

Fall 12-2021

Satellite Deceleration Using Gravity Assist from Asteroid

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SATELLITE DECELERATION USING GRAVITY
ASSIST FROM ASTEROIDS

By

Nicolas Prulhiere

A Thesis Submitted to the Faculty of Embry-Riddle Aeronautical University
in Partial Fulfillment of the Requirements for the Degree of
Master of Science in Aerospace Engineering

December 2021

Embry-Riddle Aeronautical University

Daytona Beach, Florida

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ASSIST FROM ASTEROIDS

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ACKNOWLEDGEMENTS

I would first like to thank my thesis advisor Dr. Troy Henderson, professor in the Aerospace Engineering department. He was the first to introduce me to orbital mechanics during my undergraduate degree and allowed me to pursue my goal and complete this research. I am very grateful for the committee members support and for having them push me to do more in this thesis.

I would also like to thank all of my family member for their support and help during this entire process. Having them supporting me during good and hard times, made this project possible and more enjoyable. I am especial grateful for my grandpa's support and for sharing his passion for engineering and space with me, I would have loved for him to read this thesis.

I would also like to thank my friends, Nikita Amberkar and Ben Malczewski, for supporting me and making campus fun and enjoyable during the pandemic. For sharing with me the ups and downs that is a thesis paper, and for being honest and caring.

ABSTRACT

Orbital maneuvers can be used to increase the speed of a satellite during its mission but it is also possible to use the same concept to slow it down. Most maneuvers are done around moons and planets that will allow the maneuver to be performed at a safe distance from the surface of the body. This thesis looks at using an asteroid, that can be found in the asteroid belt, to perform those maneuvers and slow down a satellite. The maneuvers used in this simulation are the reverse slingshot and a flyby maneuver where the asteroid and satellite are moving in the opposite direction. Using two different maneuvers as well as asteroids of different sizes moving at a different speed, provides a range of combinations and can find the minimum or maximum condition for the maneuver to be performed. The goal is to be able to control the satellite's final velocity without having to complete any burn or impulse maneuver. The objective can also be reached by performing the maneuver multiple times, since the exact number in the asteroid belt is constantly changing, using more than one for a single mission wouldn't be impossible. The asteroid found in the asteroid belt still holds many mysteries and or is rich in resources, making them the target of many researchers and missions, and finding yet another use for them would make them that much more valuable. As asteroids are clustered together, finding just the perfect asteroid for a mission may not always be the easiest thing, as such having a satellite in the asteroid belt collecting data at the same time as completing maneuver can prove to be much more beneficial than such completing the maneuver to just reduce fuel cost. Even if both maneuvers are not physically possible, they can still be implemented in other missions.

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NOMENCLATURE

e	Eccentricity
ρ	Density
F	Force
G	Gravitational constant
r	radius (distance between two objects)
R	Radius from center of mass to outer shell
v_{esc}	Escape velocity
m	Mass of satellite or smallest object
M	Mass of asteroid or larger body
Δv	Change in velocity
V	Volume
θ	Angle of approach
\emptyset	Deflection angle in the asteroid frame

ABBREVIATIONS

LEO	Low Earth Orbit
MEO	Medium Earth Orbit
HEO	High Earth Orbit
GSE	Geosynchronous Orbit
GEO	Geostationary Orbit

1. Introduction

Space exploration is continually moving forward to discover something new and innovations need to be made to make these discoveries possible. Innovations in satellites, capsules, rovers, and even rocket engines are made every year to reach and study our solar system. With these innovations, the math and science used to plan and execute missions are also constantly changing. New algorithm and maneuvers become possible because the technology allows it but also because no two missions are identical and as such new algorithms and flight paths need to be created.

1.1. Gravity Assist

The discoveries made in space change and improve our lives on earth and get us closer to walking among the stars in the hope of living on a different planet or even discovering life. To make these discoveries satellites and astronauts are sent to space with specific missions in mind, and no two missions are the same. This constant need to send the object in space for a mission, or even just to have a new satellite in place, is very expensive since leaving earth is no easy task. Although it has been done multiple times and will be done a lot more, the cost of sending just one pound of equipment in space is 10 000 dollars (Beon & Dunbar, 2008). That value has constantly been changing, and innovation by private companies has been able to bring that number down to about \$2000/kg (Jones, 2018), but this is only possible as long as the weight of the satellite is small enough for the rocket. If the payload is too large then multiple launches will be required and the cost-effectiveness will go down, this is how the ISS was sent in space.

One way to save money or make the payload include more equipment without raising the price is to decrease the amount of fuel required. Fuel is used to adjust the satellite's

course or to complete maneuvers to reach a certain goal. New engine thrusters are being designed to reduce fuel consumption, or to use different energy sources, all to reduce fuel quantity while also improving effectiveness if possible. One way to save fuel is to complete an orbital maneuver to gain speed without using thruster, this is collected by the orbital slingshot maneuver. This maneuver has been used on many occasions such as for Voyager 1 and 2, Galileo, and Cassini (NASA, 2018), they used the sun, Earth, Saturn, and Venus to increase their speed and reach their destination. This is one way to reduce fuel consumption but the same thing can also be done in reverse to slow down a satellite, this would be a reverse slingshot maneuver.

1.2. Similar Applications

The asteroid belt present in our solar system has a huge variety of different asteroids, from size, shape, and even density and composition. Some of the asteroids can even be a source of metal and minerals that can be worth in the thousands of quadrillion dollars (Whitt, 2021). This makes going to an asteroid much more profitable and as such a target for a future mission, however, even if some of these asteroids are worth a fortune, identifying which one is also a complicated task. Asteroids of irregular shape and size will have an irregular gravitational field, this will lead to difficult orbits and also difficulties in landing. There are a few large asteroids in our asteroid belt that have started to be more spherical, such as Ceres and Vesta, but the majority are still different from one another.

Asteroids are also a threat to earth as they can enter the earth's gravitational force and if they don't burn up upon re-entry, then the impact could be devastating. Weapons may be used to prevent this from happening but one of the most effective defenses that have

been theorized is the use of asteroids to defend earth. If an asteroid can be captured and kept in orbit around the earth, it can be used to deflect another asteroid. This has been theorized to get an asteroid in the earth-sun system, which means that the asteroid wouldn't need to orbit around the earth (Tan et al., 2021). Some have even gone as far as to try and bring the asteroid into the Earth-Moon system (Tan et al., 2017), but with the mass of the asteroid being so massive, no thruster on earth could provide the energy required to slow it down. To remedy this problem, the use of gravity assists from the moon, and other large body, to provide sufficient deceleration to keep an asteroid in earth's orbit (Bao et al., 2015).

1.3. Satellite Deceleration with Maneuver Around Asteroid

Gravitational assist has been proven to work to increase the target speed or to slow it down in certain conditions. The goal of this paper is to broaden the possibilities to use gravitational assist with an asteroid to slow down a satellite. The maneuver is mostly done with large celestial bodies to ignore the mass difference and because they also have greater speeds allowing for a larger change in velocity. Asteroids are more common in our solar system and have a wide range of masses, they are also located between the earth and some of the planets that are farther out in our system.

The objective is to find if an orbital maneuver around an asteroid can provide a change in velocity that is significant and that is physically possible. If such maneuver can be done around an asteroid, then, this would offer a wide range of possibilities of flight path and maneuvers that satellites can use. This would also decrease the fuel required for any mission in or going through the asteroid field. To increase the possibilities, two different maneuver trajectories will be used to see what are their advantages and

disadvantages.

The first trajectory is the reverse slingshot maneuver that is commonly used, this is expected to be the most effective maneuver to slow down a satellite. The second maneuver is similar to a flyby, with the asteroid and satellite moving in the opposite direction. Using these two maneuvers to change the satellite's speed to the desired value, by varying the size of the asteroid and the angle of approach of the maneuver. By varying the size of the asteroid, if the maneuver has a maximum or minimum for which asteroid can be used, then that limit can be found and the asteroid that can be used would be able to be identified in the asteroid belt.

The simulation will also figure out different combinations of the two maneuvers that can be used to change the asteroid's velocity by a designated amount. This would be similar to performing multiple maneuvers with different asteroids while going through the asteroid belt. This could be for a mission in the asteroid belt or when a satellite is on the way back and need to slow down before reaching the earth. If an asteroid can be used to change the velocity of a satellite during their mission, even if this may increase the duration of the mission, this would save space and weight and as such would reduce the cost of sending equipment in space for some missions.

2. Literary Review

There is still a lot to discover about asteroids and how they come to exist, but a lot has also been discovered from pictures and observations astronomers have made, to meteor fragments that have entered the earth's atmosphere. On the other hand, we have become very proficient at sending satellites in orbit or to other planets. Satellites are now used for a range of different applications, from communication to weather recording and also military uses. Each satellite is designed uniquely for one mission, this can be long-term communication or to complete a fly-by of a planet in the system. Satellites in earth's orbit may be similar or identical, if their missions are the same, but the ones used for exploration often have the latest technological advances.

2.1. Asteroids

Asteroids are celestial bodies that are smaller than planet embryos. Most of the time, a mass that isn't orbiting another planet, will be called an asteroid. This means that a planet embryo size asteroid will still be viewed and considered as an asteroid for calculations or observations. Meteorite is a term used to describe an asteroid when it enters the atmosphere or hits the surface of a planet. All asteroids are formed from the cosmic dust that floats and orbits solar systems (O'Brien & Sykes, 2011) at the beginning. It is possible for asteroids to be ejected from their solar system because of the gravitational influence of another body in that system. As more dust accumulates, the asteroid starts to form and will create a greater force of attraction that will gather more dust, as shown in Figure 2.1. The asteroid will continue to grow as more dust and smaller asteroids are pulled by the gravitational force (Perez & Nava, 2017). Sometimes asteroids will collide and break apart instead of combining. This will happen because not all asteroids have the

same composition, or because of their speed. Two or more asteroids can have velocities in opposite direction and the energy of the collision can be dissipated by the fragmentation of the two bodies (Entertainment Close-up, 2011).

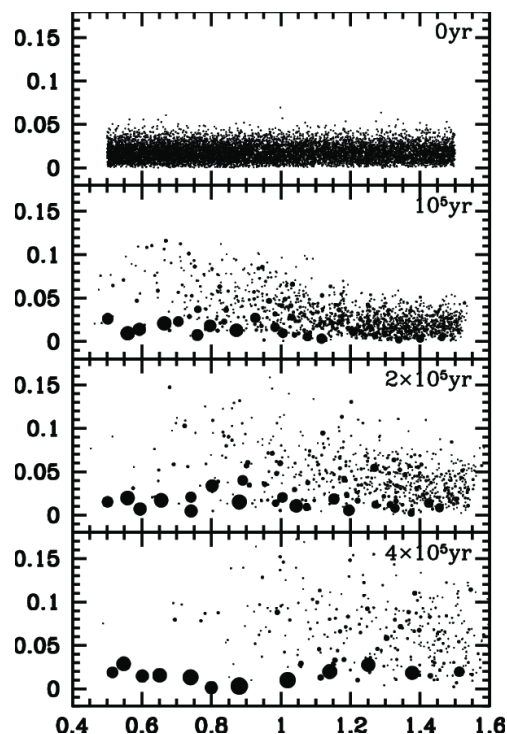


Figure 2.1 Evolution of dust particle to asteroids (Crovisier & Fulchignoni, 2021).

As the size of the asteroid increases, the amount of material required for it to continue to grow will increase at an exponential rate (O'brien & Sykes, 2011). This means that asteroids will reach a growth limit if all they have as material is cosmic dust. The most efficient way for an asteroid to grow is to collide with smaller asteroids and consume them in the process (e.g., Greenberg et al., 1978; Wetherill & Stewart, 1989; Kokubo & Ida, 1996; Weidenschilling et al., 1997). This will also mean that asteroids, which are born in an asteroid belt, will have more chances to grow into planet embryos, and possibly planets later, than asteroids that are born by themselves between stars or planets

(O'Brien & Sykes, 2011). This process can take approximately 5 Myr for the asteroid to become a planet embryos (Kleine et al., 2005). An asteroid can also reach this size much quicker if more asteroids are present and no gas giants or planets have gravitational influence over in the asteroid belt.

As asteroids grow, the gravitational force they exert will increase and make them more spherical. That means that asteroids will start with irregular shapes and masses (Erickson, 2021). Two asteroids will never be the same and will always have different characteristics. The gravitational field that they generate will also be irregular. All asteroids accumulate cosmic dust to form, but this dust is full of different elements. When observing an asteroid, it is only possible to determine its composition by observing its surface. To get more accurate information regarding the internal structure, a mission where a sample is collected and brought back to be analyzed needs to be completed.

The asteroid belt of our solar system has five major types of asteroids. The first is the E-type that is mostly in the inner belt, the asteroids closest to the sun (Vanderbilt). They have a composition close to chondrites or small mineral granules. Then, there is the S-type and M-type, located in the center of the belt. They will be mostly composed of stone, iron, or metals. The V-type is found around Vesta, one of the largest asteroids in the belt (Britt et al., 2003). They are made of basalt, similar to volcanic rock. The asteroids found mostly in the outer belt are C-type; they will be more carbon rich. C-type asteroids can contain hydrates, because of their distance from the sun (Vanderbilt). There are other types of classification such as O, K, T and more, but those are used to classify asteroids that have been more extensively studied, and where more information and data has been collected from observations or sample analyzes, to be able to make more

accurate structural estimation (Carry, 2012). See Figure 2.2 for different type of asteroid and how they are related.

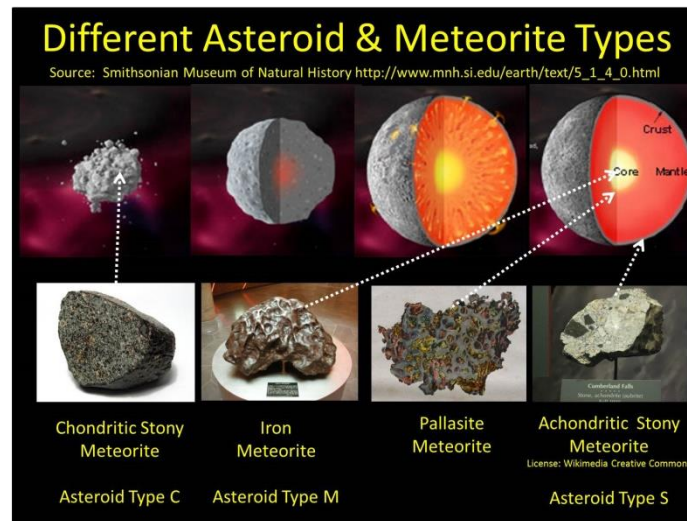


Figure 2.2 C type asteroids to S type (Mertzge, 2013).

With so many unknown variables, the density of an asteroid is given as the bulk density, which can help determine the type of the asteroid (Carry, 2012). As such, C-type will have a density of 1.4 g/cm^3 , S-type of 2.69 g/cm^3 and M-type of 4.7 g/cm^3 (Britt et al., 2003). A more accurate value of the bulk density can be calculated based on the elements that are present, and the percentage at which they are present (Chambers, & Wetherill, 2000). This can be used to find the mass of asteroid coming into the solar systems and find out if a mission to that asteroid is feasible and if valuable information, such as the presence of ice, can be collected and brought back. The presence of ice in asteroids would make long distance space exploration much easier since some resources could be collected during transit. Water is a critical element for the survival of humans in space and can be used for propellant. If most of the important material required to live in

space can be found and mined in asteroids, then they could become pit stops in space. Asteroids are also much better to visit because they have a smaller sphere of influence. The amount of energy needed for a spacecraft to leave a planet's influences increases with the mass of the body. Ceres is the largest asteroid in the asteroid belt and its mass is only 1.3% that of the moon (Carry, 2012). With such difference in mass between the asteroid and other bodies in the solar system, it would be possible to make multiple stops at different asteroids, and use the same amount of energy that would be used to make a single stop on a planet.

2.2. Asteroid Belt

The main asteroid belt of our system is located between 2.1 au and 3.3 au, between Mars and Jupiter (Levison et al., 2009), see Figure 2.3. The majority of meteoroids that come into our atmosphere or hit the moon come from this belt. When asteroids in the belt collide with one another some smaller fragments are expelled in all directions, some in the direction of the earth. Thankfully the meteoroids are small and will break apart or disintegrate when entering the atmosphere. The current asteroid belt only has about 1% of its initial mass left (Chambers et al, 2010). The belt has been present since before the creation of all of the planets in our system. During the beginning of the formation of the solar system, many planetary embryos formed and disappeared in the belt (O'brien et al., 2011). As the gas giant, Jupiter formed, more asteroids were influenced by its gravitational presence and would have their orbit altered. These changes in orbit would make the asteroid leave the solar system or they would fall in the sun and be consumed (Walsh et al., 2011). Material of the asteroid belt is also lost during the growth of larger asteroids. During a collision, some amount of material from both asteroids will be lost

(4). All of the mass lost from these different processes cannot be recuperated and as such the belt will only have its total mass decrease over time.

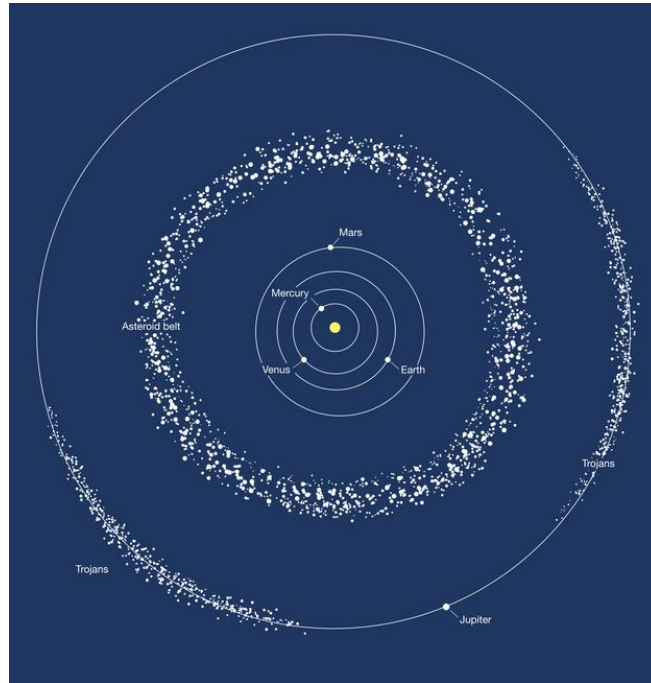


Figure 2.3 Main asteroid belt and outer belt (Wittke, 2020).

Even if the current asteroid belt is only a small percentage of what it used to be, there are still more than one million asteroids of one kilometer or more in diameter, in the belt (4). There are even smaller asteroids, with a diameter as small as a few centimeters, which numbers are still undetermined. Over the years more asteroids are categorized and given a name but there are only about five thousand of them, which have made it to this point (4). Observing asteroid takes time, and resources, which yield very little result that, can be beneficial to mankind. Optical tests and data collection need to be done using powerful telescopes and satellites in a region with little light pollution, also the time frame during which this experiment, see Figure 2.4, can be conducted on an asteroid is

very short and the next chance may only appear years later. Some asteroids can't be observed from earth because they are hidden by the sun or another celestial body, in these situations, satellites need to be sent to complete the observation. The creation and evolution of the asteroid belt have been a fascination for many scientists and observers in their endeavor to understand the creation of our solar system (Walsh et al., 2012). The goal is to understand the reason how the solar system came to its present state, to be able to predict what will happen in the years to come. Different models have been used to explain certain parts of the asteroid belt, such as what is the contributing factor in the depletion of material in the asteroid belt (Chambers et al., 2010). Another model will look at how planetary embryos come to be and what is required for them to survive and become planets (Walsh et al., 2012).

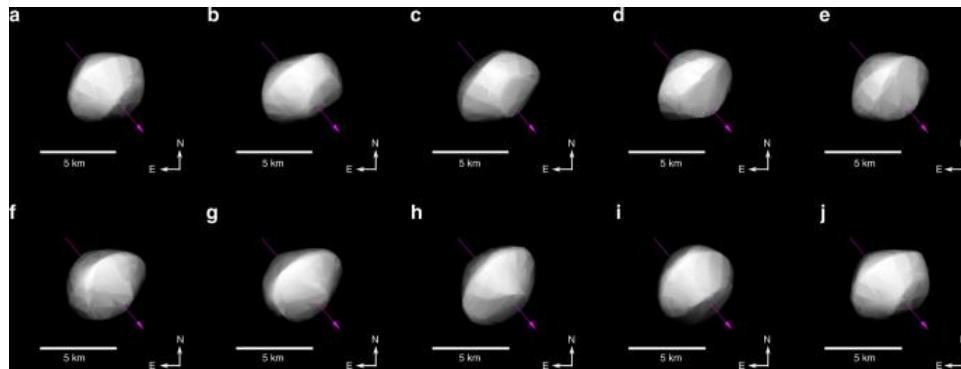


Figure 2.4 Observation of Phaethon (Takir et al., 2020).

The two largest asteroids currently in the belt in the solar system are Ceres and Vesta, see Figure 2.5. As the largest asteroid, Ceres has a diameter of about 1000 km (Drummond et al., 2017). In comparison, the diameter of the earth's moon is 3476.2 km, with such a great difference in size the difference in volume and mass will be

exponentially greater (Williams, 2020). The slingshot orbital maneuver is mostly used with large planets or stars since they will provide a greater velocity increase. Vesta is the second-largest asteroid in the belt with a diameter of about 525 km, making it smaller in mass by a factor of almost 8 (Burbine et al., 2020). As the two largest body in the belt, they are unable to get near each other without influencing one another and sending them self out of orbit, see Figure 2.6.

The asteroid present in the belt will only continue to decrease in size but will increase in number. With the variation in size, density, velocity, and position in the belt, it is much easier to find the perfect asteroid needed for a specific type of mission. Even an asteroid such as Ceres, which is close to being a planet embryo or moon, is still classified as a C-type asteroid (Rivkin et al., 2011). Ceres also has a very spherical appearance and gravitational field and has been observed to have multiple craters on its surface from other asteroids (Burbine et al., 2020). In comparison, Vesta is still irregular in shape but possesses its own asteroid family (O'brien et al., 2011). An asteroid family is a group of asteroids that will originate from a larger asteroid and will remain with orbital properties similar to one another (Burbine et al., 2020). The family around Vesta can help it grow by protecting it from other asteroids or be assimilated into Vesta. The asteroid family can be very useful for the mission since multiple asteroids with similar characteristics will be present within a certain region.

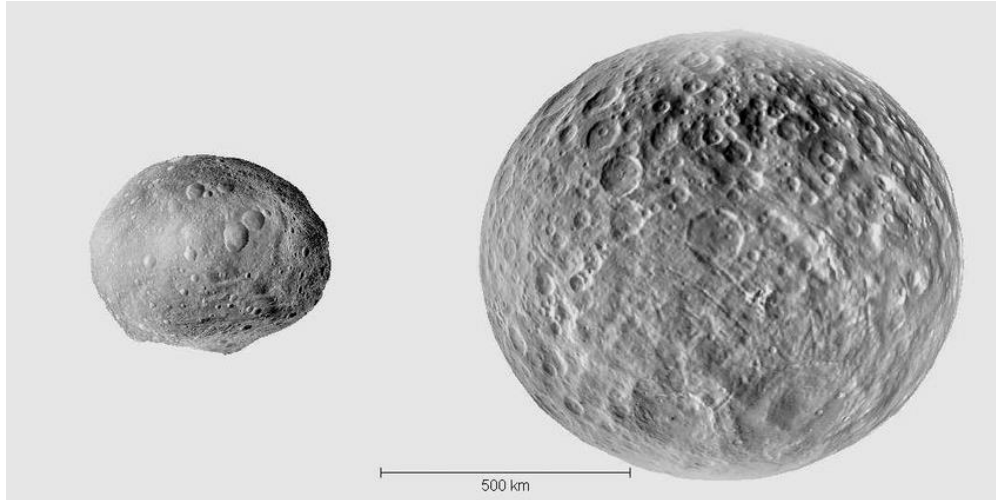


Figure 2.5 Vesta (left) and Ceres (right) size difference (Soumbatov-Gur, 2019).

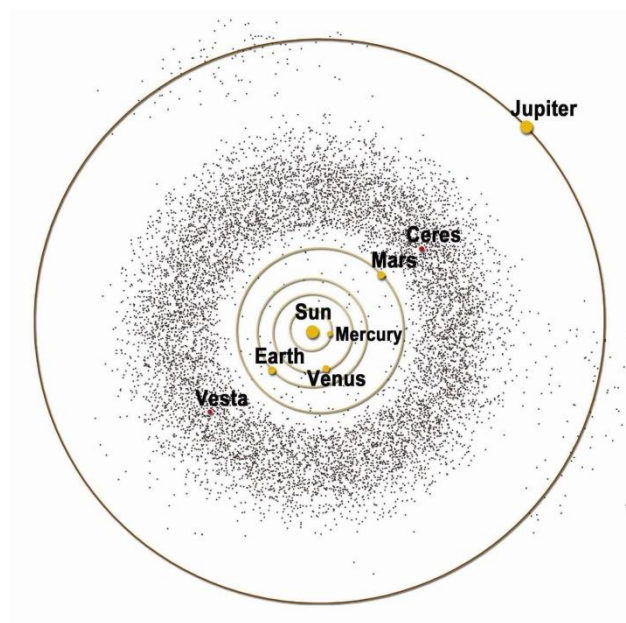


Figure 2.6 Location of Ceres and Vesta in asteroid belt (Administrator, 2020).

2.3. Sphere of Influence

Every element with a mass will have its gravitational force. This means that everything attracts one another at all times. A small object such as a coin will exert a force on the earth or moon. The force is based on the mass of the two objects and the

distance between them (Martinez-Sanchez, 2012).

$$F = G \frac{m_1 m_2}{r^2} \quad [2.1]$$

In Equation (2.1) G is the gravitational constant, m is the mass of the object, and r is the distance between the two centers of masses. The greater the distance between the objects the weaker the force will be. As the attraction force between two objects decreases, the force from another object can become the dominant force. This means that at all time the sun and the moon exert a gravitational force on everyone and everything. The reason why these forces don't have an impact on planes or other objects on earth is that the distance between the moon or sun and the objects on earth is so large that the force that the earth exerts on that object is greater (Mukhopadhyay, 2002). The other forces are so small in comparison to the ones from the earth, that they can be ignored during calculation. All masses will have an area in which they will be the dominant force, this will be their sphere of influence. This area is very small for objects on earth but has become more influential in space. Once an object leaves the sphere of influence of the planet it was on, it is then under the influence of the sun (Figure 2.7).

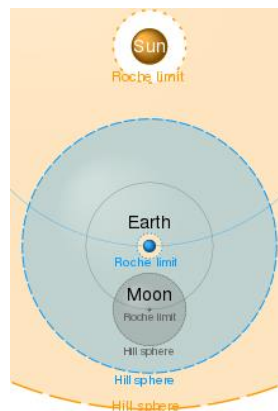


Figure 2.7 Sphere of influence (hill sphere) of earth, moon, and sun with rocket limits (n.d, 2021).

To know the size of the sphere of influence of a planet or object in space will help determine its trajectory. The influence of planets and moon has been used to correct trajectories in missions (Zaslavskii et al., 2017). The sphere of influence is the gravitational field of the object. This field is dependent on the shape and the more irregular the mass distribution is the more irregular the gravitational forces (Burov et al., 2018). Different methods are used to estimate the gravitational field such as using the spherical harmonic model (Hao et al., 2020). Another useful method is to use point mass or assumption that the object is spherical and as such the sphere of influence is constant. As the size of the object increases the gravitational force it generates will make it more spherical (Williams, 2019). This is the reason all planets and the moon in the solar system are spherical and have an axis of rotation. Even uneven masses, such as Vesta, will have two poles and an axis of rotation (Greicius, 2007), see Figure 2.8.

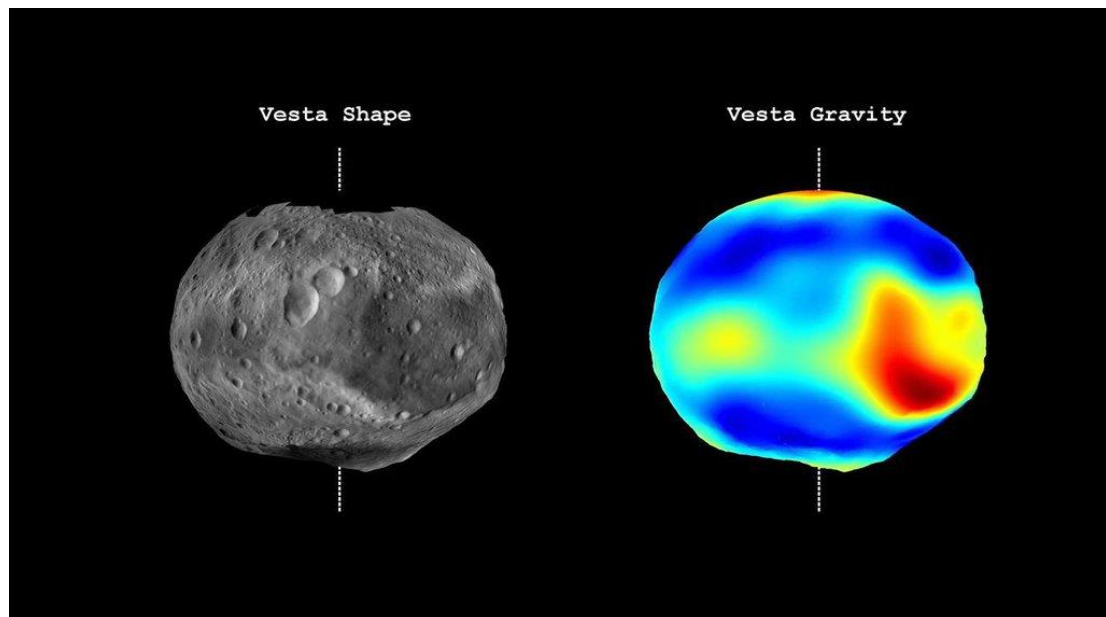


Figure 2.8 Vesta shape and its gravitational field (NASA, 2015).

2.4. Escape Velocity

The gravitational field makes leaving that object's influence harder. The greater the mass is, the stronger the force will be, and a higher escape velocity will be needed. The escape velocity doesn't depend on the size of the object, it depends on the mass of the object in which the sphere of influence is dominating (Vasiliev & Fedorov, 2014), as shown in Equation (2.2).

$$v_{esc} = \sqrt{\frac{2mG}{R}} \quad [2.2]$$

The gravitation constant is the same as from Equation (2.1), and R is the distance between the center of mass and the outer shell. This is proven with the escape velocity on earth being 11.2 km/s and the moon is 2.4 km/s (Encyclopaedia, 2017). This escape velocity is the minimum speed required to escape any gravitational field but for some planets, such as earth, other requirements are necessary to escape its gravitational field. When a planet creates an atmosphere or is a gas giant than the object will also need to overcome the drag force. If the radius from the surface to the crust and the radius of the sphere of influence are close, then the escape velocity will be very small. On the other hand, the greater the difference between the sphere of influence and the radius of the masse will make the velocity required increase exponentially. The size of the object trying to leave to planets influence doesn't impact the escape velocity but the shape and weight will impact the amount of energy needed to allow the object to reach the velocity required.

2.5. Orbital Mechanics

Orbital mechanics is related to anything moving in space. When analyzing the path of an object in space and predicting its trajectory and future behavior, then this is orbital

mechanics. Everything in the universe is orbiting around something; this can be a planet, a star, or even a black hole. This means that the earth is rotating around the sun, just like the moon is rotating around the earth, but at the same time, the sun is rotating inside our galaxy, the Milky Way (Helmi, 2020). There are other galaxies that also have solar systems. Even if everything is rotating and in constant motion, an object will be predominantly influenced by the mass in which they enter the sphere of influence. Thanks to Newton's first law, we know that once an object is in motion it will stay in motion unless another force acts upon it (Hecht, 2015). In space, the only force present is the dominant gravitational force and if the object had any initial momentum in any direction, then the force of attraction and the momentum will make the object enter in an orbit around the point of attraction.

When an object enters a stable orbit around an object it will have one of two shapes: circular or elliptical. The most common stable orbit is elliptical; having an orbit to be a perfect circle naturally is almost impossible and with the many variables to control getting an object in a circular orbit is near impossible. It is possible to have an elliptical orbit that will be almost a circular orbit, especially in space where one or two kilometers is very small compared to the scale of planets and stars. The orbit of an object that enters the sphere of influence of another body but has enough energy to escape, without completing a full revolution, will have a parabolic or hyperbolic trajectory. Most of these trajectories will be considered hyperbolic since a parabolic trajectory is a special type of hyperbolic trajectory. It is also possible to predict if the path of an object, which will be pulled by the mass' gravitational force, hits the surface of the mass. This system will be considered unstable and needs to be avoided. The path of an object in motion will be

expressed based on the eccentricity e of the system. Table 2.1 shows the orbital path that corresponds with eccentricity value.

Table 2.1

Eccentricity value with type of trajectory.

$e = 0$	Circular
$0 < e < 1$	Elliptical
$e = 1$	Parabolic
$e > 1$	Hyperbolic

The orbit also has two important points: perigee and apogee. Perigee is the point that is closest to the mass the object is orbiting around. It is also the point where the magnitude of the velocity vector is the highest. Apogee is the point that is the farthest and where the velocity is the smallest (Curtis, 2020). An ellipse also has two focus points, one would be where the earth is and the other is symmetric from the center of the ellipse, see Figure 2.9.

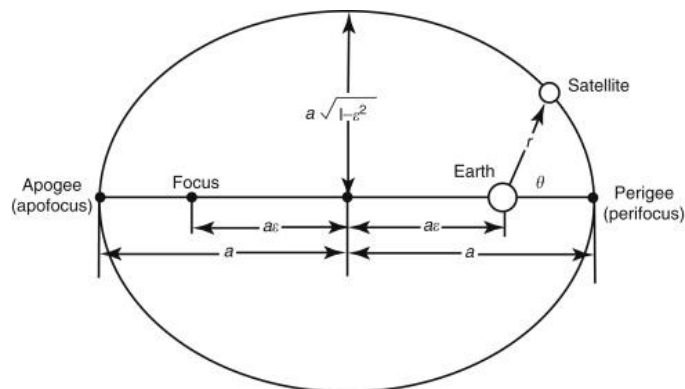


Figure 2.9 Elliptical orbit diagram (Peterson, 2003).

Any object in an elliptical or circular orbit will stay in that orbit until another force is acted upon; this is the case for every satellite in orbit as well as planets and moons. Since the gravitational force is dependent on the mass of the two objects, the radius of the orbit is different. The moon has a much larger orbital radius than smaller satellites. One of the most used and interesting orbits around the earth is Low Earth Orbit (LEO), it is also the easiest orbit to reach since the amount of energy required increases the farther from the earth the orbit is (Schettino, 2019). Anything will be in LEO if their altitude is less than 2000 km, after which they will be in Medium Earth Orbit (MEO). At an altitude of 35786 km, the orbit becomes High Earth Orbit (HEO) (Wilson & Schaub, 2021). The next boundary is the sphere of influence, at which point it cannot orbit the earth. Orbits can also be characterized by specific traits, such as Geosynchronous orbit (GSO), the period of the orbit is the same as earth. Another very particular orbit is the Geostationary orbit (GEO), this orbit has no inclination (Marmet, 2015). The inclination of a satellite is the angle between the satellite's orbit and the equator. Most satellites will be in LEO or MEO and will be pushed to HEO when they are deconditioned. Figure 2.10 show the different orbits and location satellites used.

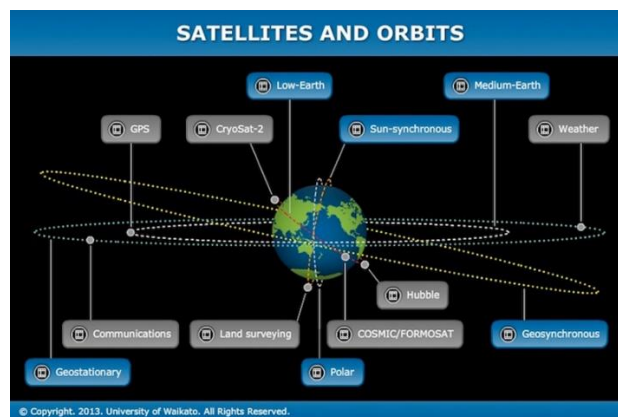


Figure 2.10 Earth orbits & satellites locations (University of Waikato, 2013).

Sometimes satellites need to move from one orbit to another, to do so an orbital maneuver needs to be executed. Different methods have been devised to complete these changes in orbit depending on if the goal is to use the least amount of energy, the smallest amount of time, least number of steps, or most precise. For each of these reasons different maneuvers can be used; such as using impulse thrust that will provide a quick change in velocity or a constant low thrust that will provide the same change in velocity but over a longer duration. For orbit transfer within the same sphere of influence, have been analyzed and solution using change in inclination, eccentricity, and velocity to optimize the transfer (Zaborsky, 2019). When doing a planetary transfer, some of the same methods may be applied but only the ones that don't use the change in inclination. Since all planets in the solar system have an inclination, from the sun, the transfer would need to change the eccentricity of the satellite's orbit so that it can reach the orbit of the target. One of the most common and low-energy cost method is the Hohmann transfer (Mabsout, 2009). This method only requires two impulses, one to push the satellite out of the earth's influence and the second to keep the satellite inside the target's gravitational pull (Figure 2.11).

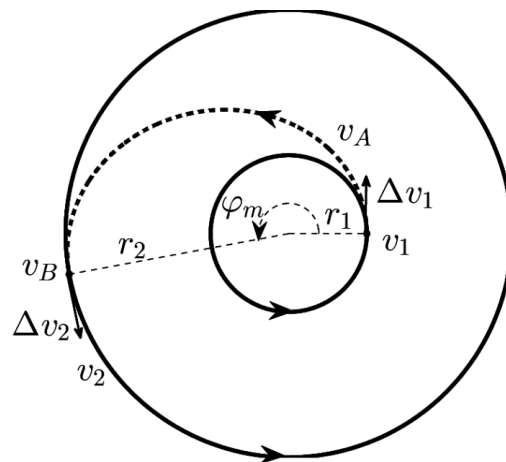


Figure 2.11 Hohmann transfer with change in velocity vectors (Gurfil, 2015).

This method is very efficient but needs to be executed at the right time because if the timing is off by a few seconds then the transfer can fail and the satellite will not be able to come back since the planet will have moved. This transfer method is also very long since the change in velocity is only so that the apogee of the new elliptical orbit crosses with the target's orbit as shown in Figure 2.11. From this maneuver, others have been created and used to shorten the transferee time.

Using a greater impulse or a long burn will provide a greater velocity change making the trip shorter. The draw-back from this method is that more fuel needs to be used for the initial acceleration and to decelerate the satellite once it reaches its target. This added fuel will mean that the satellite is heavier during take-off, and more weight means more rocket propellant is needed. Another method is the use of multiple impulses, instead of only two. This method also has the advantage of having a less linear flight path, meaning that corrections or adjustments can be made as well as the shortest path can be used at any time (Broucke & Prado, 1996). Flight paths have been analyzed and personalized for every mission; this also means that many algorithms are used to optimize the maneuvers to reduce the cost or time to perform them (Betts, 1998).

With everything that needs to be sent in space, ways to save weight by decreasing the amount of fuel sent with satellites, needed to be found. Another way for satellites to gain speed without having to use thrust is to get a gravitational assist of a larger body in space. This is also called the slingshot maneuver; the celestial body provides a boost to the satellite and the larger the mass difference is the greater the increase in speed is (Lanbunsky, 2001). During a hyperbolic trajectory, no energy is lost from when the satellite enters the sphere of influence and when it leaves, but both the satellite and the

body attract each other and will influence one another because of their respected initial momentum. This means that as the satellite gains speed, the planet slows down but because of the mass difference, the change to the planet is so small that it can be completely neglected (Wiegert, 2014). This method has been used for long-distance travel where extra fuel would not be possible. Since the mass of the object used for the gravitational assist is important, when trying to gain the most speed, the star at the center of the system is used. This is used to plan the exploration of multiple galaxies and solar systems (Nicholson & Forgan, 2013) and theorized to reach velocities that would be very difficult to reach using only thrusters and fuel.

The slingshot maneuver can be used to accelerate an object but if done in the opposite direction it is possible to slow it down. This has been studied to use the earth's gravitation field to slow down asteroids and keeps them in the earth's orbit (Eismont, 2013). The possibility that an asteroid hit the earth, and possibly destroys human civilization, remains constantly present. There are no weapons on earth that can destroy an asteroid before it hits the earth, but it may be possible to intercept it and cause a collision that will make it change trajectory. With all the asteroids available in space, being able to use one as a defense mechanism is possible. The amount of energy required to slow down an asteroid when it comes in the earth's sphere of influence is too great for one or more thruster to do while being cost-efficient. One method that has been theorized is to slow the asteroid down doing gravity assistance with the moon, making the burden on the thruster decrease significantly (Ledkov, 2015).

2.6. Satellites

Satellites have become very important to our daily life, for weather, to

communication, to military intelligence. Everything that relies on the internet or cellular service needs to communicate with the satellites in orbit. The more coverage a company wants the more satellites they will need (Future System incorporated 4 Professional Drive, 1977). More and more companies have started to send satellites that they will lend or use for themselves. The satellites used for communication, GPS, and other commercial uses need to be linked with each other and need to cover as much of the earth's surface as possible (Bektas & Int, 2015). Military satellites need to be able to survey a certain region or country for a long duration of time without being easy to notice. Wars are now fought with information and every country wants to know everything about their enemies. Every country has military satellites that they use to spy and everyone knows that but can't do anything about it.

Weather and observation satellites have been used to analyze whether patterns to predict natural disasters. Observation of forest fire and other catastrophes have been done to find a solution and help people in the best way possible (Kyzirakos et al., 2013). More and more satellites are placed in orbit by different countries for different reasons while rarely communicating with other countries about the purpose of the satellite. This results in multiple satellites in space with the same purpose but being used and send by different countries. As a result, the earth is constantly orbited by more than 3300 Satellites (USC, 2021). This only includes the satellites that have been registered and observed but it is also possible that every country has a few secret satellites. With the number only increasing every year, the limit to what earth can handle is ever closer (Alexandra, 2018), this can be seen in Figures 2.12 and 2.13.

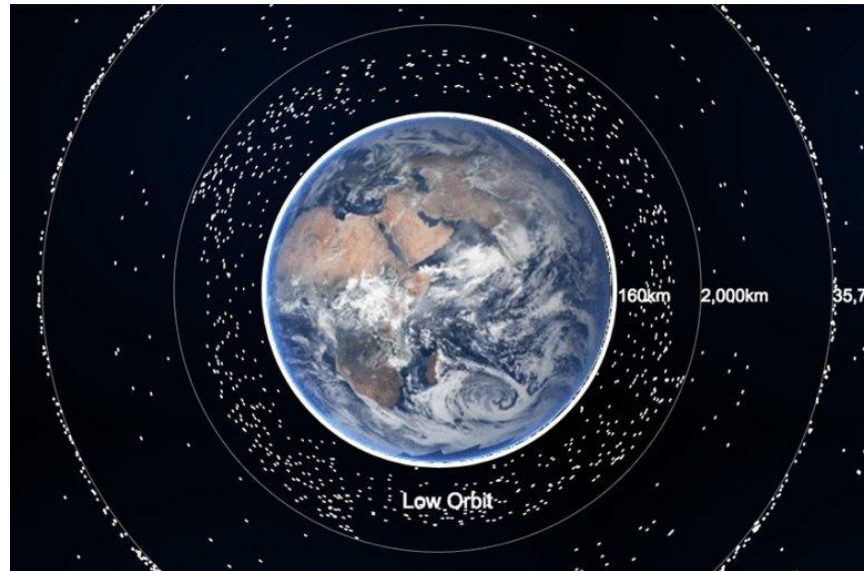


Figure 2.12 Satellite location in orbits (Nigmatulina & Abazid, 2017).

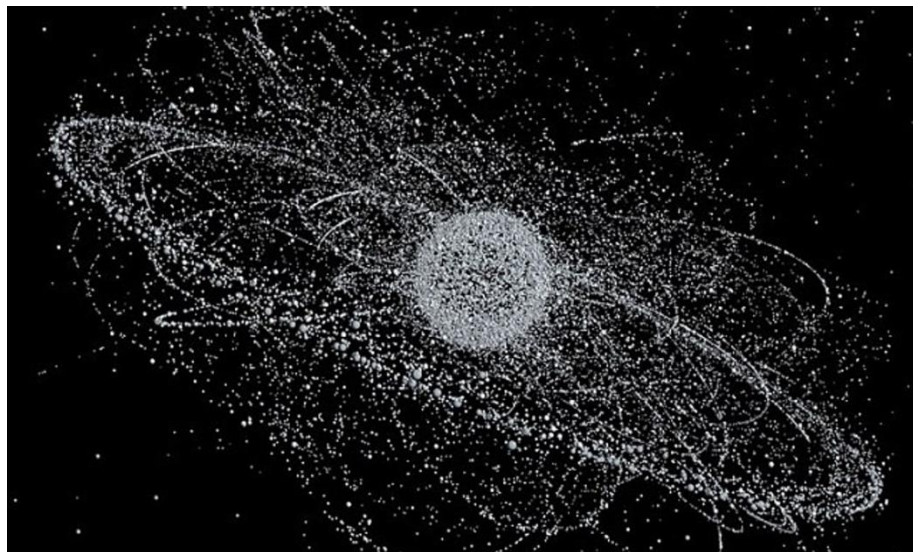


Figure 2.13 Space junk and satellites orbiting earth (Patel, 2018).

Although the space available is still very large, the number of debris that has been sent in space has made it so that the risk of collision, for satellites, has increased drastically. The biggest worry is the threat that space junk would damage or destroy satellites. Although the size of the object in space isn't very big, the speed at which they

travel is so high, between 7 km/s and 10 km/s (Makihara & Kondo, 2018), make them very destructive. At such speed, the object will cost serious damage to the satellite even if the debris disintegrates on impact. An object can move at hypervelocity in space, because of the lack of air resistance, making them the fastest projectile that the satellites need to waistband impact from.

Even the smallest object will become a very dangerous weapon at this speed as can be seen in Figure 2.14. The impact form figure comes from a projectile of only a few millimeters in diameters. The larger the debris is the more energy it will have and the more dangerous it will be. Because of the escape velocity required on earth, satellites in orbit will have a velocity close to 10 km/s, and satellites in deep space will have a velocity closer to 15km/s, this is done so that the spacecraft has more than enough energy to travel and will not do too influenced by other celestial bodies (Vasiliev & Fedorov, 2014).

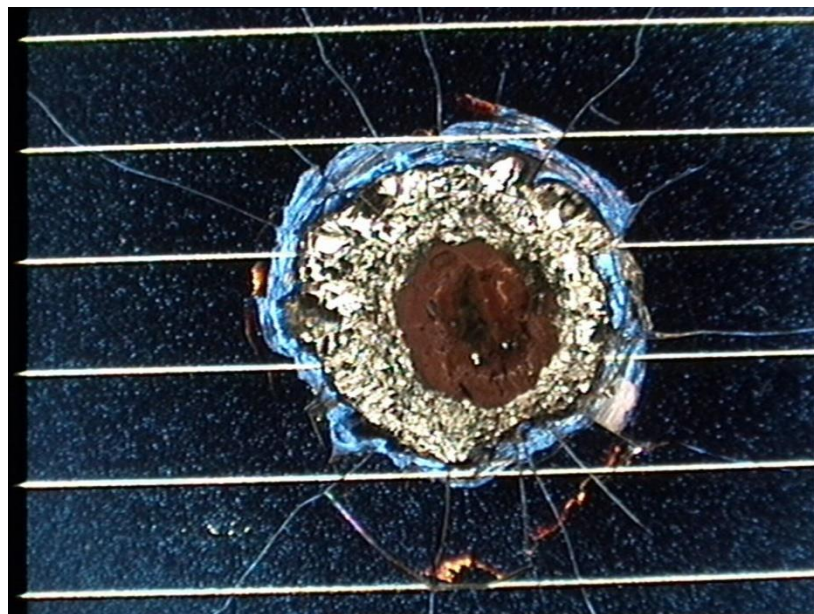


Figure 2.14 Hubble impact from space debris (ESA, 2020).

Satellites are also used to observe our star system and the galaxies around us. To do so satellites equipped with the latest technology and innovation are sent on the edge of earth's influence to capture images and data for a long time, see Figure 2.15. One such example is the Hubble telescope; it is used to observe the depth of space (NASA, 2019). The Hubble satellite is one of the largest and most technologically advanced satellites used, to complete a mission the satellite need to be pointed in a direction and keep pointing in that same direction to get a longer exposure time. These missions are complex and need to be done over a long time (Fuentes et al., 2010), see Figure 2.16. By having the satellite still orbit the earth, it is possible to do more accurate maneuvers with little to no delay between sending the command and when the telescope receives it. This short relay time allows multiple missions to be executed consecutively; the proximity to earth also allows it to receive maintenance from the International Space Station (ISS). All of this combined make the Hubble telescope the most versatile and useful satellite in earth orbit for observation and discoveries.



Figure 2.15 Hubble Space Telescope (ESA, 2020).

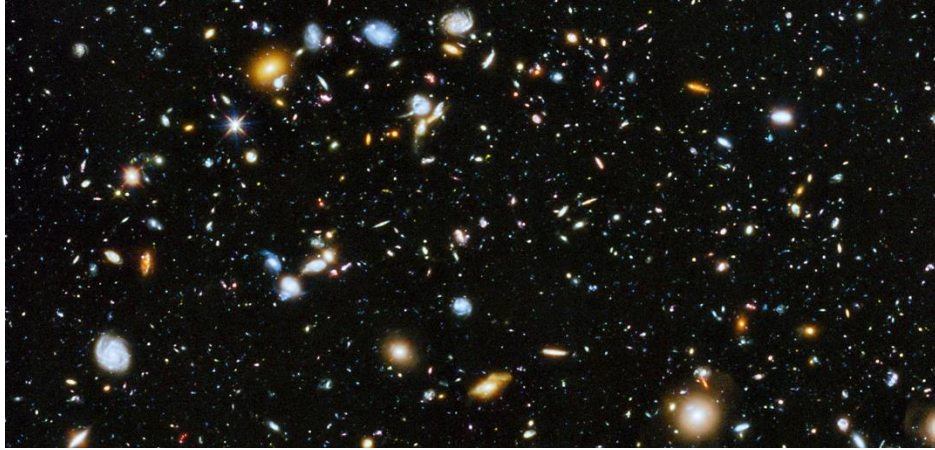


Figure 2.16 Photo of the darkest spot in the sky by Hubble Telescope (Hille, 2018).

Satellites are also sent to orbit other planets or moons to analyze the history of the celestial body. Missions have already been sent to most planets in the solar system, Mars has been of special interest because of its proximity. After the Apollo program and the landing on the moon, the next goal is to get on another planet and Mars is the closest one. Even if Mars is the next goal, much of the information that is needed to make the trip comes from analyzing the trip to the moon (Matthew, 2016). The difference between sending a satellite and a person to the red planet is that we don't know every variable yet to make the mission possible. More missions are planet to know more about Mars, satellites but also rovers, are sent to collect data (Vago et al., 2012). The trip to Mars will expose astronauts to the danger of long-term space travel resulting in understanding the environment they will live in (Michelle, 2016).

Satellites have been used to explore and understand the universe by conducting experiments and completing observations. Multiple missions have already been done towards planets, the sun, and even asteroids, the design of the satellites changed from an orbiter to a lander and even impactors. Some missions have even been about ways to

expulse waste from the earth, mostly nuclear waste, into space (Kim et al., 2015). Space exploration will continue to be of great importance and will need to go further into the unknown, to do so the mission will be longer and more complex but solutions have been found to help resolve some of these problems (Bao et al., 2020).

Missions into outer space will vary between getting in orbit, impacting the surface, and doing a flyby with the target. Orbiters are used for long period observation, once inside the planets gravitational influence they will enter a stable orbit and start the mission they are given. This can be to deliver a rover on the surface of the planet, for this, the satellite will need to reach the lowest orbit possible. If the objective of the mission is to take a picture or collecting data of the surface of the planet, the satellite will enter into a higher orbit to have a wider field of view at all times. These missions take time and effort since everything needs to be planned and anticipated before the satellite is even built (Lara et al., 2010). The time delay between when the earth gets the information and can find a solution and send the commands to the satellite, a few hours will have gone by (Levesque, 2006). The satellite needs to be given every command before arrives at his target and also need to be able to make decisions in some situations, the orbital maneuvers need to be simplified and easy to execute to reduce the propagation of error (Scheeres et al., 2001).

An impactor satellite has to reach its target surface to be able to collect data of its structure, this can be done by hitting the target and breaking up parts of its surface, or it can probe beneath the surface (Asif, 2018). This type of satellite is mostly used for asteroids and smaller celestial bodies, larger ones such as moons and planets will have rovers sent for long-term missions. By using impactors on asteroids, it becomes possible

to completely break them apart and study the core of asteroids and learn more about how they form. Impactors need to hit the target with a certain amount of speed and at a specific angle to get the best result possible, this requires extensive research on the target as well as precise orbital control.

Orbiters and impactors will execute some maneuvers after leaving the earth's influence, orbiters need to slow down to stay in orbit, and impactors need to adjust the angle of approach. Satellites tasked with completing a flyby don't need to complete extra manual maneuvers to complete their missions.

Flyby mission will provide data or pictures of a planet as it passes by but will also be able to visit other planets and moons, this has been done for Saturn and its moons (Wolf, 1996). These types of missions have the satellite follow a set trajectory that will require no extra maneuvers. The satellite will only have the equipment to collect and send data and will not need thrusters or fuel. They can complete slingshot maneuvers over stars and planets to change trajectories and get an increase in speed (Gong, 2015).

3. Methodology

For the simulation to be a proof of concept, all asteroids will be of constant density and will be spherical. This will provide an even sphere of influence and if the maneuver isn't possible with this simplified asteroid, then it would also be impossible with an irregular asteroid. An asteroid with an irregular shape may have areas with a gravitational field strong enough to complete the maneuver but if the area isn't large enough to complete the entire maneuver, then it would be impossible.

3.1. Volume and Mass Determination

The volume of the asteroid is used to determine the mass of the asteroid. By varying the density and the radius to be able to represent all different asteroids, the objective is to create an array of asteroids with different masses and sizes that can be found in the asteroid belt. The variation of density and radius is to vary the mass and the sphere of influence of the asteroid, this will indicate the influence of both the density and radius when completing the maneuver. As seen previously, the size of an object in space has no impact on the force it generates around the object around. The distribution of the mass will have an impact on the shape of the gravitational field. To remedy this problem and the irregular field of influence that would come from it, the assumption that the asteroid is spherical makes simplification better. Working with this assumption, calculating the volume of the different asteroids using the volume calculation of a sphere.

$$V = \frac{4\pi r^3}{3} \quad [3.1]$$

With the initial estimate using the range of the radius from 1km to 500km by increasing in increments of 1km. This will give a total of 500 different volumes, using each of them with the density desired will give the masses that represent the asteroids.

$$M = V * \rho \quad [3.2]$$

Using Equation 3.1 and 3.2 with the variation of density and radius continually will create a 32 by 500 array, that represent all different asteroids. This array is then used in the different simulations to have all data with the same parameters.

3.2. Satellite Parameters

After simulating the asteroid, some parameter needs to be identified for the satellite. The satellite's minimum velocity used in this experiment is set to the escape velocity from the earth. This is done to establish a baseline as to what the asteroid velocity can be. This value is much smaller than what a satellite orbiting the sun would have around the earth but this is still a value that can be found in the satellite had multiple stops or a more complex mission. The greater the velocity of the satellite is the more maneuver needs to slow down and stop it. On the other hand, a maximum velocity needs to be designated to set a range. The maximum velocity can be defined based on how much the structure of the satellite can handle but since all that matters is the mass of the satellite, the maximum velocity is based on an orbit around the sun. When a satellite enters an elliptical orbit around a body the law of conservation of energy can be used to find the velocity at different points in the orbit. Assuming that the farthest distance from the sun and the asteroid is the distance to the outer edge of the asteroid belt, at the same time we are assuming that the eccentricity of the orbit to be 0.9. Using these assumptions, the maximum velocity will be designated as the velocity in that orbit at the inner edge of the asteroid belt.

$$a = \frac{r_a}{(1+e)} \quad [3.3]$$

$$v = \sqrt{GM_s \left(\frac{2}{r} - \frac{1}{a} \right)} \quad [3.4]$$

The semi-major axis is defined using Equation 3.3 and used in Equation 3.4 to define the velocity. Using Equation 3.4, the maximum velocity is defined to be 166.95 km/s and the velocity at the apoapsis to be 13.4 km/s, this is above the set minimum value and as such is in the simulation range.

Configuring the mass of the satellite is more varied since all satellites are different based on their mission parameters. The orbiters around the earth are built by different countries for military, commercial, and research. Over three thousand three hundred satellites that fall in one of those categories have been sent in orbit with masses from 2 kg to 3000 kg and this is only from satellites information that is accessible to the public. This doesn't include any of the satellites sent in space, which include orbiters, impactors, and landers. Over time heavier and more complex satellites have been sent into space, the heaviest was landers was for the Apollo mission of about 15100kg. On the other hand, the largest rover sent was Perseverance, with a mass of only a little over 1000 kg. Using all of the satellites launched to select a satellite that can be used in the simulation and with a mass large enough so that the mass difference between the asteroid and satellite isn't negligible. The satellite used is similar to Galileo making the mass used in the simulation of 2223 kg. A satellite of this would be able to complete a different objective such as orbiting an asteroid, landing on one, or even dropping a rover and coming back.

3.3. Deceleration with Every Asteroid

To decelerate the asteroid without the use of thruster, orbital mechanics properties need to be used. The conservation of energy and momentum is used to find out the difference in velocity. As an object moves in space it will stay in motion until an external force act on it. This includes objects going into orbit around a body. This means that

The first maneuver will offer a greater change in velocity as the maneuver can be seen as an inverse slingshot maneuver. Expressing Equation 3.6 for the different directions of the velocity vectors and then solving for v_2 in terms of v_1 and U_1 .

$$v_2 = \sqrt{\frac{v_1^2 \left(1 - \frac{m}{M}\right) - 2v_1 U_1}{\frac{m}{M} - 1} + \left(\frac{\frac{m}{M} v_1 + U_1}{\frac{m}{M} - 1}\right)^2 - \frac{\frac{m}{M} v_1 + U_1}{\frac{m}{M} - 1}} \quad [3.7]$$

The satellite can also approach at an angle, at this point the satellite velocity will be divided into a x and y component.

$$v = \sqrt{v_x^2 + v_y^2} \quad [3.8]$$

$$v_x = v * \cos(\theta) \quad [3.9]$$

$$v_y = v * \sin(\theta) \quad [3.10]$$

With trajectory 1, the y component of the satellite is the same before and after the maneuver combining Equation 3.8 to 3.10 to find the final velocity at any angle.

$$v_2 = \sqrt{\left(\sqrt{\frac{v_1 \cos(\theta) (v_1 \cos(\theta) \left(1 - \frac{m}{M}\right) + 2U_1)}{\frac{m}{M} + 1} + \left(\frac{v_1 \cos(\theta) \frac{m}{M} - U_1}{\frac{m}{M} + 1}\right)^2 - \frac{v_1 \cos(\theta) \frac{m}{M} - U_1}{\frac{m}{M} + 1}} \right)^2 + (v_1 \sin(\theta))^2} \quad [3.11]$$

Using Equation 3.11 to find the final velocity by changing the mass of the asteroid, the velocity of both the asteroid and velocity, and the angle of approach to find the different decelerations. The x component of the velocity also needs to be calculated separately to find out if the direction of the velocity, if the x-component becomes negative, then the direction of the satellite also changed.

Following the same process for trajectory 2 to find the final velocity of the satellite gives us Equation 3.12.

$$v_2 = \sqrt{\frac{v_1(v_1(1-\frac{m}{M})+2U_1\cos(\theta))}{\frac{m}{M}+1} + \left(\frac{U_1\cos(\theta)+\frac{m}{M}v_1(\sin^2(\theta)-\cos^2(\theta))}{\frac{m}{M}+1}\right)^2} - \frac{U_1\cos(\theta)+\frac{m}{M}v_1(\sin^2(\theta)-\cos^2(\theta))}{\frac{m}{M}+1}$$

[3.12]

Contrary to trajectory 1, trajectory 2 provides a change in the x and y direction for the satellite. With Equation 3.11 and Equation 3.12 every change in velocity can be calculated by varying the angle of approach between 0 and 35 degrees with every asteroid and at every velocity.

3.4. Multiple Deceleration to Complete Stop

A single maneuver will not always bring the satellite velocity to the desired values, as such multiple maneuvers may be required. There are hundreds of thousands of different possibilities based on the variation in asteroids and their speed. To simplify and organize multiple maneuvers only certain variables will be changed at a time. Doing so the initial multiple maneuver testing will be done by having the satellite perform the same maneuver around the same asteroid. This will determine the number of maneuvers required to decrease the velocity to its lowest value before the direction of the satellite velocity changes. The lowest velocity value also provides and limit to the asteroid and maneuver, decreasing the option of asteroids and speed that can be used when the satellite is at lower speeds.

The next step is to complete multiple maneuvers using only one of the trajectories. Only two maneuvers will be used and then more, the more maneuvers are, the more accurate the final velocity can be. As the satellite velocity decreases the asteroids criteria that can be used will decrease. Multiple maneuvers will be done by first varying the speed of the asteroid, then the angle of approach, and finally a combination of both. The satellite velocity will always start at the maximum velocity of 166.95 km/s, any velocity

lower would be simulated already.

The multiple maneuver simulation cannot be done with trajectory 1 multiple times without extra maneuvers or assistance to redirect the direction of the asteroid. The asteroid in the asteroid field rotates in the same direction and once the satellites perform the maneuver it will travel in the opposite direction of the asteroid and if trajectory 1 is done again then the satellite will gain an increase in velocity and not a deceleration. A combination of reverse-slingshot and slingshot maneuvers can be done, where the deceleration is greater than the velocity gain, to reach the desired velocity using the same trajectory maneuver. The most effective method to use the first trajectory is as the first maneuver to have the final velocity in the opposite direction as the asteroid, allowing the satellite to then perform the second trajectory and potentially complete a slingshot maneuver at the end to rotate in the same direction as the asteroids.

3.5. Hyperbolic Path Verification

Although the change in velocity isn't related to the sphere of influence of the asteroid or its location in the asteroid belt, the possibility to perform the maneuver depends on those criteria. Finding out if the maneuver is possible is just as important as if the maneuver is possible. Any of the two-maneuver used in this simulation have a hyperbolic trajectory.

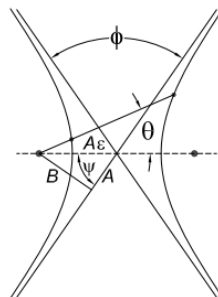


Figure 3.2 Hyperbola flight path (Borsche, Iervolino, et al., 2020).

Using the equation for hyperbola with the relation with eccentricity as well as the trigonometric between A, B, and $A\epsilon$, allows us to get the following equations.

$$e^2 = 1 + \frac{B^2}{A^2} \quad [3.13]$$

$$\frac{1}{r} = \frac{1 + \epsilon \cos(\theta)}{A(e^2 - 1)} \quad [3.14]$$

$$\frac{1}{e} = \sin\left(\frac{\theta}{2}\right) \quad [3.15]$$

The perigee of the mass and the rotating body,

$$d = A(e - 1) = A\left(\frac{1}{\sin\left(\frac{\theta}{2}\right)} - 1\right) \quad [3.16]$$

can be expressed in the frame of reference of the body and the with velocity of the relative to the body and not the sun gives use the following equations:

$$\sin\left(\frac{\theta}{2}\right) = \frac{A}{A+d} \quad [3.17]$$

The relative velocity is dependent on the trajectory and as such is different in trajectory 1 and 2, the angle of approach also needs to be taken into account to find the correct relative velocity. The maximum distance the satellite can have with the satellite is based on the size of the sphere of influence and the minimum distance is just over the surface of the asteroid.

4. Results and Analysis

The variation of the size of the asteroid provides a wide range of simulations, with the variation of the speed of both the asteroid and the satellite, simulation can only have one variable at a time. The same process is done with both trajectories, when one variable is held constant the same value will be used with the different asteroid and trajectories.

Different angles will be used for every simulation, to show any trend related to the angle of approach.

4.1. Deceleration with Trajectory 1

The first trajectory needs to be done with both large and small asteroids to find where the mass of the asteroid becomes irrelevant. This is important to find the optimal asteroid that can be used for this trajectory. The smaller asteroid is used to show if a similar maneuver can be used with such asteroid or if the bigger asteroids provide a greater deceleration.

4.1.1. Deceleration with Asteroid from 1km to 500km

Following a single maneuver, the variation of the speed of both the asteroid and the satellite as well as the angle of approach, allows us to find a pattern and what are the variables with an impact on the velocity change. For all of the results in this section, the density of the asteroid will remain constant at 1.3 kg/m^3 . The initial result to look at is with the velocity of the satellite remaining constant at 166 km/s and having the speed of the asteroid changing from 7 km/s to 20 km/s . The angle of approach changes from 0.1 degrees up to 45 degrees. After 45 degrees the y-axis satellite's velocity component will be greater and since it isn't impacted by the maneuver, and as such, the velocity change would decrease significantly.

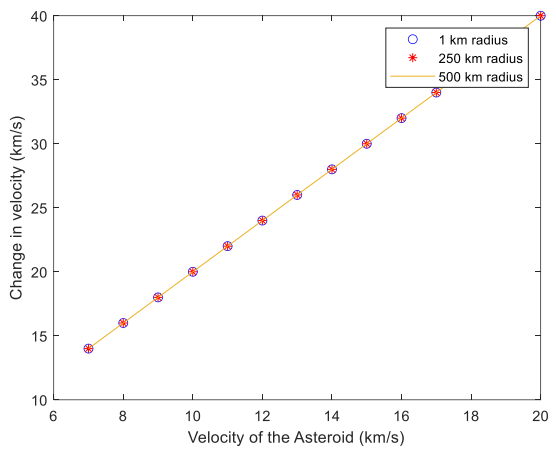


Figure 4.1.1 Change in Velocity at a 0.1° Approach Angle.

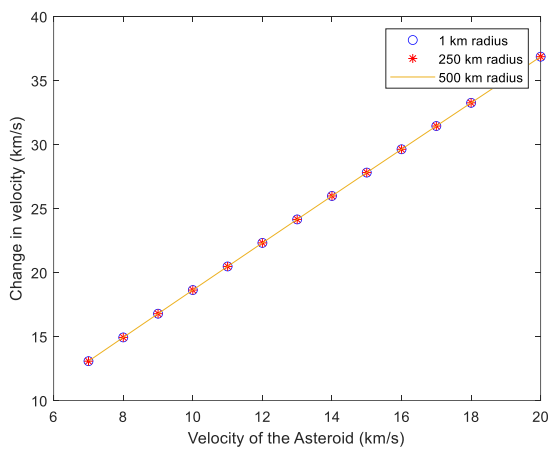


Figure 4.1.2 Change in Velocity at a 20° Approach Angle.

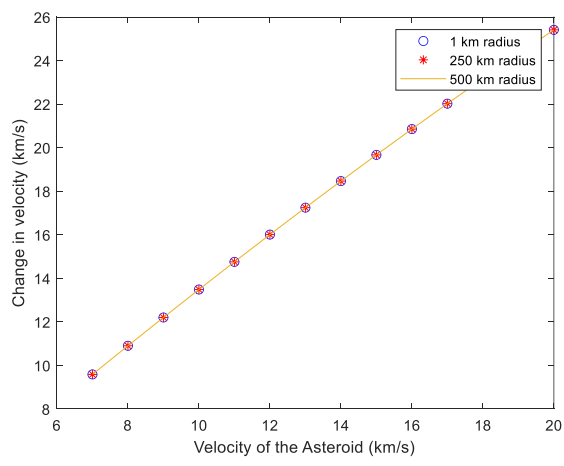


Figure 4.1.3 Change in Velocity at a 45° Approach Angle.

From the figures above, the result shows that the mass of the asteroid has little to no impact on the change in velocity. The difference in velocity change from a 1 km radius and a 500 km radius asteroid is measured in $\mu\text{m/s}$, this is possible because the difference between the satellite and the asteroid is so large that it is almost negligible.

The graphs also show that the smaller the angle of approach is the greater the deceleration but at greater angles, the change can be more accurate. Next looking at the final velocity of the satellite by varying the speed from 7 km/s to 166 km/s and keeping the speed of the asteroid constant at 10 km/s.

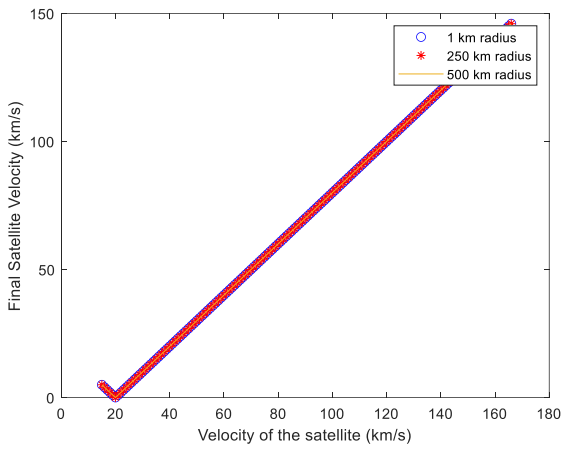


Figure 4.1.4 Satellite final velocity at a 0.1° Approach Angle.

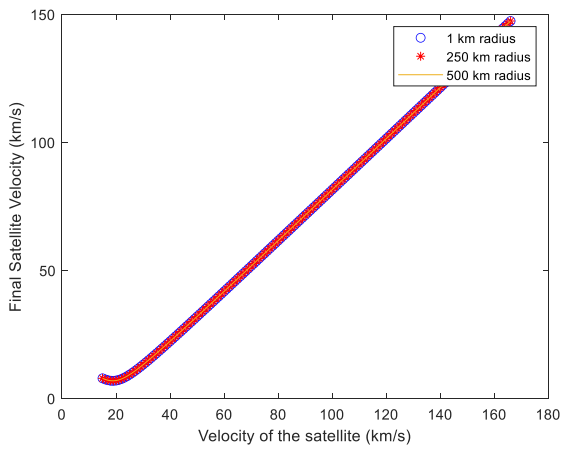


Figure 4.1.5 Satellite final velocity at a 20° Approach Angle.

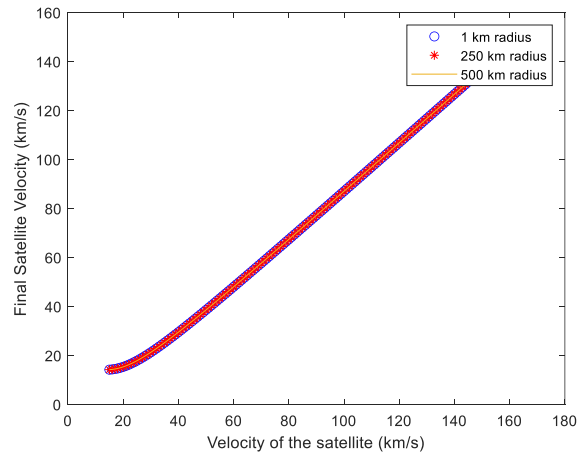


Figure 4.1.6 Satellite final velocity at a 45° Approach Angle.

When the satellite is at velocities that are close to that of the asteroid the maneuver may cause the final velocity to be in the same direction as the asteroid. The graphs show the final magnitude velocity and as such will not be able to show if the satellite is moving in the positive or negative x-axis. Figures 4.1.4 show that, at a small angle, the maneuver should be done when the satellite velocity is greater than 20 km/s, as it is the minimum velocity where the maneuver can be completed correctly. Figures 4.1.5 - 4.1.6 show that with a greater angle of approach it is impossible to cause the satellite to stop and drift in space because of the maneuver. This is related to the y velocity component that is not affected by the maneuver, the smaller the angle, the more negligible that component becomes.

4.1.2. Deceleration with Asteroid Smaller than 1km

Smaller asteroids are more common in the asteroid belt but have less gravitational force but for the experiment, it is assumed that the maneuver is possible and that all that is important is the final velocity. This data is mostly compared to the data from the larger

asteroid and to find out the most efficient asteroid to use for the maneuver and when the size and mass of the asteroid start to play a role. The figure below shows the change provided by the maneuver and the graph next to it shows the same thing but of only the 10 smallest asteroids and zoomed in to see the effect of the different mass more clearly.

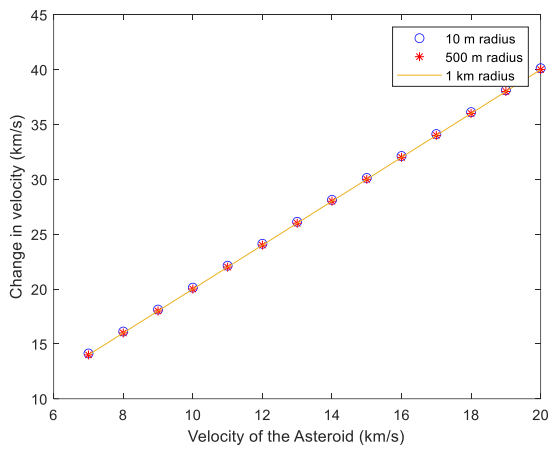


Figure 4.1.7 Change in Velocity at a 0.1° Approach Angle.

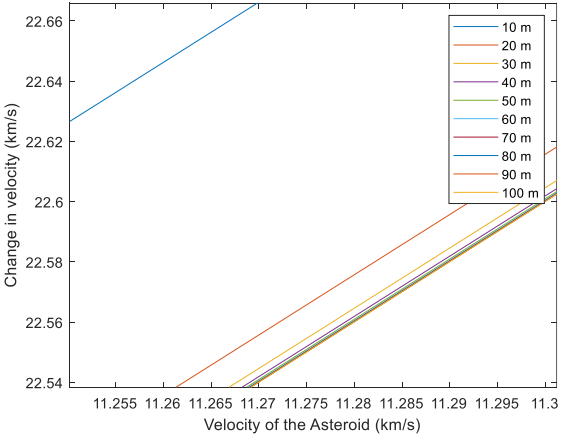


Figure 4.1.8 Change in velocity at a 0.1° Zoomed in

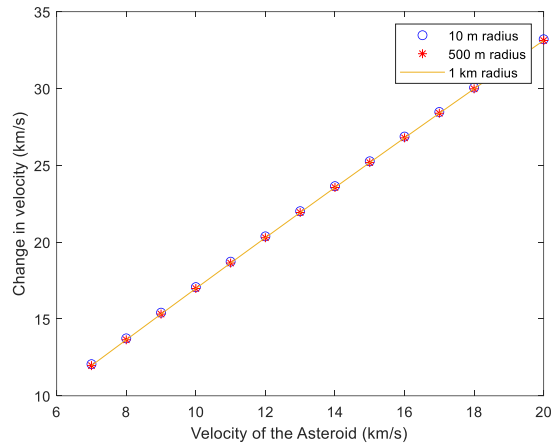


Figure 4.1.9 Change in Velocity at a 30° Approach angle.

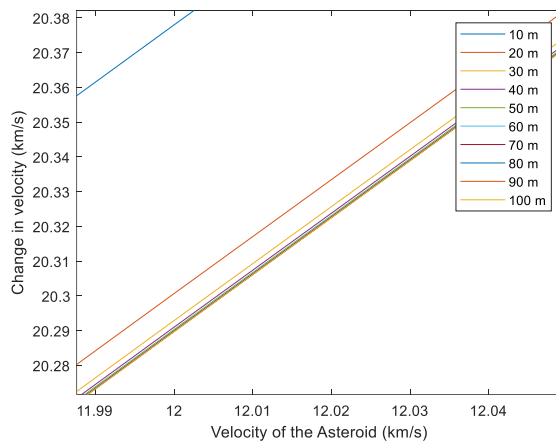


Figure 4.1.10 Change in velocity at a 30° Zoomed in.

As shown in Figures 8 and 10, the velocity difference of the smallest asteroid is a little greater than the rest with a magnitude measured in m/s, however as the asteroid starts to get closer to a radius of 30 m the difference is measured in cm/s and after passing the 50 m radius mark, it gets closer to $\mu\text{m/s}$. This rapid increase in change can be explained because the variable used to change the mass is the radius. Instead of increasing the mass by a constant value, increasing the radius will provide a greater change since the radius is related to the volume and mass as a cube function. As the mass

increases at a cubic rate, the mass difference with the satellite becomes negligible.

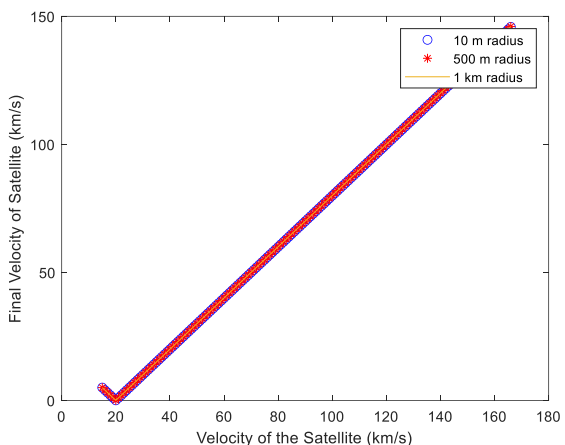


Figure 4.1.11 Satellite final velocity at a 0.1° Approach Angle.

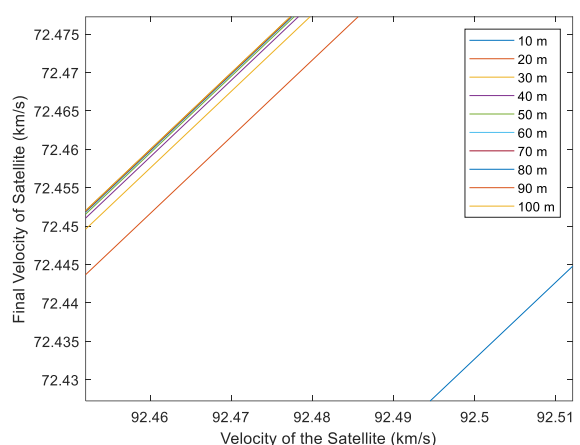


Figure 4.1.12 Satellite final velocity at a 0.1° Zoomed in.

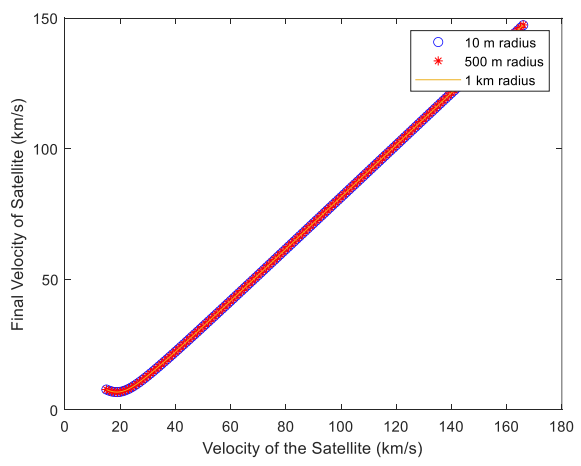


Figure 4.1.13 Satellite final velocity at a 20° Approach Angle.

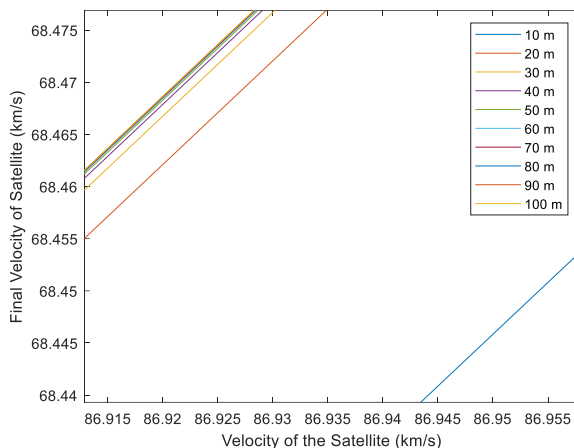


Figure 4.1.14 Satellite final velocity at a 20° Zoomed in.

Figures 11 - 13 shows the final velocity based on the initial velocity of the satellite and once again the difference in mass is only significant from a radius of 10 m to 40 m. The values for the change in velocity as well as the final velocity are almost the same as when the asteroid has a radius of 1 km or 500 km. The same thing can be said to the angle of approach. However, with a small asteroid, the mass difference has an impact on the result of the final velocity. The greater the angle of approach is the smaller the change in velocity is but at the same time as the smaller angle of attack will provide a greater change in velocity especially to the smaller asteroid. One part that isn't taken into account also includes the effect that the satellite has on the asteroid. The greater the mass difference is the less the asteroid is affected but a small asteroid will get changes in its velocity that may not always be negligible or may impact their angular direct relative to the sun. This is taken into account in the feasibility of the maneuver with the sphere of influence and size of the asteroid.

4.2. Deceleration with Trajectory 2

The simulations for the second trajectory are identical to the ones used for trajectory, but using the second equation derived. This will make the differentiation of which asteroid to use for the different maneuvers, each trajectory provides different results for a different combination.

4.2.1. Deceleration with Asteroid from 1km to 500km

The second trajectory used provides less deceleration but has the advantage to be completed multiple times in succession. For all simulations where the satellite velocity is held constant, the value is kept at the maximum of 166 km/s. This is done to find the minimum or maximum velocity change possible, the simulation is also run with the satellite velocity changing allowing for a comparison at certain values. The initial simulation is trying to find the different impacts of speed and size of the asteroid in the maneuver. During every simulation the angle of attack is always one of the variables, this allows the two simulation and set of data to be related and comparable.

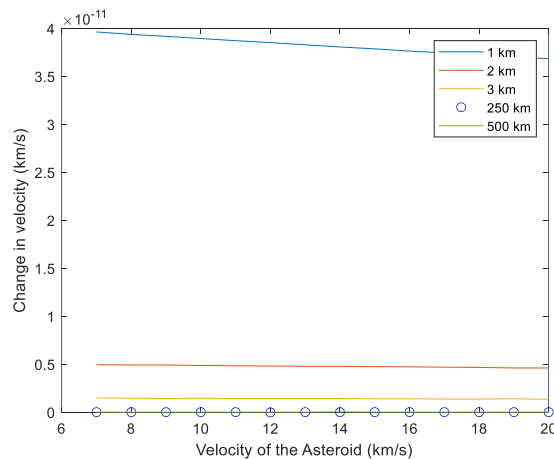


Figure 4.2.1 Change in Velocity at a 1° Approach Angle.

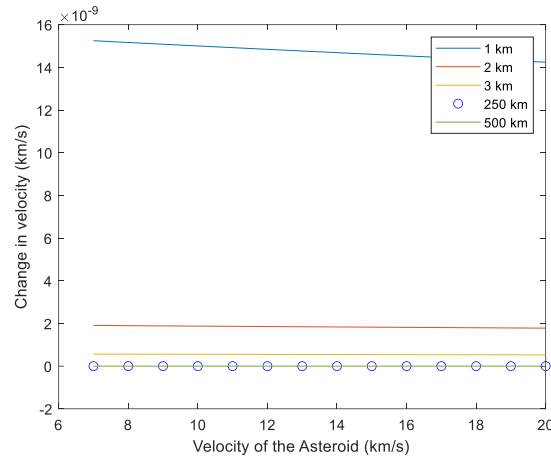


Figure 4.2.2 Change in Velocity at a 20° Approach Angle.

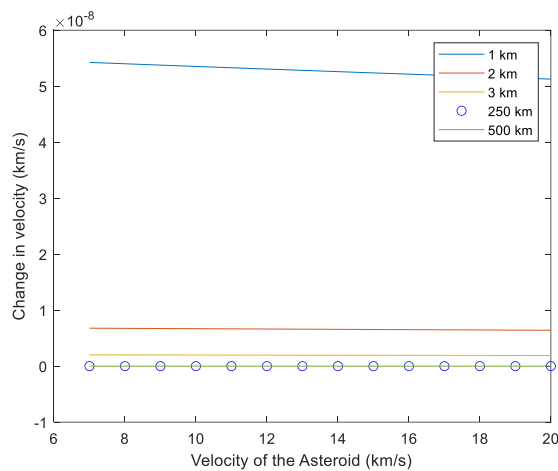


Figure 4.2.3 Change in Velocity at a 40° Approach Angle.

As shown in the figures above, the maneuver provides a small change in velocity measured in micrometers and millimeters per second. However, the change in velocity increase as the angle of attack increases. As such this maneuver is more beneficial if the angle of attack is closer to 40 degrees or higher. The graphs also show that smaller asteroids will provide greater changes in velocities, however, the speed of the asteroid doesn't affect the deceleration the asteroid can provide. With such a small change regardless of the asteroid's speed, the final satellite velocity would be almost identical to

the original, regardless of the satellite velocity, as seen below.

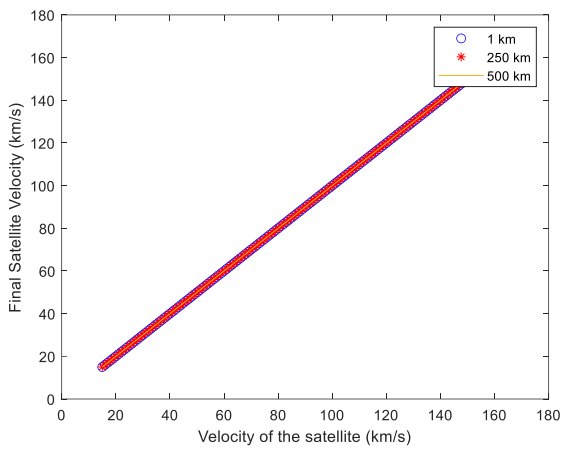


Figure 4.2.4 Satellite final velocity at a 1° Approach Angle

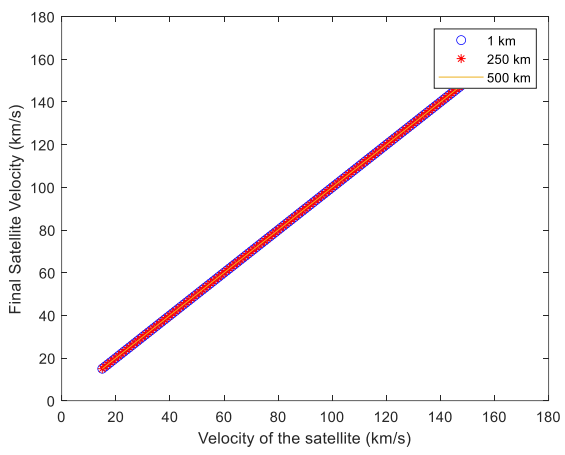


Figure 4.2.5 Satellite final velocity at a 40° Approach Angle

Although the asteroid has a constant speed of 10 km/s, the satellite velocity remains close to the same after the maneuver regardless of the satellite speed. This makes the variation in asteroids less important, and that contrary to trajectory one, performing this maneuver multiple times at a small approach angle can quickly bring the same result as doing it at one large angle. Since the size and speed of the asteroid, especially for a large asteroid, is of little importance, then finding and creating a flight path would be much easier.

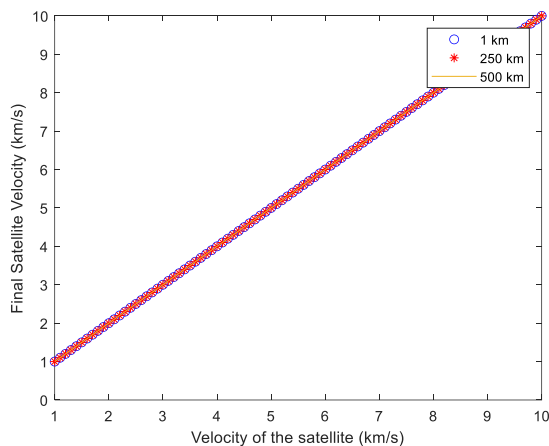


Figure 4.2.6 Low Velocity at a 1° Approach Angle.

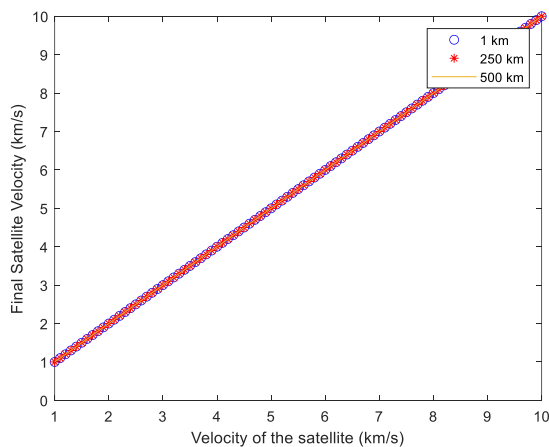


Figure 4.2.7 Low Velocity at a 40° Approach Angle.

Completing the same simulation but this time with the satellite velocity at a lower value that could be found after an initial deceleration had already been applied. This shows the efficiency of the maneuver to provide smaller decelerations that trajectory 1 wouldn't always be able to provide. The figures above show the maneuver with a constant asteroid velocity of 10 km/s, and once again the velocity change is very small. The size of the asteroid also provides little difference in the efficiency of the maneuver, but this also means that small adjustments can be done even at low velocities. Precious is important during orbital maneuvers and especially when it comes to observing or landing on an asteroid. The use of asteroid in the asteroid field to complete multiple small maneuvers can allow the satellite's velocity to change by a few cm/s.

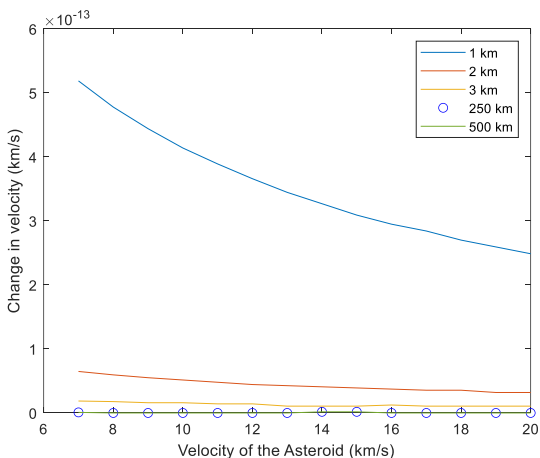


Figure 4.2.8 Change of Low Satellite Velocity at a 1° Approach Angle.

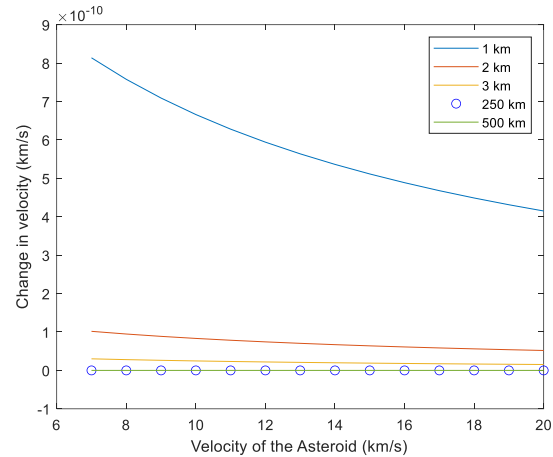


Figure 4.2.9 Change of Low Satellite Velocity at a 40° Approach Angle.

Figures 4.2.8 and 4.2.9 show that, even at a satellite speed of 5 km/s, the maneuver provides a change in velocity in the micrometer per second, however, the speed of the asteroid plays a more important role. The slower the smallest asteroid is, the greater the change in velocity, but the increase in asteroid speed has more of a quadratic relation to the deceleration contrary to the more linear relation with higher satellite velocities. This means that if a very small change in velocity is desired then the speed of the asteroid does need to be taken into account, and can limit the asteroid available.

4.2.2. Deceleration with Asteroid from 1km to 500km

Trajectory 2 was little impacted by the size of the asteroid and provided little to no deceleration making the use of large asteroids less reliable but smaller asteroids are still available. Trajectory 1 showed that a smaller asteroid can also be used to complete an orbital deceleration and that the Mass difference with small asteroids isn't negligible. Once again, the smaller the asteroid the greater the magnitude in slowing down the satellite. Figure 4.2.10 shows that the smallest asteroid can provide a change in velocity in mm/s after one maneuver with a small angle of approach. As the asteroid gains in size,

the impact decreases significantly. Figures 4.2.11 and 4.2.12, show that the angle of approach also plays an important role in the total deceleration provided. A maneuver around the smallest asteroid, with a satellite velocity of 166 km/s, at a high angle of approach can decelerate the satellite by a few m/s.

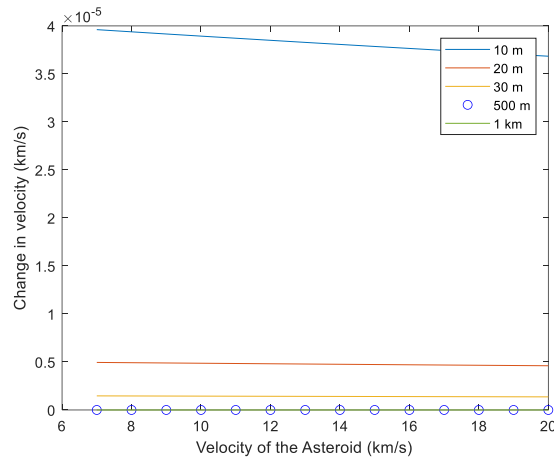


Figure 4.2.10 Change in Velocity at a 1° Approach Angle.

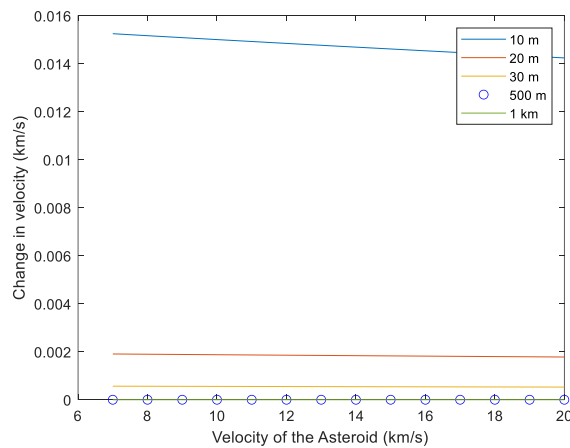


Figure 4.2.11 Change in Velocity at a 20° Approach Angle.

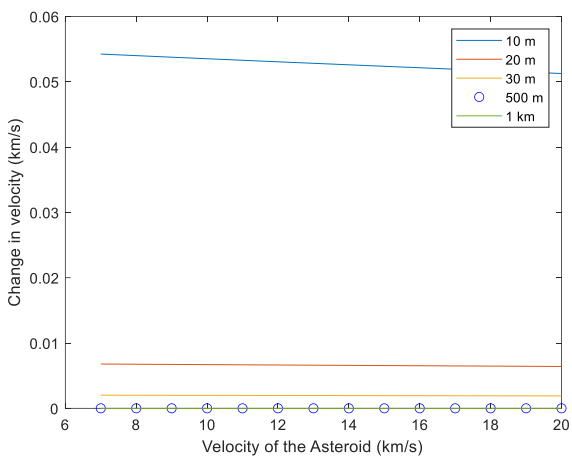


Figure 4.2.12 Change in Velocity at a 40° Approach Angle.

The change in velocity is constant regardless of the velocity of the asteroid meaning that the velocity difference is too large to have a significant impact on the change in velocity. The close-up graph figure below shows that, when the radius of the asteroid reaches close to 50 m, the mass difference with the satellite become negligible and the maneuver can then be executed with any asteroids and have similar results.

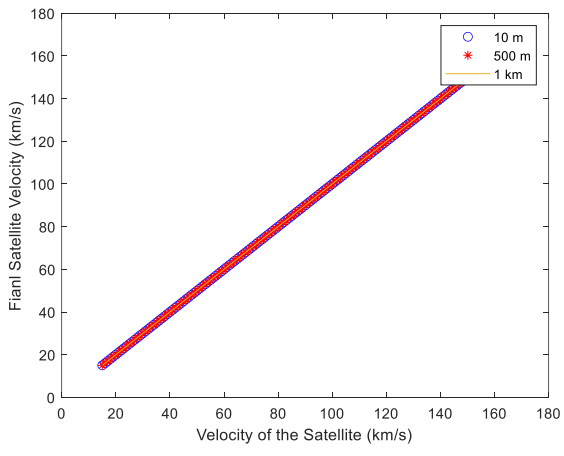


Figure 4.2.13 Satellite final velocity at a 1° Approach Angle.

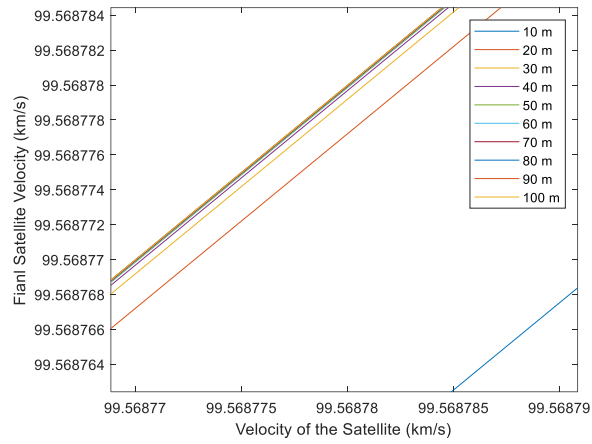


Figure 4.2.14 Satellite final velocity at a 1° Zoomed in.

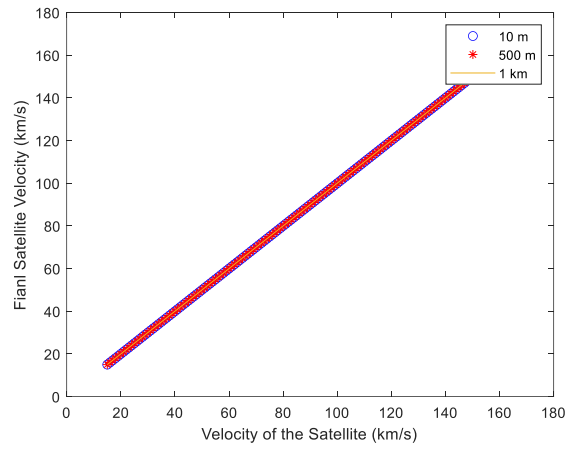


Figure 4.2.15 Satellite final velocity at a 30° Approach Angle.

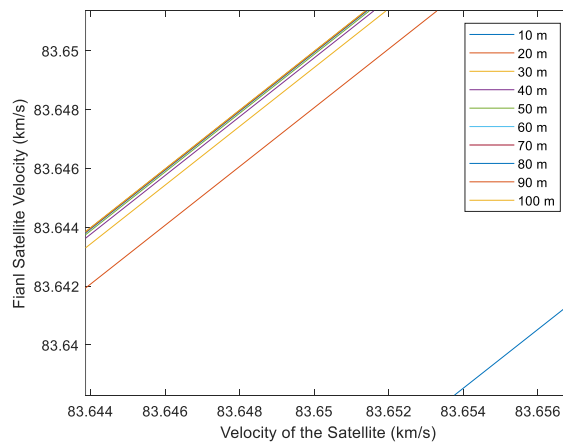


Figure 4.2.16 Satellite final velocity at a 30° Zoomed in.

With the greater impact small asteroids have with this maneuver, it would become much more useful and viable when the asteroid velocity is low or close to the final desired value. The figures below show that even at low satellite velocities, the asteroid's size doesn't impact the deceleration provided contrary to the angle of approach. The greater the angle of approach is the larger the deceleration is but at the same time the more significant the difference between the small asteroids is. The change in velocity difference between two small asteroids will increase as the angle of attack grows, making the variation of small asteroids and angle of approach more versatile and flexible.

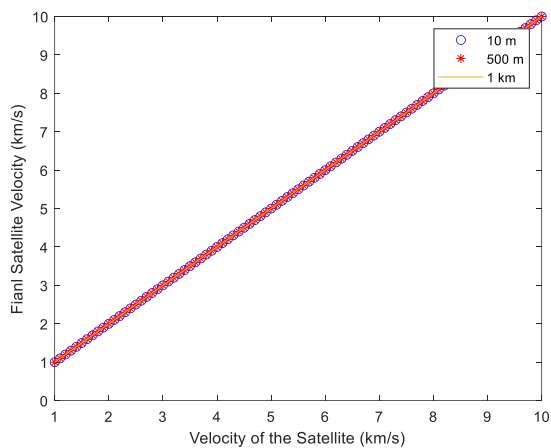


Figure 4.2.17 Low Velocity at a 1° Approach Angle.

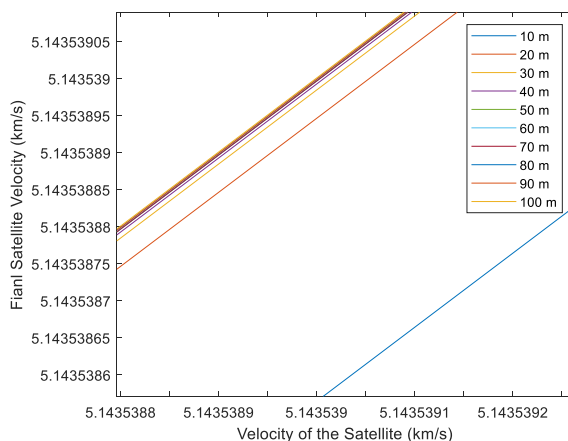


Figure 4.2.18 Low Velocity at a 1° Approach Angle Zoomed in.

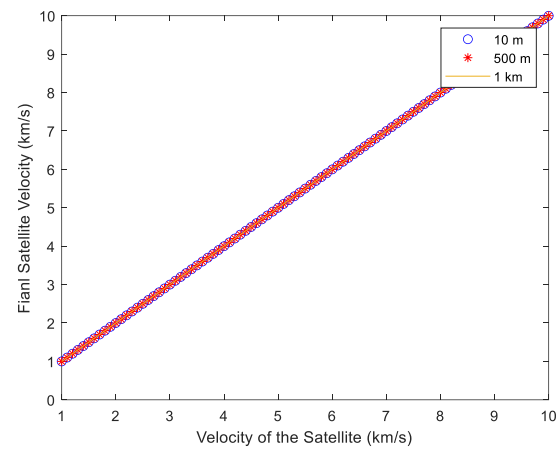


Figure 4.2.19 Low Velocity at a 40° Approach Angle.

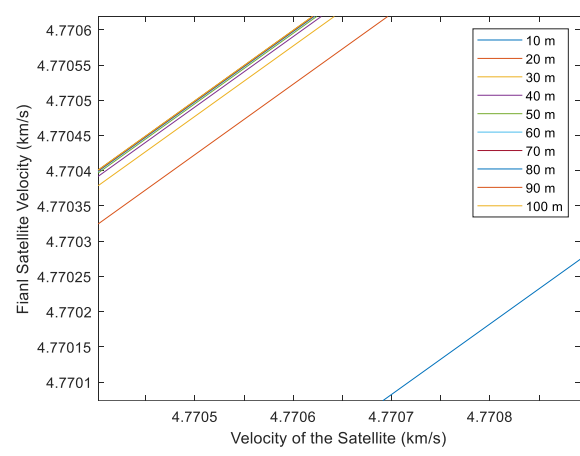


Figure 4.2.20 Low Velocity at a 40° Approach Angle Zoomed in.

At lower satellite velocities, the maneuver doesn't provide change by more than a few cm/s regardless of the satellite's velocity. At these values, the size of the asteroid is only significant when it is smaller than 40 m in radius. The angle of approach is also important and can be used to achieve the same result with a small asteroid of a different size.

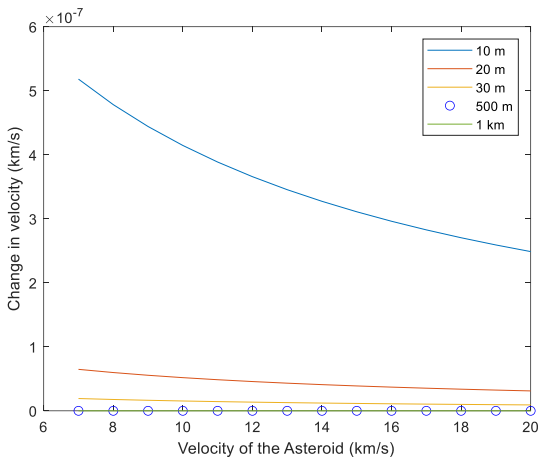


Figure 4.2.21 Change of Low Satellite Velocity at a 1° Approach Angle.

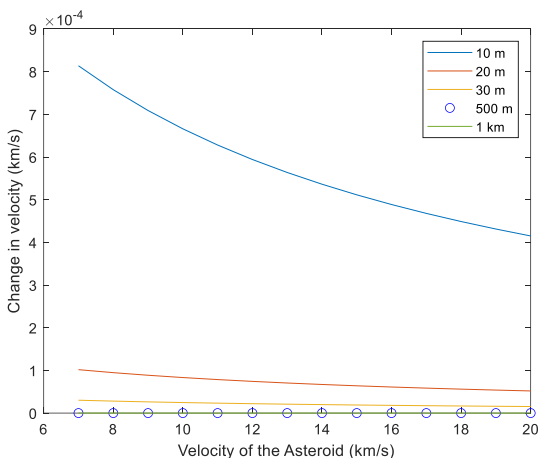


Figure 4.2.22 Change of Low Satellite Velocity at a 40° Approach Angle.

At lower satellite velocity the change in velocity also decreases in magnitude but this allows for more precise velocity adjustment. With the increase in asteroid velocity providing significantly less change, this allows for a wider range of small asteroids, at a different speed, to be used to reach the desired goal. Contrary to when the satellite has a velocity of 166 km/s when the satellite is at small or close to the desired value, then completing a few, such as 5 or 6, maneuvers can bring the satellite down by a couple of m/s or km/s.

The table above shows the number of maneuvers required to decrease the satellite's speed to the minimum by completing the same maneuver multiple times. This means that for each maneuver the same angle of approach and asteroid velocity is used. This means that a satellite will need to perform the maneuver 13 times with an asteroid moving at 7 km/s, with an angle of attack of 0° . The final satellite velocity will not always be zero but if the maneuver is done one more time, then the final direction of the satellite will be opposite to what is desired.

Table 4.1 also shows that when the asteroid's velocity is high the amount of maneuver required is decreases. The number of maneuvers isn't a steady decrease from right to left or up to down, this is because the final velocity isn't the same for all of them. This means that some of the combinations that have more maneuvers may have lower final velocities than others.

The same thing is done with trajectory 2 but instead of doing until the satellite velocity reaches its minimum, the satellite will complete a fixed number of maneuvers and the final change in velocity will be shown. This is more important if the goal is to adjust the satellite's speed, and since trajectory 2 provides only a small amount of deceleration, it would take an eternity to bring the satellite's velocity to its minimum.

The number of maneuvers used is a thousand, which is a larger number but that is with a small asteroid that can be found in the hundreds of thousands in the asteroid belt.

Table 4.2

Change in Velocity after 1000 Maneuvers with 1 km Asteroid.

		Angle of Approach (degrees)									
		4	8	12	16	20	24	28	32	36	40
Asteroid Velocity (km/s)	7	2.13e-11	9.28e-11	2.08e-10	3.70e-10	5.77e-10	8.24e-10	1.11e-09	1.43e-09	2.13e-11	9.28e-11
	8	2.13e-11	8.88e-11	1.98e-10	3.50e-10	5.47e-10	7.81e-10	1.05e-09	1.36e-09	2.13e-11	8.88e-11
	9	2.13e-11	8.38e-11	1.88e-10	3.32e-10	5.18e-10	7.42e-10	1.00e-09	1.30e-09	2.13e-11	8.38e-11
	10	2.13e-11	7.81e-11	1.77e-10	3.16e-10	4.93e-10	7.06e-10	9.55e-10	1.24e-09	2.13e-11	7.81e-11
	11	1.87e-11	7.51e-11	1.70e-10	3.01e-10	4.68e-10	6.73e-10	9.13e-10	1.18e-09	1.87e-11	7.51e-11
	12	1.79e-11	7.10e-11	1.63e-10	2.87e-10	4.47e-10	6.43e-10	8.73e-10	1.13e-09	1.79e-11	7.10e-11
	13	1.79e-11	6.99e-11	1.56e-10	2.76e-10	4.29e-10	6.18e-10	8.38e-10	1.09e-09	1.79e-11	6.99e-11
	14	1.59e-11	6.62e-11	1.49e-10	2.64e-10	4.12e-10	5.92e-10	8.04e-10	1.04e-09	1.59e-11	6.62e-11
	15	1.61e-11	6.36e-11	1.42e-10	2.5e-10	3.97e-10	5.69e-10	7.73e-10	1.00e-09	1.61e-11	6.36e-11
	16	1.45e-11	6.16e-11	1.37e-10	2.44e-10	3.82e-10	5.47e-10	7.46e-10	9.73e-10	1.45e-11	6.16e-11
	17	1.42e-11	5.72e-11	1.31e-10	2.37e-10	3.67e-10	5.29e-10	7.20e-10	9.39e-10	1.42e-11	5.72e-11
	18	1.42e-11	5.68e-11	1.27e-10	2.2e-10	3.55e-10	5.11e-10	6.96e-10	9.07e-10	1.42e-11	5.68e-11
	19	1.42e-11	5.35e-11	1.22e-10	2.20e-10	3.42e-10	4.93e-10	6.73e-10	8.76e-10	1.42e-11	5.35e-11
	20	1.42e-11	5.29e-11	1.177e-10	2.12e-10	3.33e-10	4.77e-10	6.49e-10	8.50e-10	1.42e-11	5.29e-11

The Table above, shows the total change in velocity of a thousand maneuvers with an asteroid size of 1 km and the initial satellite velocity of 10km/s. As to be expected even after such a large amount of maneuvers the velocity has changed by a few $\mu\text{m/s}$ or less. This makes the use of a 1 km asteroid impossible, since increasing the amount of maneuver will only increase the mission time and risk while providing little to no change.

Table 4.3

Change in Velocity after 1000 Maneuvers with 10 m Asteroid.

		Angle of Approach (degrees)									
		4	8	12	16	20	24	28	32	36	40
Asteroid Velocity (km/s)	7	4.061e-05	0.00016	0.00036	0.00064	0.00099	0.00142	0.00191	0.00246	0.00307	0.00372
	8	3.884e-05	0.00015	0.00034	0.00061	0.00095	0.00136	0.00183	0.00236	0.00295	0.00359
	9	3.726e-05	0.00014	0.00033	0.00059	0.00091	0.00130	0.00176	0.00227	0.00284	0.00346
	10	3.579e-05	0.00014	0.00032	0.00056	0.00088	0.00125	0.00169	0.00219	0.00274	0.00334
	11	3.449e-05	0.00013	0.00030	0.00054	0.00084	0.00121	0.00163	0.00212	0.00265	0.00323
	12	3.317e-05	0.00013	0.00029	0.00052	0.00081	0.00117	0.00158	0.00204	0.00256	0.00313
	13	3.194e-05	0.00012	0.00028	0.00050	0.00078	0.00113	0.00152	0.00198	0.00248	0.00304
	14	3.084e-05	0.00012	0.00027	0.00049	0.00076	0.00109	0.00147	0.00191	0.00241	0.00295
	15	2.980e-05	0.00011	0.00026	0.00047	0.00073	0.00105	0.00143	0.00186	0.00233	0.00286
	16	2.885e-05	0.00011	0.00025	0.00045	0.00071	0.00102	0.00138	0.00180	0.00227	0.00278
	17	2.7974e-05	0.00011	0.00025	0.00044	0.00069	0.00099	0.00134	0.00175	0.00220	0.00270
	18	2.7129e-05	0.00010	0.00024	0.00043	0.00067	0.00096	0.00131	0.00170	0.00214	0.00263
	19	2.6328e-05	0.00010	0.00023	0.00041	0.00065	0.00093	0.00127	0.00165	0.00208	0.00256
	20	2.5576e-05	0.00010	0.00022	0.00040	0.00063	0.00091	0.00123	0.00161	0.00203	0.00250

When comparing the results from Table 4.3 with Table 4.2, it is clear that using a smaller asteroid is much more beneficial even after a thousand maneuvers. Although the magnitude of the change is small in comparison to what is possible using trajectory 1, the multitude of maneuvers with a very small asteroid is much more probable and during the maneuvers, the satellite can be collecting data. By combining the two types of maneuvers to bring the satellite to the desired velocity is possible but may require time and some extra assistance. The table below has for goal to bring the satellite velocity down to 20 km/s from 166 km/s, this would simulate a missing where one of the goals is to match an

asteroid speed to observe it longer.

Table 4.4

Targeted velocity Complex Maneuver Configuration.

Final velocity: 19.9987 km/s	Trajectory 1	Trajectory 2
Number of maneuvers	5	554
Angle of Approach (degrees)	15.64°, 7.158°, 12.16°, 40.548°, 26.614°	25
Asteroid Velocity (km/s)	20.615, 16.2, 18.758, 15.36, 12.45	5
Asteroid Radius (km)	500, 250, 100, 10, 50	0.1

Table 4.5

Targeted velocity Complex Maneuver Configuration.

Final velocity: 19.9997 km/s	Trajectory 1	Trajectory 2
Number of maneuvers	6	837
Angle of Approach (degrees)	19.64°, 26.158°, 32.16°, 40.548°, 26.614°, 31.264°	35
Asteroid Velocity (km/s)	20.615, 16.2, 18.758, 15.36, 12.45, 7.26	6.2
Asteroid Radius (km)	500, 250, 100, 10, 50, 362	0.2

Table 4.6

Targeted velocity Complex Maneuver Configuration.

Final velocity: 19.9998 km/s	Trajectory 1	Trajectory 2
Number of maneuvers	4	472
Angle of Approach (degrees)	5.314°, 1.36°, 10.05°, 9.351°	36.53
Asteroid Velocity (km/s)	20.615, 16.2, 19.758, 17.36	4.65
Asteroid Radius (km)	500, 250, 100, 10	0.152

Tables 4.4 to 4.6 show some of the combinations of maneuvers possible to reach the desired final velocity. The number of maneuvers will change greatly based on the starting

velocity as well as the precision of the final value. Trajectory 2 is mostly used to remove 1 or 2 km/s and as such if the targeted speed has to fall in a range instead of a specific value, then the use of maneuver using the second trajectory may not even be needed. This combination of maneuvers is more effective than using a single maneuver multiple times and also has more application depending on the final goal. Although these maneuvers may take time, using that time to observe the asteroid while doing the maneuver may provide more benefit and better use of resources.

4.4. Hyperbolic Path Possibility

To be able to find if the maneuver is possible, the first part is to find the sphere of influence of the asteroid. This will indicate the range in which the maneuver can be performed. A body's gravitational field is related to its mass but also its distance from the center of the field it belongs. All asteroids in the simulation are in the influence of the sun and are not impacted by the other asteroid or celestial bodies around. In this simulation, the asteroid is in the asteroid belt found in the solar system and any asteroid can present anywhere in the belt's boundary. Figure 4.4.1 shows the distance from the surface of the asteroid to the edge of its sphere of influence for every radius and density in the inner edge of the belt. This is the closest distance between the sun and the belt, this is also the smallest field the asteroid can have.

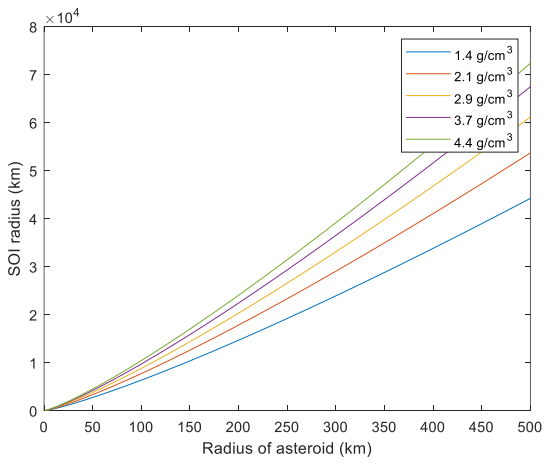


Figure 4.4.1 Sphere of Influence at the Inner Edge of Asteroid Belt.

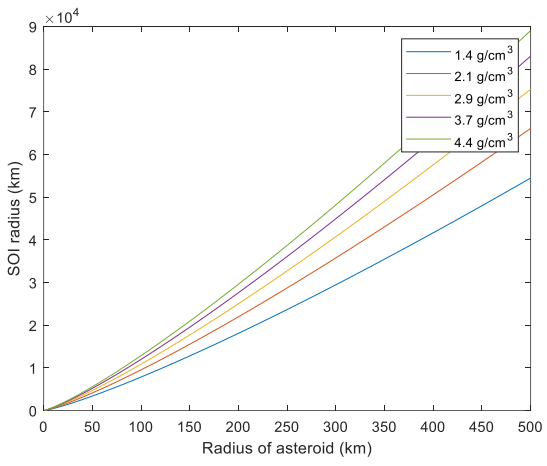


Figure 4.4.2 Sphere of Influence in the Middle of Asteroid Belt.

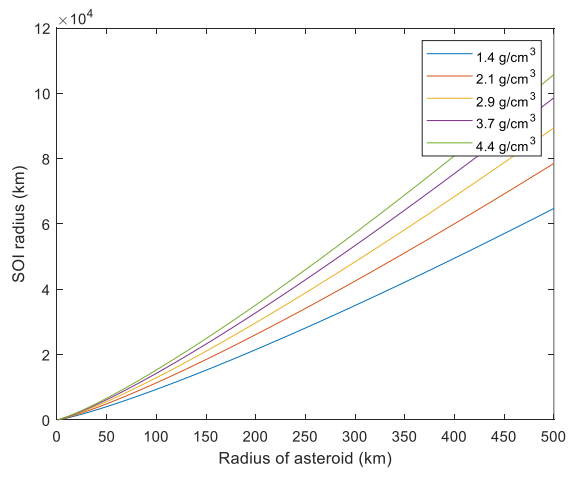


Figure 4.4.3 Sphere of Influence at the Outer Edge of Asteroid Belt.

Figure 4.4.2 is in the middle of the belt, where asteroids are generally located and also where the larger asteroids are located. Figure 4.4.3 is from the farthest distance and as such give the asteroid their largest sphere of influence possible. Asteroids found in the outer edge of the belt can also be attracted by larger bodies such as Jupiter or other asteroids traveling through our solar system.

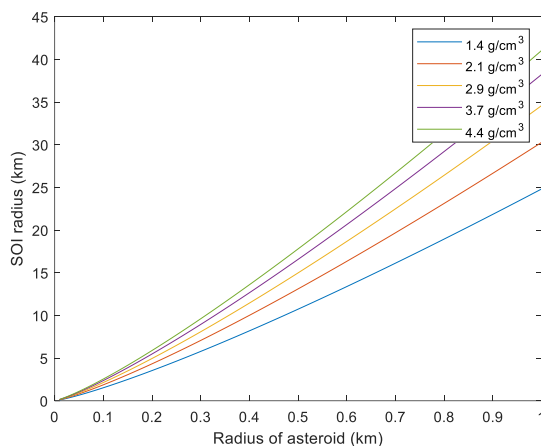


Figure 4.4.4 Sphere of Influence of Small Asteroids at the Inner Edge of Asteroid Belt.

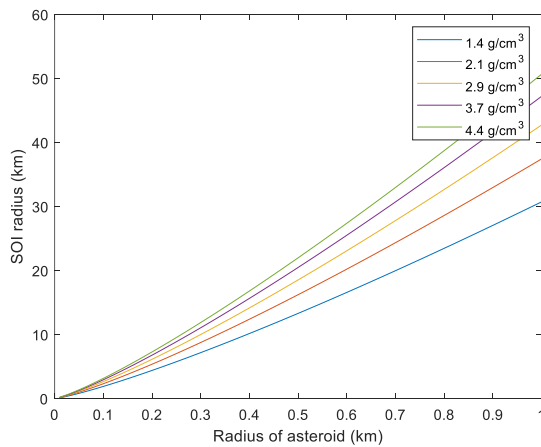


Figure 4.4.5 Sphere of Influence of Small Asteroids in the Middle of Asteroid Belt.

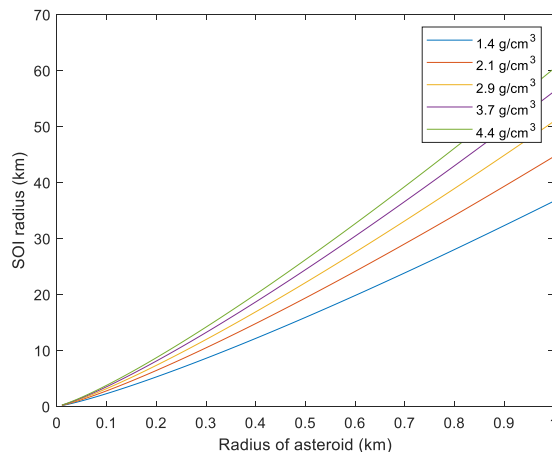


Figure 4.4.6 Sphere of Influence of Small Asteroids at the Outer Edge of Asteroid Belt.

The figures above show the sphere of influence for the smaller asteroids at the same location as the other asteroid. These graphs show that the smaller the asteroid the smaller the field's radius, reducing the range of maneuver possible but also making them more likely to be impacted by the influence of a larger asteroid around. The number of asteroids with a radius of less than 1km is almost impossible to determine since asteroids collide with one another all the time and sometimes stay in the gravitational field of the larger asteroid that surrounds them. Asteroids also have more irregular shapes but if the maneuver can't be done with a spherical asteroid, then the possibility to do it in an irregular gravitational field is even smaller.

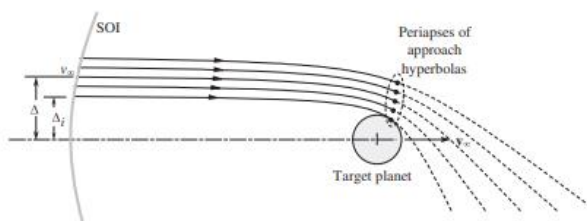


Figure 4.4.7 Different approaches for hyperbolas with different distance from the surface (Curtis, 2021).

Figure 4.4.7, taken from the book *Orbital Mechanics for Engineering Student* by

Howard Curtis, Shows the relation between the distance of the satellite from the surface of the body it is orbiting and the angle of deflection. The figures below should show that the deflection angle increases as the satellite gets farther from the surface. This also meant that if the maneuver isn't possible when the satellite is close to the asteroid, then it would also be impossible at any further distance.

4.4.1. Trajectory 1 Angle Possibilities

The maneuver can be calculated without having to take into account the feasibility of such maneuver around such a small body. In the maneuver simulation the asteroid and satellite can be considered point mass but in reality, if the only way for the satellite to complete this maneuver has it going through the surface of the asteroid, then it becomes impossible.

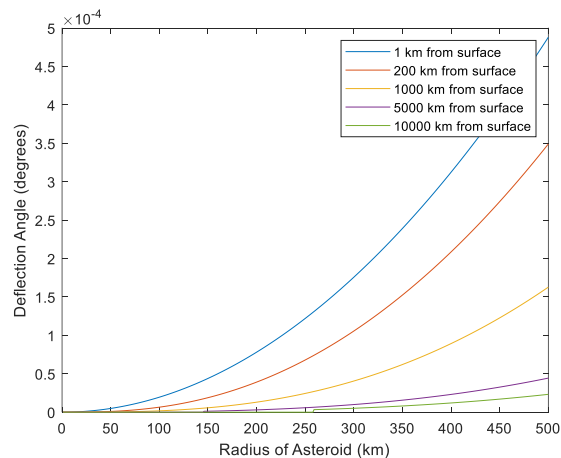


Figure 4.4.8 Deflection Angles Possible with 0.1° Angle of Approach on the Inner Edge of Asteroid Belt.

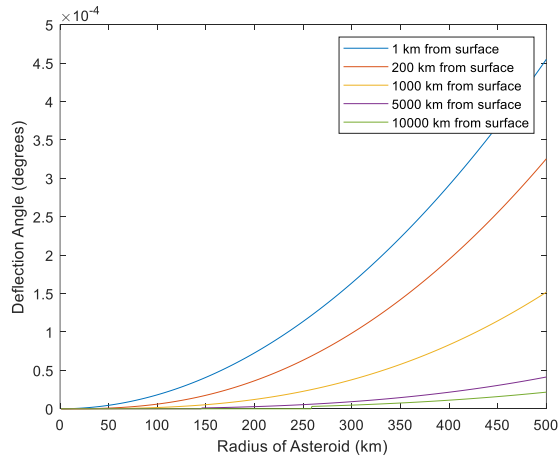


Figure 4.4.9 Deflection Angles Possible with 40° Angle of Approach on the Inner Edge of Asteroid Belt.

Figures 4.4.8 and 4.4.9 show the deflection angles possible for the different asteroids when the satellite velocity is 166 km/s and the asteroid has a speed of 20 km/s. The further out the maneuver is performed the smaller the deflection angle will be, and as such the less likely the maneuver is possible. The distance from the surface of the asteroid to the satellite is to the perigee, this means that it is impossible to use the outer edge of the sphere of influence as the limit because the satellite wouldn't start the maneuver in the sphere of influence. This also explains why as the distance from the surface increases only larger asteroids have a deflection angle. For the maneuver to be possible the deflection angle needs to be equal to,

$$\phi = 180 - 2\theta \quad [4.1]$$

This is from the geometry of the hyperbola trajectory. To be able to understand when and where it is possible to complete the maneuver, different satellite and asteroid velocities need to be simulated at different angles, and in different parts of the asteroid belt.

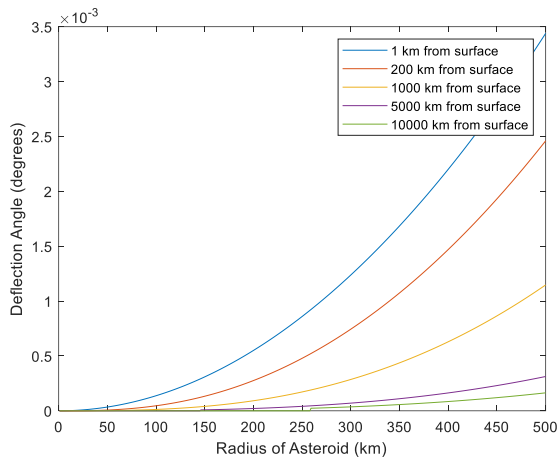


Figure 4.4.10 Deflection Angles Possible with 0.1° Angle of Approach on the Inner Edge of Asteroid Belt.

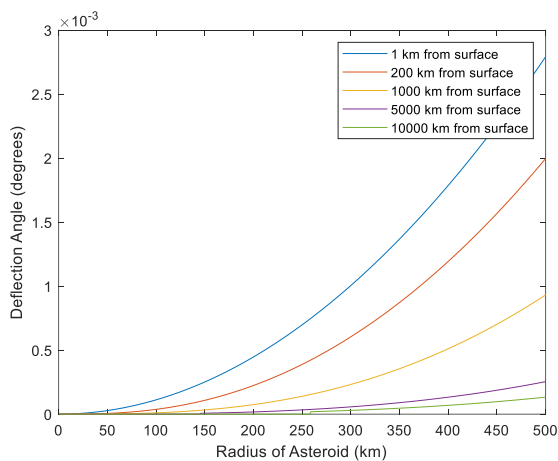


Figure 4.4.11 Deflection Angles Possible with 40° Angle of Approach on the Inner Edge of Asteroid Belt.

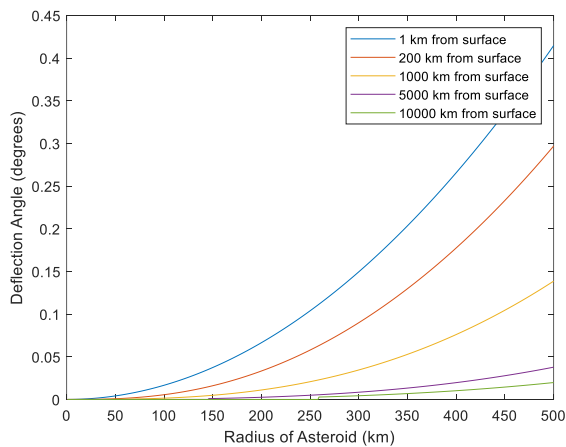


Figure 4.4.12 Deflection Angles Possible with 0.1° Angle of Approach on the Inner Edge of Asteroid Belt.

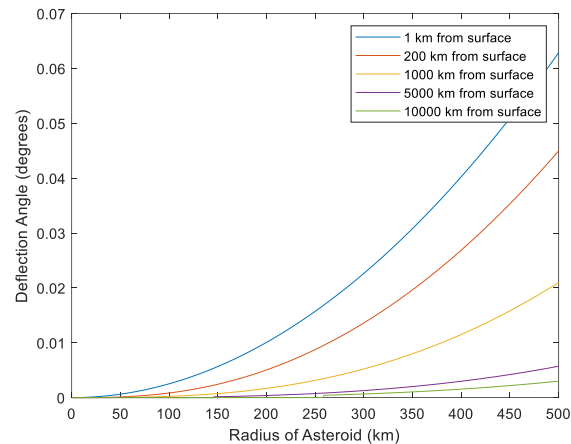


Figure 4.4.13 Deflection Angles Possible with 40° Angle of Approach on the Inner Edge of Asteroid Belt.

Figures 4.4.10 and 4.4.11 show the deflection angle at the same asteroid velocity and the same location in the asteroid belt but with a velocity of 75 km/s. In these graphs, the deflection angle is still very small but it shows that as the satellite's velocity gets closer to the asteroid's velocity, the greater the maximum deflection angle is. Figures 4.4.12 and 4.4.13 show the same thing but with a satellite velocity of 15 km/s.

The graphs also show very clearly that the smaller the angle of approach the greater the deflection angle, this is related to the relative satellite velocity in the asteroid frame. The greater the angle of approach the greater the satellite velocity in the asteroid frame and as such the smaller the deflection angle possible.

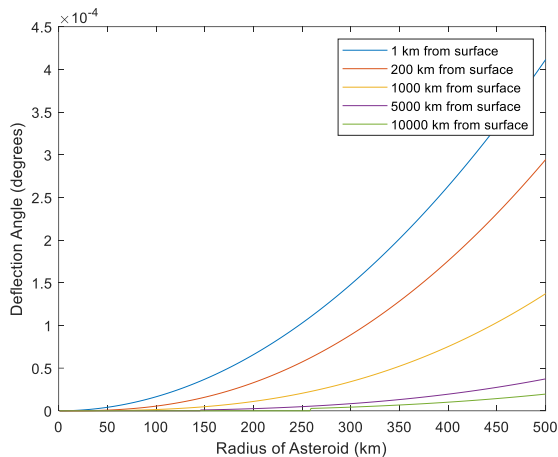


Figure 4.4.14 Deflection Angles Possible with 0.1° Angle of Approach on the Inner Edge of Asteroid Belt.

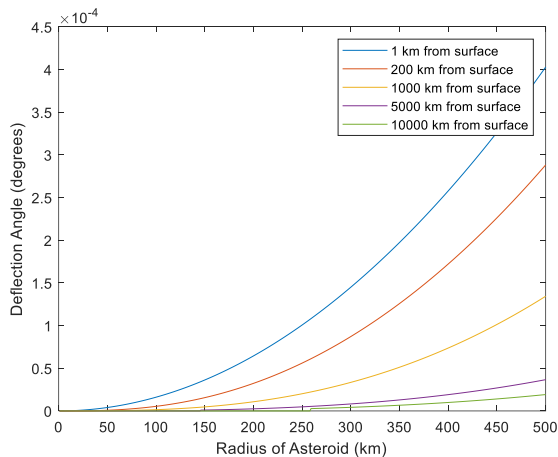


Figure 4.4.15 Deflection Angles Possible with 40° Angle of Approach on the Inner Edge of Asteroid Belt.

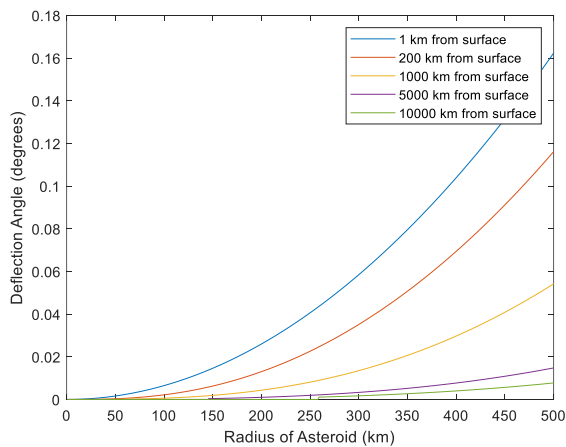


Figure 4.4.16 Deflection Angles Possible with 0.1° Angle of Approach on the Inner Edge of Asteroid Belt.

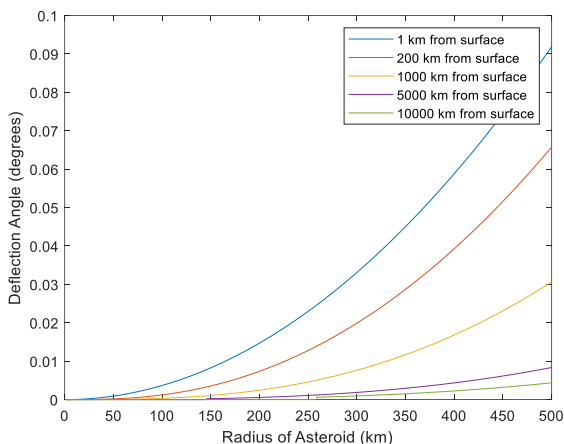


Figure 4.4.17 Deflection Angles Possible with 40° Angle of Approach on the Inner Edge of Asteroid Belt.

Figures 4.4.14 and 4.4.15 show the deflection angle with a satellite speed of 166 km/s but the asteroid speed is only 7 km/s. This is similar in Figures 4.4.16 and 4.4.17 but with a satellite velocity of only 15 km/s. The graphs show that the smaller the relative velocity is the larger the deflection angle, but it is also better to have a lower asteroid velocity.

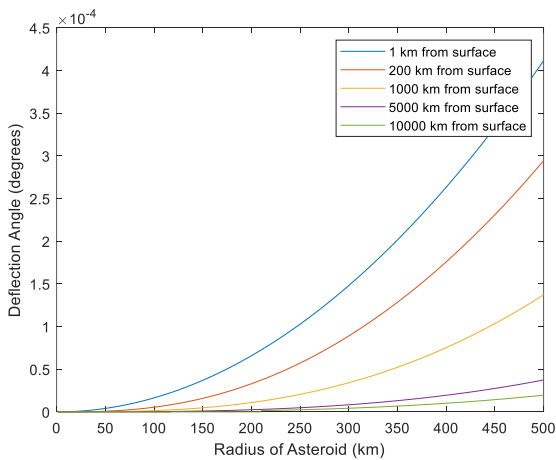


Figure 4.4.18 Deflection Angles Possible with 0.1° Angle of Approach in the Middle of Asteroid Belt.

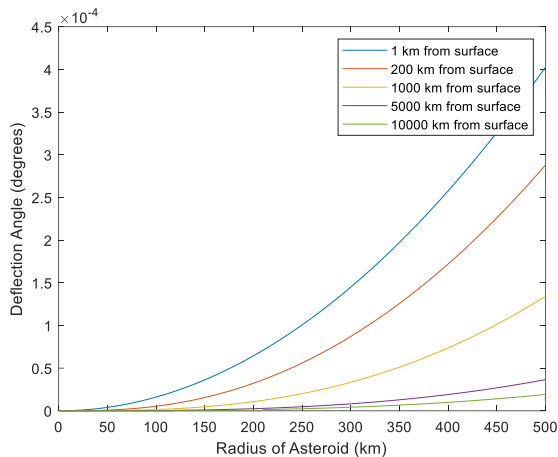


Figure 4.4.19 Deflection Angles Possible with 40° Angle of Approach in the Middle of Asteroid Belt.

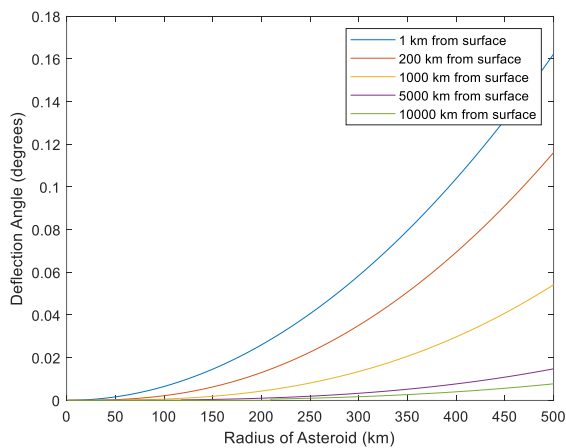


Figure 4.4.20 Deflection Angles Possible with 0.1° Angle of Approach in the Middle of Asteroid Belt.

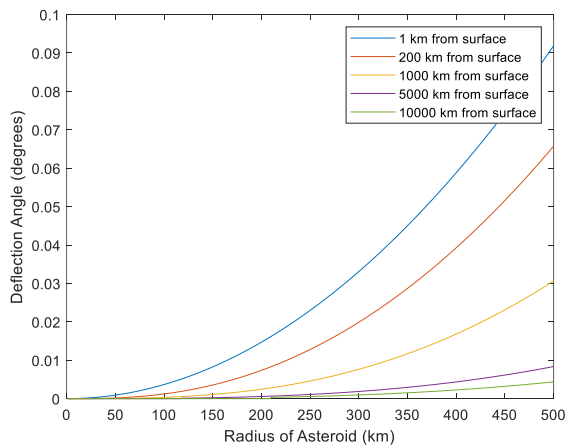


Figure 4.4.21 Deflection Angles Possible with 40° Angle of Approach in the Middle of Asteroid Belt.

Figures 4.4.18 - 4.4.21, show the maneuver in the middle of the asteroid belt and with an asteroid velocity of 7 km/s, Figures 4.4.18 and 4.4.19 are with a satellite velocity of 166 km/s and the other two are with a velocity of 15km/s. When comparing with the asteroid in the inner edge of the asteroid belt, there are little to no changes with the deflection angle. The difference in the asteroid belt only allows smaller asteroid to have a larger sphere of influence allowing more maneuver to be performed, this means that it has no impact on maneuvers done close to the surface of the asteroid.

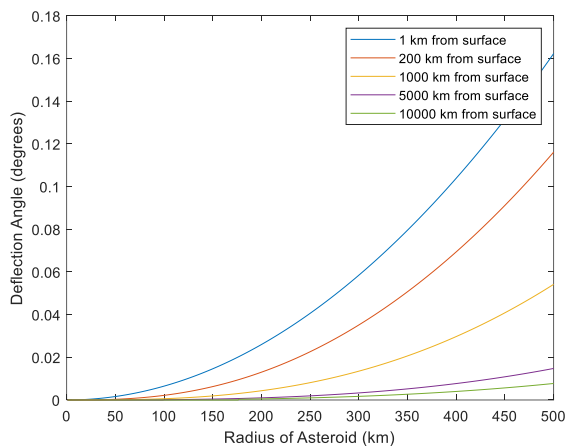


Figure 4.4.22 Deflection Angles Possible with 0.1° Angle of Approach on Outer Edge of Asteroid Belt.

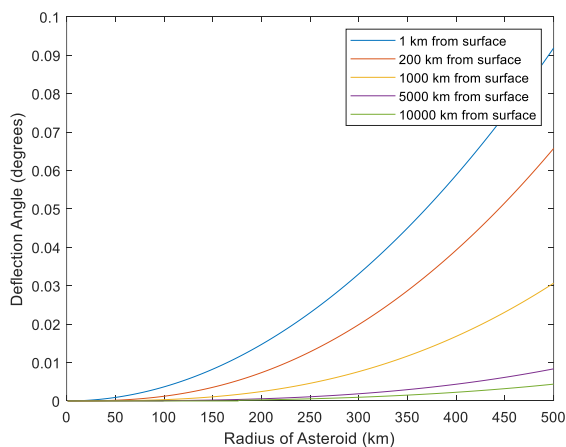


Figure 4.4.23 Deflection Angles Possible with 40° Angle of Approach on Outer Edge of Asteroid Belt.

Figures 4.4.22 and 4.4.23 look almost identical as Figures 4.4.20 and 4.4.21 but are of two different locations in the asteroid belt. This means that anywhere in the asteroid belt the largest approach angle is obtained when the satellite passes near the asteroid surface. Since small asteroids all have small to no angle of deflection in the figure above, it is expected to be similar with even smaller asteroids even at small satellite velocity.

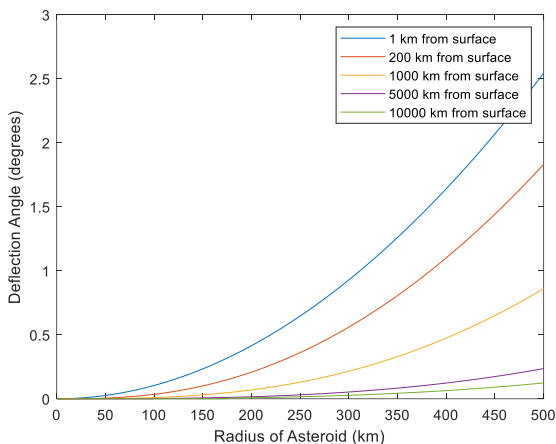


Figure 4.4.24 Deflection Angles Possible with 0.1° Angle of Approach on Outer Edge of Asteroid Belt.

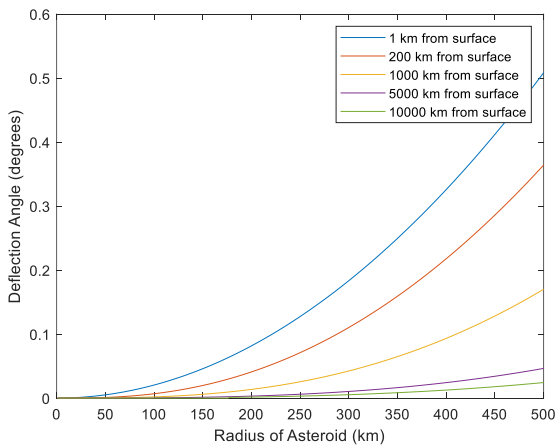


Figure 4.4.25 Deflection Angles Possible with 40° Angle of Approach on Outer Edge of Asteroid Belt.

Figures 4.4.24 and 4.4.25 represent the maneuver with an asteroid velocity of 7 km/s and a satellite velocity of 5 km/s. This represents the small satellite velocity maneuver

and also show that the largest deflection angle achievable is with low satellite velocity.

4.4.2. Trajectory 2 Angle Possibilities

Contrary to trajectory 1, where the deflection angle needs to be large for the maneuver to be possible, in trajectory 2 the deflection angle needs to be small. This doesn't mean it needs to be to the power of 10^{-2} or lower but between 0.1 and 35. This would mean that the maneuver is possible and that the satellite will not hit the surface of the asteroid or miss the sphere of influence.

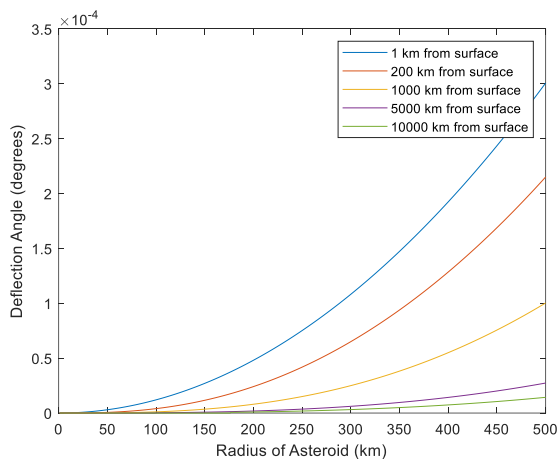


Figure 4.4.26 Deflection angle for High velocity at 0.1° Angle of Approach.

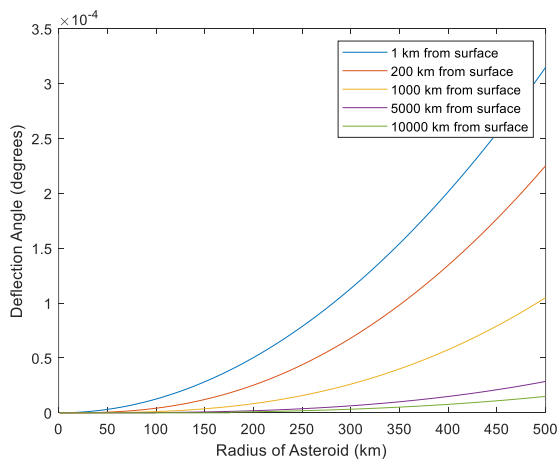


Figure 4.4.27 Deflection angle for High velocity at 40° Angle of Approach.

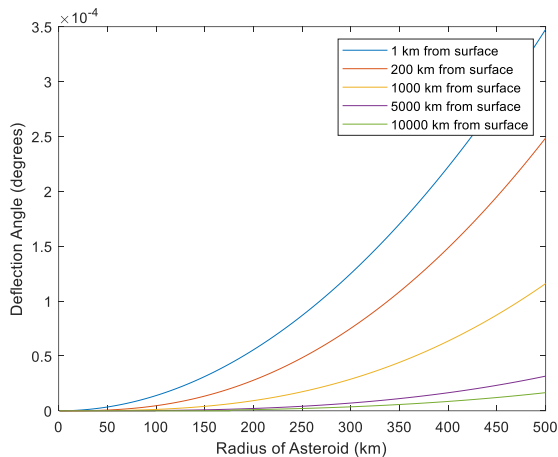


Figure 4.4.28 Deflection angle for High velocity at 0.1° Angle of Approach.

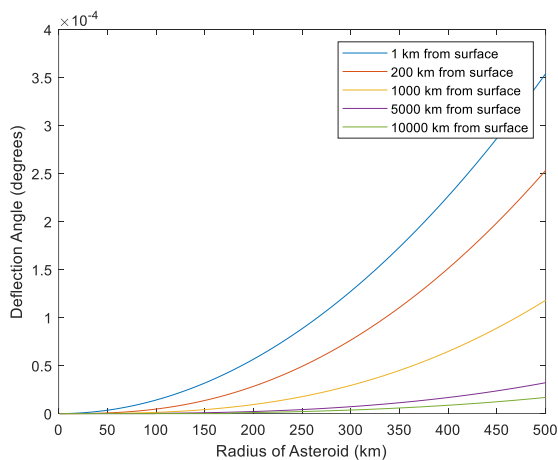


Figure 4.4.29 Deflection angle for High velocity at 40° Angle of Approach.

For Figures 4.4.26 to 4.4.29, the satellite velocity is kept constant at 166 km/s and the distance from the sun is kept at the outer edge of the asteroid belt. For Figures 4.4.26 and 4.4.27, the asteroid has a speed of 20 km/s and 7 km/s for the other two graphs. As seen with trajectory 1 the location of the asteroid in the asteroid belt offers only a small change, and as such, all simulations for trajectory 2 are at the same distance from the sun.

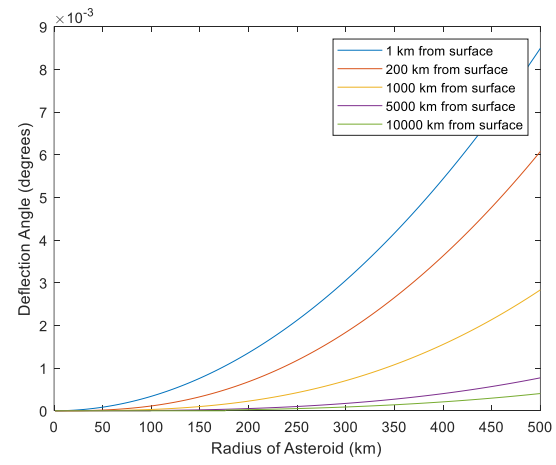


Figure 4.4.30 Deflection angle for Satellite Escape velocity at 0.1° Angle of Approach.

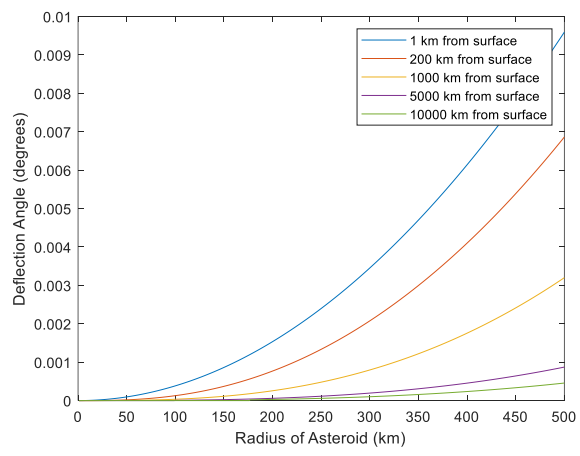


Figure 4.4.31 Deflection angle for Satellite Escape velocity at 40° Angle of Approach.

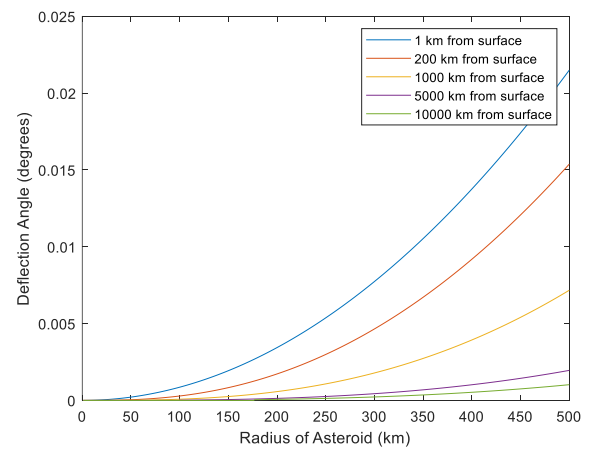


Figure 4.4.32 Deflection angle for Satellite Escape velocity at 0.1° Angle of Approach.

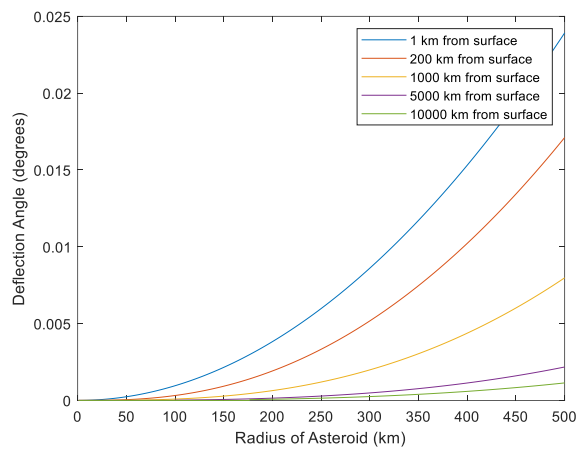


Figure 4.4.33 Deflection angle for Satellite Escape velocity at 40° Angle of Approach.

The figures above are with a satellite velocity of 15 km/s, and once again the first two are with an asteroid velocity of 20 km/s and the last two of 7 km/s. The figures show that the deflection angle of the trajectory when the asteroid velocity is small, then the maneuver is possible only with the larger asteroid and with a perigee of 1 km or less.

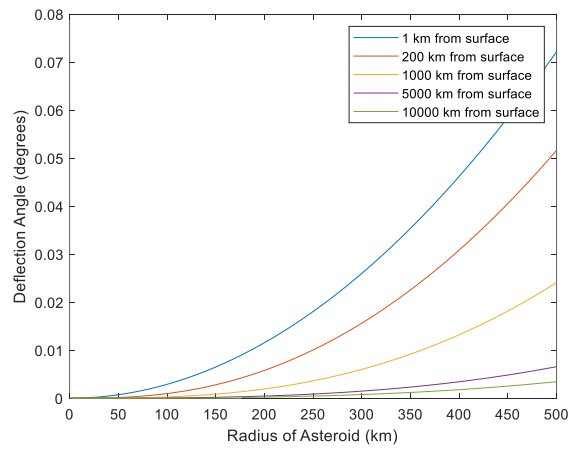


Figure 4.4.34 Deflection angle for Low velocity at 0.1° Angle of Approach.

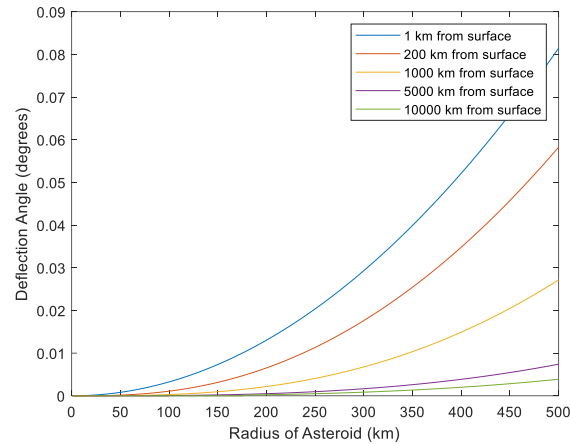


Figure 4.4.35 Deflection angle for Low velocity at 40° Angle of Approach.

The final trajectory configuration is by having the satellite velocity low and keeping the asteroid velocity at 7km/s. This is shown in the figures above when the satellite velocity is 2 km/s. Competing for a complex maneuver or a fly by maneuvering around something as small as an asteroid is much more complicated because the gravitational force is much weaker. Gravitational maneuvers are mostly done on planet size bodies because their gravitation force allows for any combination of flight paths and orbits. The asteroid may be faster and greater in number but individually, are more restricted because of their weaker attraction forces. Trajectory 1 can be completed with an asteroid but only for a larger asteroid of more than 300 km in radius and the perigee needs to be around 1km or less. The speed of the asteroid also needs to be as low as possible and the satellite's velocity needs to be closer to 10 km/s or lower.

5. Conclusion and Discussion

The maneuvers will provide a deceleration to the satellite but with a different amount. The feasibility of the maneuver shows that the asteroid doesn't meet the criteria for the maneuver to be completed successfully. There is still much more that needs to be done with different parameters, but some applications can be found from these simulations.

5.1. Application of Proof

The use of orbital maneuvers to slow down a satellite is theoretically possible but doesn't necessarily meet the criteria to physically complete the maneuver. The reverse slingshot maneuver will provide a deceleration as long as the mass of the asteroid is greater than the satellite. There are no satellites currently in space that would have a greater mass than the asteroid used in this simulation, around which a maneuver can be performed. The fly-by maneuver can easily be accomplished and also slow down the satellite, but the amount is very small and large angle of approach are required. After looking at these two different trajectories, that will provide a change in velocity, one was found to provide the larger deceleration and the other was easier to complete and can be done in succession.

The first trajectory was the reverse slingshot that provided the greatest deceleration. Although the size and density of the asteroid were varied, this provided little to no change because the mass difference was too large even with an asteroid of 1 km in radius and with a density of 1.3 g/cm³. This shows the difference in magnitude everything in space is, similarly the speed was measured in km/s. To find out how small an asteroid needed to be for the mass difference to have an impact on the change in velocity, the smallest radius used was 10 m. At such size the maneuver was influenced by the mass of the asteroid,

however as the radius increased to 50 m, then the result started to converge. This meant that any asteroid with a mass similar to the mass of 100 m radius would ignore the mass difference influence with any satellite. The current largest satellite is the International Space Station (ISS) and the mass difference between it and a spherical asteroid of 100 m radius would be so small that it can be considered negligible.

With the mass of the asteroid providing no difference. The speed of the asteroid and the angle of approach become the next major factor to influence the deceleration. The speed of the asteroid proved to be the most important factor to maximize the change in velocity, the angle of approach provides a better precision and flexibility. This means that by changing the angle of approach it is possible to have a small change in velocity even with the fastest asteroid. As the range of asteroid velocity was from 7 km/s to 20 km/s, this also means that if the satellite's velocity was lower than the asteroid, there was the possibility to inverse the satellite's final direction. The final magnitude of the velocity may be lower than what it was initially but if the final orientation is just as important then a different maneuver may need to be used at lower velocities.

The second trajectory resembles more a fly-by trajectory, this maneuver didn't provide a significant change in velocity and was more effective with smaller asteroids. The mass difference played a much more important role in trajectory 2, as it reduces the change in velocity significantly. When the maneuver was performed with a significant mass difference, the change in speed was measured in micrometer per second, on the other hand, the smaller the asteroid mass the greater the change in velocity. This was even more significant when the satellite's speed was closer to the asteroid speed or when the satellite was slower.

Contrary to the first trajectory, the slower asteroid provided a greater change in velocity and as the angle of approach increased so did the deceleration. This shows a result that is the opposite of the first trajectory but at the same time is a good thing. This means that with a combination of the two trajectories any final velocity can be achieved and with multiple different flight path combinations. The second trajectory also works better with an asteroid that has the mass of a 10 m radius asteroid, asteroids of that size and mass are in greater number than larger bodies in the asteroid belt, meaning that finding asteroids in a different location in the belt shouldn't be as difficult. Although the fly-by maneuver has its advantages, the change in velocity that it provides can be in $\mu\text{m/s}$ or m/s . To be able to change the satellite's speed by a few km/s the maneuver may need to be done a few hundred or even a thousand times, depending on the asteroid size, speed, and angle of approach.

By combining the two different trajectories and using different asteroid speeds and angles of approach it is possible to reach a desired final velocity regardless of the starting velocity. One of the drawbacks is that completing the first maneuver multiple times in succession is impossible. All asteroids in the belt rotate in the same direction and for the reverse slingshot to be completed the satellite needs to move in the same direction as the asteroid and after the maneuver will be going the opposite direction. This is useful in completing the flyby maneuver, as the asteroid and satellite need to be going in the opposite direction, but to complete multiple reverse slingshots, the satellite would need to change its direction after each maneuver. This can be done by performing a slingshot maneuver around a slower asteroid that would provide a lower gain than what was previously lost, or by using thrusters or other mechanical assists on the satellite.

Even if mathematically these maneuvers provide a change in the satellite's velocity the maneuver still needs to be physically possible. This is done by finding out the deflection angle of a hyperbola, this also makes sure that the satellite will not hit the surface of the asteroid as it performs the maneuver. As the results show, for the reverse slingshot maneuver, the asteroid is too small for the maneuver to be done. The satellite would need to go through the asteroid, which is impossible, or the asteroid needs to be bigger or faster. This means that the maneuver is possible for moons or planets, and could be possible with an asteroid passing through our solar system at higher velocities. The result also shows that even a normal slingshot maneuver wouldn't be possible for the asteroid in question.

The second trajectory is possible at a small angle of approach and with an asteroid that has a velocity that is small or close to the satellites. The maneuvers with a large angle of approach are not possible as the satellite would need to go through the asteroid. For the few angles of approach at which the maneuver is possible the satellite would need to be very close to the surface of the asteroid and as such this would increase the risk during the maneuver. For every maneuver and asteroid configuration, the greatest angle of deflection is when the satellite is close to the surface, this is only at the perigee of the maneuver and as long as the asteroid's shape isn't completely irregular then this should be possible.

5.2. Future Experiments

The gravitational deceleration using a reverse slingshot maneuver does provide a significant result as long as the celestial body's speed is great enough, the only major difference from the asteroid is that the mass needs to be larger by a few orders of

magnitude. Although the maneuver is not possible or easy to implement, this still provides data and information that can help. The future approach can look at doing the same thing but this time with planets and moons, these bodies should be large enough for the maneuver to be possible and still provide a change in velocity that would be equivalent to a thrust burn. For future missions, with the objective to explore distance systems, the use of this maneuver may be used to slow the probe using a planet or the star of that system. Long-term missions may not always be great for fuel, as they can degrade and corrode the satellite over time, the reverse slingshot can be used to reduce the amount of fuel but would be unable to eliminate the use of fuel all together.

The fly-by trajectory may still be used in the asteroid belt, as it is more effective with small asteroids. There is still a lot to learn and discover by observing asteroids, future missions have goals to land on an asteroid and collect samples to bring back, others have tried to observe then to understand their structure and relationship with the creating of planets. There will always be a mission to the asteroid belt. Some of these missions may be long-term observation and during these missions, the satellite can use the flyby maneuver to slow down over a long period of time, to finally come back down to earth. Missions that take advantage of the slow deceleration would need minimum fuel, for course adjustment, and be able to continuously send back data.

A single mission that makes use of both trajectories may be impossible but they still have their use separately for different missions. Some more research to see if different shapes and sizes of the asteroid may also slow down a satellite, because of their irregular gravitation field. Some may also try to find maneuvers that can be done around Ceres and Vesta, and what the conditions to do them are. When completing these maneuvers, the

influence of the satellite on the larger body is neglected, but the larger satellite may have an impact on an asteroid, from a gravitation field, or will complete the maneuver.

Asteroid still have a lot of uses and application that have yet to be discovered, and this is a small part of the answer to these mysteries.

Some works have touched on these topics, but much more is still to be discovered to be able to freely move around in space. Space travel is still very expensive and innovations have been made on earth to drive these costs down, but there may also be other ways to save money in space.

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