Constraints on Violations of Lorentz Symmetry from Gravity Probe B

James M. Overduin  
*Towson University*

Ryan D. Everett  
*Towson University*

Quentin G. Bailey  
*Embry-Riddle Aeronautical University, baileyq@erau.edu*

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We use the final results from Gravity Probe B to set new upper limits on the gravitational sector of the Standard-Model Extension, including for the first time the coefficient associated with the time-time component of the new field responsible for inducing local Lorentz violation in the theory.

The minimal pure-gravity sector of the Standard-Model Extension (SME) is characterized by nine independent coefficients $\bar{s}^{AB}$ corresponding to the vacuum expectation values of a new tensor field whose couplings to the traceless part of the Ricci tensor induce spontaneous violations of local Lorentz symmetry.\textsuperscript{1} These coefficients are assumed to be constant in the asymptotically flat (Minkowski) limit. Most are constrained either individually or in various combinations by existing experiments and observations,\textsuperscript{2} but no limits have yet been placed on the $\bar{s}^{TT}$ coefficient.

Gravity Probe B (GPB) was a satellite experiment launched in 2004 to measure the geodetic and frame-dragging effects predicted by General Relativity (GR). As shown by Bailey and Kostelecky in 2006,\textsuperscript{3} the orientation of a gyroscope in orbit around a spinning central mass like the earth is sensitive to seven of the nine $\bar{s}^{AB}$ coefficients, including $\bar{s}^{TT}$. Following earlier preliminary work,\textsuperscript{4} our goal here is to calculate the resulting constraints using the recently released final results from GPB.\textsuperscript{5}

Within GR the geodetic and frame-dragging precession rates of a gyro-
scope with position $\vec{r}$ and velocity $\vec{\dot{r}}$ in orbit around a central mass $M$ with moment of inertia $I$ and angular velocity $\vec{\omega}$ are:

$$
\vec{\Omega}_{GR}^{g} = \left( \frac{3GM}{2c^2r^3} \right) \vec{r} \times \vec{\dot{r}}, \quad \vec{\Omega}_{GR}^{fd} = \left( \frac{GI}{c^2r^3} \right) \left[ \frac{3}{r^2} (\vec{\omega} \cdot \vec{r}) - \vec{\omega} \right].
$$

Fig. 1. Experimental results are expressed in GPB coordinates ($\hat{e}_{GS}, \hat{e}_{NS}, \hat{e}_{WE}$). Theoretical SME predictions are derived in the $(\hat{n}, \hat{\sigma}, \hat{z})$ system. Both are ultimately referred to Sun-centered inertial coordinates $(\hat{x}, \hat{y}, \hat{z})$, where $\hat{x}$ points toward the vernal equinox.

The combined precession $\vec{\Omega}_{GR} = \vec{\Omega}_{G,GR} + \vec{\Omega}_{fd,GR}$ causes the unit spin vector $\hat{S}$ of the gyroscope to undergo a relativistic drift $\vec{R} \equiv d\hat{S}/dt = \vec{\Omega}_{GR} \times \hat{S}$. Averaging over a circular, polar orbit of radius $r_0$ around a spherically symmetric central mass, one obtains

$$
\vec{R}_{G,GR} = -3(GM)(3)\frac{3/2}{2c^2r_0^{5/2}} \hat{e}_{NS}, \quad \vec{R}_{fd,GR} = -\frac{GI\omega \cos \delta_{GS}}{2c^2r_0^3} \hat{e}_{WE},
$$

where $\hat{e}_{GS}$ points toward the guide star (located in the orbit plane at right ascension $\alpha_{GS}$ and declination $\delta_{GS}$), $\hat{e}_{WE}$ is an orbit normal pointing along the cross-product of $\hat{e}_{GS}$ and the unit vector $\hat{z}$ (aligned with the earth’s rotation axis) and $\hat{e}_{NS}$ is a tangent to the orbit directed along $\hat{e}_{WE} \times \hat{e}_{GS}$ (Fig. 1). The choice of polar orbit orthogonalizes the two effects so that $\vec{R}_{G,GR}$ points entirely along $\hat{e}_{NS}$ and $\vec{R}_{fd,GR}$ points entirely along $\hat{e}_{WE}$.

For GPB with guide star IM Pegasi, $r_0 = 7018.0$ km, $\delta_{GS} = 16.841^\circ$, $R_{G,GR} = 6606.1$ mas/yr (including oblateness) and $R_{fd,GR} = 39.2$ mas/yr where mas=milliarcsecond. The final joint results for all four gyros indicate that $R_{NS,obs} = 6601.8 \pm 18.3$ mas/yr and $R_{WE,obs} = 37.2 \pm 7.2$ mas/yr with 1σ uncertainties. Thus the NS and WE components of relativistic drift rate may deviate from the predictions of GR by at most $\Delta R_{NS} < |R_{G,GR} - R_{NS,obs}| = 22.6$ mas/yr and $\Delta R_{WE} < |R_{fd,GR} - R_{WE,obs}| = 9.2$ mas/yr.
Within the SME, Lorentz-violating terms introduce an additional “anomalous” relativistic drift $\Delta \vec{R}$ whose components along $\hat{n}, \hat{\sigma}$ and $\hat{z}$ are given by Eqs. (158-160) of Ref. 3. Here $\hat{n} \equiv \hat{\sigma} \times \hat{z}$ and $\hat{\sigma} = -\hat{e}_{WE}$ is an orbit normal (Fig. 1). These equations may be expressed in the form

$$\Delta \vec{R} = \left( \begin{array}{c}
\frac{1}{2} \omega_{GS} (\vec{s}^{XY} - \vec{s}^{ZX}) \sin 2\alpha_{GS} + \omega_{GS} \vec{s}^{XY} \cos 2\alpha_{GS} \\
- \omega_{GS} \vec{s}^{XY} \sin 2\alpha_{GS} - \vec{s}^{ZX} \cos 2\alpha_{GS} \\
\omega_{WE} (\vec{s}^{YX} \cos \alpha_{GS} - \vec{s}^{YZ} \sin \alpha_{GS})
\end{array} \right),$$

where $\omega_{GS} = \omega_{WE} = \frac{\alpha}{4} (1 - 3I/5M_r^2) R_{E,GR}$ = 4603 mas/yr, $\omega_{T} = \frac{\alpha}{4} (1 - I/3M_r^2) R_{E,GR}$ = 4503 mas/yr, $\omega_{NS} = \frac{\alpha}{4} (1 + 9I/M_r^2) R_{E,GR}$ = 1904 mas/yr and $\alpha_{GS} = 343.26^\circ$. To transform to GPB coordinates, we reflect across the orbit plane and rotate about $\hat{\sigma}$ by $\delta_{GS}$. The resulting drift rates along the GS, NS and WE axes are

$$\Delta \vec{R} = \left( \begin{array}{c}
\omega_{GS} \frac{1}{2} (\vec{s}^{XY} - \vec{s}^{ZX}) \sin 2\alpha_{GS} + \vec{s}^{XY} \cos 2\alpha_{GS} \cos \delta_{GS} \\
- \vec{s}^{ZX} \sin \alpha_{GS} \sin \delta_{GS} + \vec{s}^{YZ} \cos \alpha_{GS} \sin \delta_{GS} \\
- \omega_{GS} \vec{s}^{XY} \sin 2\alpha_{GS} - \vec{s}^{ZX} \cos 2\alpha_{GS} \\
\omega_{WE} \frac{1}{2} (\vec{s}^{XX} - \vec{s}^{YY}) \sin 2\alpha_{GS} \sin \delta_{GS} - \vec{s}^{YY} \cos 2\alpha_{GS} \sin \delta_{GS} \\
- \vec{s}^{XX} \sin \alpha_{GS} \cos \delta_{GS} + \vec{s}^{YZ} \cos \alpha_{GS} \cos \delta_{GS}
\end{array} \right).$$

Numerically,

$$\Delta R_{GS} = 1215 \vec{s}^{XX} + 3674 \vec{s}^{YY} + 384 \vec{s}^{XZ} - 1215 \vec{s}^{YY} + 1277 \vec{s}^{YZ},$$

$$\Delta R_{NS} = -4503 \vec{s}^{YT} - 158 \vec{s}^{XX} - 1050 \vec{s}^{YY} - 1746 \vec{s}^{YX},$$

$$\Delta R_{WE} = -368 \vec{s}^{XX} - 1112 \vec{s}^{XY} + 1260 \vec{s}^{ZX} + 368 \vec{s}^{YY} + 4219 \vec{s}^{YZ},$$

where $\Delta R_{NS} < 22.6$ and $\Delta R_{WE} < 9.2$ from GPB (all units in mas/yr). The SME can accommodate precessions greater than those predicted by GR, unlike other extensions of the standard model where Einstein’s theory is a limiting case.\(^6\) GPB does not constrain the GS component, since the gyro spin axes point along this direction by design. The GS and WE components are linear combinations of $\vec{s}^{XY}, \vec{s}^{XZ}, \vec{s}^{YZ}$ and $(\vec{s}^{XX} - \vec{s}^{YY})$, so they are superseded in any case by existing constraints, which read:\(^2,7\)

$$|\vec{s}^{XY}| < (0.6 \pm 1.5) \times 10^{-9},$$

$$|\vec{s}^{XZ}| < (2.7 \pm 1.4) \times 10^{-9},$$

$$|\vec{s}^{YZ}| < (0.6 \pm 1.4) \times 10^{-9},$$

$$|\vec{s}^{XX} - \vec{s}^{YY}| < (1.2 \pm 1.6) \times 10^{-9},$$

$$|\vec{s}^{XX} + \vec{s}^{YY} - 2\vec{s}^{XZ}| < (1.8 \pm 38) \times 10^{-9}.$$
Thus in practice the only new GPB constraint on the SME comes from the NS component of Eqs. (5), associated entirely with geodetic precession in standard GR. It reads:

$$|\bar{\eta}^{TT} + 0.035\bar{\eta}^{XX} + 0.23\bar{\eta}^{XY} + 0.39\bar{\eta}^{YY}| < 5.0 \times 10^{-3}. \quad (11)$$

To get seven conditions on seven unknowns, we supplement Eqs. (6-11) with the requirement that $\bar{\eta}^{AB}$ be traceless,

$$|\bar{\eta}^{TT} - \bar{\eta}^{XX} - \bar{\eta}^{YY} - \bar{\eta}^{ZZ}| = 0.3$$

Inverting, we then find that

$$\bar{\eta}^{TT} < 4.4 \times 10^{-3}, \quad \bar{\eta}^{XX}, \bar{\eta}^{YY}, \bar{\eta}^{ZZ} < 1.5 \times 10^{-3}.$$ 

This constitutes the first experimental upper bound on $\bar{\eta}^{TT}$. (Other tests such as light deflection are also sensitive to this coefficient at similar levels of precision.)

It also lifts a degeneracy between other existing limits, allowing us to extract individual upper bounds on $\bar{\eta}^{XX}, \bar{\eta}^{YY}$ and $\bar{\eta}^{ZZ}$.

One should also look at the effect of $\bar{\eta}^{AB}$ on the equation of motion for the gyroscope. This has the effect of rescaling Newton’s gravitational constant $G$, increasing our sensitivity to $\bar{\eta}^{TT}$ and strengthening our limits by about 5%. If the actual orbit is not perfectly circular, as was the case for GPB (whose gyros remained in essentially perfect free fall around a non-spherically symmetric Earth), then additional $\bar{\eta}^{AB}$-dependent terms are also introduced in the leading-order (GR) expressions for geodetic and frame-dragging precession, Eqs. (1). These do not significantly alter the NS or geodetic constraint from GPB, but they do strengthen the WE or frame-dragging constraint so that it may potentially become competitive with existing limits. We will report on these results elsewhere.

References