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Notes on Rules of Thumb for Pilots

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Introduction

Pilot's rules of thumb are a curation of information primarily related to aircraft maneuvers. These rules consist of simple algebraic equations that can be easily calculated during flight. These rules of thumb are found to be part of flight instructions and handbooks for pilots, such as Grohmann (2015), McElroy (2015), and Parma (2016). Currently, the available rules are presented without physical proof or adequate citations to the corresponding references. However, each rule could be verified when it was proposed. In addition, these rules are generally presented without a valid range, such as velocity or altitude ranges. Consequently, it is unclear whether the rule can be applied to a reciprocating general aviation airplane, turboprop airplane, or turbofan airplane.

This paper presents the background theory for these rules. Most rules can be explained by a first-order approximation of the background theory. Subsequently, the valid range of these rules is presented by analyzing the prediction accuracy of the rule, which is defined as the relative error of the rule compared to the theoretical values.

These rules of thumb provide a good prediction capability for various altitude or velocity ranges; however, a few rules have not been adequately addressed for use within the valid range.

Table 1

Value of Independent Variable x for Each Linearization Equation to Obtain the Specified Accuracy

	Specified accuracy (Relative error)	
Linearization	5%	10%
Equation (1)	0.29	0.46
Equation (2)	0.40	0.57

Method

Five rules are discussed in the following section, and each rule is explained in one subsection. In each subsection, except for Rule 1, a basic equation (theory) for the rule is presented. Subsequently, this equation is linearized (if possible) to generate a first-order approximation of the theory. Next, the theory, its first-order approximation, and the rule of thumb are plotted in the same graph to help visualize the valid range of the rule, and the accuracy of the rule is discussed. A relative error of 10% was used to identify the valid range under the assumption that 10% would be permitted in the initial estimate obtained using mental calculations during flight.

To linearize the basic equation, a standard method is used. For example,

$$(1 + x)^{\alpha} \approx 1 + \alpha x \quad (|x| < 1) \tag{1}$$

or

$$\tan^{-1} x \approx x \quad (|x| < 1), \tag{2}$$

where both equations can be obtained by considering the first term of the Taylor series expansion of the original function. The values of the independent variable x used to obtain the specified relative errors are listed in Table 1.

The mathematical notation and abbreviations used in this study are summarized in the last sections of this paper. The altitude of the analysis in this study was limited to the troposphere (0–33,000 ft) for simplicity.

Target Rules of the Present Study

Although there are several rules of thumb, only a few have been discussed here. Relatively simple rules, such as the unit conversion rules (Celsius to Fahrenheit in temperature, meter to foot in length, etc.) and angle-to-length or angle-to-velocity conversion rules using trigonometric functions (1-in-60 rule for course correction, rules for descent rates with 3-degree glideslope, crosswind correction, etc.) are not included in this article. The rules related to takeoff and landing distance, velocity, or weight (such as the 10/20 rules) are interesting; however, they will be addressed in the future and are not included here. Only four rules related to turn radius (Rule 2), bank angle (Rule 3), pressure change with altitude (Rule 4), and TAS change with altitude (Rule 5) are discussed in this paper. Although Rule 1 is a simple unit conversion rule, it is included because it is required to explain Rule 2. The verification processes are detailed to verify other rules that are not included in this study.

Results and Discussion

Rule 1: Mach Number Times 10 Equals Nautical Miles per Minute

The speed of sound varies with temperature, which in turn varies with altitude. The relative error of Rule 1 was calculated using the relationship between the temperature and altitude of a standard atmosphere and was found to be minimum at approximately 25,000 ft; at this altitude, the speed of sound a was approximated as $a = 603$ kt. Subsequently, the true air speed V_{TAS} in units of [nm/min] is obtained as follows:

$$V_{\mathrm{TAS}}\,[\mathrm{nm/min}] = \left(\frac{V_{\mathrm{TAS}}\,[\mathrm{kt}]}{60}\right) \approx \left(\frac{a}{60}\right)\left(\frac{V_{\mathrm{TAS}}\,[\mathrm{kt}]}{a}\right) \approx 10 \times M, \tag{3}$$

where $M = V_{\mathrm{TAS}}/a$ is the Mach number.

The magnitude of the relative error of Rule 1 is less than 3% in the altitude range of 18,000 to 33,000 ft (typical cruise altitude for airliners equipped with turboprop or turbofan engines). The accuracy of Rule 1 worsens as the altitude decreases, and the relative error reaches a value of approximately 9% near the mean sea level.

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Rule 2: Turn Radius Equals Mach Number Times 10 Minus 2

Rule 2 can be expressed as

$$R\ [\mathrm{nm}] = 10 \times M - 2, \tag{4}$$

which can also be written as

$$R\ [\mathrm{nm}] = V_{\mathrm{TAS}}\ [\mathrm{nm/min}] - 2, \tag{5}$$

by using Rule 1. Equations (4) and (5) use the true air speed (TAS) instead of the ground speed (GS); hence, the wind velocity is assumed to be zero in these rules.

Turn radius R [m] can be expressed as

$$R\ [\mathrm{m}] = \frac{{V_{\mathrm{GS}}}^2}{\mathrm{g} \tan \phi}, \tag{6}$$

by considering the vertical and horizontal balance of the forces acting on the aircraft (refer to a typical elementary textbook on aircraft performance, e.g., Brandt (2015)). Here, V_{GS} [m/s] is the ground speed, $\mathrm{g} = 9.8\ \mathrm{m/s^2}$ is the gravitational acceleration, and ϕ [rad] is the bank angle. The AIM-JAPAN Editorial Association (2020) assumes that the bank angle $\phi = 25°$ in Rule 2; hence, Equation (6) can be rewritten as

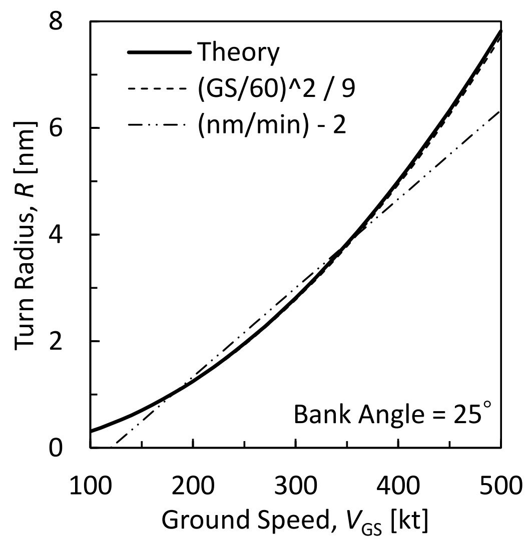
$$R[\mathrm{nm}] = \frac{1}{8.88}(V_{\mathrm{GS}}\ [\mathrm{nm/min}])^2 \approx \frac{1}{9}\left(\frac{V_{\mathrm{GS}}\ [\mathrm{kt}]}{60}\right)^2, \tag{7}$$

which is a derivative of Rule 2. The denominator is 10.99, instead of 8.88, assuming $\phi = 30°$ (this case is presented in Figure 2).

Equations (5), (6), and (7) are plotted in Figure 1. Equation (7) (thin dashed line) yields an approximation close to the theoretical values (Equation (6), thick solid line) for all ranges of the ground speed; however, Equation (7) is difficult to calculate mentally during flight. Meanwhile, Equation (5) (thin dashed line with two dots) can be easily calculated. As shown in Figure 1, Equation (5) can be considered as a linearization of Equation (6) around a ground speed of approximately $V_{\mathrm{GS}} = 270$ kt. The turn radius calculated using Equation (5) can help approximate the theoretical values (Equation (6)) within a relative error of approximately 10% in the speed range of 170–420 kt, which can cover the typical flight path of turboprop and turbofan airliners.

Figure 1

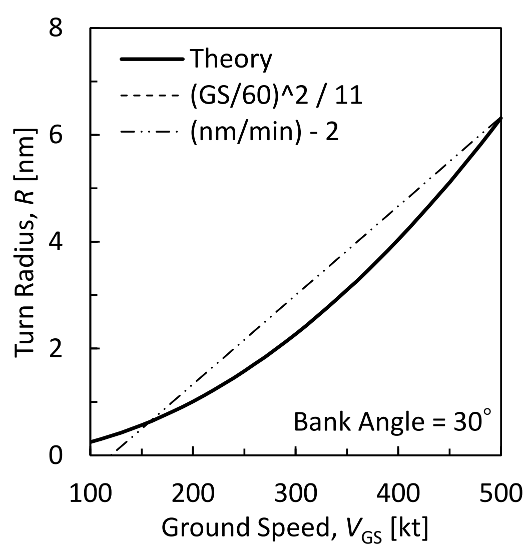
Comparison of Rule 2 with Theoretical Values (bank angle = 25°)



McElroy (2015) and Grohmann (2015) presented Rule 2 using a bank angle of 30°. They described that Rule 2 should be used in a 'high-speed range,' that is, greater than 200 kt. In this case, as observed in Figure 2, the relative error is greater than 20% in the airspeed range of 180–370 kt, with a maximum error of 38% at approximately 240 kt. Therefore, Rule 2 provides comparatively accurate results for a bank angle of 25°, rather than 30°.

Figure 2

Comparison of Rule 2 with Theoretical Values (bank angle = 30°)



Note. The dashed line nearly coincides with the solid line.

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For a standard rate turn (SRT), the turn radius R [m] can be directly calculated from the ground speed V_{GS} [m/s] and angular velocity ω_{SRT} [rad/s] as $R = V_{\mathrm{GS}}/\omega_{\mathrm{SRT}}$, where the angular velocity of the SRT is $\omega_{\mathrm{SRT}} = \pi/60$. By converting the units, the following equation is obtained with neglecting the wind velocity ($V_{\mathrm{TAS}} = V_{\mathrm{GS}}$):

$$R\ [\mathrm{nm}] = V_{\mathrm{TAS}}\ [\mathrm{kt}]/179. \tag{8}$$

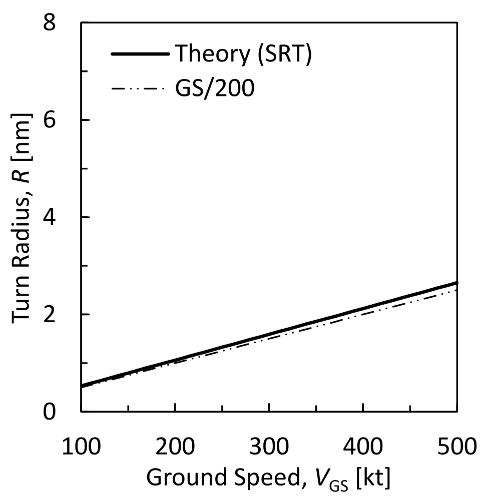
McElroy (2015) and Grohmann (2015) presented a 'low speed range' rule for SRT that states:

$$R\ [\mathrm{nm}] = V_{\mathrm{TAS}}\ [\mathrm{kt}]/200. \tag{9}$$

Equations (8) and (9) are plotted in Figure 3. Equation (9) is referred to as the 'low-speed rule', but this approximation maintains a relative error of 6% for all speed ranges. However, the bank angle in SRT exceeds 45° for speeds greater than approximately 360 kt; such a large bank angle is not practical for commercial airliners and general aviation.

Figure 3

Comparison of a Variant of Rule 2 with Theoretical Values (SRT: standard rate turn)



Rule 3: Bank Angle Equals KTAS Divided by 10, Times 1.5

Equation (6) can be rewritten as

$$\phi\ [\mathrm{rad}] = \tan^{-1}\left(\frac{V_{\mathrm{GS}}\omega}{\mathrm{g}}\right), \tag{10}$$

where ω [rad/s] is the angular velocity of the turn, and the relationship $V_{\mathrm{GS}} = R\omega$ is used to obtain Equation (10) from Equation (6). Equation (10) can be approximated by considering the first-order term of the series expansion of $\tan^{-1}$ (i.e., using Equation (2)) as follows:

$$\phi\ [\mathrm{rad}] \approx \frac{V_{\mathrm{GS}}\omega}{\mathrm{g}} \tag{11}$$

when $(V_{\mathrm{GS}}\omega/\mathrm{g}) < 1$. Because the angular velocity for the SRT is $\omega_{\mathrm{SRT}} = 3\ \mathrm{deg/s} \approx 0.052\ \mathrm{rad/s}$, the condition $(V_{\mathrm{GS}}\omega_{\mathrm{SRT}}/\mathrm{g}) < 1$ holds true when $V_{\mathrm{GS}} < 188\ \mathrm{m/s} \approx 366\ \mathrm{kt}$.

Substituting the constants into Equation (11), neglecting the wind velocity ($V_{\mathrm{TAS}} = V_{\mathrm{GS}}$), and converting the physical units, Rule 3 for SRT is obtained as follows:

$$\phi\ [\mathrm{deg}] \approx \frac{(V_{\mathrm{TAS}}\ [\mathrm{kt}] \times 0.51)(3\ °/\mathrm{s})}{(9.8\ \mathrm{m/s^2})} \approx \frac{(V_{\mathrm{TAS}}\ [\mathrm{kt}])}{10} \times 1.5, \tag{12}$$

Here, a factor of 0.51, is used to convert the unit from [kt] to [m/s].

Another form of Rule 3 is obtained by manipulating Eq. (12) as follows:

$$\phi\ [\mathrm{deg}] \approx \frac{(V_{\mathrm{TAS}}\ [\mathrm{kt}])}{10} + \frac{(V_{\mathrm{TAS}}\ [\mathrm{kt}])}{20}, \tag{13}$$

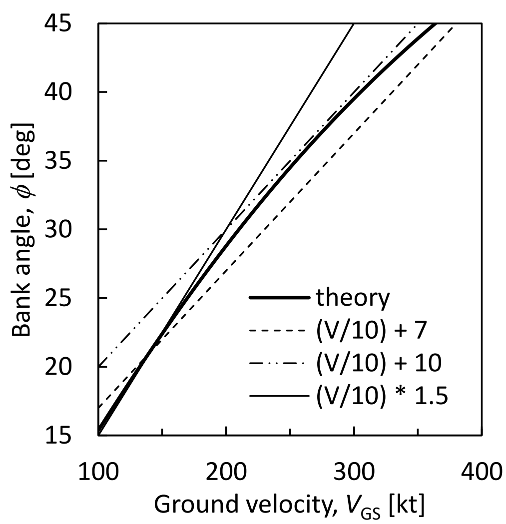
and substituting $V_{\mathrm{TAS}} = 140\ \mathrm{kt}$ in the second term of Equation (13) as follows:

$$\phi\ [\mathrm{deg}] \approx \frac{V_{\mathrm{TAS}}\ [\mathrm{kt}]}{10} + 7, \tag{14}$$

which is accurate if V_{TAS} is approximately 140 kt.

Figure 4

Comparison of Rule 3 and its Variants with Theoretical Values



Equations (10), (12), and (14) are plotted in Figure 4. Rule 3 (thin solid line) exhibits greater accuracy for a low-speed range and provides a relative error within

10% for $V_{\mathrm{TAS}}\ (= V_{\mathrm{GS}})$ less than 260 kt. The variant of Rule 3, (V/10) + 7 (Equation (14), thin dashed line) provides a relative error within 10% for all speed ranges, as shown in Figure 4. The AIM-JAPAN Editorial Association (2020) suggests using the (V/10) + 7 rule for velocities less than 200 kt; further, in such cases, the relative error improves and lies within 6%. Note that the accuracy of rule (V/10) + 7 abruptly decreases at speeds less than or equal to 100 kt, which is out of the range in Figure 4. Another variant of Rule 3, (V/10) + 10, is plotted in Figure 4 by a dashed line with two dots and exhibits improved accuracy for a higher speed range.

Rule 4: Pressure Decreases by 1 inHg for Each 1000 ft Elevation

The variation in atmospheric pressure p [kPa] along altitude Z [m] is expressed by the International Civil Aviation Organization (1993) as follows:

$$\frac{p(Z)}{p_0} = \left[1 + \frac{\beta}{T_0} Z\right]^{-\frac{g}{R\beta}}, \tag{15}$$

where $R = 287\ \mathrm{J/kg/K}$ is the gas constant of dry air, $\beta = -0.0065\ \mathrm{K/m}$ is the vertical temperature gradient, and $p_0 = 103.1\ \mathrm{kPa}$ and $T_0 = 288\ \mathrm{K}$ are the atmospheric pressure and temperature at the mean sea level, respectively. Because the difference between the geopotential and geometric altitudes is negligible under 10 km (approximately 33,000 ft), Z is treated as the geometric altitude thereafter. By considering the first-order term of a series expansion of Equation (15) (i.e., by using Equation (1)), we obtain

$$\frac{p(Z)}{p_0} \approx 1 + \frac{\beta}{T_0}\left(-\frac{g}{R\beta}\right) Z, \tag{16}$$

when $\beta Z / T_0 < 1$. By substituting the constants in Equation (16) and changing the units, Equation (16) becomes

$$p\ [\mathrm{inHg}] \sim 29.92 - 1.08 \times Z\ [\mathrm{kft}]. \tag{17}$$

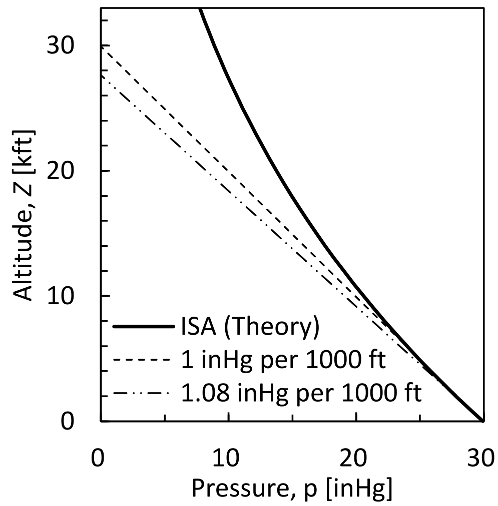
where [kft] is the unit kilofoot. Conversely, Rule 4 can be expressed as

$$p\ [\mathrm{inHg}] \sim 29.92 - 1 \times Z\ [\mathrm{kft}]. \tag{18}$$

Equations (15), (17), and (18) are plotted in Figure 5. Although the accuracy of Equation (17) is higher than that of Equation (18) when the altitude is close to the sea level ($Z = 0$), Equation (18) shows better approximation capability over a wide range of $Z < 10{,}000$ ft, as can be observed in Figure 5. Rule 3 is valid up to 10,000 ft, where the relative error is approximately 3%. A few textbooks (e.g., Federal Aviation Administration (2016)) adequately address this valid range of altitude, while others do not.

Figure 5

Comparison of Rule 4 with Theoretical Values



Rule 5: TAS Increases by 2% of IAS for Each 1000 ft Elevation

We can derive the relationship between altitude, Z, and density, ρ, of the standard atmosphere by substituting the equation of state for an ideal gas, $p = \rho RT$, into Equation (15). The result is

$$\frac{\rho_0}{\rho(Z)} = \left[1 + \frac{\beta}{T_0} Z\right]^{\frac{g}{R\beta}+1}, \tag{19}$$

where $\rho_0 = 1.225$ [kg/m^3] is the air density at mean sea level. Therefore, the relationship between TAS and equivalent air speed (EAS) can be expressed as

$$\frac{V_{\text{TAS}}}{V_{\text{EAS}}} = \sqrt{\frac{\rho_0}{\rho}} = \left[1 + \frac{\beta}{T_0} Z\right]^{\frac{1}{2}\left(\frac{g}{R\beta}+1\right)}, \tag{20}$$

by combining the definitions of the TAS and EAS with Equation (19).

It is assumed that the difference between the EAS and the calibrated air speed (CAS) is negligible. The assumption that EAS = CAS is discussed later in this section. The difference between the IAS and CAS is primarily caused by the position error. The position error is generally small (1%–3% of the airspeed) in the cruise configuration and is negligible. Therefore,

$$\frac{V_{\text{TAS}}}{V_{\text{CAS}}\ (\text{or } V_{\text{IAS}})} \approx \frac{V_{\text{TAS}}}{V_{\text{EAS}}} \approx 1 + \frac{\beta}{T_0}\frac{1}{2}\left(\frac{g}{R\beta} + 1\right) Z, \tag{21}$$

by considering the first-order term of a series expansion of Equation (20) (i.e., using Equation (1)). By substituting the constants into Equation (21) and changing the units, Equation (21) becomes

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$$\frac{V_{\mathrm{TAS}}}{V_{\mathrm{CAS}}\ (\text{or } V_{\mathrm{IAS}})} \approx 1 + 0.015 \times (Z\ [\mathrm{kft}]) \approx 1 + 0.02 \times (Z\ [\mathrm{kft}]), \qquad (22)$$

which is Rule 5.

Equations (20) and (22) are plotted in Figure 6. The ‘1.5% rule’ is superior to the ‘2% rule’ under altitudes of 10,000 ft, although the ‘2% rule’ is valid for a wider range of altitudes. The relative error of the ‘2% rule’ (Rule 5) was investigated on the spreadsheet (not shown here) for cases where IAS = 100, 200, and 300 kt, and the relative error was found to be within 5% for an altitude range of 0–33,000 ft. This error includes the compressibility error (EAS = CAS assumption) and the linearization error owing to the truncation of the series expansion of functions.

Figure 7 shows the effects of the EAS = CAS assumption for CAS = 300 kt, which corresponds to a Mach number of 0.84, at an altitude of 33,000 ft. In this case, CAS differs from EAS by approximately 6%. Upon neglecting the EAS = CAS assumption, the relative error of Rule 5 becomes the dashed line in Figure 7; the dashed line represents the linearization error. Although the EAS = CAS assumption is incorrect, particularly for high-speed cruises at high altitudes, the assumption provides better accuracy to Rule 5, because the direction of the error is opposite to that of the linearization error.

Figure 6

Comparison of Rule 5 with the Theoretical Values

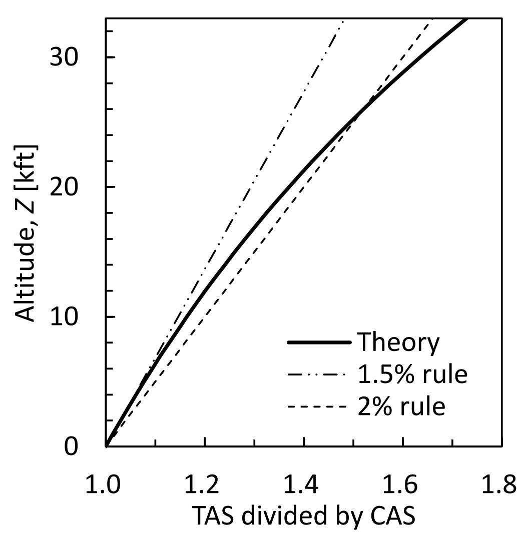
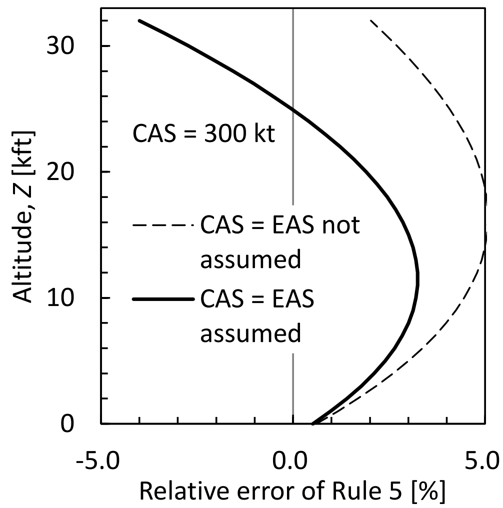


Figure 7

Effect of Assuming EAS = CAS on the Calculation of Relative Rrror of Rule 5



Conclusions

The valid ranges and accuracies of the five rules of thumb for pilots are discussed. Typically, the background theory of the rule is a nonlinear function, and the rule of thumb is a first-order approximation of the background theory. These rules of thumb provide a good approximation of the background theory for various altitudes or velocities. However, a few rules have not been adequately addressed to use with valid ranges in a few instructions; caution must be exercised when using such rules. The usefulness of these rules of thumb may differ from pilot to pilot, depending on their airplane category and the mission of the flight. Pilots may more safely rely on recent glass cockpit, electronic flight bag, or electronic flight computer information when available, which will provide more accurate information than these traditional rules of thumb. Although the rules of thumb are not as accurate as other tools, they may still be valuable when preparing for an emergency case, such as electric power loss. When a pilot has a question about the accuracy of a rule in his/her daily pilot life, discussing the question and checking the valid range of the rule with other pilots, technicians, or engineers will help resolve the question.

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Abbreviations

CAS	Calibrated air speed
EAS	Equivalent air speed
GS	Ground speed
IAS	Indicated air speed
ISA	International standard atmosphere
KIAS	Knots indicated air speed
KTAS	Knots true air speed
MSL	Mean sea level
SRT	Standard rate turn
TAS	True air speed

Nomenclature

g	[m/s^2]	9.8	Acceleration of gravity
M	[-]		Mach number
p_0	[kPa]	101.3	ISA pressure at MSL
R	[J/kg/K]	287	Gas constant of the dry air
T_0	[K]	288.15	ISA temperature at MSL
V	[m/s]		Velocity. The unit is [kt] or [nm/min] in the context of pilot's rules of thumb.
Z	[m]		Geometric altitude. The unit is [kft] (kilofoot) in the context of pilot's rules of thumb.
β	[K/m]	-0.0065	ISA vertical temperature gradient

γ	[-]	1.4	Specific heat ratio of a diatomic ideal gas
π	[-]	3.1415	Ratio of the circumference of a circle to its diameter
ρ_0	[kg/m^3]	1.225	ISA density at MSL
ϕ	[rad]		Bank angle. The unit is [°] in the context of pilot's rules of thumb.
ω	[rad/s]		Angular velocity

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