rotation) and shape. Blades are made of different materials that determine their mass, but most common ones are made of wood and metal. Next, it is obvious that blades rotate with respect to an axis located at their root or the root of the hub. Lastly, regarding the shape, most physics books apply the thin rod approximation for this calculation given that the actual airfoil shape brings more complexity.
3.2.1.1 Thin Rod Inertia Approximation

The following Figure represents the blade as a thin rod of radius $r$ and length $R$.

![Figure 3.6: Thin rod representation with rotation axis at the center of mass.](image)

Its mass and moment of inertia are respectively given by

\[ m = \rho \pi r^2 R \] (3.3)

\[ I_{cm} = \frac{1}{12} mR^2 \] (3.4)

For the full blade case, where it rotates about the axis located at the root, it is required to apply the parallel axis theorem discussed in subsection 2.3.1 to account for the axis shift. Recall that applying this theorem, the moment of inertia becomes

\[ I = I_{cm} + md^2 \] (3.5)

![Figure 3.7: Thin rod representation with rotation axis at the center of mass.](image)
Replacing equation 3.4 and d in equation 3.5, after simplifying, the inertia for this first case can be expressed as follows:

\[ I(R) = \frac{1}{3}mR^2 \]  

(3.6)

Similarly, to obtain the inertia expression for the second case, where a hub section is considered, the parallel axis theorem can be applied to the diagram in Figure 3.8.

![Figure 3.8: Thin rod representation with hub section.](image)

The inertia of the blade is then

\[ I(R)_{hub} = m\left(\frac{R^2}{3} + r_0^2 + r_0R\right) \]  

(3.7)

### 3.2.1.2 Numerical Solution to Blade Inertia Variation

The following list is the set of assumptions and blade characteristics associated with this approximation. The material was chosen to be aerospace grade carbon fiber based on the available blades for PAVER. The radius of the thin rod representing the blade corresponds to the maximum airfoil chamber measured off the blade. For the torque calculation, it was assumed a time constant of 0.077sec, and the reference rpm change was chosen to be 200rpm with limits \(N_1\) and \(N_2\), which are within the rotational speed range of the rotors used for PAVER.

- 2 blades rotor
- Carbon fiber aerospace grade \(\rho = 0.05\text{lbs/in}^3\)
- Hub mass neglected (linking arm)
- Thin rod of \( r = 0.175 \text{in} \)
- \( N_1 = 1400 \text{rpm} \)
- \( N_2 = 1600 \text{rpm} \)
- \( \tau = 0.077 \text{sec} \)

The following equation reveals the required acceleration to produce the desired rotational speed change within a certain time. In this particular case, the angular acceleration turns out to be 272 rads/s\(^2\). Note that the numerical constant in the equation is for unit conversion.

\[
\alpha = \frac{0.10472(N_2 - N_1)}{\tau}
\]  

(3.8)

From Newton second’s law, it is known that torque can be calculated as \( T = I\alpha \).

Then running equations 3.6 and 3.7 through this expression it is possible to calculate the required torque associated with each inertia change as the length of the blade is increased. Similarly, the power required is given by \( P = T\Delta\omega \).

### 3.2.1.3 Motor Dimensions

Electric motor design has several parameters at play. Equation 3.9 outputs the torque a motor is capable to do based on the design characteristics listed below (Hanselman, 2006).

\[
|T| = 2N_m N B_g L_{st} R_{roi} i
\]  

(3.9)

where

- \( N_m \) number of magnets
- \( N \) number of turns of the coil
• $B_g$ magnetic flux density in the gap

• $L_{st}$ axial rotor length

• $R_{ro}$ radius of the rotor

• $i$ current

A similar but empirical equation for motor sizing is presented below. However, this approximation is only valid for radial flux motors (Hanselman, 2006).

$$T = k (2R_{ro})^2 L_{st} \quad (3.10)$$

Comparing equation 3.9 and 3.10 it can be seen that $k$ is analogous to $N_m N B_g i$, while $2R_{ro}$ is the diameter of the rotor.

In the following Figure is shown the motor used for PAVER with its dimensions, which were used to calculate the constant $k$ and run size variations applying equation 3.10. The variable, in this case, is the set of torque values calculated previously. Recall that those values are based on the blade’s inertia increments.

![Figure 3.9: Pyro 750-50 XL dimensions.](image)

Solving equation 3.10 for $k$ requires to know the rated torque of the motor, which by manufacturer specification the value is 1.43Nm (1.055lb-ft), leading to a $k$ value of 31.58lb/ft³. Given that $L_{st}$ and $R_{ro}$ can be calculated while keeping one or the other as a constant, and considering that a motor cannot be infinitely wide or long, the calculation for motor sizing was done for $\Phi^2 L_{st}$ as one variable. That is:
3.2.2 Motor Weight Approximation

In the previous section, it was addressed the change in the dimensional size of the motor as the torque and inertia increased based on rotor blade length. However, in aviation, a heavy motor presents one of the biggest drawbacks leading to less payload and other design issues. This section derives an approximation for the change in weight of an electric motor based on the blade size variation.

The first step in this process is based on obtaining a relationship between motor weight and the peak torque for which the motor is rated. This was achieved by gathering electric motors data and performing a simple curve fitting.

Table 3.2 contains the characteristics of five different EMRAX motors, which were extracted from the company website. This electric motor manufacturer is a Slovenian company, which is known for developing high-efficiency motors for aerospace applications. The line of motors they offer promises to be lightweight, powerful, reliable and without vibrations and noise. EMRAX has tested different prototypes including axial and radial flux motors, resulting in axial flux being the best option for aviation (EMRAX, 2017). Recall that the empirical equation 3.10 only applies to radial flux motors, hence, it cannot be used in this case.

<table>
<thead>
<tr>
<th>EMRAX</th>
<th>188</th>
<th>208</th>
<th>228</th>
<th>268</th>
<th>348</th>
</tr>
</thead>
<tbody>
<tr>
<td>Casing diameter [in]</td>
<td>7.40</td>
<td>8.19</td>
<td>8.98</td>
<td>10.55</td>
<td>13.70</td>
</tr>
<tr>
<td>Axial length [in]</td>
<td>3.03</td>
<td>3.35</td>
<td>3.39</td>
<td>3.58</td>
<td>4.21</td>
</tr>
<tr>
<td>Weight - air cooled [lb]</td>
<td>14.99</td>
<td>20.06</td>
<td>26.46</td>
<td>43.87</td>
<td>85.98</td>
</tr>
<tr>
<td>Peak power [HP]</td>
<td>93.87</td>
<td>107.28</td>
<td>134.10</td>
<td>308.44</td>
<td>402.31</td>
</tr>
<tr>
<td>Peak torque [lb·ft]</td>
<td>66.38</td>
<td>103.25</td>
<td>169.56</td>
<td>368.78</td>
<td>737.56</td>
</tr>
</tbody>
</table>

While these motors vary in size and performance characteristics they all have the same look, as an example, the following Figure displays the EMRAX 228.
3.2.3 **Thrust - Weight - Rotor Size Analysis**

In aviation one of the well-known parameters is the thrust-to-weight ratio \( T/W \), which is a non-dimensional quantity that reflects performance. In gas powered aircraft and rotorcraft, this value varies as fuel is consumed. However, in this case, where an electric motor acts as the propulsion source, the aircraft gross weight does not change with time. Also, for this study, it is more meaningful to analyze the thrust being produced by a rotor against the weight of the motor required to spin the blades, plus the weight of the rotor itself.

This process requires to apply the BET developed in subsection 2.2.4 to generate a thrust vector based on varying blade length and chord, in this case. Recall that BET involves a non-dimensional parameter known as rotor solidity, which depends on the blade chord. Since both dimensional characteristics are changing, the chord was set up as a function of blade length by means of the rotor blade aspect ratio, meaning that this parameter was kept constant in this part of the study.

\[
c = \frac{2R}{AR} \quad (3.12)
\]

The reference \( AR \) used in equation 3.12 was taken from the SAB blade shown in Figure 3.11. The calculation for the weight of the motor was done based on the correspondent torque to each blade increment, as demonstrated in the previous section.
The set of assumptions for this analysis are listed below:

- $AR = 24.8$
- $M = 0.5$ based on aviation standard atmosphere at sea level (adiabatic index $k = 1.4$, gas constant $R = 286.9 J/K\cdot Kg$, temperature $T = 288.15 K$)
- Airfoil NACA 0012
- $\theta = 13^\circ$ collective pitch

### 3.2.4 Latency Variation

Up to this point in this chapter, calculations were based on the assumption that the acceleration of a motor from 1400rpm to 1600rpm has to be such that the time constant remains 0.077sec. In this analysis, it is desired to show how this latency changes while maintaining constant torque, which was selected based on the peak torque from EMRAX 228. The reason for choosing this motor out of the five available was that performance wise is the average of them, with a torque of 230Nm (169.56lb·ft). Combining equation 3.8 and Newton’s second law for rotation it is possible to derive the following expression which leads to the desired data.

$$
\tau = \frac{\Delta \omega I(R)}{T}
$$

### 3.3 PAVER CATIA Design

The inertia matrix representing PAVER for the rotational system was obtained from the CAD model built in CATIA. Figure 3.12 shows the aircraft with its inertial axes located at the center of mass. The correspondent materials were assigned to each part and include mainly Chromoly 4130, carbon fiber aerospace grade, and aluminum 6061-T6.
The following inertia matrix corresponds to the design as shown, and it was automatically generated by the software, which means it may not be 100% true to the real model. For this reason it is expected that the controller may not display the same behavior as the simulation in future real implementation.

\[
J = \begin{bmatrix}
241.969 & 0.216 & -23.798 \\
0.216 & 343.63 & 0.009 \\
-23.798 & 0.009 & 570.012
\end{bmatrix} \text{ slug \cdot ft}^2
\quad (3.14)
\]

Figure 3.12: PAVER CATIA model with inertial axes.

3.4 Aircraft Modeling Methods

3.4.1 Zero Torque Combination Analysis

Newton’s third law states that when a body exerts a force on another one, the latter produces a reaction force of equal magnitude and opposite direction. The same principle applies to a motor with a rotating shaft, where the reaction force is called torque. Particularly, PAVER is equipped with eight electric motors and each of them produce a torque reaction that is transferred to the aircraft structure. This effect is of great importance in terms of stability and control given that it is required to maintain zero net
torque conditions in all operations, except when yawing is desired. Given the number of motors, there are seventy possible combinations with clockwise and counterclockwise rotation. These combinations were analyzed and discarded based on three different criteria:

- Maintain zero yawing during pure pitch and roll rotation
- Redundant combinations
- Structure

Given that in this study the controller operates based on variable collective pitch and constant rpm, the aircraft pitch and roll is produced by differential thrust, while yawing is produced by differential torque. Referring to Figure 3.13, in order to produce negative pitch, the net thrust generated by motors \( M_1, M_2, M_3, \) and \( M_4 \) has to be lower than the net thrust produced by motors \( M_5, M_6, M_7, \) and \( M_8 \). Likewise, to produce positive roll, motors \( M_1, M_2, M_5, \) and \( M_6 \) have to produce more thrust than motors \( M_3, M_4, M_7 \) and \( M_8 \). Note that despite the thrust differential, the net thrust of the eight motors remains constant as long as there is no altitude change commanded.

![Figure 3.13: Motors layout.](image)
Any thrust change in the motors produces torque, hence, analyzing the pitch command, it turns out that the requirement for zero torque is to have two forward and aft counter-rotating pairs. Figure 3.14 shows the combinations that were discarded due to the pitch command.

![Combination selection due to pitch at zero torque.](image)

Figure 3.14: Combination selection due to pitch at zero torque.

Similarly, rolling requires two left and right counter-rotating pairs, which leads to eliminate the seventeen combinations from the remaining ones as shown in Figure 3.15.
The next step is to remove redundant combinations, which leaves nine ones.

When torque is produced by the motors, it is transferred to the structure. Therefore, it is vital to remove any combination that could compromise the structure of the aircraft. From the remaining combinations, the first one to the left shown in Figure 3.17 is the only one that would cause structural problems. Note that along the spar, there are two clockwise rotating motors on one side and two counterclockwise motors on the other side. This issue would cause spar bending on the X-Y plane.
3.4.2 PAVER Non-linear Differential Equations

This subsection summarize the 12 non-linear differential equations that conform the mathematical model of PAVER in the hovering phase of flight.

Rotational Equations:

$$
\begin{align*}
\dot{p} & = [J_{xx} \ J_{xy} \ J_{xz}]^{-1} \left\{ -p \ J_{xy} \ J_{xz} \ J_{yz} \right\} - \frac{1}{m} \left( \begin{array}{c} p \\ q \\ r \end{array} \right) \times \left( \begin{array}{c} 1 \ J_{xy} \ J_{xz} \ J_{yz} \\ J_{xy} \ J_{yy} \ J_{yz} \\ J_{xz} \ J_{yz} \ J_{zz} \end{array} \right) \right\} + \left[ M_{x} \ M_{y} \ M_{z} \right] \\
\dot{q} & = [J_{yy} \ J_{yx} \ J_{yz}]^{-1} \left\{ -q \ J_{xy} \ J_{xz} \ J_{yz} \right\} - \frac{1}{m} \left( \begin{array}{c} p \\ q \\ r \end{array} \right) \times \left( \begin{array}{c} 1 \ J_{xy} \ J_{xz} \ J_{yz} \\ J_{xy} \ J_{yy} \ J_{yz} \\ J_{xz} \ J_{yz} \ J_{zz} \end{array} \right) \right\} + \left[ M_{x} \ M_{y} \ M_{z} \right] \\
\dot{r} & = [J_{zz} \ J_{zy} \ J_{zy}]^{-1} \left\{ -r \ J_{xy} \ J_{xz} \ J_{yz} \right\} - \frac{1}{m} \left( \begin{array}{c} p \\ q \\ r \end{array} \right) \times \left( \begin{array}{c} 1 \ J_{xy} \ J_{xz} \ J_{yz} \\ J_{xy} \ J_{yy} \ J_{yz} \\ J_{xz} \ J_{yz} \ J_{zz} \end{array} \right) \right\} + \left[ M_{x} \ M_{y} \ M_{z} \right] 
\end{align*}
$$

(3.15)

Kinematics Equations:

$$
\begin{align*}
\dot{\phi} & = \begin{bmatrix} \sin \phi & \cos \phi \\ 0 & 0 \end{bmatrix} \dot{\theta} \\
\dot{\psi} & = \begin{bmatrix} 0 & 0 \end{bmatrix} \dot{\phi}
\end{align*}
$$

(3.16)

Translational Equations:

$$
\begin{align*}
\ddot{x} & = 0 - \frac{F_{z}}{m} \left[ \begin{array}{c} \cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi \\ \cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi \\ \cos \phi \cos \theta \end{array} \right] \\
\ddot{y} & = 0 - \frac{F_{z}}{m} \left[ \begin{array}{c} \cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi \\ \cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi \\ \cos \phi \cos \theta \end{array} \right] \\
\ddot{z} & = g \left[ \begin{array}{c} \cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi \\ \cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi \\ \cos \phi \cos \theta \end{array} \right]
\end{align*}
$$

(3.17)

Navigation Equations:

$$
\begin{align*}
\dot{x} & = \begin{bmatrix} c \theta \psi & s \theta \psi c \psi - c \phi s \psi & c \phi s \psi c \psi + s \phi s \psi \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} \\
\dot{y} & = \begin{bmatrix} c \theta \psi & s \phi s \psi + c \phi \psi & c \phi s \psi - s \phi \psi \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} \\
\dot{z} & = \begin{bmatrix} -s \theta & s \phi \psi & c \phi \psi \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix}
\end{align*}
$$

(3.18)
3.5 Control Development

3.5.1 PAVER Full NLDI

A full NLDI control architecture can be developed using a cascade control structure. This technique subdivides the whole system in two loops, which are called outer and inner loop. The outer loop contains the guidance and navigation control laws, hence, the translational equations of motion, while the inner loop contains the attitude control laws and the rotational equations of motion.

In a cascade control strategy, a set point or desired value is fed to the outer loop and the signal is processed by each controller to obtain the desired final response of the aircraft. This requires for each control stage to be faster than the preceding one to allow enough time to process the new signal. Therefore, the outer and inner loop are internally split between a slow mode and fast mode controller. Note that implicitly, the inner loop as a whole must be faster than the outer loop.

All these concepts can be easily understood by following the full NLDI control architecture displayed in Figure 3.18. Here, the red box depicts the inner loop, which houses the angular rates controller (fast mode) and the Euler angles controller (slow mode). Similarly, the green box represents the outer loop containing the velocity controller (fast mode) and the position controller (slow mode).

The set points, in this case, can be generated by the autopilot for autonomous navigation and altitude hold, or by the pilot. Additionally, this particular controller has been designed with an independent altitude control, which relieves the pilot from maintaining the desired altitude while controlling the aircraft in the X-Y plane. This is achieved by utilizing the decoupled altitude signal from the autonomous navigation module, which simply adds the $X$, $Y$ coordinates if full auto-nav is desired.
The result from the controller, based on the set point, is a set of forces and moments required to produce the desired behavior or reaction of the aircraft. Given that this particular controller is based on collective pitch change at constant rpm, the actuator controller contains the algorithm to generate the appropriate collective pitch angle signal. At this point, since this is a simulation-based research, the collective pitch signal has to be fed to the motor-rotor system, which was developed in sections 2.1 and 2.2. Hence, the rotational and translational subsystems will generate the new states reproducing the aircraft behavior.

### 3.5.1.1 Rotational Subsystem - Angular Rates NLDI

Angular rates controllers are essential in all aerospace systems to achieve overall aircraft stability. All controllers require signals of the current states to compute the error with the desired value and generate the stabilizing command. The response signals are obtained through sensors like accelerometers or gyroscopes. The real-time angular rate that the aircraft is experiencing at a given time is sensed by gyroscopes, which present a significant advantage given that this kind of sensors is inexpensive, accurate and easy to operate.
To develop an angular rate NLDI controller refer to equation 2.31 which describes the rotational dynamics of a rigid body.

\[
\dot{\omega} = J^{-1} \{-\omega \times (J \omega) + \bar{M}(t)\}
\] (3.19)

Recall that \(\bar{M} \in \mathbb{R}^{3 \times 1}\) is the vector containing the moments acting on the body, while \(\omega \in \mathbb{R}^{3 \times 1}\) represents the angular rates and \(J \in \mathbb{R}^{3 \times 3}\) the time-invariant inertia matrix, which is important to derive the inversion from equation 3.19. Solving this equation for the moments acting on the body results in the control law that generates the required moments to stabilize the rotational system.

\[
\bar{M}(t) = J \{v(t) - J^{-1}[-\omega \times (J \omega)]\}
\] (3.20)

Here, \(v(t)\) is the virtual controller producing the feedback linearization. For simplification, let \(f(\omega) = J^{-1}[-\omega \times (J \omega)]\), then equation 3.20 resembles the general expression for the dynamic inversion derived in subsection 2.5.3 with \(g(x)^{-1} = J\) resulting in

\[
\bar{M}(t) = J \{v(t) - f(\omega)\}
\] (3.21)

Combining equation 3.19 and 3.21 the rotational system closed loop dynamics are:

\[
\dot{\omega} = v_\omega(t)
\] (3.22)

Given that for this study the pseudo controller was chosen to be a PID, in the case of the angular rate NLDI \(v(t) = [v_p \ v_q \ v_r]^T\). It was found that to achieve stability in this system it was enough to implement a PI controller. Then the virtual controller can be
expressed as follows (Perez Rocha et al., 2016)(Wang and Zhang, 2012):

\[ v_{\omega}(t) = \begin{bmatrix} K_p (p_{ref} - p) + K_i \int (p_{ref} - p) \, dt + \dot{p}_{ref} \\ K_p (q_{ref} - q) + K_i \int (q_{ref} - q) \, dt + \dot{q}_{ref} \\ K_p (r_{ref} - r) + K_i \int (r_{ref} - r) \, dt + \dot{r}_{ref} \end{bmatrix} \] (3.23)

Note that the vector equation 3.23 has a parameter added at the end of each component: \( \dot{p}_d, \dot{q}_d, \dot{r}_d \). This is known as feed forward, and takes care of large changes in the set-points or desired signals by making the control response even faster (Perez Rocha, 2016).

The controller gains can be tuned until the desired response is obtained or they can be calculated to meet a required natural frequency and damping applying equations 3.24, 3.25 and 3.26.

\[ K_{p_p} = 2\xi_p \omega_{n_p} \] (3.24)

\[ K_{p_q} = 2\xi_q \omega_{n_q} \] (3.25)

\[ K_{p_r} = 2\xi_r \omega_{n_r} \] (3.26)

### 3.5.1.2 Rotational Subsystem - Euler Angles NLDI

When it comes to the attitude of an aircraft, the variables involved are the Euler angles. This portion of the controller is the slow mode of the inner loop, hence, the attitude controller is the one that generates the desired angular rates which are fed to the fast mode dynamics described in the angular rates NLDI derivation. With that condition in mind, the equation to calculate the set-point for the angular rates controller can be obtained by inverting equation 2.24 from subsection 2.4.3 as follows. Let \( \Theta = [\phi \ \theta \ \psi] \), then the kinematics equation can be written as (Yuan et al., 2009):
Here, $g(\Theta)$ represents the kinematics matrix, which allows to obtain the inversion and generate the required control law.

$$\dot{\Theta} = g(\Theta)\omega$$  \hspace{1cm} (3.27)

Expanding equation 13 results in the following expression

$$\omega_d(t) = g(\Theta)^{-1}v_\Theta(t)$$  \hspace{1cm} (3.28)

where $v_\Theta(t)$ is the linear controller based on a simple proportional gain, which in the simulation environment demonstrated to be enough to attain the required stability dynamics for this phase of the inner loop controller. Combining the fast mode and slow mode of the inner loop dynamics it is possible to generate the attitude controller schematic as shown in Figure 3.19:

![Figure 3.19: Inner loop schematic.](image)

### 3.5.1.3 Translational Subsystem NLDI

As mentioned before, the outer loop of the full NLDI architecture corresponds to the guidance phase of the controller, and it is composed by a slow mode and fast mode stage. In this case, the velocity control corresponds to the fast mode whereas the position control corresponds to the slow mode. The feedback linearization of the translational dynamics is produced by the inversion of equation 2.35. Then the non-linearities associated with the
system are suppressed and stabilization can be achieved by simple control techniques as discussed in the development of the attitude controller. The process to generate the inversion is based on performing a variable change and rearrange the terms to obtain an invertible system of the form \( \dot{x} = f(x) + g(x)u(t) \).

Given that the propulsion plant is fixed in the z-axis, the only force acting on the body is thrust \( F_z \), which implies that the forward and lateral motion of the aircraft is produced by pitch and roll respectively, while altitude is controlled by the net thrust produced by the eight motors. The attitude change is generated by differential thrust, hence, the inputs to the system are desired pitch \( \theta_{ref} \) and roll \( \phi_{ref} \), whereas for altitude is \( F_{z_{ref}} \). This means that the outer loop creates the set points for the attitude controller, which aligns with the cascade control strategy being applied.

The first step for the inversion, in this case, is the variable change. Therefore, consider the following equation, which is the expanded version of equation 2.35:

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{z}
\end{bmatrix} =
\begin{bmatrix}
0 \\
0 \\
g
\end{bmatrix}
+ \begin{bmatrix}
-\frac{F_z}{m} (\cos \phi \sin \theta \cos \psi) - \frac{F_z}{m} (\sin \phi \sin \psi) \\
-\frac{F_z}{m} (\cos \phi \sin \theta \sin \psi) + \frac{F_z}{m} (\sin \phi \cos \psi) \\
-\frac{F_z}{m} \cos \phi \cos \theta
\end{bmatrix}
\]

Replacing the input variables in equation 3.30 yields,

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{z}
\end{bmatrix} =
\begin{bmatrix}
0 \\
0 \\
g
\end{bmatrix}
+ \begin{bmatrix}
-\frac{F_z}{m} (\cos \phi \sin \theta_{ref} \cos \psi) - \frac{F_z}{m} (\sin \phi_{ref} \sin \psi) \\
-\frac{F_z}{m} (\cos \phi \sin \theta_{ref} \sin \psi) + \frac{F_z}{m} (\sin \phi_{ref} \cos \psi) \\
-\frac{F_{z_{ref}}}{m} \cos \phi \cos \theta
\end{bmatrix}
\]

To obtain the invertible system the inputs representing \( u(t) \) are factorized and equation 3.31 can be rearranged as follows:
\[
\begin{align*}
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{z}
\end{bmatrix} &= \begin{bmatrix} 0 & -\frac{Fz}{m} (\cos \phi \cos \psi) & -\frac{Fz}{m} \sin \psi \\
0 & -\frac{Fz}{m} (\cos \phi \sin \psi) & \frac{Fz}{m} \cos \psi \\
g & 0 & 0
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} + \begin{bmatrix}
\sin \theta_{ref} \\
\sin \phi_{ref} \\
F_{z,ref}
\end{bmatrix}
\end{align*}
\]  

(3.32)

Similarly to the previous systems inversions, solving for the inputs vector,

\[
u(t) = g(x)^{-1} [v_r(t) - f(x)]
\]  

where \(v_r(t)\) is the virtual controller for the aircraft guidance algorithm. As a result, the feedback linearization for longitudinal, lateral and vertical motion is given by equations 3.33, 3.34, and 3.35 (Perez Rocha, 2016)(Ireland et al., 2015).

\[
\theta_{ref} = \sin^{-1} \left( \frac{m (v_x(t) \cos \psi - v_y(t) \sin \psi)}{F_{z,d} \cos \phi} \right)
\]  

(3.33)

\[
\phi_{ref} = \sin^{-1} \left( \frac{m (v_x(t) \sin \psi - v_y(t) \cos \psi)}{F_{z,d}} \right)
\]  

(3.34)

\[
F_{z,ref} = \frac{m (v_z(t) - g)}{\cos \phi \cos \theta}
\]  

(3.35)

The second order dynamics governing the system’s outer loop are:

\[
v_r(t) = \begin{bmatrix}
K_{P_x} K_{P_z} (x_{ref} - x) - K_{P_z} \dot{x} \\
K_{P_y} K_{P_z} (y_{ref} - y) - K_{P_z} \dot{y} \\
K_{P_z} K_{P_z} (z_{ref} - z) - K_{P_z} \dot{z}
\end{bmatrix}
\]  

(3.36)

For a specific performance based on natural frequency and damping, the controller can be expressed as:

\[
v_r(t) = \begin{bmatrix}
\omega_{nx}^2 (x_{ref} - x) - 2 \xi_x \omega_{nx} \dot{x} \\
\omega_{ny}^2 (y_{ref} - y) - 2 \xi_y \omega_{ny} \dot{y} \\
\omega_{nz}^2 (z_{ref} - z) - 2 \xi_z \omega_{nz} \dot{z}
\end{bmatrix}
\]  

(3.37)
3.5.2 Decoupled Control Input

Equations 3.38 and 3.39 show the control allocation used in the system. To decouple attitude moments and net thrust, it was assumed that the motors effort is split equally by eight. Note that thrust is applicable to altitude, roll, and pitch whereas yaw depends on the net torque produced by each motor-rotor. Heading control turns out to be more complicated due to the principle hovering multirotors apply to generate yawing moment. In a simple quadcopter, each motor counter-acts each other by having counter-rotating pairs, which means that the net torque around the z-axis is zero. To produce a yawing moment in such setup, it is required to produce a differential torque by changing the rotational speed (or collective pitch) of one pair and increasing it in the other one. This will also produce a change of thrust in each motor, however, the net thrust must remain constant unless coupled with a desired altitude change (ElKholy, 2014).

\[
\begin{align*}
\mathbf{U}(t) &= \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} h \\ \phi \\ \theta \\ \psi \end{bmatrix} = \begin{bmatrix} F_{z,ref} \\ M_{x,ref} \\ M_{y,ref} \\ M_{z,ref} \end{bmatrix} \\
\mathbf{U}(t) &= \begin{bmatrix} f_1 + f_2 + f_3 + f_4 + f_5 + f_6 + f_7 + f_8 \\ r_{1x} \cdot f_1 + r_{2x} \cdot f_2 + r_{3x} \cdot f_3 + r_{4x} \cdot f_4 + r_{5x} \cdot f_5 + r_{6x} \cdot f_6 + r_{7x} \cdot f_7 + r_{8x} \cdot f_8 \\ r_{1y} \cdot f_1 + r_{2y} \cdot f_2 + r_{3y} \cdot f_3 + r_{4y} \cdot f_4 + r_{5y} \cdot f_5 + r_{6y} \cdot f_6 + r_{7y} \cdot f_7 + r_{8y} \cdot f_8 \\ \tau_1 - \tau_2 - \tau_3 + \tau_4 + \tau_5 - \tau_6 - \tau_7 + \tau_8 \end{bmatrix}
\end{align*}
\] (3.38)

(3.39)

The mathematical models of the motor, the rotor and the aircraft (PAVER) were built and tested in MATLAB Simulink. While the aircraft was made fully autonomous, pilot-in-the-loop was also built in the system. Therefore, the only piece of hardware used in the simulation was a joystick Logitech Extreme 3D PRO.
3.6 MATLAB Simulink Interface

3.6.1 Overview

Figure 3.20 displays the simulation interface. For practical purposes a semi-traditional aviation “six-pack” was implemented to easily monitor aircraft behavior such as airspeed, attitude, altitude, vertical speed, and heading. Usually, in most aircraft, there is also a turn coordinator, which is not included in the simulation.

The control module houses the NLDI algorithm and generates the individual collective pitch signal for each motor. The plant module contains the motor model and motors controller, which process the signal and outputs the stabilizing and control forces and moments. Also, this module includes the aircraft model based on the rigid body equations, which were classified as translational and rotational subsystems. The outputs from this module include twenty-one states from the subsystems and twelve states from the NLDI and motor controller, for a total of twenty-five states. The state feedback and routing module distributes the states in the system for controller feedback input, monitoring and Flight Gear visualization.

Next, the control panel indicates the actual forces and moments being fed in the plant, and the control switches allow to toggle heading control, altitude hold and autopilot. The motor model switch was used for testing purposes allowing to run the simulation with or without the electric model of the motor, which basically adds a small lag to rpm response when torque demand occurs.

3.6.2 Pilot-in-the-Loop

PAVER is meant to be a vehicle that provides effective and safe passengers transportation. Therefore, initially, this vehicle is not meant to fly without a pilot.

Recall from Figure 3.18 that the pilot inputs are distributed in the control architecture such that the controller can help the pilot achieve the desired aircraft behavior. For the position case, where the pilot can control movement in the X-Y plane during hovering, the signal is fed into the velocity controller, hence, the pilot simply tells the controller in
which direction wants to accelerate. This is because after various test runs, and control tuning iterations it was concluded that the pilot cannot accurately stop the aircraft in the desired position by only creating opposite pitch or bank. The problem associated with this behavior is related to the big inertia of the aircraft and its limited ability to create enough stabilizing moments.

Once the pilot reaches the desired position, as soon as the joystick is neutralized the controller will safely bring the aircraft to a stop and remain hovering if the altitude hold switch is engaged or the throttle is properly adjusted.

The manual altitude control is one of the simplest in this system, given that the pilot operates the throttle that talks directly to the set of eight motors increasing or decreasing the net thrust.

As explained in subsection 3.5.2, heading control has a complex operation given that it is based on differential torque and there are issues associated with it. Regardless, for testing purposes, a pilot yawing control was included signaling the yaw angular rate controller located in the inner loop.
4. RESULTS

4.1 Electric Motor Modeling Experimental Data and Results

4.1.1 Transmission Experimental Data and Factor Calculation

The number of teeth for each gear are listed in Table 4.1.

Table 4.1: Transmission gear size (teeth number).

<table>
<thead>
<tr>
<th>( N )</th>
<th>( N_1 )</th>
<th>( N_2 )</th>
<th>( N_3 )</th>
<th>( N_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_1 )</td>
<td>22</td>
<td>60</td>
<td>19</td>
<td>68</td>
</tr>
</tbody>
</table>

From Table 4.1 and equation 3.1 the theoretical transmission factor results in 0.1025.

The experimental approximation was made with a tachometer yielding the following results:

Table 4.2: Transmission Experimental Data.

<table>
<thead>
<tr>
<th>Motor rpm</th>
<th>Rotor rpm</th>
</tr>
</thead>
<tbody>
<tr>
<td>659.05</td>
<td>72.115</td>
</tr>
<tr>
<td>836.15</td>
<td>90.542</td>
</tr>
<tr>
<td>1019.3</td>
<td>109.99</td>
</tr>
<tr>
<td>1282</td>
<td>135.89</td>
</tr>
<tr>
<td>1440.3</td>
<td>157.69</td>
</tr>
<tr>
<td>1753.5</td>
<td>187.98</td>
</tr>
<tr>
<td>1956.7</td>
<td>208.6</td>
</tr>
<tr>
<td>2930.3</td>
<td>309.5</td>
</tr>
</tbody>
</table>

The linear regression of the data is shown in Figure 4.1, and it results in a transmission factor of 0.1047, which differs by 2.15% with the theoretical value calculated above.
4.1.2 Electrical Parameters Experimental Results

Table 4.3 contains the experimental data recorded from the tachometer and the oscilloscope used in the motor test bed experiment.
Table 4.3: rpm and Back EMF Values.

<table>
<thead>
<tr>
<th>Motor rpm</th>
<th>Back EMF [V]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2150</td>
<td>4.19</td>
</tr>
<tr>
<td>2418</td>
<td>4.66</td>
</tr>
<tr>
<td>2690</td>
<td>5.28</td>
</tr>
<tr>
<td>2948</td>
<td>5.775</td>
</tr>
<tr>
<td>3380</td>
<td>6.615</td>
</tr>
<tr>
<td>3648</td>
<td>7.17</td>
</tr>
<tr>
<td>3952</td>
<td>7.825</td>
</tr>
<tr>
<td>4384</td>
<td>8.59</td>
</tr>
<tr>
<td>4672</td>
<td>9.23</td>
</tr>
<tr>
<td>5005.2</td>
<td>9.82</td>
</tr>
<tr>
<td>5305.8</td>
<td>10.44</td>
</tr>
<tr>
<td>5561.8</td>
<td>10.7855</td>
</tr>
<tr>
<td>5847.8</td>
<td>11.405</td>
</tr>
<tr>
<td>6091.2</td>
<td>12.01</td>
</tr>
<tr>
<td>6427.4</td>
<td>12.685</td>
</tr>
<tr>
<td>6740</td>
<td>13.27</td>
</tr>
<tr>
<td>7087.4</td>
<td>14.025</td>
</tr>
<tr>
<td>7296.8</td>
<td>14.54</td>
</tr>
<tr>
<td>7453.2</td>
<td>14.75</td>
</tr>
<tr>
<td>7738</td>
<td>15.39</td>
</tr>
<tr>
<td>8093.6</td>
<td>15.925</td>
</tr>
<tr>
<td>14896.8</td>
<td>29.45</td>
</tr>
</tbody>
</table>

Figure 4.2 shows the rpm and Back EMF data distribution along with a trend line. The data regression resulted in equation 4.1, which leads to parameter $K_B$ required in equations 2.1 and 2.2:

$$EMF = 0.001985N - 0.088319$$  \hfill (4.1)
Figure 4.2: Motor rpm vs Back EMF.

The remaining electric parameters like the motor winding resistance $R_m$ and inductance $L_m$ were obtained by means of an LCR meter, resulting in 0.0158 Ohms and 8E-6 H respectively.

### 4.1.3 Mechanical Parameters Experimental Results

The following Table lists the mechanical parameters and the resulting inertia.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density, $\rho$ [lb/ft$^3$]</td>
<td>86.4</td>
</tr>
<tr>
<td>Rod radius, $r$ [ft]</td>
<td>0.014583</td>
</tr>
<tr>
<td>Single blade span, $R_t$ [ft]</td>
<td>2.083</td>
</tr>
<tr>
<td>Hub, $h$ [ft]</td>
<td>0.5</td>
</tr>
<tr>
<td>Single blade inertia, $J_R$ [lb·ft$^2$]</td>
<td>0.607</td>
</tr>
<tr>
<td>Total rotor inertia [lb·ft$^2$]</td>
<td>1.214</td>
</tr>
</tbody>
</table>
4.1.4 Motor Characterization Summary

A summary of the motor characterization parameters with their correspondent value is presented in Table 4.5. The units correspond to the SI system to maintain the relationship between the torque and back EMF constants.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Armature resistance, $R_m$ [Ohms]</td>
<td>0.0158</td>
</tr>
<tr>
<td>Motor winding inductance, $L_m$ [H]</td>
<td>8E-6</td>
</tr>
<tr>
<td>Back EMF, $K_B$ [V/ rad]</td>
<td>0.0189</td>
</tr>
<tr>
<td>Inertia of the armature, $J_m$ [Kg $\cdot$ m$^2$]</td>
<td>0.0001264</td>
</tr>
<tr>
<td>Motor torque, $K_T$ [N$\cdot$m/A]</td>
<td>0.0189</td>
</tr>
<tr>
<td>Rotor inertia, $J_R$ [Kg $\cdot$ m$^2$]</td>
<td>0.02317706</td>
</tr>
<tr>
<td>Total inertia, $J_t$ [Kg $\cdot$ m$^2$]</td>
<td>0.0233</td>
</tr>
</tbody>
</table>

4.2 Rotor Modeling Data Calculation

4.2.1 Airfoil and Rotor Characteristics

Table 4.6: NACA 0012 airfoil characteristics.

<table>
<thead>
<tr>
<th>$C_{l_{a}}$ [1/rad]</th>
<th>$2\pi$</th>
<th>$C_{d_{0}}$</th>
<th>0.011</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$ [in]</td>
<td>2.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.7: Rotor characteristics.

<table>
<thead>
<tr>
<th>$N_b$</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>0.0513</td>
</tr>
<tr>
<td>$R$ [in]</td>
<td>31</td>
</tr>
</tbody>
</table>

4.2.2 Rotor Modeling Results and Plots

For the simulation of the aircraft being analyzed in this study, the thrust, power, and torque were designed to run via look-up tables as a function of collective pitch. Therefore, it is required to have a vector for each parameter.

From experimental data on the same rotor, it is known that the thrust produced at 2100rpm and 12° collective pitch is approximately 50lbf (Gehlot, 2017). Therefore, solving for $\theta_b$ in equation 2.12 and solving equation 2.6 for a certain thrust range, 0 to 60lbf in this case, it is possible to generate the correspondent collective pitch vector. Note that in this case, equation 2.6 is not incremental, but it accounts for the net $C_T$ and thrust. In addition, aviation standard atmosphere was assumed for the calculation, which means
that air density is 0.00238 slug/ft\(^3\).

The data obtained from this operation is shown in Table 4.8.
Table 4.8: Thrust and thrust coefficient vs collective pitch data at 2100rpm.

<table>
<thead>
<tr>
<th>𝜏 [lb]</th>
<th>𝐶_𝐓</th>
<th>𝜃 [deg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.0001</td>
<td>0.81</td>
</tr>
<tr>
<td>2</td>
<td>0.0002</td>
<td>1.22</td>
</tr>
<tr>
<td>3</td>
<td>0.0004</td>
<td>1.57</td>
</tr>
<tr>
<td>4</td>
<td>0.0005</td>
<td>1.89</td>
</tr>
<tr>
<td>5</td>
<td>0.0006</td>
<td>2.18</td>
</tr>
<tr>
<td>6</td>
<td>0.0007</td>
<td>2.46</td>
</tr>
<tr>
<td>7</td>
<td>0.0009</td>
<td>2.72</td>
</tr>
<tr>
<td>8</td>
<td>0.0010</td>
<td>2.98</td>
</tr>
<tr>
<td>9</td>
<td>0.0011</td>
<td>3.23</td>
</tr>
<tr>
<td>10</td>
<td>0.0012</td>
<td>3.47</td>
</tr>
<tr>
<td>11</td>
<td>0.0014</td>
<td>3.71</td>
</tr>
<tr>
<td>12</td>
<td>0.0015</td>
<td>3.94</td>
</tr>
<tr>
<td>13</td>
<td>0.0016</td>
<td>4.17</td>
</tr>
<tr>
<td>14</td>
<td>0.0017</td>
<td>4.39</td>
</tr>
<tr>
<td>15</td>
<td>0.0019</td>
<td>4.61</td>
</tr>
<tr>
<td>16</td>
<td>0.0020</td>
<td>4.83</td>
</tr>
<tr>
<td>17</td>
<td>0.0021</td>
<td>5.05</td>
</tr>
<tr>
<td>18</td>
<td>0.0022</td>
<td>5.26</td>
</tr>
<tr>
<td>19</td>
<td>0.0024</td>
<td>5.47</td>
</tr>
<tr>
<td>20</td>
<td>0.0025</td>
<td>5.68</td>
</tr>
<tr>
<td>21</td>
<td>0.0026</td>
<td>5.89</td>
</tr>
<tr>
<td>22</td>
<td>0.0027</td>
<td>6.09</td>
</tr>
<tr>
<td>23</td>
<td>0.0029</td>
<td>6.30</td>
</tr>
<tr>
<td>24</td>
<td>0.0030</td>
<td>6.50</td>
</tr>
<tr>
<td>25</td>
<td>0.0031</td>
<td>6.70</td>
</tr>
<tr>
<td>26</td>
<td>0.0032</td>
<td>6.90</td>
</tr>
<tr>
<td>27</td>
<td>0.0034</td>
<td>7.10</td>
</tr>
<tr>
<td>28</td>
<td>0.0035</td>
<td>7.30</td>
</tr>
<tr>
<td>29</td>
<td>0.0036</td>
<td>7.49</td>
</tr>
<tr>
<td>30</td>
<td>0.0037</td>
<td>7.69</td>
</tr>
<tr>
<td>31</td>
<td>0.0039</td>
<td>7.88</td>
</tr>
<tr>
<td>32</td>
<td>0.0040</td>
<td>8.07</td>
</tr>
<tr>
<td>33</td>
<td>0.0041</td>
<td>8.27</td>
</tr>
<tr>
<td>34</td>
<td>0.0042</td>
<td>8.46</td>
</tr>
<tr>
<td>35</td>
<td>0.0044</td>
<td>8.65</td>
</tr>
<tr>
<td>36</td>
<td>0.0045</td>
<td>8.84</td>
</tr>
<tr>
<td>37</td>
<td>0.0046</td>
<td>9.03</td>
</tr>
<tr>
<td>38</td>
<td>0.0047</td>
<td>9.21</td>
</tr>
<tr>
<td>39</td>
<td>0.0048</td>
<td>9.40</td>
</tr>
<tr>
<td>40</td>
<td>0.0050</td>
<td>9.59</td>
</tr>
<tr>
<td>41</td>
<td>0.0051</td>
<td>9.77</td>
</tr>
<tr>
<td>42</td>
<td>0.0052</td>
<td>9.96</td>
</tr>
<tr>
<td>43</td>
<td>0.0053</td>
<td>10.14</td>
</tr>
<tr>
<td>44</td>
<td>0.0055</td>
<td>10.33</td>
</tr>
<tr>
<td>45</td>
<td>0.0056</td>
<td>10.51</td>
</tr>
<tr>
<td>46</td>
<td>0.0057</td>
<td>10.69</td>
</tr>
<tr>
<td>47</td>
<td>0.0058</td>
<td>10.87</td>
</tr>
<tr>
<td>48</td>
<td>0.0060</td>
<td>11.06</td>
</tr>
<tr>
<td>49</td>
<td>0.0061</td>
<td>11.24</td>
</tr>
<tr>
<td>50</td>
<td>0.0062</td>
<td>11.42</td>
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The data shown in the table above correspond to the following plot. Since is not the
coefficient but the dimensional thrust the meaningful parameter for the simulation, the plot is based on dimensional values only.

![Figure 4.3: Rotor thrust vs Collective pitch.](image)

Next, the power data displayed in table 4.9 comes from equation 2.14 as a function of $C_T$, which corresponds to the collective pitch vector generated in the previous operation.
Table 4.9: Thrust coefficient and Power coefficient vs collective pitch data at 2100rpm.

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For practical purposes, the power plot shown in Figure 4.4 was made in horsepower.
In subsection 2.2.4 it was shown that power is directly proportional to the torque being produced, and the link between the two parameters is the angular velocity. As a result, from the power data obtained in the previous step, it is possible to calculate the torque being produced by the rotor applying \( Q = \frac{P}{\Omega} \). Note that the power and torque graphs do not cross the y-axis at zero, this is due to the induced power associated with the zero-lift drag coefficient in equation 2.14.
Table 4.10: Power and Torque vs Collective pitch data at 2100rpm.

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The correspondent plot for the torque data shown in the table above is displayed in
the following Figure.

![Figure 4.5: Rotor torque vs Collective pitch.](image)

### 4.3 Motor Size and Rotor Blade Inertia Variation

#### 4.3.1 Inertia and Torque Increment Associated with Blade Length

The data obtained from the blade’s inertia calculations is reflected in Figure 4.6. The non-hub case assumes that the lifting section of the blade starts at the axis of rotation, while the hub case assumes a 6in section at the root, which is not contributing to the thrust production. Both cases present a rapid increment of the blade’s inertia which translates into higher torque required to maintain the assumption of 77ms as shown in Figure 4.7.
Figure 4.6: Non-hub and hub case - Blade length vs. Inertia.
Figure 4.7: Non-hub and hub case - Blade length vs. Torque.

4.3.2 Motor Size Variation Associated with Blade Length

The two plots shown Figure 4.8 present the results for the hub and non-hub cases of the motor volume variation as the length of the blade is increased. Recall that this portion of the research is based on equation 3.11, which is experimental, and it is only valid for
radial flux motors.

Figure 4.8: Non-hub and hub case - Blade length vs. Motor size.

4.3.3 Motor Weight Variation Associated with Rotor Blade Size

Based on the motors peak torque and their correspondent weight as given by Table 3.2, it was developed a relationship between those parameters as shown in Figure 4.9. The
curve fit that was utilized to obtain equation 4.2 relating motor weight and torque is based on the statistical power law, where a quantity varies as a power of the other one. The reason for choosing this method in this approximation is that it provides the best correlation, which in this case is 0.988.

\[ W_t = 0.7614T^{0.7062} \]  \hspace{1cm} (4.2)

![Figure 4.9: EMRAX motors weight vs rated peak torque.](image)

Based on equation 4.2 it is possible to run the torque values that correspond to each blade length increment. This relationship is shown by Figure 4.10, where it becomes clear that as the blade size increases, the weight of the motor to drive such rotor increases very rapidly.
Figure 4.10: Motor weight variation with rotor diameter increments.

4.3.4 Thrust-to-Weight Ratio Variation Associated with Rotor Blade Size

The test run under the constraints listed in subsection 3.2.3 lead to the thrust and weight curves shown in Figure 4.11. This graph shows that both parameters meet at a rotor diameter value of approximately 32ft, where the thrust-to-weight ratio is 1. Also, this result is confirmed in Figure 4.12, where the ratio is compared with the rotor diameter variation.
Figure 4.11: Thrust and weight curves vs Rotor diameter.

The thrust-to-weight ratio behavior displayed in Figure 4.12 starts with a rotor diameter of 2in, and the graph on the right shows a ratio range from ten to one. Scaling-up a rotor is possible while the ratio is kept above one, otherwise the motor-rotor assemble would not be able to lift itself. However, as the rotor size is increased the ratio is reduced very rapidly. It also has to be considered that this approximation does not include the weight of batteries or electric energy sources that usually account for a significant weight percentage.
Figure 4.12: Thrust-to-weight ratio vs Rotor Diameter.

Given that the blade size varies, the tangential speed at the tip will vary as a function of the blade length. For this reason, a constant tip Mach number was maintained throughout the experiment, which implies that rpm will be changing with each length increment. Figure 4.13 shows how the rotational speed of the rotor changed as the
diameter increased.

Figure 4.13: rpm variation at M0.5.

In small consumer drones, depending on the size, the rpm ranges from approximately 60,000rpm to 10,000rpm. Also, medium RC helicopters of approximately 5ft run at 2000rpm, while a common transport helicopter with gas engine operates bellow 1000rpm
with a rotor diameter above 30ft.

4.3.5 Latency Increment Associated with Rotor Blade Size

As seen in Figure 4.14, the time it takes to produce a variation in the motor rotational speed increases as the size of the rotor increases. This result was expected, given that changing the size of the rotor increases its inertia, which translates in higher torque required to spin the blades. Recall that the motor EMRAX 268, which has a peak torque of 169.56lb-ft, was utilized for test. As result, the latency curve is revealing how this particular motor would behave with the rotors of different size: a very small rotor would give a fast response, but it may not be suitable for the motor weight. Similarly, an oversized rotor would be able to handle the weight of this motor but the response time would be compromise.

Recall from section 2.2 that helicopters operate at constant rpm and change collective pitch to produce the required thrust for control and lifting purposes. This latency study justifies why helicopters operate this way.

Figure 4.14: Latency change with rotor diameter increments at constant torque.
4.4 Hovering Stability and Control Results

This section condenses the results of one of the main goals in this research work, which is achieving stable hovering flight for this particular rotorcraft. This includes a pure hovering case and CG envelope test.

The stability of an aircraft is mainly based on its attitudes states, which reveal the aircraft behavior. While PAVER is meant to be a tiltrotor, in the hovering phase of flight, which occurs prior to the transition stage, its stability behavior is not like an airplane. Hovering assumes static flight, hence airplane dynamics modes such as phugoid, short period or dutch roll do not apply in this case. However, they must be considered once the aircraft transitions to horizontal flight.

4.4.1 NLDI Tuning Process

When a cascade control strategy is utilized, it is important to keep in mind the response speed of each stage. This topic was discussed in subsection 3.5.1, where it was stated that the controller has an inner loop housing the attitude controller, and an outer loop containing the navigation controller. Each of these loops is subdivided into slow mode and fast mode. If the latter is not stabilized the rest of the system would be unstable. Therefore, the tuning process must start at the most inner fast mode controller and progress to the outer levels. From the NLDI architecture shown in Figure 3.18, the inner fast mode of the whole system is the angular rates controller, hence, the tuning starting point. Table 4.11 contains the correspondent gain values that were selected to stabilized each stage.

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<th>Speed controller</th>
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4.4.2 Pure Hover Attitude States and Moments

In this first test, the aircraft is acting fully autonomous, which includes not only automatic stability but also automatic navigation. This implies that the system will not only stabilize the vehicle but it will also maintain its initial position and heading. The only translation should occur in the z-axis while ascending or descending to achieve hovering flight, while the movements in the x-axis and y-axis should be close or equal to zero. Recall that for a helicopter in translational flight (X-Y plane), the rotor equations change, azimuthal symmetry is no longer valid and the inflow is not perpendicular to the rotor disk plane. Since this portion of rotor modeling falls outside the scope of this research, it is important to force the aircraft to maintain the initial hovering spot.

For the rotorcraft to be stable, it is required that the attitude states converge to zero. Therefore, any undesired variation of the attitudes states must be corrected by the controller (NLDI). In all tests, the simulation was set so that PAVER starts with the motors running at operational rpm and zero altitude. With the vehicle stable on the ground, at \( t = 15\text{sec} \) it was commanded to hover at 10ft from the ground with a climb rate limit of 500ft/min. Figure 4.15 shows the commanded altitude change and the controller response in the climb rate to accelerate the aircraft in the upward direction. It is important to point out that the derivation of the aircraft model was made based on aviation standard body axes. Referring back to Figure 2.11 in subsection 2.4.1, as seen from the pilot position, the x-axis is positive forward, the y-axis is positive rightward and the z-axis is positive downward. However, for practical purposes, the z-axis plots were inverted to display positive altitude and climb rate going up. From the plots below, it becomes clear that the navigation stage of the controller works appropriately and as expected by design (roughly 10ft in 2sec).
In section 2.2, it was explained how rotorcraft vary collective pitch to produce a change in thrust while keeping rpm constant by means of a governor. Hence, it is also important in this analysis to look at the performance of motor model developed in section 2.1. In this case, the rpm are kept constant by an electric speed controller (ESC), which was simulated as a PID with rpm feedback control. Each of the eight rotors are likely to
have different collective pitch variation, which depends on the controller command to maintain stability and control of the aircraft. This translates in a torque demand applied to each motor affecting its rotational speed. The drop in the rpm graph occurs when the climb is commanded and more thrust is required to accelerate upwards.

Figure 4.16: Average rpm and average torque from the eight motors.

The following graph shows the net thrust demand generated by the altitude change.
and the system response, which is almost perfect. This may not be 100% realistic given that there is a dynamic response to the thrust change associated with collective pitch variation. This factor was not modeled but recommended for future research.

Figure 4.17: Net thrust tracking.

Next, the attitudes states behavior during this test are shown in figures 4.18, 4.19 and 4.20. From the graphs, the oscillations peak seem to be high, but the scale of the vertical axis is very small.

When the altitude change is injected into the system, the vertical acceleration produces attitude variations due to the asymmetric inertia matrix. Here, the attitude controller or inner loop has to correct and compensate for these oscillations keeping the attitude states stable. There are two factors to keep in mind when analyzing the attitude states plot: the large inertia matrix of the aircraft and control power available. Also, while the units are in degrees and degrees per second, it is important to note the order of the y-axis scale, which is significantly small. This means that the controller is strong and fast enough to keep the vehicle stable while it accelerates to its hovering altitude.
Figure 4.18: Pitch and pitch rate tracking.
Similarly to pitch and roll, heading also converges to zero leading to a stable state. However, due to the control input utilized to produce yaw rate variation, and consequently a heading change, the response of this state is slightly different. Recall that this aircraft was modeled based on the same principles that govern the controls of quadcopters or octocopters. As discussed, in the previous chapter, heading is the only one state that is not
controlled by thrust but by differential torque produced by the rotors. By design, pitch, roll and yaw are decoupled as shown by equation 3.39, therefore net torque is zero when a variation of pure pitch, roll or net thrust is desired.

Figure 4.20: Heading and yaw rate tracking.

The next portion of the analysis refers to the stabilizing moments produced by the controllers and the response of the system. In this analysis, it is important to consider
motor model because when the controller signals a moment, it translates to collective pitch angle. In section 2.2, it was discussed how collective pitch produces thrust by keeping rpm constant, which consequently increases the torque on the shaft. If the motor is not capable of coping with this load, it will produce an rpm drop, which means that thrust will decrease. In turn not generating the required moment.

By looking at figures 4.21, 4.22, and 4.23 all three stabilizing moments response is stable, which correlates with the stable states discussed previously. Particularly, pitching moment has an offset that is attributed to the asymmetric CG location in the x-axis. This issue is discussed in the CG envelope test section. The rolling moment plot presents a perfect tracking, which is expected considering the symmetry of the motors location with respect to the X-Z. Lastly, the yawing moment is linked to the net torque acting on the body, hence, the response comes from the net torque generated by the eight rotors, and presents a convergent signal.

Figure 4.21: Stabilizing pitching moment.
4.4.3 Hovering CG Variation Test

In the previous section, the aircraft was tested with the CG and inertia matrix matching the inertia values in the controller. In other words, the NLDI was tuned to be stabilized in that particular configuration. If an object is placed at any point on the frame,
it will not only change the total mass of the vehicle but also its CG, inertia and moment arms with respect to the center of mass. To test the robustness of the controller four hovering cases were considered: three of them placing a simulated cubic object of approximately 26lbs far forward, far aft and on the farthest left location possible as shown in figures 4.24, 4.31 and 4.38. The object weight for this trials was selected estimating the weight of the equipment and miscellaneous items that have not been accounted for in the CAD model. For the fourth case, the weight of the object was increased to 140lbs and located were the pilot would be sitting. This allowed to study the behavior of the controller when the overall mass is highly increased, significantly reducing the thrust-to-weight ratio and control authority.

These variations represent some of the non-linearities or non-modeled objects that the controller would be handling as disturbances. The key of these tests is that the original inertia values used in the controller’s inversion and tuning process are kept constant, while the body inertia values and CG change.

The states to be observed and analyzed are attitude states, which reveal if the controller is able to handle the disturbance in each case while also complying with the desired altitude command.

**Light Object Forward Location**

- Total weight: 289.161lbs
- CG variation: \[
\begin{bmatrix}
6.0740 & 0 & 1.0690
\end{bmatrix}
\text{in}
\]
- Inertia variation:
\[
\begin{bmatrix}
23.5192 & 0 & -130.7873 \\
0 & 766.5777 & 0 \\
-130.7873 & 0 & 743.5411
\end{bmatrix}
\text{lb}\cdot\text{ft}^2\]
Figure 4.24: Light object placed forward.
Figure 4.25: Pitch and pitch rate tracking - Light weight forward.
Figure 4.26: Bank angle and roll rate tracking - Light weight forward.
Figure 4.27: Heading and yaw rate tracking - Light weight forward.
Figure 4.28: Stabilizing pitching moment - Light weight forward

Figure 4.29: Stabilizing rolling moment - Light weight forward.
Figure 4.30: Stabilizing yawing moment - Light weight forward.

**Light Object Aft Location**

- Total weight: 289.161 lbs

- CG variation: $\begin{bmatrix} -5.0560 & 0 & -2.1040 \end{bmatrix}$ in

- Inertia variation:
  $$
  \begin{bmatrix}
  89.6689 & 0 & -214.2467 \\
  0 & 604.4208 & 0 \\
  -214.2467 & 0 & 515.2666 \\
  \end{bmatrix} \text{ lb-ft}^2$$
Figure 4.31: Light object placed aft.
Figure 4.32: Pitch and pitch rate tracking - Light weight aft.
Figure 4.33: Bank angle and roll rate tracking - Light weight aft.
Figure 4.34: Heading and yaw rate tracking - Light weight aft.
Figure 4.35: Stabilizing pitching moment - Light weight aft.

Figure 4.36: Stabilizing rolling moment - Light weight aft.
Figure 4.37: Stabilizing yawing moment - Light weight aft.

**Light Object Left Location**

- Total weight: 289.161lbs

- CG variation: \[
\begin{bmatrix}
0.6560 & -1.2130 & 1.0690
\end{bmatrix}
\text{in}
\]

- Inertia variation: \[
\begin{bmatrix}
53.1193 & 16.0548 & -14.1244 \\
16.0548 & 32.2062 & 26.0931 \\
-14.1244 & 26.0931 & 38.8018
\end{bmatrix}
\text{lb-ft}^2\]
Figure 4.38: Light object placed left.
Figure 4.39: Pitch and pitch rate tracking - Light weight left.
Figure 4.40: Bank angle and roll rate tracking - Light weight left.
Figure 4.41: Heading and yaw rate tracking - Light weight left.
Figure 4.42: Stabilizing pitching moment - Light weight left.

Figure 4.43: Stabilizing rolling moment - Light weight left.
Figure 4.44: Stabilizing yawing moment - Light weight left.

**Heavy Weight Test**

- Total weight: 403.462lbs

- CG variation: 
  \[
  \begin{bmatrix}
  12.4480 & 0 & 3.5850 \\
  75.3515 & 0 & -233.9693 \\
  -233.9693 & 0 & 820.3727 \\
  \end{bmatrix}
  \text{in}
  \]

- Inertia variation: 
  \[
  \begin{bmatrix}
  0 & 887.7450 & 0 \\
  -233.9693 & 0 & 820.3727 \\
  \end{bmatrix}
  \text{lb-ft}^2
  \]
Figure 4.45: Heavy weight placed at the pilot location.
Figure 4.46: Pitch and pitch rate tracking - Heavy weight.
Figure 4.47: Bank angel and roll rate tracking - Heavy weight.
Figure 4.48: Heading and yaw rate tracking - Heavy weight.
Figure 4.49: Stabilizing pitching moment - Heavy weight.

Figure 4.50: Stabilizing rolling moment - Heavy weight
Each of the test cases have revealed that the controller is able to handle those disturbances keeping the aircraft hovering stabilized. Note that in each case the attitude states converge at zero, while the altitude is kept constant. In three out of four trials and in the pure hovering test, the rolling moment displayed high accuracy tracking. Refer to figures 4.42, 4.43, and 4.44. These are the plots of the moments response for the light object at the left location. Except for this test, the rest were run with a symmetric y-axis location for the object. This means that the item was placed at different positions only in the X-Z plane resulting in a y-axis symmetric inertia matrix variation and no change in the y component of the CG.

When the object was placed towards the left, the CG was moved from the y-axis causing the motors position to be non-symmetric with respect to the X-Z plane. This means that the moment arm of each motor on the left is shorter than the ones on the right, which translates into an unbalanced effort to generate the stabilizing moments. As a result, the plot of the rolling moment shows a biased error given that the controller is constantly trying to correct for the lack of equilibrium.

This unbalance is also occurring in all tests for the pitching moment, and the key of
the issue becomes obvious in the aft placement test. In this trial, the object placed far aft caused the original CG to move aft reducing the original non-symmetric position of the motors with respect to the Y-Z plane, which resulted in a reduction of the biased error.
5. CONCLUSIONS

5.1 Hovering Stabilization and Motor-Rotor Scaling

The future of aviation holds greener and safer airspace with new aircraft that promise to also provide an alternative for efficient urban transportation. Most of the designs that can meet the future demand resemble scaled-up consumer drones. PAVER is an idea inspired in this prospect, and it was used as a test bed for the research of propulsion and controllability problems affecting the scaling process.

Thrust variation based on collective pitch and constant rpm control was proposed as a solution to the lack of controllability associated with a scaled up fix pitch propeller. Also, this approach solves the problem of the energy required to spin scaled up rotors. However, there are limitations based on the size and weight of the motor and thrust production as the motor-rotor assembly is scaled-up.

To test this solution, a mathematical model of the motors, the rotors, and the aircraft was derived and simulated in MATLAB Simulink. Then a hovering feedback linearization NLDI controller, which commands collective pitch angles, was designed and implemented in the simulation. This controller is limited to an accurate modeling for real implementation, and it has limited disturbance rejection.

However, the simulation tests revealed that an NLDI controller is capable of holding PAVER in hovering flight obeying pilot commands and yielding the vehicle stable. Additionally, the controller’s behavior against non-linearities/non-modeled equipment was successfully completed. This included CG and inertia variations as well as a significant increase of the aircraft weight by an external object to test control authority margin.

This study also included an analysis of motor size and weight in contrast with rotor size. It is known that helicopters operate at constant rpm and vary collective pitch to change thrust. This is due to the latency of the engine to produce changes in the rotational speed of the rotor given its large inertia. This section revealed that as the blade scaled up the rotor’s inertia and torque required increases at a faster rate for each inch of blade increment. Applying the empirical design equation for radial flux motors to the torque
results from the blade, it was shown that the motor’s size increases significantly to produce enough torque to drive a larger rotor. As a consequence, this also translates into an important increase of the motor’s weight, which was demonstrated based on the electric motors produced by EMRAX.

It was successfully proved that if the motor does not have the power required, the latency to generate rotational velocity variation will increase very rapidly for each increment of the rotor diameter. The second part of this analysis included thrust as a variable, allowing to obtain thrust-to-weight ratio values. Naturally, increasing the blade size translates into more thrust, which means the rotor can handle more weight. The research demonstrated that this ratio decreases very rapidly due to the high rate of change of the overall weight, mainly due to the motor, and it reaches a value of one for a rotor of 32ft at a Mach number of 0.5 and a pitch angle of thirteen degrees. These results demonstrate that, while collective pitch at constant rpm offers a solution to scaling problems, the electric motor size and weight present a limitation where the thrust production is not enough to overcome the total weight of the assembly. This limitation is aggravated if the weight of the airframe, electric energy source and other aircraft components are considered.

5.2 Future Work and Recommendations

The solution to the scaling problem presented in this research was proved to be viable under certain constraints, which present a limit to which a consumer drone can be scaled up. However, this limit can be optimized by implementing the best power plant and rotor design. The following considerations for future research will enable to improve and accurately test the new ideas.

A NLDI controller is simple yet powerful, but due to its limitations, it is better for it to be used as a baseline controller augmented by a more complex control system. For this particular aircraft, given that it is a tiltrotor where the motors and the wings together are tilting to transition into horizontal flight, it is highly recommended to implement an adaptive controller on top of the NLDI. This will allow to increase control reliability and
efficiency while handling the change in the aircraft inertia associated with the tilting feature.

For a deeper analysis on the hovering stability characteristics, it is highly recommended to improve the three basic mathematical models: aircraft, rotor, and motor. Once the vehicle is built, obtaining a high fidelity inertia matrix can be accomplished by practical experimental methods, and it will significantly improve the simulation fidelity. Next, upgrading the rotor model may include: adding 3D parameters, such as tip losses and empirical factors, and dynamic inflow. Also, it is important to consider the dynamic response to thrust change with variation of collective pitch.

Lastly, the motor model assumes an infinite energy source given that it lacks the battery model. Therefore, adding this system to emulate the batteries feeding the motor will allow to estimate the hovering time and energy required/consumption. Also, the fact that energy is not infinite implies that the voltage decreases with time, which will impact the response of the motor to a rotor torque change due to collective pitch variation.

Considering the nature of PAVER and its ultimate goal, it will require an energy source to feed all eight motors featuring lightweight, endurance, and high power. Also, given that an electric motor can be designed and built based on its application varying several parameters, it is strongly recommended to research and design a motor specific to this particular aircraft. Furthermore, if this is accomplished in conjunction with a specific rotor design, it is possible to come up with an innovative aircraft that will comply with future aviation demands.
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