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CONSTRAINTS ON VIOLATIONS OF LORENTZ SYMMETRY FROM GRAVITY PROBE B

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We use the final results from Gravity Probe B to set new upper limits on the gravitational sector of the Standard-Model Extension, including for the first time the coefficient associated with the time-time component of the new field responsible for inducing local Lorentz violation in the theory.

The minimal pure-gravity sector of the Standard-Model Extension (SME) is characterized by nine independent coefficients $\bar{s}^{\rm AB}$ corresponding to the vacuum expectation values of a new tensor field whose couplings to the traceless part of the Ricci tensor induce spontaneous violations of local Lorentz symmetry.[1] These coefficients are assumed to be constant in the asymptotically flat (Minkowski) limit. Most are constrained either individually or in various combinations by existing experiments and observations,[2] but no limits have yet been placed on the $\bar{s}^{\rm TT}$ coefficient.

Gravity Probe B (GPB) was a satellite experiment launched in 2004 to measure the geodetic and frame-dragging effects predicted by General Relativity (GR). As shown by Bailey and Kostelecky in 2006,[3] the orientation of a gyroscope in orbit around a spinning central mass like the earth is sensitive to seven of the nine $\bar{s}^{\rm AB}$ coefficients, *including* $\bar{s}^{\rm TT}$. Following earlier preliminary work,[4] our goal here is to calculate the resulting constraints using the recently released final results from GPB.[5]

Within GR the geodetic and frame-dragging precession rates of a gyro-

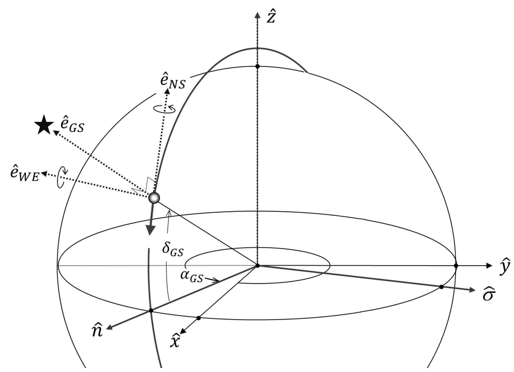

Fig. 1. Experimental results are expressed in GPB coordinates $(\hat{e}_{\rm GS}, \hat{e}_{\rm NS}, \hat{e}_{\rm WE})$. Theoretical SME predictions are derived in the $(\hat{n}, \hat{\sigma}, \hat{z})$ system. Both are ultimately referred to Sun-centered inertial coordinates $(\hat{x}, \hat{y}, \hat{z})$, where $\hat{x}$ points toward the vernal equinox.

scope with position $\vec{r}$ and velocity $\vec{v}$ in orbit around a central mass M with moment of inertia I and angular velocity $\vec{\omega}$ are:

$$\vec{\Omega}_{\rm g,GR} = \left(\frac{3}{2}\frac{GM}{c^2 r^3}\right)\vec{r}\times\vec{v} \quad , \quad \vec{\Omega}_{\rm fd,GR} = \frac{GI}{c^2 r^3}\left[\frac{3\vec{r}}{r^2}(\vec{\omega}\cdot\vec{r}) - \vec{\omega}\right]. \qquad (1)$$

The combined precession $\vec{\Omega}_{\rm GR} = \vec{\Omega}_{\rm g,GR} + \vec{\Omega}_{\rm fd,GR}$ causes the unit spin vector $\hat{S}$ of the gyroscope to undergo a relativistic drift $\vec{R} \equiv d\hat{S}/dt = \vec{\Omega}_{\rm GR} \times \hat{S}$. Averaging over a circular, polar orbit of radius r_0 around a spherically symmetric central mass, one obtains

$$\vec{R}_{\rm g,GR} = -\frac{3(GM)^{3/2}}{2\,c^2 r_0^{5/2}}\,\hat{e}_{\rm NS} \quad , \quad \vec{R}_{\rm fd,GR} = -\frac{GI\omega\cos\delta_{\rm GS}}{2\,c^2 r_0^3}\,\hat{e}_{\rm WE}, \qquad (2)$$

where $\hat{e}_{\rm GS}$ points toward the guide star (located in the orbit plane at right ascension $\alpha_{\rm GS}$ and declination $\delta_{\rm GS}$), $\hat{e}_{\rm WE}$ is an orbit normal pointing along the cross-product of $\hat{e}_{\rm GS}$ and the unit vector $\hat{z}$ (aligned with the earth's rotation axis) and $\hat{e}_{\rm NS}$ is a tangent to the orbit directed along $\hat{e}_{\rm WE} \times \hat{e}_{\rm GS}$ (Fig. 1). The choice of polar orbit orthogonalizes the two effects so that $\vec{R}_{\rm g,GR}$ points entirely along $\hat{e}_{\rm NS}$ and $\vec{R}_{\rm fd,GR}$ points entirely along $\hat{e}_{\rm WE}$.

For GPB with guide star IM Pegasi, $r_0 = 7018.0$ km, $\delta_{\rm GS} = 16.841°$, $R_{\rm g,GR} = 6606.1$ mas/yr (including oblateness) and $R_{\rm fd,GR} = 39.2$ mas/yr where mas=milliarcsecond. The final joint results for all four gyros indicate that $R_{\rm NS,obs} = 6601.8 \pm 18.3$ mas/yr and $R_{\rm WE,obs} = 37.2 \pm 7.2$ mas/yr with 1σ uncertainties.[5] Thus the NS and WE components of relativistic drift rate may deviate from the predictions of GR by at most $\Delta R_{\rm NS} < |R_{\rm g,GR} - R_{\rm NS,obs}|$ $= 22.6$ mas/yr and $\Delta R_{\rm WE} < |R_{\rm fd,GR} - R_{\rm WE,obs}| = 9.2$ mas/yr.

Within the SME, Lorentz-violating terms introduce an additional "anomalous" relativistic drift $\Delta\vec{R}$ whose components along $\hat{n}, \hat{\sigma}$ and $\hat{z}$ are given by Eqs. (158-160) of Ref. 3. Here $\hat{n} \equiv \hat{\sigma} \times \hat{z}$ and $\hat{\sigma} = -\hat{e}_{\rm WE}$ is an orbit normal (Fig. 1). These equations may be expressed in the form

$$\Delta\vec{R} = \begin{pmatrix} \frac{1}{2}\,\omega_{\rm GS}(\overline{s}^{\rm YY} - \overline{s}^{\rm XX})\sin 2\alpha_{\rm GS} + \omega_{\rm GS}\overline{s}^{\rm XY}\cos 2\alpha_{\rm GS} \\ \omega_{\rm T}\overline{s}^{\rm TT} + \omega_{\rm NS}(\overline{s}^{\rm XX}\sin^2\alpha_{\rm GS} - \overline{s}^{\rm XY}\sin 2\alpha_{\rm GS} + \overline{s}^{\rm YY}\cos^2\alpha_{\rm GS}) \\ \omega_{\rm WE}(\overline{s}^{\rm YZ}\cos\alpha_{\rm GS} - \overline{s}^{\rm XZ}\sin\alpha_{\rm GS}) \end{pmatrix}, \quad (3)$$

where $\omega_{\rm GS} = \omega_{\rm WE} = \frac{5}{6}(1 - 3I/5Mr_0^2)\,R_{\rm g,GR} = 4603$ mas/yr, $\omega_{\rm T} = \frac{3}{4}(1 - I/3Mr_0^2)\,R_{\rm g,GR} = 4503$ mas/yr, $\omega_{\rm NS} = \frac{1}{12}(1 + 9I/Mr_0^2)\,R_{\rm g,GR} = 1904$ mas/yr and $\alpha_{\rm GS} = 343.26°$. To transform to GPB coordinates, we reflect across the orbit plane and rotate about $\hat{\sigma}$ by $\delta_{\rm GS}$. The resulting drift rates along the GS, NS and WE axes are

$$\Delta\vec{R} = \begin{pmatrix} \omega_{\rm GS}\big[\frac{1}{2}(\overline{s}^{\rm YY} - \overline{s}^{\rm XX})\sin 2\alpha_{\rm GS}\cos\delta_{\rm GS} + \overline{s}^{\rm XY}\cos 2\alpha_{\rm GS}\cos\delta_{\rm GS} \\ \qquad - \overline{s}^{\rm XZ}\sin\alpha_{\rm GS}\sin\delta_{\rm GS} + \overline{s}^{\rm YZ}\cos\alpha_{\rm GS}\sin\delta_{\rm GS}\big] \\ -\,\omega_{\rm T}\overline{s}^{\rm TT} - \omega_{\rm NS}(\overline{s}^{\rm XX}\sin^2\alpha_{\rm GS} - \overline{s}^{\rm XY}\sin 2\alpha_{\rm GS} + \overline{s}^{\rm YY}\cos^2\alpha_{\rm GS}) \\ \omega_{\rm WE}\big[\frac{1}{2}(\overline{s}^{\rm XX} - \overline{s}^{\rm YY})\sin 2\alpha_{\rm GS}\sin\delta_{\rm GS} - \overline{s}^{\rm XY}\cos 2\alpha_{\rm GS}\sin\delta_{\rm GS} \\ \qquad - \overline{s}^{\rm XZ}\sin\alpha_{\rm GS}\cos\delta_{\rm GS} + \overline{s}^{\rm YZ}\cos\alpha\cos\delta_{\rm GS}\big] \end{pmatrix}. \quad (4)$$

Numerically,

$$\begin{aligned} \Delta R_{\rm GS} &= 1215\overline{s}^{\rm XX} + 3674\overline{s}^{\rm XY} + 384\overline{s}^{\rm XZ} - 1215\overline{s}^{\rm YY} + 1277\overline{s}^{\rm YZ}, \\ \Delta R_{\rm NS} &= -4503\overline{s}^{\rm TT} - 158\overline{s}^{\rm XX} - 1050\overline{s}^{\rm XY} - 1746\overline{s}^{\rm YY}, \\ \Delta R_{\rm WE} &= -368\overline{s}^{\rm XX} - 1112\overline{s}^{\rm XY} + 1269\overline{s}^{\rm XZ} + 368\overline{s}^{\rm YY} + 4219\overline{s}^{\rm YZ}, \end{aligned} \quad (5)$$

where $\Delta R_{\rm NS} < 22.6$ and $\Delta R_{\rm WE} < 9.2$ from GPB (all units in mas/yr). The SME can accommodate precessions greater than those predicted by GR, unlike other extensions of the standard model where Einstein's theory is a limiting case.[6] GPB does not constrain the GS component, since the gyro spin axes point along this direction by design. The GS and WE components are linear combinations of $\overline{s}^{\rm XY}, \overline{s}^{\rm XZ}, \overline{s}^{\rm YZ}$ and $(\overline{s}^{\rm XX} - \overline{s}^{\rm YY})$, so they are superseded in any case by existing constraints, which read:[2,7]

$$|\overline{s}^{\rm XY}| < (0.6 \pm 1.5) \times 10^{-9} \quad (6)$$

$$|\overline{s}^{\rm XZ}| < (2.7 \pm 1.4) \times 10^{-9} \quad (7)$$

$$|\overline{s}^{\rm YZ}| < (0.6 \pm 1.4) \times 10^{-9} \quad (8)$$

$$|\overline{s}^{\rm XX} - \overline{s}^{\rm YY}| < (1.2 \pm 1.6) \times 10^{-9} \quad (9)$$

$$|\overline{s}^{\rm XX} + \overline{s}^{\rm YY} - 2\overline{s}^{\rm ZZ}| < (1.8 \pm 38)\ \times 10^{-9} \quad (10)$$

Thus in practice the only new GPB constraint on the SME comes from the NS component of Eqs. (5), associated entirely with geodetic precession in standard GR. It reads:

$$|\overline{s}^{\rm TT} + 0.035\overline{s}^{\rm XX} + 0.23\overline{s}^{\rm XY} + 0.39\overline{s}^{\rm YY}| < 5.0 \times 10^{-3}. \qquad (11)$$

To get seven conditions on seven unknowns, we supplement Eqs. (6-11) with the requirement that $\overline{s}^{\rm AB}$ be traceless, $|\overline{s}^{\rm TT} - \overline{s}^{\rm XX} - \overline{s}^{\rm YY} - \overline{s}^{\rm ZZ}| = 0$.[3] Inverting, we then find that

$$\overline{s}^{\rm TT} < 4.4 \times 10^{-3} \quad , \quad \overline{s}^{\rm XX}, \overline{s}^{\rm YY}, \overline{s}^{\rm ZZ} < 1.5 \times 10^{-3}.$$

This constitutes the first experimental upper bound on $\overline{s}^{\rm TT}$. (Other tests such as light deflection are also sensitive to this coefficient at similar levels of precision.[8]) It also lifts a degeneracy between other existing limits, allowing us to extract individual upper bounds on $\overline{s}^{\rm XX}, \overline{s}^{\rm YY}$ and $\overline{s}^{\rm ZZ}$.

One should also look at the effect of $\overline{s}^{\rm AB}$ on the equation of motion for the gyroscope.[3] This has the effect of rescaling Newton's gravitational constant G, increasing our sensitivity to $\overline{s}^{\rm TT}$ and strengthening our limits by about 5%.[9] If the actual orbit is not perfectly circular, as was the case for GPB (whose gyros remained in essentially perfect free fall around a non-spherically symmetric Earth), then additional $\overline{s}^{\rm AB}$-dependent terms are also introduced in the *leading-order* (GR) expressions for geodetic and frame-dragging precession, Eqs. (1). These do not significantly alter the NS or geodetic constraint from GPB, but they do strengthen the WE or frame-dragging constraint so that it may potentially become competitive with existing limits. We will report on these results elsewhere.[9]

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