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## TESTING LORENTZ SYMMETRY WITH GRAVITY

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In this talk, results from the gravitational sector of the Standard-Model Extension (SME) are discussed. The weak-field phenomenology of the resulting modified gravitational field equations is explored. The application of the results to a variety of modern gravity experiments, including lunar laser ranging, Gravity Probe B, binary pulsars, and Earth-laboratory tests, shows promising sensitivity to gravitational coefficients for Lorentz violation in the SME.

### 1. Introduction

At the present time, a comprehensive and successful description of nature is provided by general relativity and the Standard Model of particle physics. It is expected, however, that a single underlying unified theory would merge them at the Planck scale. To date, a completely satisfactory theory remains elusive. Experimental clues about this underlying theory are lacking since direct measurements at the Planck scale are infeasible at present.

An alternative approach is to look for suppressed new physics effects coming from the underlying theory that are potentially detectable in modern sensitive experiments. One promising class of signals satisfying this criteria are minuscule violations of Lorentz symmetry.<sup>1</sup> For describing the observable signals of Lorentz violation, the effective field theory known as the Standard-Model Extension (SME) provides a useful tool.<sup>2,3</sup>

Much of the theoretical and experimental work on the SME has involved the the Minkowski-spacetime limit. Experimental studies have included ones with photons<sup>4</sup>, electrons<sup>5</sup>, protons and neutrons<sup>6</sup>, mesons<sup>7</sup>, muons<sup>8</sup>, neutrinos<sup>9</sup>, and the Higgs.<sup>10</sup> Though no compelling evidence for Lorentz violation has been found, only about half of the possible signals involving light and ordinary matter have been experimentally investigated, while

some other sectors remain largely unexplored. The subject of the talk will be a recent SME-based study of gravitational experiments searching for violations of local Lorentz invariance. For a more detailed discussion, the reader is referred to Ref. 11.

## 2. Theory

The gravitational couplings in the SME action are presented in Ref. 3. The geometric framework assumed is a Riemann-Cartan spacetime, allowing for torsion. For simplicity, attention is restricted to the Riemann-spacetime limit. In this limit, the effective action of the pure-gravity minimal SME is written

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} [(1-u)R + s^{\mu\nu} R_{\mu\nu}^T + t^{\kappa\lambda\mu\nu} C_{\kappa\lambda\mu\nu}] + S'. \quad (1)$$

Here  $R$  is the Ricci scalar,  $R_{\mu\nu}^T$  is the trace-free Ricci tensor, and  $C_{\kappa\lambda\mu\nu}$  is the Weyl conformal tensor. The leading Lorentz-violating gravitational couplings are controlled by the coefficients for Lorentz violation  $u$ ,  $s^{\mu\nu}$ , and  $t^{\kappa\lambda\mu\nu}$ . Equation (1) contains 20 independent coefficients, of which 1 is in  $u$ , 9 are in the traceless  $s^{\mu\nu}$ , and 10 are in the totally traceless  $t^{\kappa\lambda\mu\nu}$ .

It is known that explicit Lorentz violation, whereupon the coefficients for Lorentz violation in Eq. (1) are nondynamical functions of spacetime, is generally incompatible with Riemann spacetime.<sup>3</sup> Spontaneous Lorentz violation, however, evades this problem<sup>12</sup> and is the approach adopted to analyze Eq. (1). In this scenario the coefficients  $u$ ,  $s^{\mu\nu}$ , and  $t^{\kappa\lambda\mu\nu}$  are dynamical fields that acquire vacuum expectation values denoted  $\bar{u}$ ,  $\bar{s}^{\mu\nu}$ , and  $\bar{t}^{\kappa\lambda\mu\nu}$ . The general matter action  $S'$  in Eq. (1) therefore includes the dynamics for ordinary matter as well as the coefficients for Lorentz violation.

To construct the field equations associated with the action (1), while taking into account the unknown dynamics of the coefficient fields  $u$ ,  $s^{\mu\nu}$ , and  $t^{\kappa\lambda\mu\nu}$ , represents a challenging theoretical task. In the case of weak-field gravity, however, a set of modified field equations can be obtained under mild assumptions,<sup>11</sup> which then determine the leading corrections to general relativity arising from Lorentz violation. In particular, the dominant terms in the post-newtonian metric can be determined. From the post-newtonian metric an effective classical lagrangian for  $N$  point-like bodies can be derived. This lagrangian provides the basis for studies of orbital experiments probing the coefficients  $\bar{s}^{\mu\nu}$ , while the post-newtonian metric is used to describe experiments probing spacetime geometry.

It is standard to compare a given post-newtonian metric with the

Parametrized Post-Newtonian (PPN) metric.<sup>13,14</sup> It turns out that the match can only be achieved when the SME coefficients  $\bar{s}^{\mu\nu}$  are assumed isotropic in a special coordinate frame, resulting in only one rotational scalar coefficient (taken as  $\bar{s}^{00} = \bar{s}^{jj}$ ) remaining. This isotropic assumption is not generally adopted in SME studies and so the relationship between the SME and the PPN is one of partial overlap.

### 3. Lunar laser ranging

The primary observable in lunar laser ranging experiments are oscillations in the Earth-Moon distance. High sensitivity is achieved by timing laser pulses reflecting off of one or more of the five reflectors on the lunar surface.<sup>15</sup> Appropriate application of the effective classical lagrangian yields the Lorentz-violating corrections to the Earth-Moon coordinate acceleration. Ideally, a computer code would be used that includes the standard dynamics of the Earth-Moon system and effects from the pure-gravity sector of the minimal SME.

It is useful, however, to perform a perturbative analysis that extracts the dominant oscillation frequencies and corresponding amplitudes for Earth-Moon separation oscillations driven by Lorentz violation. The radial corrections  $\delta r$  arising from the Lorentz-violating terms in the acceleration take the generic form

$$\delta r = \sum_n [A_n \cos(\omega_n T + \phi_n) + B_n \sin(\omega_n T + \phi_n)]. \quad (2)$$

The dominant amplitudes are denoted  $A_n$  and  $B_n$  and the corresponding phases are  $\phi_n$ . For example, one oscillation occurs at twice the mean orbital frequency  $\omega$  with amplitudes given by  $A_{2\omega} = -\frac{1}{12}(\bar{s}^{11} - \bar{s}^{22})r_0$  and  $B_{2\omega} = -\frac{1}{6}\bar{s}^{12}r_0$  where  $r_0$  is the mean Earth-Moon distance. The coefficients  $\bar{s}^{11} - \bar{s}^{22}$  and  $\bar{s}^{12}$  are combinations of the standard Sun-centered frame coefficients  $\bar{s}^{JK}$ , and depend on them through angles describing the orbit. This angular dependence indicates that it may be useful to consider artificial satellite orbits of varying orientation, in order to attain sensitivity to coefficients that may elude the lunar orbit.

For lunar laser ranging, at least 5 independent combinations of coefficients for Lorentz violation can be measured. Using standard lunar values and assuming ranging precision at the centimeter level,<sup>15</sup> the estimated experimental sensitivities are parts in  $10^{10}$  on combinations of coefficients in  $\bar{s}^{JK}$  and parts in  $10^7$  on the coefficients  $\bar{s}^{TJ}$ . An analysis studying the dominant Earth-Moon oscillations using 30+ years of data has recently

been performed and has achieved roughly this level of sensitivity.<sup>16</sup> The new Apache Point Observatory Lunar Laser-ranging Operation (APOLLO), may substantially improve these sensitivities.<sup>17</sup>

#### 4. Gyroscope experiment

In general relativity there are two well-known types of precession of the spin of a freely falling test body in the presence of a massive spinning body like the Earth.<sup>18</sup> These two types of spin precession are the geodetic precession about an axis perpendicular to the body's orbit and the gravitomagnetic precession about the spin axis of the Earth. In the context of the pure-gravity sector of the minimal SME there is an additional precession effect that occurs due to Lorentz violation.

Ultimately the dominant measurable effects controlled by the SME coefficients reveal themselves in the secular evolution of the gyroscope spin  $\vec{S}$ , described by  $d\vec{S}/dt = gv_0\vec{\Omega} \times \vec{S}$  where  $g = GM_{\oplus}/r_0^2$  is the mean value of the gravitational acceleration at the orbital radius  $r_0$  and  $v_0$  is the mean orbital velocity. The precession vector  $\vec{\Omega}$  is split into two pieces via  $\Omega^J = \Omega_E^J + \Omega_S^J$ , with the first term containing precession due to conventional effects in general relativity, and the second term containing contributions from the coefficients for Lorentz violation. The latter is given by

$$\Omega_S^J = \frac{9}{8}(\bar{s}^{TT} - \bar{s}^{KL}\hat{\sigma}^K\hat{\sigma}^L)\hat{\sigma}^J + \frac{5}{4}\bar{s}^{JK}\hat{\sigma}^K, \quad (3)$$

where the result is written in the Sun-centered frame,  $\hat{\sigma}$  is a unit vector normal to the orbital plane, and contributions from the Earth's inertia have been suppressed.

The result (3) gives contributions to the precession about the orbital angular momentum axis  $\hat{\sigma}$  and the Earth's spin axis  $\hat{J}$ . In addition, however, there is a qualitatively new precession about the axis defined by  $\hat{n} = \hat{\sigma} \times \hat{J}$ , that is due entirely to Lorentz violation controlled by the  $\bar{s}^{JK}$  coefficients. Data from the Gravity Probe B (GPB) experiment could potentially measure the combinations of coefficients occurring in Eq. (3).<sup>19</sup> If the spin precession vector in three orthogonal directions can be extracted, including the  $\hat{n}$  direction, then attainable sensitivity to  $\bar{s}^{JK}$  coefficients is expected to be at the  $10^{-4}$  level, given GPB projected sensitivities.

#### 5. Binary pulsars

A particularly useful testing ground for general relativity is the binary-pulsar system.<sup>20,13</sup> In particular, such systems contain compact objects and

high orbital velocities which make them appropriate for studies of strong-field gravity. Pulsar timing data from binary pulsar systems also offers the possibility of probing SME coefficients for Lorentz violation.

The Einstein-Infeld-Hoffman (EIH) lagrangian describes the post-newtonian dynamics of such systems and represents a standard approach.<sup>13,21</sup> To obtain the key features arising from Lorentz violation, however, a point-mass approximation suffices and appropriate use can be made of the effective classical lagrangian. The basic orbit can be modeled as a perturbed elliptic two-body problem, where six standard orbital elements are used to describe the orbit:  $a$ ,  $e$ ,  $l_0$ ,  $i$ ,  $\Omega$ , and  $\omega$ . Ultimately a pulsar timing formula is used to model the number of pulses received as a function of arrival time. The timing formula receives modifications due to Lorentz violation from two sources. First, the orbital elements, with the exception of  $a$ , acquire secular Lorentz-violating corrections. Second, the timing formula itself involves an explicit dependence on combinations of coefficients for Lorentz violation.

Some simple estimates of sensitivities reveal that, for example, data from the binary pulsar system PSR 1913 + 16 could yield sensitivities<sup>22</sup> to Lorentz violation at the level of  $\bar{s}_e \lesssim 10^{-9}$  and  $\bar{s}_\omega \lesssim 10^{-11}$  where  $\bar{s}_e$  and  $\bar{s}_\omega$  are the combinations of coefficients relevant for the orbital elements  $e$  and  $\omega$ . These combinations of coefficients will also change with the orientation of the binary pulsar system.

## 6. Other tests

Other types of gravitational experiments have been explored for their merits in probing the SME coefficients  $\bar{s}^{\mu\nu}$ .<sup>a</sup> In particular, Earth-laboratory tests studying gravitational interactions between either two controlled masses or between a test body and the Earth could be used.

One prediction is a newtonian potential between two point masses that is modulated by an anisotropic term  $\hat{x}^j \hat{x}^k \bar{s}^{\hat{j}\hat{k}}$ , where the unit vector  $\hat{x}$  points between the two masses. The SME coefficients  $\bar{s}^{\hat{j}\hat{k}}$  are taken in the Earth-laboratory frame of reference. Although the inverse-distance behavior of the usual newtonian potential is maintained, the associated force is generally misaligned relative to the unit vector  $\hat{x}$ . It is conceivable that experiments studying short-range tests of gravity might be used to probe these coefficients.<sup>23</sup> Currently, an analysis of this type is underway.<sup>24</sup>

<sup>a</sup>Analysis of the classic tests is in Ref. 11.

When considering the effects of the Earth's gravity on test bodies near the surface of the Earth, a modified local gravitational acceleration arises. In the local laboratory frame of reference this acceleration has a vertical ( $\hat{z}$ ) component which is time dependent on sidereal day and year time scales. Experiments with gravimeters are ideally suited for probing such a time variation.<sup>13</sup> An analysis using gravimeter data to extract measurements on combinations of coefficients occurring in this modified acceleration is currently underway.<sup>25</sup> In addition, the local acceleration in the horizontal directions  $\hat{x}$  (south) and  $\hat{y}$  (east) receives modifications from Lorentz violation.

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