Active Control of Coherent Structures in an Axisymmetric Jet

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ACTIVE CONTROL OF COHERENT STRUCTURES IN AN AXISYMMETRIC JET

By

Michael Marques Goncalves

A Thesis Submitted to the Faculty of Embry-Riddle Aeronautical University
In Partial Fulfillment of the Requirements for the Degree of
Master of Science in Aerospace Engineering

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Embry-Riddle Aeronautical University
Daytona Beach, Florida
ACTIVE CONTROL OF COHERENT STRUCTURES IN AN AXISYMMETRIC JET

By

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This Thesis was prepared under the direction of the candidate’s Thesis Committee Chair, Dr. Vladimir Golubev, Department of Aerospace Engineering, and has been approved by the members of the Thesis Committee. It was submitted to the Office of the Senior Vice President for Academic Affairs and Provost, and was accepted in the partial fulfillment of the requirements for the Degree of Master of Science in Aerospace Engineering.

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Finally, I must acknowledge the support of my family, their immeasurable love and care made this possible.

"The farther you go, the harder it is to return"
ABSTRACT

The primary objective of this work is to develop high-fidelity simulation model for jet noise control predictions and quantify the sound reduction when an external source frequency mode excitation is imposed on the jet flow. Whereas passive approaches using mixing devices, such as chevrons, have been shown to reduce low-frequency noise in jet engines, such approaches incur a performance penalty since they result in a reduced thrust. To avoid a performance penalty in reducing jet noise, the current work investigates a open-loop active noise control (ANC) system that utilizes a unsteady microjet actuator on the nozzle lip in the downstream direction to produce a desired effect on the jet flow-field dynamics thereby directly affecting the source source. In contrast to the passive approach, the proposed open-loop control design will utilize a local flow excitation device that can be turned off when not needed or adjusted according to the desired control signal. To make it feasible, the effectiveness of every forcing frequency mode has to be mapped for a certain jet velocity. This analysis considers an axisymmetric round jet at supersonic and subsonic speeds. Current studies are verified against previous low-order simulations conducted using Linearized Euler Equations (LEE), and compare qualitatively achieved noise reduction results against available experimental data. High-fidelity analysis, such as Detached-Eddy Simulations (DES), was implemented using OpenFOAM, an open source CFD software. Results show that some excited frequency modes reduced the far-field jet noise by around 2 dB, supporting the use of unsteady microjet actuators as a jet noise reduction technology.
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<td>$\dot{m}$</td>
<td>Mass flow rate</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Dynamic viscosity</td>
</tr>
<tr>
<td>$\mu_o$</td>
<td>Reference dynamic viscosity</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Kinematic viscosity</td>
</tr>
<tr>
<td>$\Psi$</td>
<td>Tensor field</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Simulation time</td>
</tr>
<tr>
<td>$\tau_{ij}$</td>
<td>Viscous stress term</td>
</tr>
<tr>
<td>$A$</td>
<td>Nozzle exit area</td>
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<tr>
<td>$c_0$</td>
<td>Speed of sound</td>
</tr>
<tr>
<td>$D$</td>
<td>Nozzle diameter</td>
</tr>
<tr>
<td>$e_t$</td>
<td>Total energy</td>
</tr>
<tr>
<td>$f$</td>
<td>Frequency</td>
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<tr>
<td>$M_c$</td>
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<td>$NPR$</td>
<td>Nozzle Pressure Ratio</td>
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<td>Reference sound pressure</td>
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<tr>
<td>$p_{RMS}$</td>
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<td>$Pr$</td>
<td>Prandtl number</td>
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<td>Sound Pressure Level</td>
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<tr>
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<tr>
<td>$T_{ij}$</td>
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1. Introduction

Federal Aviation Administration (FAA) restrictions on aircraft noise have become even more rigorous in order to avoid the negative effects that aircraft has in community noise on airfield operation and population around main air traffic areas (Chen et al., 1992). Moreover, sonic fatigue on the aircraft may also be reduced. On the engine noise source breakdown for departure situations (Nesbitt, 2019), jet noise plays a significant role and any technique that mitigates its intensity should result in a lowered overall jet engine noise, better if integrated with fan noise reduction approaches, see Figure 1.1. This fact led researchers to a scientific race for technology capable to suppress jet noise.

Historically, jet noise has been reduced by increasing the engine bypass ratio thus reducing the overall jet exit velocity. However, as bypass ratio increases so does the engine cross sectional area, and there are limits on practical air vehicle design, especially for military vehicles desiring to maintain performance and stealth condition. Engine and airframe manufacturers have sought various ways to reduce jet noise by redesigning the nozzle.

Previous work had shown that passive mixing devices, such as chevrons and asymmetric nozzle shape, have been found to reduce the low frequency sound that dominates noise at downstream angles (Burak et al., 2009), although that is often accompanied by a penalty in the high frequency noise radiated to sideline angles. Noise comparisons should be performed on the basis of constant thrust. However, even then, the performance penalty will cause increased fuel burn and may reduce the operating envelope of the flight vehicle. Especially for military air vehicles, the need for reduced noise operations may be limited to only certain portions of the mission profile. There remains a need for a jet noise reduction technology that can be adjusted or even shut off at will.

In the following, the sources of radiated sound that constitute jet noise are identified
and reviewed. After, the conventional and innovative techniques to mitigate jet noise are reviewed according to the timeline. Finally, computational approaches related to this work are reviewed and discussed.

**Figure 1.1** Typical departure engine noise distribution (Nesbitt, 2019).

### 1.1. Jet Noise Components

The fact that exhaust gas flow that comes from a jet engine, either supersonic or not, is a dominant source of noise does not infer enough about the noise generation and propagation. Turbulent flows are complex to explain, similarly is the noise generated from it. Although a precise interpretation of noise source is still pending, several studies were able to classify and replicate some patterns that explain the physical mechanism of sound generation and radiation from turbulent jets. This section provides a summary of the primary jet noise components.

#### 1.1.1. Turbulent Mixing Noise and Mach Wave Radiation

It is known that the unsteady flow fluctuations in the jet shear layer due to the turbulent mixing of high speed gases and static outer atmosphere is the cause of jet exhaust noise. Small turbulent structures generated around the nozzle are responsible for high frequency noise and downstream of the jet plume, large turbulent structures will develop
and are responsible to propagate low frequency noise. If the flow-field presents any
shockwave solution, probably that those turbulent structures will interact with the shock
and promote secondary source of noise. In order to understand how researchers developed
to this point, it is important to acknowledge that since jet engine was invented in 1940’s
during war there have been concerns with the high sound levels generated by the exhaust
gases.

By 1950, encouraged by the novel in jet noise suppression Sir James Lighthill
published a two-part pivotal work that established the origin of what we denominate as the
field of Aeroacoustics. Lighthill’s acoustic theory recalls the Navier-Stokes Equations into
an acoustic wave propagation equation with equivalent source terms and found that
fluctuations in flow-field quantities due to turbulence generate sound sources that behave
as quadrupoles (Lighthill, 1952; Lighthill, 1954), see Equation 1, where \( \rho \) is the fluid
density, \( c_0 \) is the ambient speed of sound and \( T_{ij} \) is known as Lighthill stress tensor,
defined in Equation 2.

The physical meaning for the Lighthill stress tensor is the divergence between fluid
flow and acoustic wave propagation stresses and requires knowledge of the unsteady flow
variables at all locations in the flow. At that time, this could be hard to obtain through
experiments, so Lighthill manipulates the Green’s function to find a formal solution using
a volume distribution of acoustic sources and this is known as Lighthill’s acoustic analogy.
His work presumes that acoustic sources may move independently of fluid-flow, even in
different speeds.

However, this acoustic analogy has its accuracy limited to the known unsteady
flow-field and does not consider acoustic sources interaction with surfaces or the flow
itself. Also, its application works best in the supersonic flow regimes due to the linearity
of the equations. Lighthill’s theory is linked to turbulent mixing noise and states by the
second derivative in space of the Lighthill stress tensor that these source types are
quadrupoles. Later, the Lighthill’s analogy was expanded to consider acoustic sources moving towards an observer (Ffowcs Williams & Lighthill, 1963).

\[
\frac{\partial^2 \rho}{\partial t^2} - c_0^2 \nabla^2 \rho = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}
\]

\[
T_{ij} = \rho v_i v_j + \left( p - c_0^2 \rho \right) \delta_{ij} + \tau_{ij}
\]

Lighthill’s non-dimensional analysis using the proposed analogy through volume distribution gave the estimate that the jet acoustic power radiates scale with the 8th power of the mean jet velocity \(U_j^8\) known as the 8th Power Law. For higher speed jets, Ffowcs Williams predicted that the far-field noise scales with \(U_j^3\). At this time, researchers would agree that the jet turbulence was formed solely by small eddy structures.

Later investigation showed that noise radiated to 30° angle in downstream direction was distinct from 90° angle in frequency and amplitude, they conclude that an additional source might exist (Laufer et al., 1976). Although Lighthill predicted that noise radiated by turbulent structures in the jet had a preferred peak angle specified on Equation 3, where \(M_c\) is convective Mach number of the large scale turbulent eddies in the jet shear layer with respect to the ambient speed of sound. Distinguished work from early 70’s reported the existence of large coherent turbulent structures in a jet shear layer as an addition to the classical small-scale turbulence model (Crow & Champagne, 1971), this would be confirmed later by other studies (Brown & Roshko, 1974).

The identification of three families of instability waves supported by supersonic jets, among them, Kelvin-Helmholtz instability waves are responsible for the development of large-scale turbulent structures in jet flows, which is a product of nonlinear instabilities generated near the nozzle exit and develop into vortex structures (Tam & Hu, 1989). Theoretical work showed that the vortex pairing causes vortex growth and entrainment which result in spreading of the shear layer (Michalke, 1971). The large-scale vortex
structures grow in the flow direction while the fine-scale structures have a more uniform size throughout the mixing layer. This was confirmed by experiments (Freyimuth, 1966).

\[ \theta_{\text{max}} = \sin^{-1} \left( \frac{1}{M_c} \right) \] (3)

On previous paper, authors are convicted that the physical sources of jet noise are the fine-scale turbulence and the large turbulence structures of the jet flow, having the large-scale structures as the dominant sources of jet noise (Tam et al., 2008). Figure 1.2 demonstrates the concept of large coherent turbulent structures towards the end of the potential core that generate noise that propagates in the downstream direction. In the other hand, small-scale turbulence structures are all around the jet length, but the noise they generate propagates more towards the sideline.

![Figure 1.2 Schematic diagram of the sources of jet noise radiating to the side line and the downstream directions (Tam et al., 2008).](image)

Considering an axisymmetric laminar jet, the vortex structures formed in the shear layer are ring-shaped around the jet axis, as Figure 1.3 illustrates. These vortices propagate downstream while dissipate until they behave as a 3D turbulent structure with considerable rotational components (Raman et al., 1994). The initial region of the jet, known as jet potential core transitions to a disordered region in consequence of the collapsed rotational flow in the jet shear layer. The length of this transition decreases at
higher Reynolds number. Consequently, the shear layer is fully turbulent immediately downstream of the nozzle lip at high Reynolds number cases, as Figure 1.4 depicts.

The pattern that large coherent structures follow inside the jet potential core are shown up on experiments as evidence that might exist a preferred frequency for these noise sources. There is indications that experiments concluded in preferred Strouhal number ($St_D = f D / U$) from $St_D = 0.25$ to $St_D = 0.5$ (Ho & Huerre, 1984) and these discrepancies were further explained (Gutmark & Ho, 1983). They ensure that a low level spatially coherent disturbances which is dependent on each test facility can cause this spreading rate found in previous experiments. Therefore, a jet source was excited at higher amplitude using a loudspeaker at various frequencies (Crow & Champagne, 1971), a classical idea that comes from Lighthill’s work. They found that the preferred frequency, which amplified the most of the sound, was $St_D = 0.3$.

![Figure 1.3](image)  
*Figure 1.3* Jet instabilities for a low Reynolds number axisymmetric jet visualized with smoke tracer (Michalke, 1965; adapted from Wille, 1963).

In order to identify the noise sources of a jet issuing from a nozzle, the investigator must know that large-scale structures are of the same order as the jet diameter, while the small turbulent eddies are much smaller. This two-noise source model has been supported and utilized by several computational researchers (Mankbadi & Liu, 1984; Tam et al., 1996; Viswanathan, 2004). There is an accomplished support to the two-noise source model by developing two distinct similarity spectra, one for small turbulent scales and other for large structures and compared them with measured spectra from a large number
of experiments (Laufer et al., 1976). Outstanding agreement was found for analogous comparisons for subsonic jet noise as well. Finally, researchers performed a comprehensive experiment study to further confirm the model (Tam et al., 2008).

1.1.2. Shock-Associated Noise and Screech Tones

In constrast to subsonic regime, on an imperfectly expanded supersonic jet there is the presence of a semi-periodic shock cell structure that extends downstream of the nozzle exit. The large coherent structures interact with the shock waves, giving rise to a noise component that propagates upstream and extend over a wide range of frequencies, hence it is called broadband shock-associated noise (BBSAN). The BBSAN is initially investigated in early 70’s, as researchers provided the first comprehensive model explaining BBSAN with the assumption that the points at which the shock-expansion waves reflect from the shear layer represent acoustic sources. They observed that the eddy convection time between the shock cells was responsible for the relative phasing of the shock-associated

*Figure 1.4* Evolution of jet instability with advancing Reynolds number. Parts (a)-(d) span the Reynolds number interval from around $10^2$ to $10^3$ (Crow & Champagne, 1971).
noise and developed a model to predict the peak frequency and amplitude of BBSAN (Harper-Bourne & Fisher, 1973).

Later, researchers postulated that the wavenumber range of the generated disturbances is broad, because the range of turbulent scales is broad as well (Tam & Tanna, 1982). The phase velocities of these disturbances are different and they could also be subsonic and disperse quickly. However, those at supersonic velocities will radiate noise to the far-field. These waves might be thought as Mach waves at different angles due to different phase velocity between them. This sustain the hypothesis that BBSAN is the superposition of disturbances with supersonic phase velocities generated by the weak interaction of large-scale turbulent structures with the shock cell pattern and its contribution to the radiated noise at downstream angles is considerably small compared with the noise generated by large-scale turbulence.

Additionally, the interaction of the large-scale turbulent structures with the shock cells generates upstream travelling waves that may interact with the nozzle exit, thus exciting the initial thin shear layer at same frequency. This instability grows as it propagates downstream and interacts with the shock cells again giving rise to new acoustic waves, establishing a feedback mechanism that generates a high-amplitude, acoustic tone referred to as screech, also found that, to be maintained, this feedback loop requires sufficient amplitude on upstream waves to excite an instability wave large enough to modify the initial shear layer (Tam et al., 1986). Researchers are not able to explain clearly physics behind screech tones yet and its behavior is highly dependent on scale, design and operating conditions. Also, recent computational investigation has shown that it is highly sensitive to the boundary conditions at the nozzle exit as this feedback process is amplified by thicker nozzle lips and higher jet temperatures (Shen & Tam, 2000).

A first successful tentative to explain screech noise response in jets was proposed (Powell, 1956). This model features swirl disturbances originated from the nozzle lip that
will interact with the shock-cell pattern of the jet plume generating acoustic waves. These in turn will impinge back on the nozzle, forcing more disturbances to appear and that will close a feedback loop of resonant nature for a fundamental frequency and its harmonics, see Figure 1.5.

![Figure 1.5 Schematic diagram of resonant screech loop (solid lines) and associated phenomena (dashed lines) (G. Raman, 1999).](image)

Although several authors, following Powell’s feedback loop approach, provided critical analysis of jet screech research, their model will not agree to be simple and powerful enough to predict the screech noise amplitude. Special cases will demonstrate that this phenomena is not persistent and can be more complicated than it seems. High-fidelity CFD simulations can capture flow field that represents screech tones, but no computational study was broad enough to explain the non-linearity of tonal noise intensity brought by the screech phenomena at different mach number and nozzle configurations.

Recent work on jet screech noise has shown that, different from Powell, a supersonic jet can manifest neutral upstream propagating wave modes. This finding is surprising but not unphysical and their existence is confined to very narrow frequency bands (Tam & Hu, 1989). Many aspects of the problem, however, remain open, including the phase lag term on aeroacoustic feedback process, that can be related to a possible co-existence of multiple
feedback and receptivity acoustic propagation paths (Weightman et al., 2017). The dynamic response of coupled fluid-acoustic systems prevails unpredictable due to lack of dedicated research.

1.2. Conventional Noise Reduction Concepts for Jets

Effective devices on jet noise reduction demands exhaustive research and deep knowledge on physics behind noise generation and propagation. Historically, jet noise reduction has been considered as a project outcome related to engine thrust, bypass ratio and combustion temperature. However, community noise restrictions has become rigorous enough that researchers have begun a scientific race for jet noise suppression looking for further alternatives that can reduce jet noise, but they must not be detrimental to aircraft performance.

Jet noise reduction mechanisms split up between active or passive control method. In passive method, such as bypass flow, chevrons and nozzle corrugation, a permanent design modification alters the flow in order to affect far field acoustics, whereas active methods such as fluid injection, plasma actuation and microjet excitation can be switched off as the need arises and act in response to an open-loop or closed-loop feedback control system for an automatic and optimized application. Noise reduction technologies must be proven effective before they are implemented on production aircraft.

1.2.1. Bypass Flow

Traditional turbofan engines have a primary core flow that passes inside the engine for thrust purposes together with a secondary bypass flow that remains at ambient temperature through the cycle, enhancing engine cooling. Before exhaust, the mixing between both flows bring jet temperature down and this drastically reduces the noise radiation when compared to turbojet engines at similar thrust delivery (Saravanamuttoo et al., 2008). The increase in the ratio between core and bypass flow, called bypass ratio, over the last decades has resulted in dramatic suppression in jet noise for turbofan engines.
Although modern engines achieved impressive results in noise reduction, making them quieter became very hard.

Since the early 1990s, NASA focus research efforts on the Ultra High Bypass (UHB) engine cycle to determine its potential for noise reduction and other performance goals. The UHB refers to a bypass ratio greater than 13. Recently completed test was reported and showed that first generation designs were able to reduce noise of fan tip-case interaction by as much as 2 dB in certain flight regimes and soft Vanes were able to reduce the rotor-stator interaction noise source up to 1 dB over the fan operating speed envelope (Christopher, 2009). Fan performance losses around 4% were detected.

The complexity in design, build and test for these new generation of bypass engines and its limited application due to the fan duct size and weight held back its development. Recent numerical simulations of F404 nozzle found less than 1 dB Overall Sound Pressure Level (OASPL) of noise suppression when jet has bypass flow (Liu & Ramamurti, 2019).

1.2.2. Chevrons and Nozzle Corrugations

Chevron technology is one of the most popular applications that went through intensive testing and validation to be implemented on full-scale aircraft. Generally, these are serration cuts at the nozzle lip that slightly penetrate the jet flow in order to generate a streamwise vorticity that makes noise from large-scale structures weaker. Investigation of the effect of chevron nozzles on turbofan engines concluded that the far-field results pointed for reduction on OASPL at all observer angles for certain operating conditions (Callender et al., 2005). However, this noise reduction was being limited to increased higher frequency noise.

Noise suppression mechanism on turbofan engine nozzle with chevrons is due to the enhancement of the mixing between core and bypass flows (Callender et al., 2006). Also, they suggest that the increase in high frequency noise can be attributed to increased turbulence kinetic energy (TKE) near the nozzle exit and that early mixing increases the
fine-grained turbulence, thus increasing the momentum thickness, that in turn can also delay the large-scale turbulence structures development. Similar results were further reported (Alkislar et al., 2007; Heeb et al., 2013). The final statement is that low frequency noise reduction by chevron nozzles comes at the cost of increased high frequency noise. A well-known example of chevron nozzle industry application can be found on the Boeing 787 engine.

Another type of passive technology as a nozzle deformation is the corrugated nozzle. Acoustic investigation on a F-404 showed that a hard-walled interior corrugation can be effective on jet noise reduction, but performance penalties might limit the depth or number of corrugations or lobes (Seiner et al., 2005). Alternatively, there is an effort to develop fluid corrugation that can play the aeroacoustic effects of hard-walled corrugations, with the advantage of being able to be switched off when performance is preferable over noise attenuation. There is a concept that uses distributed injection of low mass flow rate (5% of the core mass flow) and could reduce OASPL over 5 dB (Powers et al., 2018). Later reports on BBSAN reduction indicated 7 dB OASPL reduction (Cuppoletti, 2018). Figure 1.6, compares the hard-walled corrugation against the fluidic corrugation configuration.

Figure 1.6 Aircraft engine nozzle with (a) hard-walled corrugations and (b) fluidic inserts (Powers et al., 2018).
1.2.3. Fluid Injection

With the objective to provide further details in fluid injection, this section introduces the idea that fluidic injection is a secondary jet source that interacts with the main jet flow to modify the flow field and, hence the acoustics of the primary jet. Literature review on fifty years of research into fluid injection and each concept is categorized as aqueous or gaseous injection (Henderson, 2010). Water injection affects the flow field through a two-phase flow where droplets evaporate and transfer momentum to the main jet, whose purpose is to reduce jet velocity and radiated noise from turbulence structures.

Differently, air injection works by means of vortices generated by the interaction of the secondary flow into the primary jet shear layer, this disturbance will develop while travels downstream and suppress radiated noise of many types. Another major difference between water and air injection is the practicability. The use of water injection in aircrafts is not practical because of the need to carry and pressurize the water, now for the gaseous injection, the ability to use compressor bleed air as the injection medium on an aircraft, has attracted more researcher to investigate gas injectors than water injectors.

Fluidic injection have been investigated since late 50’s when researchers attempted to suppress jet noise using water injection (Kurbjun, 1958), therefore the coupling between chevrons and fluidic injection came later when investigators proposed experiment using air injection that can mimic chevrons behavior in the flow (Henderson et al., 2006), however, those can be turned off or adjusted as the mission demands.

Since water injection is clearly harder to maintain in flight, its application attains to ground operation, as it is applied on rocket launch. Water injection experiments concluded that this mechanism reduces jet velocity through momentum transfer and jet temperature through evaporation, as it is a liquid of higher density and heat capacity than air (Krothapalli et al., 2003). Also, it is applied effectively to reduce noise of over-pressured
jets at a mass flow rate (MFR) over 100% of the primary jet. A recent investigation provided numerical guidance through water injection that enhance jet mixing to mitigate noise in a launch pad situation (Salehian et al., 2018).

Nonetheless, air injection can be supplied by engine intake and compressors. Also, it is lighter than water which allows air vorticity to be generated with less energy than water. The idea that counter-rotating vortices are created in primary jet that alters mixing characteristics and turbulence of it was later introduced (Alkislar et al., 2007) and the nozzle concept with fluidic chevrons can reduce low frequency peak noise even further compared to geometrical chevrons, but will not increase high frequency mode amplitude (Henderson et al., 2006). Recent computational work analyzed the effectiveness of a noise reduction strategy based on GE F400-series engines (Coderoni et al., 2019), see Figure 1.7. From design, the idea is to have injectors blowing air into the diverging section of the nozzle to emulate the effect of interior corrugation, proposed previously by other researchers (Powers, 2015). A maximum noise reduction of about 3 dB is obtained along the direction of maximum sound propagation.

Figure 1.7 Sketch of the nozzle geometries: (a) baseline nozzle and (b) nozzle with fluid injection. Only the internal geometry of the nozzle is shown (Coderoni et al., 2019).
1.3. Unsteady Jet Noise Actuators

Active Noise Control (ANC) consists in small input disturbances introduced by an actuator to the flow field to modify itself its directivity pattern. ANC over jet flow has been extensively studied both numerically and experimentally for several applications. Now, unsteady actuators can excite the flow at a desired frequency and extensive work has been done on jet excitation via speakers. This idea was introduced by Lighthill in his distinguished aeroacoustics investigation in early 50’s and he presented a physical explanation for this jet excitation mechanism (Lighthill, 1952).

There is a summary of the advantages and disadvantages of each active control technique they propose: piezoelectric actuators, zero-net mass-flux (ZNMF) and plasma actuators (Cattafesta & Sheplak, 2011). A feedback control suitable to achieve a large frequency band and high amplitude is necessary to obtain desired response in flight to reduce jet noise.

1.3.1. Localized Arc Filament Plasma Actuator

In recent years, the use of electromagnetic source for flow control has been considered (Samimy et al., 2007). Generally, experimental approaches and engineering applications using plasma arc filament, RF, microwave and laser-induced have been used to modify fluid flow behaviour. The plasma-assisted flow control primarily focus on viscous drag reduction and boundary layer separation control at low Mach regime, as well shock wave displacement and wave drag reduction on supersonic flows. This review will focus on plasma actuation to induce thermal heating (Joule effect) to the flow.

Different from approaches that focus in momentum transfer from high-power lasers and microwave beams immersed into the flow, acoustic manipulation will require no more than a localized thermal effect that can induce temperature and pressure perturbations that in turn can mitigate noise propagation. This approach is not limited to low-speed flows and
still requires low energy input, so it can be integrated to a flying vehicle (Gaitonde, 2012).

In the last decade, several experimental tests were performed using Localized Arc Filament Plasma Actuators (LAFPA), shown on Figure 1.8. The setup consists in 8 pair of electrodes equally displaced around the nozzle lip that can switch on and off as desired to achieve not just axisymmetric excitation mode, but also other azimuthal flapping modes, like the 
\( m = 1 \) mode that is imposed by firing the actuators in sequence. Also, a duty cycle is applied to the input signal and the excitation amplitude is temperature-fluctuation based that goes from ambient temperature of 300 K up to 1500 K on the plasma region. Experimental data (Samimy et al., 2007) was compared to numerical results (Gaitonde & Samimy, 2011; Kim et al., 2009).

Primary studies were focused on near-field analysis and showed that constant excitation will not affect the potential core, but unsteady excitation using LAFPA will significantly reduce that and this can be seen through centerline Mach number plot; unsteady cases decline faster than constant and baseline ones (Gaitonde & Samimy, 2011). Also, researchers provided SPL spectra at near-field for several different frequencies and observes that the largest peak corresponds to axisymmetric excitation at \( St = 0.3 \) (Gaitonde, 2012).

The \( m = 1 \) mode at \( St = 0.3 \) signal is most prominent; and \( St = 2 \) both at \( m = 1 \) and axisymmetric modes reduce each pressure azimuthal mode response to values below the baseline. Note that the peak frequency for baseline case is \( St = 0.6 \).

1.3.2. Microjet Actuator

Finally yet importantly, fluidic injection had develop into microjet actuators. Although there’s no strict definition of a ’microjet’, it can be related to actuators that operate around 1% of primary jet MFR, usually they are generated by piezo-electric components such as air driven ones, but it can be a mechanical system of plenum chambers and control valves that are supplied by air bleed from the engine’s compressor or any other high pressure air
supply. Impinging microjets effects on acoustic signal were investigated for a STOVL aircraft hovering in close proximity to the ground (Alvi et al., 2003).

Similarly to the shock associated noise, when the jet plume impinges on the ground, acoustic waves are generated, will travel through the medium, reach back on the nozzle and excite the jet shear layer. This generates even more instability waves, which evolve into large-scale structures, thus closing the feedback loop. As final result, a higher noise of the impinging jet compared to a free jet case, also lift-loss and structural over-stress were reported due to this circumstance (Krothapalli et al., 1999). The proposed solution is the use of microjets to attenuate the upward-moving acoustic waves effects by providing a shielding for the shear layer, however, researchers believe that the most significant reason for the weaker impinging feedback is due to the redirection of large-scale structures in the shear layer into streamwise vorticity as a result of microjet injection. Also, it is important to note that frequency and amplitude play a major role on the efficiency of this technique and the control design has to be accurate to avoid unexpected amplification of the impinging flow feedback effect.

Experimental observation showed that a fully expanded free jet at Mach 1 and an under-expanded at Mach 1.36 was excited perpendicularly to the main jet flow on constant regime and unsteady at a forcing Strouhal Number of ($St_f = 0.16$), also flapping modes
were investigated for the unsteady method (Ibrahim et al., 2002). These results showed that the flapping injection has a higher spreading rate if compared to the constant or axisymmetric unsteady injection regarding the slower decay of jet centerline velocity on antisymmetric case. Even lowering the mass injection rate, the antisymmetric mode grew and persisted farther downstream of the potential core region. Also, the underexpanded case shows that the presence of the shock cell structure on the jet flow potentially destabilize the acoustic mitigation effect for the unsteady cases by triggering unexpected flow perturbations. Hence, the solution that performed better in suppressing radiated noise was the constant injection technique.

Finally, comprehensive work on several injected MFR percentages and the number of microjets, its diameter and distribution over the nozzle for high-speed subsonic jet, see Figure 1.9, showed the importance to observe that the maximum noise reduction was not obtained by using maximum number of microjets, because the manipulation of coherent structures in the flow are a coupled result from spacing, diameter, number and injection velocity that depends for each single case. For them, configurations with paired microjets seemed to be particularly efficient (Castelain et al., 2008).

![Figure 1.9 Experimental setup (a) 36 straight 1 mm exit diameter microjets and (b) 18 microjets with removable nozzles of different diameters (Castelain et al., 2008).]
1.4. Numerical Methods for Jet Simulation and Noise Prediction

The analysis of a sound source requires a flow field solution represented by unsteady fluctuations. Simple methods like RANS (Reynolds Averaged Navier-Stokes) equation system are not sufficient, once this can only solve for time-averaged quantities. A NASA Technical report is an example of several semi-empirical methods that researchers developed, before 90’s, to predict jet noise without any time-varying turbulence numerical simulation (Russell, 1984). At this time, computational power was a major issue for this field of research.

On that same year, researchers attempted to model large coherent structures for a supersonic jet by considering a radial profile of wave packets that fits linear stability theory, then unsteady Navier-Stokes Equations were simplified into ordinary differential equations (ODE’s) that can be easily solved and plugged in Lighthill’s acoustic analogy method (Mankbadi & Liu, 1984). Their work was compared to existing experiments (Lush, 1971).

On the 90’s, direct numerical simulations (DNS) began to show up on papers and technical reports. Their validation with experimental data allowed researchers to investigate Computational Aeroacoustics (CAA) as coupled aerodynamic and acoustic effects using high fidelity computational procedures. These so called "virtual laboratories" are cheaper to operate.

1.4.1. Reynolds Averaged Navier-Stokes

As the name suggests, a Reynolds averaging is applied to the Navier-Stokes Equations which makes its solution considerably easier, although it depends on empirical data to be tuned. First, consider an instantaneous velocity \( u(t) \) that decomposes into mean \( \bar{u} \) and fluctuating \( u'(t) \) components. The decomposition shown on Equation 4 is known as Reynolds decomposition. For compressible high Mach number jets, a density weighted Favre averaging is preferable to make resulting equations easier to deal with. The relation
from Equation 5 is crucial to understand Favre decomposition. Hence, the final decomposition shown on Equation 6 has mean ($\overline{u_i}$) and fluctuating ($u''_i$) parts. A direct application of this averaging process is the Favre-averaged Reynolds stress components at the form of Equation 7.

$$u(t) = \overline{u} + u'(t) \quad (4)$$

$$\overline{u}_i = \frac{\rho u_i}{\rho} \quad (5)$$

$$u_i = \overline{u}_i + u''_i \quad (6)$$

$$\tau_{ij} = -\overline{\rho u''_i u''_j} \quad (7)$$

The Navier-Stokes Equations will solve if all time averaged quantities plus averaged Reynolds stresses are employed together with a turbulence model to solve for the Reynolds stresses. These turbulence models that will require empirical data to be adjusted, which limits the accuracy of RANS (Georgiadis et al., 2006).

Due to the empirical data limitation explained before, RANS simulations are not efficient in modelling far-field pressure signal, because turbulence is being modelled at some limitation. Acoustic analogies would fit better to predict far-field noise based on a previous solved near-field using RANS method.

The method of predicting noise using RANS method starts with the simulation that results in a mean flow solution. In the following, a solution of the linearized Euler Equations can be obtained for the acoustic fluctuations through an integral technique using Green’s function and a noise source term, as Lighthill’s (detailed on section 1.1.1). Finally, a Fourier transform of the results will provide acoustic spectra and directivity.

Despite its limitations, RANS simulation remains feasible for use with more accurate
CFD applications due to faster convergence before employing more advanced numerical schemes.

**1.4.2. Large Eddy Simulation**

Due to computational cost is not common to see Navier-Stokes Equations implemented as a direct numerical simulation (DNS) as it requires time and space discretization at Kolmogorov scale, small enough to solve any flow unsteadiness, but computationally expensive. Instead, LES method will accept scales larger than Kolmogorov’s and will resolve flow instabilities for these specific scales.

The first LES supersonic jet noise computational simulation was reported (Mankbadi et al., 1994). Later, LES was then implemented to capture both the noise generation and propagation (Mankbadi et al., 2000), results were validated by comparison with available experimental data (Troutt & McLaughlin, 1982).

Also, averaging fluid variables by separating the steady components from Navier Stokes Equations gives a time-averaged set of equations, this is known as the Reynolds decomposition. Although it is not useful for noise simulation, URANS (Unsteady Reynolds Averaged Navier-Stokes) can be set to solve the flow field when grid size is not fine enough in the boundary layer. The eddy viscosity is then reduced as the model switches from URANS to LES model and this avoids early flow separation due to an artificial low shear stress in the boundary layer, called Grid Induced Separation (Spalart et al., 2006). This hybrid LES-URANS technique minimizes computational cost, does not significantly affect noise generation and is known as Detached Eddy Simulation (DES). A more sophisticated model was then presented, the Delayed Detached Eddy Simulation (DDES), that can be resistant to ambiguous grid densities and force URANS method until grid spacing is fine enough to solve for LES in the free-shear flow (Spalart et al., 2006).
1.4.3. Linearized Euler Equations

During the late 1990’s, the implementation of Linearized Euler Equation (LEE) technique to predict unsteady noise sources in the flow field (Mankbadi, Hixon, et al., 1998). Although the LEE is not capturing nonlinear acoustic source effects, the results for an axisymmetric, supersonic and fully expanded jet agreed reasonably well with available experimental data (Troutt & McLaughlin, 1982). This indicated that LEE technique can be used for accurate prediction of sound propagation in the linear flow field around the acoustic generation field. Thus, the approach can be used to extend nonlinear, near-field computation by LES or DNS, to obtain the radiated far-field sound with considerable reduction in computation time compared to direct CFD. Also, LEE accounts for refraction effects caused by nonuniform mean flow.

The main difficulty of using this method to predict sound generated aerodynamically is the artificial effects that could be generated at the computational boundaries. The boundary treatments has to be carefully selected to avoid unsteadiness behavior in the solution, but keep its wavy nature. For the specific supersonic jet case, the adopted boundaries were a simple 1-D characteristic inflow condition, acoustic radiation condition on the sides, centerline axisymmetric condition in the middle and modified outflow condition by asymptotic analysis of the LEE (Tam & Webb, 1993).

1.4.4. Near-Field Extension to Far-Field Acoustics

Once the agreement that noise is simply pressure fluctuation, measuring noise near the aircraft presents as a complex job as needs to be isolated from the hydrodynamic components of pressure fluctuation in the flow region. Also, important observer are usually at several meters away from the jet. The FAA noise regulations usually focus on keeping lower noise levels for those inside airport complex or at communities around the flight paths. In this case, the simulation has to be capable to resolve noise on the far-field
and a larger numerical grid would demand an unsustainable computational cost.

Considering that the numerical grid will enclose the nonlinear region of the solution where the noise generation takes place, a linear wave propagation model can be conceived for a extrapolation of the near-field solution to the far-field, as the noise propagates by linear process of small amplitude pressure waves. Having the noise source properly modeled, then the following step is the implementation of any technique for calculating the radiated sound. In his pioneering work, Lighthill separated the Navier-Stokes Equations into wave propagation and sound source. The acoustic field is result of a volume integral of the Lighthill Stress Tensor (LST) that can be obtained by flow field time fluctuations (Lighthill, 1952). Back in that time, the only way to gather this data was through experimental procedure. Later, a theoretical model of the noise sources by integrating the Navier-Stokes Equations radially and applying the linear stability theory was provided (Mankbadi & Liu, 1984). This allowed the calculation of the Lighthill Stress Tensor and then its associated acoustic field. Results have shown that coherent structures tend to reach the peak noise on the spectra and propagate its waves at a preferred directivity angle.

Later on, jet noise prediction using Kirchhoff formulation applied at surface that encloses the noise sources and their nonlinear effects was presented (Lyrintzis & Mankbadi, 1996). The solution of a surface integral of the wave equation exists for known pressure and its normal derivative on each superficial point. The radiation effects are thought to be linear outside of this surface.

Although this method is simpler than Lighthill’s for considering only pressure related components, it has a strong impact on the boundary location. The far the surface is, better can be the result, but harder is to calculate pressure and its normal derivative precisely due to grid size and computational cost. A more sophisticated Surface-Integral Formulation (SIF) "in which the calculated pressure on the cylindrical surface is used to obtain the far-field sound, without the need for the normal derivative of the pressure" was introduced
(Mankbadi, Shih, et al., 1998). The comparison between both integral methods can be found on Figure 1.10. As previously said, Kirchhoff results are strongly grid dependent and SIF method using only the pressure (but not its derivative) contours this issue.

Literature review from early 2000’s provided additional details about other acoustic formulations from integral formulation of Lighthill’s analogy (Lyrintzis, 2003).

Parallel to these integral methods, Lighthill’s work (Lighthill, 1952) was further extended to account for the presence of solid boundaries at arbitrary motion (Fflowcs Williams & Hawkings, 1969). This would cover rotor noise, impinging jet and even nozzle interaction feedback loop from screech tones. Although that was a strong and comprehensive model, the Fflowcs Williams-Hawkings Equation (FW-H) required not just the boundaries relative velocity, but several flow-field quantities and their derivatives over the points on a proposed surface. Hard to implement, researchers at Langley Research Center started to investigate their general solutions focused on rotor noise problem, as the objective was to validate their results in both near and far fields. The NASA report from Farassat, the Farassat 1A Formulation was presented (Farassat, 2007) and currently constitutes most commercial and open-source CFD acoustic prediction tools together with other analogy based methods. A previous paper provided details about primitive acoustic formulations from FW-H Equation (Farassat, 1981).

![Figure 1.10](image)

*Figure 1.10* Effect of grid spacing on pressure amplitude between integration methods (Mankbadi et al., 1998).
2. Methodology

Previous computational work explored the benefits of a constant injection into the jet flow in terms of noise generation (Coderoni et al., 2019). They demonstrated that the constant fluid injection has the potential of affecting the flow, breaking down the shock-cells into weaker shocks, and modifying the potential core shape and the jet development. In the other hand, the noise mitigation mechanism analyzed in this current research is the manipulation of coherent structure frequency modes through unsteady fluidic injection at the nozzle lipline in the downstream direction. It is expected that the instability waves near the nozzle lip get affected by the unsteady injection and, while while they travel downstream, they will grow into large turbulent structures at a specific periodicity and this may affect noise generation.

This Thesis presents the qualitative validation of a numerical approach that simulates axisymmetric free jet flow under excitation via unsteady microjets issuing from the nozzle lip at downstream direction. Two flow regimes are evaluated, a supersonic fully expanded Mach 1.8 case and a transonic Mach 0.9 case. A hybrid LES-URANS approach is implemented to model turbulence wherein URANS is used near wall boundaries and LES in the free shear flow. The obtained far-field noise is compared with previous experimental and numerical results and near-field quantities are qualitatively compared with previous work on LEE method (Golubev et al., 2003a) to be verified by this actual high-fidelity simulation.

In the following sections, the methodology for computational modeling of the fluid flow and aeroacoustics is thoroughly detailed. Finally, the aerodynamic and acoustic results are discussed.
2.1. Problem Statement

For the perfectly expanded supersonic case, a convergent-divergent (CD) nozzle with exit diameter of \( D = 0.04 \) m is used to simulate a Mach 1.8 jet through total pressure condition against ambient pressure denoted by nozzle-pressure-ratio of \( NPR = 5.8 \), as well for the subsonic case, a convergent nozzle with same exit diameter of \( D = 0.04 \) m produces a Mach 0.9 jet flow when subjected to \( NPR = 1.7 \) (Figure 2.1). Table 2.1 presents the inflow conditions used on both cases. The CD nozzle used for Mach 1.8 case is utilized and verified by previous computational investigation (Salehian & Mankbadi, 2020) and the convergent nozzle for Mach 0.9 is a provisional design adopted for this work, this means that none of available numerical or experimental data corresponds to this specific nozzle profile and Reynolds number, which could lead to different flow field results.

The desired scheme of several microjets distributed along the nozzle lip is represented by an annular cut in the axisymmetric nozzle (Figure 2.2, a thin cut of thickness \( t = 0.4 \) mm injects unheated air along the lipline in the downstream direction. The area ratio between microjet and nozzle exit diameter is exactly \( A_t/A_e = 0.04095 \) and the injection pressure ratio varies to achieve desired injection velocity at a specified sinusoidal waveform of peak mass flow rate ratio of \( \dot{m}_t/\dot{m}_j = 1\% \) between microjet injection and main jet (Figure 2.3). A range of forcing frequencies from Strouhal number \( St_F = 0.05 \) to \( St_F = 1 \) is analyzed, see table 2.2.

![Figure 2.1 Nozzle used in study. (a) Supersonic case (b) Subsonic case.](image-url)
Table 2.1

Inflow conditions.

<table>
<thead>
<tr>
<th></th>
<th>Mach No.</th>
<th>Pressure (kPa)</th>
<th>Temperature (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supersonic inflow</td>
<td>N/A</td>
<td>590.000</td>
<td>300.0</td>
</tr>
<tr>
<td>Subsonic inflow</td>
<td>N/A</td>
<td>171.371</td>
<td>300.0</td>
</tr>
<tr>
<td>Freestream</td>
<td>0.015</td>
<td>101.325</td>
<td>300.0</td>
</tr>
</tbody>
</table>

*Figure 2.2* Nozzle-injector setup.

*Figure 2.3* Illustration of the pulsating mass flow boundary condition used in the simulations.

### 2.2. Computational Setup

This section covers the numerical procedures on predicting the unsteady flow field on the computational grid provided, subsequently the far-field pressure fluctuation obtained using an extension technique of the nearfield analysis is considered. The review starts in the governing equations, then numerical scheme and boundary conditions are presented, in the following, computational grid and the acoustic surface integral are shown. Finally,
Table 2.2

Frequency specifications of the Microjets modeled in the simulations.

<table>
<thead>
<tr>
<th>Case</th>
<th>( f - \text{Supersonic Case (Hz)} )</th>
<th>( f - \text{Subsonic Case (Hz)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clean Nozzle</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Constant</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( St_F = 0.05 )</td>
<td>612.5</td>
<td>312.5</td>
</tr>
<tr>
<td>( St_F = 0.125 )</td>
<td>1531.25</td>
<td>781.25</td>
</tr>
<tr>
<td>( St_F = 0.25 )</td>
<td>3062.5</td>
<td>1562.5</td>
</tr>
<tr>
<td>( St_F = 0.35 )</td>
<td>4287.5</td>
<td>2187.5</td>
</tr>
<tr>
<td>( St_F = 0.5 )</td>
<td>6125</td>
<td>3125</td>
</tr>
<tr>
<td>( St_F = 0.75 )</td>
<td>9187.5</td>
<td>4687.5</td>
</tr>
<tr>
<td>( St_F = 1 )</td>
<td>12250</td>
<td>6250</td>
</tr>
</tbody>
</table>

Noise definitions applied on the analysis are introduced followed by window function and signal processing techniques that are applied on the frequency domain results for better accuracy.

2.2.1. Numerical Scheme

A density-based compressible solver is employed implementing a total variation diminishing (TVD) scheme to simulate the flow field of a supersonic, fully expanded heated jet. A DES hybrid approach is applied to simulate the turbulence fluctuations of the flow and avoid computationally expensive LES simulations to model the near-wall boundary layer by applying URANS method on that region.

In this computational research, the \( k - \omega \) SST DES turbulence model is adopted. Therefore, the computational cost is reduced compared to the full LES that requires extensive near wall treatment. For the current simulations, a statistically steady solution is achieved with the \( k - \omega \) SST RANS model first, then the DES simulations are carried out using the RANS results as an initial solution.

The rhoCentralFoam application in OpenFOAM is the solver assigned for this research. OpenFOAM is an open-source CFD software package consisting of a set of flexible C++ modules to resolve complex fluid flows. The rhoCentralFoam application is
an unsteady, compressible solver that uses semi-discrete, non-staggered, Godunov-type central scheme (Kurganov et al., 2000) and upwind-central scheme (Kurganov & Tadmor, 2000). These schemes avoid the explicit need for a Riemann solver, resulting in a numerical approach that is both simple and efficient. The directed convective fluxes mentioned above are interpolated using the van Albada scheme (van Albada et al., 1997) to provide a second-order spatial discretization that, as a TVD scheme, is appropriate for capturing flow discontinuities, such as shocks, and the limiter automatically provides high-order stable solution. The finite volume method is applied for expressing the differential equations. In the application of the finite volume to polyhedral cells with an arbitrary number of faces, each face is assigned to an owner cell and a neighboring cell (Greenshields et al., 2010).

In compressible fluid flows, properties are not only transported by the flow but also by the propagation of waves. This requires the construction of flux interpolations to consider that transport can occur in any direction (Marcantoni et al., 2012). The convective terms of the conservation equations in the forms of \( \nabla [\rho u] \), \( \nabla [u (\rho u)] \), \( \nabla [u (\rho E)] \) and \( \nabla [u \rho] \) integrated over a control volume and linearized.

### 2.2.2. Governing Equations

The governing equations for the entire computational domain are the compressible, unsteady Navier-Stokes Equation (Mankbadi, 1994), given by Equations 8 to 12.

\[
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j} (\rho u_j) = 0
\]  

(8)

\[
\frac{\partial}{\partial t} (\rho u_i) + \frac{\partial}{\partial x_j} \left[ \rho u_i u_j + p \delta_{ij} - \tau_{ij} \right] = 0
\]

(9)

\[
\frac{\partial}{\partial t} \left( \rho e_i \right) + \frac{\partial}{\partial x_j} \left[ \rho u_j e_i + p u_j + q_j - u_j \tau_{ij} \right] = 0
\]

(10)
\[ \tau_{ij} = 2\mu S_{ij} - \frac{1}{3} \frac{\partial u_k}{\partial x_k} \delta_{ij} \quad (11) \]

\[ e_t = e + \frac{u_k^2}{2} \quad (12) \]

Where \( e_t \) is the total energy, \( q_j \) is the heat flux, and \( \tau_{ij} \) is the viscous stress term, and \( S_{ij} \) is the strain rate tensor, detailed on Equation 13 and Equation 14.

\[ S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad (13) \]

\[ q_j = -C_p \frac{\mu}{Pr} \frac{\partial T}{\partial x_j} \quad (14) \]

Coefficient \( Pr \) is the Prandtl number and \( \mu \) is the laminar dynamic viscosity of air which is calculated from the Sutherland’s law of viscosity, see Equation 15.

\[ \mu(T) = \mu_o \left( \frac{T_o + T_s}{T + T_s} \right)^{3/2} \quad (15) \]

Where \( \mu_o, T_o, \) and \( T_s \) are the reference dynamic viscosity, temperature, and Sutherland’s constant respectively for air, with values of \( 1.716 \times 10^{-5} \text{ kg/m/s}, 300K, \) and \( 110.4K \) respectively.

**2.2.3. Time & Space Discretization**

The control-volume technique is used here to solve the governing equations. The conventional upwind procedure would cause the numerical dissipation to restrain the predictive capabilities of LES whenever it is of the same order of magnitude or larger than the Sub Grid Scale (SGS) dissipation (Castiglioni & Domaradzki, 2015). A Gauss limited linear scheme, second order accurate, unbounded, but more stable than pure linear is selected; according to the software itself, this convective discretization scheme is recommended for LES simulations and it is compared to Fromm’s method (Fromm, 1968).

To reconstruct the face values, the gradients of the flow variables are calculated. The
cell-based least squares approach is adopted in these simulations. The solver chooses the weighted average of upwind and central interpolations, such that it does not generate solution extrema. The bounded second-order implicit time scheme (Versteeg & Weeratunge, 2007) is used to march the solution in time, see Equation 16. The time step was selected here to ensure that the important frequencies are appropriately resolved. A constant time step of $1 \times 10^{-8}$ physical seconds is used to ensure capturing the highest needed non-dimensional frequency of $St = \frac{f_d}{U_j}$. In order to avoid losing time on the solution setup, initial RANS solution is implemented using a pseudo transient time scheme from OpenFOAM called "localEuler", a first order time scheme based on Euler implicit time scheme (see Equation 17) with cell-based time scale set by specific Local Time Stepping (LTS) solvers.

$$\frac{\partial}{\partial t} (\phi) = \frac{1}{\Delta t} \left( \frac{3}{2} \phi - 2 \phi^o + \frac{1}{2} \phi^{oo} \right)$$  \hspace{1cm} (16)

$$\frac{\partial}{\partial t} (\phi) = \frac{\phi - \phi^o}{\Delta t}$$  \hspace{1cm} (17)

For every statistical data analyzed on this work, a total of 4096 samples is collected at a sampling frequency of 200 kHz, which is similar to experimental setups. A total of 0.02048 physical seconds is simulated and this will define our minimum frequency accessible by the post-processing, see Equation 18. A conservative minimum of 25 time history points is required to resolve a single wave of the lowest possible frequency and $\tau = 0.02048 \text{ s}$. The result is $St = 0.1$ for the supersonic case and $St = 0.2$ for the subsonic case.

$$St_{min} = \frac{25D}{\tau U_j}$$  \hspace{1cm} (18)
2.2.4. Turbulence Modeling

The URANS $k - \omega$ SST turbulence model is defined by Equations 19 and 20 (Menter et al., 2003).

$$\frac{\partial(\rho k)}{\partial t} + \frac{\partial(\rho u_i k)}{\partial x_i} = \tilde{P}_k - \beta \rho k \omega + \frac{\partial}{\partial x_i} \left[ \left( \mu + \sigma_k \mu_t \right) \frac{\partial k}{\partial x_i} \right]$$  \hspace{1cm} (19)

$$\frac{\partial(\rho \omega)}{\partial t} + \frac{\partial(\rho u_i \omega)}{\partial x_i} = \alpha \rho S^2 - \beta \rho \omega^2 + \frac{\partial}{\partial x_i} \left[ \left( \mu + \sigma_\omega \mu_t \right) \frac{\partial \omega}{\partial x_i} \right] + 2(1-F_1) \rho \sigma_\omega^2 \frac{1}{\omega} \frac{\partial k}{\partial x_i} \frac{\partial \omega}{\partial x_i}$$  \hspace{1cm} (20)

The blending function $F_1$ is defined by Equation 21, $CD_{k\omega}$ by Equation 22 and $y$ is the distance to the nearest wall.

$$F_1 = \tanh \left\{ \min \left[ \max \left( \frac{\sqrt{k}}{\sigma_\omega^2 y^2}, \frac{4 \rho \sigma_\omega^2 k}{CD_{k\omega} y^2} \right) \right] \right\}^4$$  \hspace{1cm} (21)

$$CD_{k\omega} = \left( 2 \rho \sigma_\omega^2 \frac{1}{\omega} \frac{\partial k}{\partial x_i} \frac{\partial \omega}{\partial x_i}, 10^{-10} \right)$$  \hspace{1cm} (22)

The turbulent eddy viscosity is defined as Equation 23, where $S$ is the invariant measure of the strain rate and $F_2$ is a second blending function defined by Equation 24.

$$\nu_t = \frac{a_1 k}{\max(a_1 \omega, SF_2)}$$  \hspace{1cm} (23)

$$F_2 = \tanh \left[ \max \left( \frac{2 \sqrt{k}}{\beta^* \omega y^2}, \frac{500 \nu}{y^2 \omega} \right) \right]^2$$  \hspace{1cm} (24)

A limiter is used in the SST model to prevent the build-up of turbulence in stagnation regions, see Equation 25. And all constants were computed as (Menter et al., 2003):

$\beta^* = 0.09, \alpha_1 = 5/9, \beta_1 = 3/40, \sigma_{k1} = 0.85, \sigma_{\omega1} = 0.5, \alpha_2 = 0.44, \beta_2 = 0.0828, \sigma_{k2} = 1$ and $\sigma_{\omega2} = 0.856$. 
\[
\bar{p}_k = \min \left( \mu \frac{\partial U_i}{\partial x_j} \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right), 10\beta^* \rho k \omega \right)
\] (25)

The DES formulation of the \( k - \omega \) SST model (Menter et al., 2003) is achieved such that in the LES regions of the grid, the solution would reduce to a Smagorinski-like sub-grid model, such that the eddy viscosity is proportional to the magnitude of the strain tensor, and to the square of the grid spacing (Strelets, 2001). However, the only difference in the DES mode of the \( k - \omega \) SST RANS model is the dissipative term of Equation 19. This equation for the DES mode is defined as Equation 26, where \( \bar{d} \) is the length scale from Equation 27.

\[
\frac{\partial (\rho k)}{\partial t} + \frac{\partial (\rho u_i k)}{\partial x_i} = \bar{p}_k - \beta \rho k \omega \bar{d} + \frac{\partial}{\partial x_i} \left[ (\mu + \sigma_k \mu_t) \frac{\partial k}{\partial x_i} \right]
\] (26)

\[
\bar{d} = \max \left( \frac{L_t}{C_{DES} \Delta}, 1 \right)
\] (27)

Where \( \Delta \) is the local grid spacing, generally defined as \( \Delta = \max (\Delta x, \Delta y, \Delta z) \) and \( C_{DES} = 0.61 \). The shifting between URANS and LES happens when the local grid is fine enough in all directions, compared to the turbulent length scale term grows larger than 1. As consequence, \( k \) is reduced, allowing the solution to resolve turbulence and it will reduce the amount of modeled turbulent shear stress and allow the free jet region to be treated as LES.

### 2.2.5. Boundary Treatment

Inlet conditions are defined for a Nozzle Pressure Ratio of \( NPR = 5.834 \) and Nozzle Temperature Ratio of \( NTR = 1 \) and the jet was expected to be fully expanded with a \( NPR = 5.834 \). Ambient temperature is prescribed to \( T_o = 300K \) and ambient pressure is \( P_o = 101325Pa \). Adveective far-field condition was imposed on the rest of the domain boundaries, which corresponds to “waveTransmisse” boundary conditions in OpenFOAM.
The non-reflecting boundary condition used here is based on the characteristic wave relations derived from the Euler equations reformulated into an orthogonal coordinate system such that one of the coordinates is normal to the boundary. The amplitude of the incoming pressure and entropy waves are computed from the Linear Relaxation Method (LRM) (Poinset & K., 1992). This non-reflecting condition is based on the same idea of non-reflecting boundary condition without full inter-field coupling (Poinset & K., 1992).

The nozzle inner walls are prescribed as adiabatic no-slip condition, so the RANS simulations near the wall can predict the boundary layer with the specified $y^+$. On the other hand, on the nozzle outer wall and microjet walls, adiabatic slip conditions are imposed. Since the flat plate is only to reflect the acoustic wave and the microjet just need to deliver the excitation to the main flow, the no-penetration rule is enforced by imposing zero pressure gradient out of the surface and zero normal velocity.

Finally, the bottom vertex has an axial boundary condition assigned to it and will consider the construction of the grid as a $5^\circ$ angle revolved extrusion, 1 cell thick, running along the center line and straddling one of the coordinate planes (The OpenFOAM Foundation, 2017), as shown in Figure 2.4. The wedge planes must be specified as separate patches of wedge type boundary condition; this will ensure the 2-dimensional axisymmetric geometry.

### 2.2.6. Computational Grid

The computational grid used in the current simulations contains hexahedrally dominant cells. The entire computational domain extends to $80D$ downstream of the nozzle exit and $10D$ upstream of the nozzle exit, also it extends radially up to $15D$ from the axis plane. The grid spacing on nozzle walls is chosen such that it ensures $y^+$ to have a value of 20 on the wall, and to make sure the close wall calculations of boundary layer in the RANS region are accurate. This value for $y^+$ is calculated considering the isentropic flow assumption along the nozzle and using the nozzle exhaust velocity $U_j$. As it is
illustrated in Figure 2.5, the fine grid spacing on nozzle walls are gradually increased. This grid spacing is maintained and extended up to \( x/D = 10 \) in the jet axis direction to capture turbulent mixing near nozzle exit, and then it is gradually increased from \( D/100 \) up to \( D/40 \) in jet axis direction. From \( x/D = 10 \) to \( x/D = 30 \) the grid spacing maintains up to \( D/52 \). These refinement regions are illustrated by red boxes in Figure 2.6. On the subsonic case, the grid is the same, except for the nozzle shape.

![Figure 2.4](image)

*Figure 2.4* Axisymmetric geometry using the wedge patch type (The OpenFOAM Foundation, 2017).

![Figure 2.5](image)

*Figure 2.5* Planar cut of the computational grid near nozzle exit. (a) Supersonic case (b) Subsonic case.

![Figure 2.6](image)

*Figure 2.6* Planar cut of the computational domain (Supersonic case)
The FWH surface used in this study is a cylinder section from the nozzle exit extending to $y/D = 6$ up to $x/D = 30$ in the jet axis direction, enclosing the near-field region illustrated with the red box in Figure 2.6. The maximum grid spacing of $D/40$ in this region is to be used for FWH acoustic predictions. Such grid spacing on FWH surface would ensure capturing acoustic waves up to Strouhal number $St = 1.4$ and $St = 2.4$ for the supersonic and subsonic cases, respectively. This maximum frequency represents up to $86\%$ of the spectra shown in experimental results and contains the important aspects of the trend in spectral analysis of the acoustic signal, such as the peak frequency observed in experimental results and proposed excitation frequencies. From the numerical point of view, the maximum resolvable frequency is calculated based on the assumption that a minimum of 15 points (cells) per wavelength are required to capture the acoustic waves with the current numerical scheme. Such requirement has been tested for prediction of waves using second-order finite volume schemes when applied to hexahedral cells. The clean nozzle case has exactly the same grid as the excited cases, except by the injector addition on the nozzle lip that can be seen in detail on Figure 2.7.

![Planar cut of the computational grid near microjet injector](image)

*Figure 2.7* Planar cut of the computational grid near microjet injector. (a) Supersonic case. (b) Subsonic case.

### 2.2.7. Ffowes-Williams Hawkings Integral Method

Far-field acoustics is obtained using the FWH integral technique. The FWH Equation is an inhomogeneous wave equation derived by manipulating the continuity equation and the Navier-Stokes Equations. If we assume that the control surface contains all acoustic
sources, the volume integrals outside this surface can be dropped. To provide accurate, and clear documentation of the FWH formulations implement within OpenFOAM, the formulation and assumptions made for simplifications are provided here. Starting from the Equation 28 which is the FWH in its general form, assisted by Equations 29 to 31 with definitions for $U_n$, $L_i$ and $T_{ij}$, respectively.

$$\Box^2 p'(x, t) = \frac{\partial}{\partial t} \left[ (\rho_o U_n) \delta(f) \right] + \frac{\partial}{\partial x_i} [L_i \delta(f)] - \frac{\partial^2}{\partial x_i \partial x_j} \left[T_{ij} H(f) \right]$$  

(28)

$$U_n = U_i \cdot \hat{n}_i = \left[ \left( 1 - \frac{\rho}{\rho_o} \right) v_i + \left( \frac{\rho}{\rho_o} \right) u_i \right] \cdot \hat{n}_i = \left( 1 - \frac{\rho}{\rho_o} \right) v_n + \left( \frac{\rho}{\rho_o} \right) u_n$$  

(29)

$$L_i = P_{ij} \hat{n}_j + \rho u_i (u_n - v_n)$$  

(30)

$$T_{ij} = \rho u_i u_j - \sigma_{ij} + \left( p' - c^2 \rho' \right) \delta_{ij}$$  

(31)

The notation, $\Box^2$, is the wave or D’Alembertian operator in the three-dimensional space, $T_{ij}$ is the Lighthill stress tensor, $u_n$ is the fluid velocity normal to the control surface, $v_n$ is the surface velocity in the normal direction, $\rho_o$ is the free-stream density and $\rho$ or $\rho u_i$ are flow field variables previously obtained on the control surface. The Heaviside and the Dirac delta functions are denoted $H(f)$ and $\delta(f)$, respectively.

The Farassat 1A formulation of the FW-H Equations (Brentner & Farassat, 1998) is utilized such that the far field acoustic can be represented as Equations 32 to 34.

$$p'(x, t) = p''_T(x, t) + p''_L(x, t) + p''_Q(x, t)$$  

(32)

$$4\pi p''_T(x, y) = \int_{f=0} \left[ \frac{\rho_o (\bar{U}_n + \bar{U}_\hat{r})}{r(1 - M_r)^2} \right]_{ret} dS + \int_{f=0} \left[ \frac{\rho_o U_n (r M_r + c (M_r - M^2))}{r^2(1 - M_r)^3} \right]_{ret} dS$$  

(33)
Here, $U$ and $M$ are the surface motion velocity and Mach number, $r$ is the distance between source and observer. $\dot{L}_r$, $\dot{U}_n$ and $\dot{M}_r$ represent the source time derivatives. The subscripts $r$ or $n$ denote a dot product of the vector with the unit vector in the radiation direction $\hat{r}$, or the unit vector in the surface normal direction $\hat{n}$, respectively. The term $L_M = L_i M_i$, subscript "ret" refers to retarded time and term $f = 0$, represents closed surface integration on the control surface. The last term in Equation 32 is the volume integral which represent quadrupole (volume) sources in the region. The contribution of the volume integrals becomes small when the source surface encloses the source region. The above equations can be simplified for a control surface that is fixed in space, as permeable control surfaces (Lyrintzis, 2003).

The implemented FWH formulation in OpenFOAM is achieved by assuming the volume integral term can be ignored, the surface integrals are simplified as Equations 35 to 38. Where, all other terms can simplified to $U_n = U_i \hat{n}_i$, $\dot{U}_n = \frac{\partial U_n}{\partial t}$, $L_r = L_i \cdot \hat{r}_i$ and $\dot{L}_r = \frac{\partial L_r}{\partial t}$.

\[
4\pi p'_L(x, y) = \int_{f=0} \left[ \frac{\dot{L}_r}{r} \right]_{ret} dS
\]  
\[4\pi p'_T(x, y) = \int_{f=0} \left[ \frac{\rho_o \dot{U}_n}{r} \right]_{ret} dS\]  
\[
4\pi p'_L(x, y) = \frac{1}{c} \int_{f=0} \left[ \frac{\dot{L}_r}{r} \right]_{ret} dS + \int_{f=0} \left[ \frac{L_r}{r^2} \right]_{ret} dS
\]

\[
U_i = \frac{\rho}{\rho_o} u_i
\]

\[
L_i = P_{ij} \hat{n}_j + \rho u_i u_n
\]
The Farassat 1A formulation is implemented using libAcoustics (Epikhin et al., 2015). This is an open source library for OpenFOAM applications developed by UniCFD Web-laboratory (Evdokimov et al., 2020).

2.2.8. Noise Metrics

This is a brief overview of the various measures of noise used in this study. The pressure fluctuation $p'(t)$ obtained in near-field and far-field domains is normalized by a commonly-used reference pressure, $p_{ref} = 20 \mu Pa$. The spectrum of the pressure fluctuation, $p_{RMS}(f)$, is computed using a fast Fourier transform (FFT) algorithm on the pressure fluctuation time signal $p'(t)$. In the simulations, the frequency range considered is from 1220Hz to 17220Hz, equivalent to $St = 0.06$ and $St = 1.4$, respectively. The following noise metrics are then computed; sound pressure level ($SPL$) on Equation 39, normalized pressure fluctuation ($S_p(f)$) on Equation 40, overall pressure root mean square ($P_{OverallRMS}$) on Equation 41 and overall sound pressure level ($OASPL$) on Equation 42.

$$SPL(f) = 20 \log_{10} \left( \frac{p_{RMS}(f)}{P_{ref}} \right)$$ (39)

$$S_p(f) = \frac{p_{RMS}^2(f)}{\Delta f}$$ (40)

$$P_{OverallRMS} = \sum_{i=1}^{N} p_{RMS}^2(f) = \int_{0}^{\infty} S_p(f) df$$ (41)

$$OASPL = 20 \log_{10} \left( \frac{P_{OverallRMS}}{P_{ref}} \right)$$ (42)

2.2.9. Window Function & Signal Processing

After an extensive data collection either on far-field or near-field, the collected time history is converted into frequency spectrum by using Fast Fourier Transform (FFT). However, applying the FFT function on a random complex signal without the use of
averaging, results in spurious fluctuations of amplitudes between the neighboring modes of the Fourier spectrum. To obtain a smooth spectrum, a single period sample is divided into two segments. Here, each segment contains 2048 samples with hanning window and 50% overlap. The FFT is then applied on each segment and the resulting spectra is averaged. This procedure can significantly avoids ‘spectral leakage’ and reduces the amplitude of the discontinuities at the boundaries of each finite segment, see Figure 2.8. Although the frequency resolution is reduced by the amount of window blocks, the reason to apply a window relates to the length of a real signal, as it can be random and larger than the collected sample. This procedure reduces computational time and gives a fair spectral estimate. The most important parameters that will affect the final result when cutting a signal into blocks is the block length (in samples), type of window function and the overlap between blocks, see Figure 2.9

![Figure 2.8 Hann window processing effects on noise spectrum against standard FFT.](image)

The use of window functions on analyzing discrete-time signals as frequency spectrum is supported by the modulation or windowing theorem that says that discrete-time convolution of sequences (convolution sum) is equivalent to multiplication of corresponding periodic Fourier transforms, and multiplication of sequences is equivalent to periodic convolution of corresponding Fourier transforms (Oppenheim & Schafer, 1999).
3. Results & Discussion

In this section, there is a validation of mean flow profiles through comparison against available experimental data. Furthermore, a comparison of current simulation results against previous computational work (Golubev et al., 2003b) serves as a cross validation. Finally, as the main contribution of this work, an analysis of flow field when the user inputs 1% of main jet mass-flow-rate (MFR) at unsteady injection following a specific sinusoidal wave frequency is provided. This was done by comparing excited results with the clean nozzle case. Near-field results showed evidence that the disturbances was picked up by the flow.

In the following, far-field acoustics results were compared against experimental and 3D numerical results to discuss its validity, then an analysis of acoustic response was examined using available far-field data. A mere 2 dB suppression, for a basic axisymmetric injection as this, is a fair achievement (Raman, 1984; Gaitonde, 2012; Henderson, 2010).

3.1. Flow Field Validation

Mean flow shape assumptions were provided by analytic equations describing the velocity and density profiles (Dahl & Mankbadi, 2002). A cold jet issuing from a round nozzle at perfectly expanded condition was considered. With the purpose to compare analytical results and mean flow obtained from current simulations, Figure 3.1 is provided. The mean flow profiles of density (Mean(\(\rho\))) and axial velocity (Mean(\(U\))) have smooth gradients in the radial direction and smoothly transition from the potential core region to

![Figure 2.9 Windowing representation and signal overlap visual representation.](image)
the fully developed region downstream. The velocity in the radial component ($Mean(V)$) is realistic for both cases, as the flow is outward towards the shear layer and then inward, outside of the jet potential core, indicating quiescent flow entrainment. In addition, Figures 3.2 and 3.3 provide a quantitative comparison of the jet mean flow velocity along the centerline between computational work (Gaitonde, 2012) and experimental data (Samimy et al., 2007) for supersonic Mach $M = 1.8$ and subsonic Mach $M = 0.9$ cases.

The supersonic case agreed reasonably well with available data and the extended potential core observed must be a consequence of the axisymmetric assumption in the numerical methodology applied to the current simulations. The absence of helical modes at high Reynolds number regime retards the jet dissipation.

Now, the subsonic Mach 0.9 presents periodic oscillations inside the potential core and this is probably due to nozzle shape or unexpected shock solution, since the proposed nozzle profile for this case has not been tested or simulated yet. Mach number contour plot from Figure 3.4 reveals a diamond shock formation, or Mach disk, that is plausible to appear in transonic regimes.

![Figure 3.1](image)

*Figure 3.1* Cold jet mean flow profiles computed at 8 axial locations for clean nozzle Mach 1.8.
Figure 3.2 Non-dimensional mean flow velocities along the jet centerline at clean nozzle Mach 1.8 from simulations and experiments.

Figure 3.3 Non-dimensional mean flow velocities along the jet centerline at clean nozzle Mach 0.9 from simulations and experiment.

Figure 3.4 Mach number contour plot of clean nozzle Mach 0.9 case.
3.2. Comparison Against Previous LEE Results

An analysis of acoustic radiation from the source model representing spatially-growing instability waves in a round jet at high speeds was examined (Dahl & Mankbadi, 2002). The mean flow quantities were derived from analytical equations, instability waves obtained from the linear stability theory system of equations and finally a nonlinear solution for instability waves was provided using the proposed method of local energy integrals. The fact that the large-structures have a spectrum of frequencies and multiple azimuthal modes has been ignored. Furthermore, researchers suggested an extension of the previous integral energy approach by using linearized Euler Equations (LEE) instead of the instability wave equations (Golubev et al., 2003b). This has several advantages such as LEE accounts for the fully non-parallel flow defects, disturbances of multifrequency components can be simulated using a time-marching code and disturbances may be of a general nature, since the normal mode decomposition is not used in deriving the code.

An extensive work on jet issuing from a round nozzle at different Mach numbers was provided (Dahl & Mankbadi, 2002), the LEE approach considers cold jet at Mach number $M = 1.8$, excited with a helical mode $n = \pm 1$ at a single frequency corresponding to Strouhal number $St_D = 0.2$. Current research provides simulation results for a cold jet at same Mach number, excited with an axisymmetrical mode $n = 0$ at a single frequency corresponding to Strouhal number $St_D = 0.2631$ with an amplitude of 1% of main jet flow mass flow rate (MFR).

A curve based on the real part of the unsteady pressure fluctuations from the frequency spectra at each point on the lipline $(r = D/2)$, obtained through a Fast Fourier Transformation (FFT) of the unsteady flow-field solution would show the instability wave growth into coherent structures (Golubev et al., 2003b). A pressure fluctuation contour
plot was generated for current simulations using the real part of the specific frequency component, see Figure 3.5. This spectral analysis is a type of mode decomposition capable to extract coherent structures from snapshot sequences of temporally evolving data. Figure 3.6 brings a comparison between studies.

Figure 3.7 compares the development of the jet instability wave magnitude along the nozzle lipline using the linear (using locally parallel stability theory) and nonlinear analysis (Dahl & Mankbadi, 2002) against DES simulation results. The magnitude analysis considers the complex conjugate composed by a real part (shown in Figure 3.6) that governs the axial phase change of the instability wave and an imaginary part that relates to the local growth or decay of the wave. It is important at this point to note that the extracted frequency mode from Figure 3.5 does not peak at the same axial position as the pressure magnitude plot from Figure 3.7. This means that the extracted frequency mode is not the most energetic mode, but serves as a comparison between different studies.

The instability wave development on LEE achieves higher amplitude and takes longer to dissipate. Differently, DES results show high energy instability issuing from the nozzle exit that quickly develops into vortical structures, reaching its peak amplitude and decreases its amplitude rapidly while they travel downstream, until it forms a wave-packet model and eventually fade out. Unfortunately, current study did not reach further than 40 diameters away from the nozzle for the mode decomposition analysis. Although the excitation amplitude is not explicit in the paper that reports LEE results, it could be larger than 1% MFR applied on DES simulations.

Additionally, it is observed that the nonlinear effects limit the amplitude growth and shift the peak amplitude towards the jet nozzle exit. This nonlinear saturation effect corresponds to what was revealed when previous researchers compared linear stability theory and energy integral results, that the modes initially grow linearly and then saturate as a result of the nonlinear interactions and the flow divergence effects (Dahl & Mankbadi,
The nonlinear effects obtained through the energy integral theory had some qualitative agreements with experiment.

The fact that current DES simulations are axisymmetric might be an issue for appropriate comparison against excited cases at non-zero helical modes, as these components scale up at high Mach number. In a realistic 3D case, the oscillations would have higher amplitudes and would take longer to dissipate due to the presence of helical components.

Figure 3.5 Real part of pressure fluctuation spectrum for Mach 1.8 excited at $St_F = 0.2631$.

Figure 3.6 Jet instability wave at nozzle lip for Mach 1.8.
3.3. Flow Field Results

The numerical simulations are carried out for the fully expanded supersonic and subsonic cases, and the results are compared with the corresponding clean nozzle results, to provide comparison on the effect of the unsteady excitation on the flow and acoustic field.

3.3.1. Supersonic Mach 1.8 Fully Expanded Jet

In Figures 3.8 and 3.9, the instantaneous pressure fluctuation and the turbulent kinetic energy contour normalized by jet velocity squared are illustrated for each excited and clean nozzle cases. The instantaneous pressure fluctuation Figure shows that $St_F = 0.125$ and $St_F = 0.25$ significantly amplifies the downstream waves as can be seen on the dark region on the top right of the contour plot. In the other hand, $St_F = 1.00$ appears to somewhat control downstream waves and provide acoustic attenuation.

The normalized turbulent kinetic energy contour shows that $St_F = 0.05$ and $St_F = 0.125$ excitation are not suitable to decrease kinetic energy in the shear-layer region. However, $St_F = 1.00$ seems to perform way better than clean nozzle case. This suggests that less turbulence energy consists in less noise generation, consequently, less noise propagation due to turbulence structures.
Figure 3.10 shows the snapshot of the pressure fluctuations in terms of the dilatation, overlaid on top of it is the vorticity field. This Figure illustrates the microjet introducing a smaller scale of motion and enhance turbulent mixing. An enhanced analysis of the disturbances affecting the shear layer and controlling the instabilities that eventually will grow into large scale structures calls for a magnified picture of the vorticity near the nozzle exit, see Figure 3.11. An attentive comparison indicates that $St_F = 1.00$ has a steadier initial shear layer compared to other cases. The assumption relies on the idea that the excitation imposed to the flow is enough to suppress some of the natural instabilities that are present in the clean nozzle case.

This evidence justifies the purpose of such active noise control. Different from previous computational work that used momentum transfer to break shock cells and affect large scale structures (Coderoni et al., 2019), the proposed unsteady fluidic injection device has the capability to affect the instability waves in the initial shear layer and, under specific conditions, it can reduce the instability and will probably suppress sound generated by large scale structures while they travel downstream.

*Figure 3.8* Near-Field Instantaneous Pressure Fluctuation for Mach 1.8.
Figure 3.9 Turbulent Kinetic Energy normalized by jet velocity squared for Mach 1.8.

Figure 3.10 Snapshot of dilatation contour plot overlayed by Vorticity for Mach 1.8.
3.3.2. Subsonic Mach 0.9 Jet

The effect of the unsteady excitation on the flow-field is presented on Figures 3.12 and 3.13. The instantaneous pressure fluctuation and the turbulent kinetic energy contour normalized by jet velocity squared are illustrated for each excited and clean nozzle cases.

The normalized turbulent kinetic energy contour shows that $S_{T_F} = 0.75$ and $S_{T_F} = 1.00$ excitation are not suitable to decrease kinetic energy in the shear-layer region. However, $S_{T_F} = 0.35$ and $S_{T_F} = 0.05$ seems to perform way better than clean nozzle case. This suggests that less turbulence energy consists in less noise generation, consequently, less noise propagation due to turbulence structures.

Figure 3.14 shows the snapshot of the pressure fluctuations in terms of the dilatation, overlaid on top of it is the vorticity field. This Figure illustrates the microjet introducing a smaller scale of motion and enhance turbulent mixing. Repeating the enhanced analysis that was done for Mach 1.8 cases, a snapshot of the vorticity field near the nozzle exit for the Mach 0.9 cases can be seen in Figure 3.15. It seems that initial shear layer from $S_{T_F} = 0.05$ case is the one that takes longer to break into large vorticities and probably will suppress noise better than other cases when evaluating far-field acoustics.

3.3.3. Near Field Pressure Spectral Evolution

Along the investigation process, the seek for comprehensive analysis arose the need for an innovative examination of the flow field. The spectral evolution consists in
Figure 3.12 Near-Field Instantaneous Pressure Fluctuation for Mach 0.9.

Figure 3.13 Turbulent Kinetic Energy normalized by jet velocity squared for Mach 0.9.

compiling several spectra of a specific physical flow quantity, such as velocity, temperature, pressure or density, over a line of probes. In the current study, 8192 pressure fluctuation samples at each $5 \times 10^{-6}$ second were collected using a line of probes at the nozzle lipline ($r = D/2$), FFT was performed to each data set without applying any window function and the frequency components of the pressure fluctuations could be
obtained. These frequency components can be related to its specific Strouhal number frequency ($St_D$) for each specific probe axial location ($x/D$), and the outcome is a surface plot called the pressure spectral evolution.

According to far-field data that will be presented in the following section, the Mach
1.8 jet noise peak frequency at 30° line is around \( St_F = 0.25 \). At the time of this simulation, the forcing Strouhal number of \( St_F = 0.2631 \) and its subharmonic \( St_F = 0.1355 \) were picked for this analysis. Figures 3.16 to 3.18 document the downstream spectral evolution of pressure fluctuations \( p_{\text{prime}}(f) \) in the shear layer for \( St_F = 0.2631, St_F = 0.1355 \) and clean nozzle case, respectively.

The excited \( St_F = 0.2631 \) case, on Figure 3.16, can be identified by a tiny mark around \( x/D = 0 \) and \( St_D = 0.2631 \). This is a byproduct of the unsteady injection and it is expected that it will modify the unsteady pressure signature downstream, if compared to the clean nozzle case. A similar tiny mark around \( x/D = 0 \) and \( St_D = 0.1355 \) can be seen on excited \( St_F = 0.1355 \) case, on Figure 3.17.

The fact that pressure ranges from 0 to \( 10^4 \) Pa on each Figure, allows the intuitive comparison by color scales. It can be observed that high pressure peaks at \( St_D = 0.39 \) presented on clean nozzle case completely vanishes if compared to \( St_F = 0.2631 \) case. Several other local pressure peaks decrease due to the unsteady injection. However, an outcome can be observed on \( St_F = 0.2631 \) case, exactly around the \( St_D = 0.2631 \) line and \( x/D = 5 \) the pressure fluctuations increase their magnitude, but still lower than \( St_D = 0.39 \) high pressure peak from clean nozzle case. This might explain any advantages that can be obtained from unsteady jet excitation.

Contrarily, the subharmonic excitation case \( St_F = 0.1355 \) does not perform very well. Even though it can decrease the high pressure peaks at \( St_D = 0.39 \), there is a significant pressure fluctuation growth around \( St_D = 0.15 \) line from \( x/D = 5 \) to \( x/D = 15 \) that achieves higher magnitude when compared to \( St_F = 0.2631 \) and clean nozzle cases.

Although this spectral evolution analysis may be clarifying, it required extensive near-field data collection at several points and could not be replicated for several excited cases.
3.4. Far-Field Acoustics

The investigation on far-field acoustics using FWH method relies on data obtained at several probes located at three different radial distances from the nozzle exit, see Figure 3.21. There is total of 181 probes per radial location from $0^\circ$ to $180^\circ$ and the radial locations are $48D$, $80D$ and $103D$, having $D$ as nozzle exit diameter.
Figure 3.18 Shear layer pressure spectral evolution for Clean Nozzle for Mach 1.8.

The OASPL directivity plot is taken at probes ranging from 20° to 150°. This avoids the capturing of "pseudo-sounds", which are pressure fluctuations liked to the hydrodynamic components, the jet plume itself. Figure 3.19 shows OASPL directivity plot for all three radial locations provided for the supersonic case and Figure 3.20 for the subsonic case.

Figure 3.19 Jet OASPL Directivity Plot for Mach 1.8.

3.4.1. Validation

The numerical scheme verification, including the acoustic prediction using FWH through Farassat 1A formulation, continues by representing the far-field spectrum
comparison between the current supersonic case against a Mach 1.8 isolated jet experimental work (Vaughn et al., 2016) and the similarity spectra for large coherent structures (Tam et al., 1996), see Figure 3.22. Both references represent data at a radial location of 40 nozzle diameters and are being compared with current simulation data at 48 diameters. Every acoustic spectrum is taken at 30° angle downstream, considered as the peak radiation angle (Tanna, 1977). The axisymmetric numerical scheme proposed on this thesis has poor agreement when compared with these references. This is due to the fact that, at supersonic speeds, asymmetric helical modes are dominant components in sound emission and this correlation increases at higher frequencies (Hashimoto et al., 2004). The red vertical line points out for the grid cut-off frequency of the current method to be analyzed.

Figure 3.21 Representation of far-field probes location in comparison to the jet.
Experimental OASPL directivity plot for a supersonic isolated jet at 40 nozzle diameters is obtained from reference work (Vaughn et al., 2016). This data is compared with the current clean nozzle supersonic case at 48 diameters, see Figure 3.23. The subsonic case presents a multi-lobe solution, having local peaks at two different directivity locations. This behavior is explained as an effect of the non-compact source distribution due to the excessively large axial extension of the instability wave structure (Golubev et al., 2002).

![Far-field Power Spectral Density](image)

*Figure 3.22 Far-field 30° Power Spectral Density Comparison at 48D for Mach 1.8.*

### 3.4.2. Far-Field Acoustic Spectra

The objective of this work is to provide a quantitative analysis of the far-field noise due to flow perturbation over an unbounded jet. The SPL spectra comparison shown on Figure 3.24 was taken at the maximum radiation angle of 30° for three different far-field locations. Dashed lines in black represent the clean nozzle case and the red arrow points for the excitation frequency mode on each spectra. Red dashed line is the grid cut-off frequency (around $St_D = 1.4$).

Although some peaks represent the excitation frequency mode, nothing suggests that
overall noise is being reduced or increased. FAA regulations mainly focus on overall noise limitations on the perceived noise, so there is no reason to provide deeper analysis on SPL spectra as, for this case, they will only function as indication that the acoustic field responds differently when a specific frequency excitation is being imposed.

According to the OASPL directivity plot for the subsonic case presented on Figure 3.20, the multi-lobe solution has peaks on 25° and 55°. The objective of this work is to provide a quantitative analysis of the far-field noise due to flow perturbation over an unbounded jet. The SPL spectra comparison shown on Figure 3.25 were taken at the local maximum radiation angle of 25° for three different far-field distances and comparisons on Figure 3.26 were taken at the local maximum radiation angle of 55° for the same three far-field distances. Dashed lines in black represent the clean nozzle case and the red arrow points for the excitation frequency mode on each spectra. Red dashed line is the grid cut-off frequency (around $St_D = 2.7$).

Although some peaks represent the excitation frequency mode, nothing suggests that overall noise is being reduced or increased. FAA regulations mainly focus on overall noise limitations on the perceived noise, so there is no reason to provide deeper analysis on SPL.
spectra as, for this case, they will only function as indication that the acoustic field responds differently when a specific frequency excitation is being imposed.

*Figure 3.24*  SPL Spectra Comparison for Mach 1.8. (a) 103D, (b) 80D and (c) 48D.

*Figure 3.25*  25° SPL Spectra Comparison for Mach 0.9. (a) 103D, (b) 80D and (c) 48D.

### 3.4.3. Overall Sound Pressure Level Directivity

A clearer picture of the microjet device as an acoustic reduction mechanism effectiveness must be provided. The outcome on noise due to the excitation is presented in
a form of $\Delta OASPL$, which is defined on Equation 43.

$$\Delta OASPL = OASPL_{CleanNozzle} - OASPL_{Excited} \quad (43)$$

Figures 3.26 to 3.29 show the OASPL directivity comparison at far-field radial location of 103, 80 and 48 nozzle diameters for the supersonic case. Dashed lines in black represent the clean nozzle case. The analysis of each of these curves are summarized on tables 3.1 to 3.3. In all three radial locations, the $St_F = 1.00$ performs the best in attenuating noise without increasing its peak at a different directivity angle. Although every excited case provided positive results in far-field acoustic reduction at clean nozzle case peak angle, $St_F = 0.125$ had the worst performance and increased the peak OASPL at another directivity angle.

Similarly to Mach 1.8 cases, a clearer picture of the microjet device as an acoustic reduction mechanism effectiveness must be provided. The outcome on noise due to the excitation is presented in a form of $\Delta OASPL$, which is defined on Equation 43.

Figures 3.30 to 3.32 show the OASPL directivity comparison at far-field radial location of 103, 80 and 48 nozzle diameters for the supersonic case. Dashed lines in black
represent the clean nozzle case. The analysis of each of these curves for the first lobe (Lobe 1) around 25° are summarized on tables 3.4 to 3.6 and for the second lobe (Lobe 2) around 55° are summarized on tables 3.7 to 3.9. Around the first lobe, in all three radial locations, the $St_F = 0.25$ performs the best in attenuating noise without increasing its peak at a different directivity angle. Although it minimizes sound at the second lobe as well, $St_F = 0.35$ performs better in this region.
Table 3.1

OASPL reduction effectiveness at 103D for Mach 1.8.

<table>
<thead>
<tr>
<th>Case</th>
<th>(\Delta OASPL) at 28°</th>
<th>Peak Angle</th>
<th>Peak OASPL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clean Nozzle</td>
<td>0.0 dB</td>
<td>28°</td>
<td>120.9 dB</td>
</tr>
<tr>
<td>Constant</td>
<td>1.8 dB</td>
<td>36°</td>
<td>120.7 dB</td>
</tr>
<tr>
<td>(St_F = 0.05)</td>
<td>1.3 dB</td>
<td>38°</td>
<td>120.5 dB</td>
</tr>
<tr>
<td>(St_F = 0.125)</td>
<td>0.0 dB</td>
<td>30°</td>
<td>121.2 dB</td>
</tr>
<tr>
<td>(St_F = 0.25)</td>
<td>1.4 dB</td>
<td>33°</td>
<td>120.3 dB</td>
</tr>
<tr>
<td>(St_F = 0.35)</td>
<td>1.4 dB</td>
<td>36°</td>
<td>121.0 dB</td>
</tr>
<tr>
<td>(St_F = 0.5)</td>
<td>1.7 dB</td>
<td>34°</td>
<td>120.7 dB</td>
</tr>
<tr>
<td>(St_F = 0.75)</td>
<td>1.8 dB</td>
<td>33°</td>
<td>121.2 dB</td>
</tr>
<tr>
<td>(St_F = 1)</td>
<td>1.9 dB</td>
<td>36°</td>
<td>120.1 dB</td>
</tr>
</tbody>
</table>

Table 3.2

OASPL reduction effectiveness at 80D for Mach 1.8.

<table>
<thead>
<tr>
<th>Case</th>
<th>(\Delta OASPL) at 26°</th>
<th>Peak Angle</th>
<th>Peak OASPL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clean Nozzle</td>
<td>0.0 dB</td>
<td>26°</td>
<td>123.4 dB</td>
</tr>
<tr>
<td>Constant</td>
<td>2.1 dB</td>
<td>35°</td>
<td>123.3 dB</td>
</tr>
<tr>
<td>(St_F = 0.05)</td>
<td>1.6 dB</td>
<td>36°</td>
<td>122.9 dB</td>
</tr>
<tr>
<td>(St_F = 0.125)</td>
<td>0.4 dB</td>
<td>30°</td>
<td>123.8 dB</td>
</tr>
<tr>
<td>(St_F = 0.25)</td>
<td>1.8 dB</td>
<td>33°</td>
<td>123.0 dB</td>
</tr>
<tr>
<td>(St_F = 0.35)</td>
<td>1.8 dB</td>
<td>34°</td>
<td>123.4 dB</td>
</tr>
<tr>
<td>(St_F = 0.5)</td>
<td>2.1 dB</td>
<td>33°</td>
<td>123.3 dB</td>
</tr>
<tr>
<td>(St_F = 0.75)</td>
<td>2.2 dB</td>
<td>33°</td>
<td>123.7 dB</td>
</tr>
<tr>
<td>(St_F = 1)</td>
<td>2.3 dB</td>
<td>34°</td>
<td>122.5 dB</td>
</tr>
</tbody>
</table>

4. Conclusion

This objective of this work is to verify the use of a Detached-Eddy Simulation (DES) model in jet noise prediction as a open-loop approach that mitigates jet mixing noise through unsteady microjet injection. The open-loop control approach implemented in the current study is based in imposing a unsteady microjet injection at a single frequency mode on the jet and analyze its effect on the far-field SPL spectra.

Two scenarios are considered, a supersonic jet around Mach 1.8 and a subsonic jet
Table 3.3

OASPL reduction effectiveness at 48D for Mach 1.8.

<table>
<thead>
<tr>
<th>Case</th>
<th>ΔOASPL at 31°</th>
<th>Peak Angle</th>
<th>Peak OASPL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clean Nozzle</td>
<td>0.0 dB</td>
<td>31°</td>
<td>128.8 dB</td>
</tr>
<tr>
<td>Constant</td>
<td>0.5 dB</td>
<td>29°</td>
<td>128.5 dB</td>
</tr>
<tr>
<td>$St_F = 0.05$</td>
<td>1.0 dB</td>
<td>26°</td>
<td>128.6 dB</td>
</tr>
<tr>
<td>$St_F = 0.125$</td>
<td>0.3 dB</td>
<td>26°</td>
<td>129.7 dB</td>
</tr>
<tr>
<td>$St_F = 0.25$</td>
<td>0.6 dB</td>
<td>28°</td>
<td>128.7 dB</td>
</tr>
<tr>
<td>$St_F = 0.35$</td>
<td>0.6 dB</td>
<td>29°</td>
<td>128.5 dB</td>
</tr>
<tr>
<td>$St_F = 0.5$</td>
<td>0.4 dB</td>
<td>29°</td>
<td>128.7 dB</td>
</tr>
<tr>
<td>$St_F = 0.75$</td>
<td>0.5 dB</td>
<td>28°</td>
<td>129.2 dB</td>
</tr>
<tr>
<td>$St_F = 1$</td>
<td>1.0 dB</td>
<td>31°</td>
<td>128.0 dB</td>
</tr>
</tbody>
</table>

**Figure 3.30** OASPL Directivity Comparison at 103D for Mach 0.9.

**Figure 3.31** OASPL Directivity Comparison at 80D for Mach 0.9.

around Mach 0.9, both axisymmetric round nozzles and fluid is injected at the nozzle lip, coaxially, in the downstream direction. For each case, nine simulation trials were performed; first is a clean nozzle, without the injector on the nozzle lip; secondly, a
Figure 3.32 OASPL Directivity Comparison at 48D for Mach 0.9.

Table 3.4

Lobe 1: OASPL reduction effectiveness at 103D for Mach 0.9.

<table>
<thead>
<tr>
<th>Case</th>
<th>ΔOASPL at 26°</th>
<th>Peak Angle</th>
<th>Peak OASPL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clean Nozzle</td>
<td>0.0 dB</td>
<td>26°</td>
<td>99.4 dB</td>
</tr>
<tr>
<td>Constant</td>
<td>1.1 dB</td>
<td>28°</td>
<td>98.5 dB</td>
</tr>
<tr>
<td>$St_F = 0.05$</td>
<td>0.8 dB</td>
<td>24°</td>
<td>98.7 dB</td>
</tr>
<tr>
<td>$St_F = 0.125$</td>
<td>0.8 dB</td>
<td>25°</td>
<td>98.8 dB</td>
</tr>
<tr>
<td>$St_F = 0.25$</td>
<td>1.8 dB</td>
<td>23°</td>
<td>98.2 dB</td>
</tr>
<tr>
<td>$St_F = 0.35$</td>
<td>1.1 dB</td>
<td>29°</td>
<td>98.8 dB</td>
</tr>
<tr>
<td>$St_F = 0.5$</td>
<td>1.3 dB</td>
<td>23°</td>
<td>98.7 dB</td>
</tr>
<tr>
<td>$St_F = 0.75$</td>
<td>1.3 dB</td>
<td>23°</td>
<td>98.4 dB</td>
</tr>
<tr>
<td>$St_F = 1$</td>
<td>1.2 dB</td>
<td>23°</td>
<td>98.7 dB</td>
</tr>
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</table>

Table 3.5

Lobe 1: OASPL reduction effectiveness at 80D for Mach 0.9.

<table>
<thead>
<tr>
<th>Case</th>
<th>ΔOASPL at 25°</th>
<th>Peak Angle</th>
<th>Peak OASPL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clean Nozzle</td>
<td>0.0 dB</td>
<td>25°</td>
<td>102.1 dB</td>
</tr>
<tr>
<td>Constant</td>
<td>1.1 dB</td>
<td>26°</td>
<td>101.1 dB</td>
</tr>
<tr>
<td>$St_F = 0.05$</td>
<td>0.8 dB</td>
<td>24°</td>
<td>101.4 dB</td>
</tr>
<tr>
<td>$St_F = 0.125$</td>
<td>1.0 dB</td>
<td>23°</td>
<td>101.5 dB</td>
</tr>
<tr>
<td>$St_F = 0.25$</td>
<td>2.0 dB</td>
<td>22°</td>
<td>100.8 dB</td>
</tr>
<tr>
<td>$St_F = 0.35$</td>
<td>1.1 dB</td>
<td>27°</td>
<td>101.5 dB</td>
</tr>
<tr>
<td>$St_F = 0.5$</td>
<td>1.4 dB</td>
<td>22°</td>
<td>101.3 dB</td>
</tr>
<tr>
<td>$St_F = 0.75$</td>
<td>1.2 dB</td>
<td>22°</td>
<td>101.0 dB</td>
</tr>
<tr>
<td>$St_F = 1$</td>
<td>1.2 dB</td>
<td>22°</td>
<td>101.4 dB</td>
</tr>
</tbody>
</table>
constant injection of 1% MFR; and finally, seven different forcing frequencies ranging from $St_F = 0.05$ to $St_F = 1.00$ injection of 1% MFR. In addition, an extensive comparison was built focused on performing a qualitative comparison between current detached-eddy simulation (DES) and previous work using linear stability theory (LST) and linear Euler Equations (LEE). Finally, two special cases of $St_F = 0.2631$ and $St_F = 0.1355$ excitation modes were simulated and analyzed using the shear layer pressure spectral evolution plot. The main concluding remarks, and objectives of each case are summarized here.
Table 3.8

Lobe 2: OASPL reduction effectiveness at 80D for Mach 0.9.

<table>
<thead>
<tr>
<th>Case</th>
<th>ΔOASPL at 54°</th>
<th>Peak Angle</th>
<th>Peak OASPL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clean Nozzle</td>
<td>0.0 dB</td>
<td>54°</td>
<td>101.1 dB</td>
</tr>
<tr>
<td>Constant</td>
<td>−0.2 dB</td>
<td>54°</td>
<td>101.3 dB</td>
</tr>
<tr>
<td>$St_F = 0.05$</td>
<td>0.8 dB</td>
<td>53°</td>
<td>100.4 dB</td>
</tr>
<tr>
<td>$St_F = 0.125$</td>
<td>−0.2 dB</td>
<td>52°</td>
<td>101.5 dB</td>
</tr>
<tr>
<td>$St_F = 0.25$</td>
<td>0.6 dB</td>
<td>55°</td>
<td>100.5 dB</td>
</tr>
<tr>
<td>$St_F = 0.35$</td>
<td>1.3 dB</td>
<td>52°</td>
<td>100.4 dB</td>
</tr>
<tr>
<td>$St_F = 0.5$</td>
<td>0.1 dB</td>
<td>53°</td>
<td>101.1 dB</td>
</tr>
<tr>
<td>$St_F = 0.75$</td>
<td>−0.1 dB</td>
<td>54°</td>
<td>101.2 dB</td>
</tr>
<tr>
<td>$St_F = 1$</td>
<td>0.3 dB</td>
<td>57°</td>
<td>101.1 dB</td>
</tr>
</tbody>
</table>

Table 3.9

Lobe 2: OASPL reduction effectiveness at 48D for Mach 0.9.

<table>
<thead>
<tr>
<th>Case</th>
<th>ΔOASPL at 55°</th>
<th>Peak Angle</th>
<th>Peak OASPL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clean Nozzle</td>
<td>0.0 dB</td>
<td>55°</td>
<td>105.8 dB</td>
</tr>
<tr>
<td>Constant</td>
<td>0.3 dB</td>
<td>53°</td>
<td>105.8 dB</td>
</tr>
<tr>
<td>$St_F = 0.05$</td>
<td>1.1 dB</td>
<td>52°</td>
<td>104.9 dB</td>
</tr>
<tr>
<td>$St_F = 0.125$</td>
<td>0.3 dB</td>
<td>52°</td>
<td>105.8 dB</td>
</tr>
<tr>
<td>$St_F = 0.25$</td>
<td>0.9 dB</td>
<td>55°</td>
<td>104.9 dB</td>
</tr>
<tr>
<td>$St_F = 0.35$</td>
<td>1.3 dB</td>
<td>50°</td>
<td>104.6 dB</td>
</tr>
<tr>
<td>$St_F = 0.5$</td>
<td>0.5 dB</td>
<td>52°</td>
<td>105.7 dB</td>
</tr>
<tr>
<td>$St_F = 0.75$</td>
<td>0.6 dB</td>
<td>53°</td>
<td>105.3 dB</td>
</tr>
<tr>
<td>$St_F = 1$</td>
<td>0.4 dB</td>
<td>55°</td>
<td>105.5 dB</td>
</tr>
</tbody>
</table>

4.1. Remarks

According to the grid resolution adopted, the numerical scheme could resolve frequencies up to $St_D = 1.4$ for supersonic case and up to $St_D = 2.4$ for subsonic case. This is enough to show consistent noise mitigation for every case. It is likely that very low excitation frequency modes as $St_F = 0.05$ and $St_F = 0.125$ remain unresolved depending on the minimum $St_D$ that both grid resolution and numerical scheme allows, but the purpose of this work is to decrease SPL at the peak frequency, this is crucial to minimize
OASPL, so there is no reason to improve grid resolution.

The verification of the mean flow quantities for the Mach 1.8 case was successful as current simulation results agreed with available experimental data, but the Mach 0.9 case showed shock formations that might affect far-field noise measurement.

The comparison between previous LEE results (Golubev et al., 2003b) and the current DES simulation has proven to be in agreement with investigation from early 2000’s (Dahl & Mankbadi, 2002), as they indicate that nonlinear effects, present on DES simulations as well, will shift the pressure fluctuation peak towards the nozzle exit and restrain the amplitude if compared to the linear response obtained from LEE or LST codes. However, LEE results were obtained using helical mode excitation and slightly different excitation frequency, so current DES simulation must run on a 3D grid, then match frequency and peak amplitude in order provide better comparison against previous LEE results.

The verification of the supersonic case was done by comparing power spectral density against available experimental data (Vaughn et al., 2016) and similarity spectra (Tam et al., 1996). The peak frequency for the supersonic case is found to be around \( St_D = 0.25 \). Although the agreement was not clear, it is important to understand that helical modes are dominant in sound propagation for supersonic jets; this could explain why current numerical results seems lower than experimental data in comparison. Furthermore, a directivity plot shows evidence that the current numerical clean nozzle case predicts lower sound level if compared to experiments (Vaughn et al., 2016) and that this attenuation is due to helical modes that may not be present on axisymmetric simulation. On the subsonic case, the multi-lobe directivity plot could be related to the effect of the non-compact source distribution due to the excessively large axial extension of the instability wave structure, which means that a larger Fowcs-William Hawkings surface is needed to better enclose near-field nonlinearities.

Regarding flow field analysis on supersonic cases, the \( St_F = 1.00 \) seemed promising
in minimizing turbulence and noise, while $St_F = 0.125$ was the worst case, similarly for the directivity plot analysis. When unsteady 1% MFR at $St_F = 1.00$ is injected, results show far-field noise reduce around 1-2 dB when compared to clean nozzle peak OASPL directivity angle and 0.8 dB reduction when comparing OASPL at respective peak directivity angle of each case.

For the subsonic case, the flow field analysis has shown that the $St_F = 0.05$ case carried out as the best scenario in minimizing turbulence and noise. Now considering far-field results, on the first lobe, around $25^\circ$, when unsteady 1% MFR at $St_F = 0.25$ is injected, results show far-field noise reduce around 1-2 dB when compared to clean nozzle peak OASPL directivity angle and 1.3 dB reduction when comparing OASPL at respective peak directivity angle of each case. On the second lobe, around $55^\circ$, $St_F = 0.25$ case maintains beneficial results reducing around 1 dB in overall; the case of $St_F = 0.35$ also presents good results in minimizing sound on the second lobe.

In overall, the active noise cancellation technique that was investigated through numerical simulations in this work did not imply a strict relation between forcing frequencies and far-field noise suppression. Similarly to current work, microjet injection study in supersonic jet concluded that frequency spectrum and noise levels are sensitive to the injection frequency and physical characteristics (Hafsteinsson et al., 2012). Moreover, they observed that noise reduction of a steady-state injection is more robust than a pulsating injection due to its simplicity, but similar results. Finally, the jet response is still very hard to understand as it can be distinct under frequency excitations below or above the jet column mode (Samimy et al., 2018). Consequently, it is hard to accurately determine for any arbitrary configuration if far-field noise will be suppressed without comprehensive investigation.
4.2. Recommendations for Future Work

Further investigations considering the helical modes by performing 3D simulations and extensive analysis on spectral evolution at different forcing frequencies must be done to enhance comparison against previous LEE results and provide accurate estimates on far-field noise mitigation. Results shown in this work are promising, but it is hard to connect flow-field interactions with far-field noise effects. Also, a larger FWH surface and finer grid should lead to a more reliable result for Mach 0.9 case.

The current work paves the way to modelling an active, robust, nonlinear closed-loop control system targeting reduction of broadband supersonic jet noise in aircraft engines. Such Closed-Loop Active Suppression Technology (CAST) design could be based on previous analyses of the reduced-order models for jet noise predictions (Dahl et al., 2003; Dahl & Mankbadi, 2002; Golubev et al., 2003b) and recent studies in robust, nonlinear control designs applied to flutter (Ramos Pedroza et al., 2017), UAV tracking control (Golubev & MacKunis, 2014; Kazarin et al., 2016). Finally, will incorporate both low-fidelity (reduced-order models) and high-fidelity (LES) tools to demonstrate the effectiveness of the proposed technique.
REFERENCES


