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Air passenger demand forecast through the use of Artificial Neural Network algorithms

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Plans for developing different components of an airport system depend to a great extent on the levels of activity that are anticipated. To plan facilities and infrastructures of an airport or system/group of airports, and to be able to satisfy future needs, it is essential to predict the level and distribution of demand in the various components of the airport system (Transportation Research Board [TRB], 2002). Forecasting demand in an industry as dynamic and sensitive to exogenous factors as aviation is an extremely difficult task. However, it is necessary to make air traffic estimates as a preliminary step in planning and designing airport facilities, be it an airport, an airport system, or a network (Horonjeff et al., 2010; Wells & Young, 2004).

Understanding future demand patterns enables the airport planner to evaluate the future performance of the airport and, thereby, recommend consistent development programs so that the costs associated with these development plans are estimated and the sources and level of income to support future capital investments are projected (Ashford et al., 2011). Demand forecasting is a basic requirement to develop an airport master plan or an airport system plan at the regional or national level, thus understanding the entire airport network of a region or country (International Civil Aviation Organization [ICAO], 1987; Janic, 2021, 2009).

To assess the characteristics of future demand, it is necessary to develop reliable predictions of airport activity. Many factors will affect demand, so planners preparing demand forecasts or updating existing forecasts should consider, in addition to historical aeronautical data (air traffic), local socio-economic data (historical series) such as national wealth, purchasing power of the inhabitants, demographics (population), industrial production, consumer price index, an exchange rate (of the local currency against the US dollar), etc. These indicators have a great influence on the behavior of air traffic demand (García Cruzado, 2013; Horonjeff et al., 2010; ICAO, 2006; Rodríguez et al., 2020).

So, the objective of this research is to make a short and medium-term forecast of air passenger demand. For this, Colombia (a complete network of airports as a whole) has been used as a case study, with the particularity that the analysis includes data on demand for 2020, which has been severely affected by the COVID-19 pandemic. This analysis will serve to estimate as a complementary result (but of great interest approximate date of recovery of both the volume of demand and its growth trend to the pre-pandemic period (2019). To achieve this objective, and as a calculation tool, a model derived from Artificial Neural Networks of the ConvLSTM2D type (<Conv> of Convolutional and <LSTM2D> of long-short-term memory 2-dimensional) is developed. This type of architecture is a hybrid between Convolutional Neural Networks (CNN), very useful for the extraction of invariant patterns in their spatial position, and Recurrent Neural Networks (RNN), very appropriate for the extraction of patterns within their temporal context, as it is the case of time series forecast (Hermans & Schrauwen, 2013; Malhotra et al., 2015; Millstein, 2018; Sewak et al., 2018; Yang et al., 2015). These prediction techniques, based on Machine Learning/Deep Learning (ML/DL), can incorporate more elements of analysis for pattern extraction and, therefore, potentially be more efficient (Ketkar & Moolayil, 2021). Another advantage of ConvLSTM2D networks, compared to classical methods based on autoregression, is that they admit a more complex multivariate treatment (several input features) (Calin, 2020; Pedrycz & Chen, 2020). Finally, when applying this type of neural model (ML/DL ConvLSTM2D) to the case study of the present research, it comes to represent the abstract knowledge model, inferred from the learning of historical patterns in the time series of air traffic, which predicts the future evolution of these time series.

Literature Review

In the scientific literature, there are various approaches based on ANN for forecasting passenger demand and air travel. The first is a hybrid neural model with data preprocessing by decomposition of variables, which allows for improving the performance of the network and, thus, optimizing the results in the forecast of demand (Alekseev & Seixas, 2009). The second is the Ensemble Empirical Mode Decomposition (EEMD) based on Support Machines Vector (SMV), with a modeling framework that incorporates slope-based methods to constrain the problem of the final effect that occurs during the process of change of the EEMD; in other words, an EEMD-Slope-SVMs model (Bao et al., 2012).

The third is the use of Backpropagation Neural Network (BNN) to improve the accuracy of the demand forecast (Chen et al., 2012; Kuo & Chen, 2010). The fourth is a hybrid approach VMD-ARMA/KELM-KELM, which consists of the Variational Mode Decomposition (VMD) in an Autoregressive Moving Average (ARMA) model and in a Kernel Extreme Learning Machine (KELM). This means that the VMD is first adopted to break down the original data into various functions, in order to reduce its complexity. Then ARMA and KELM models are used to forecast the stationary and non-stationary components, respectively, and the final result is integrated by another KELM model, which incorporates the forecast results of all components (Jin et al., 2020).

The fifth approach consists of the combined use of Machine Learning and Support Vector Regression (SVR) (Plakandaras et al., 2019; Sulistyowati et al., 2018). The sixth is the Nonlinear Vector Autoregression Neural Network (NVARNN) approach, based on MIV. This approach consists of using: (1) a Mean Impact Value (MIV) method from a neural network to identify and extract input variables, and (2) NVARNN to deal with the irregularity and volatility of the time series (Sun et al., 2019). The seventh is the ensemble forecast modeling based on Singular Spectrum Analysis (SSA), this implies that the original time series is decomposed into three components: trend, seasonal oscillations, and irregular components. The trend is predicted using a Generalized Regression Neural Network (GRNN), while seasonal oscillations are predicted using Radial Basis Function Network (RBF Network) (Xiao et al., 2015).

The eighth approach consists of using a Multilayer Perceptron Architecture (MLP) with a feedback propagation algorithm (Dingari et al., 2019). The ninth entails using evolutionary metaheuristic algorithms (Mostafaeipour et al., 2018); the tenth, uses time-delayed feedback neural networks (Blinova, 2007), and, finally, the eleventh uses recurrent neural networks (RNN) based on long-short-term memory (LSTM) (Gupta et al., 2019).

Data and Methodology

In the methodology, before the development of the model, the data (or dataset) related to the annual historical time series of each of the six independent variables (or selected features) were prepared. These are the aeronautical variables (national/domestic passenger or international passenger), and the socioeconomic variables (at the national level) usually used in the calculation of forecasts in air transport (Díaz Olariaga & Girón, 2020; Rodríguez et al., 2020), which are: GDP (Gross Domestic Product), population, IPI (Industrial Production Index), IPC (Consumer Price Index), and TRM (US Dollar-Colombian Peso official exchange rate). The data of this research correspond to the country-case study selected, Colombia; historical data cover a continuous period of 42 years (1979-2020); aeronautical data were obtained from the statistical system of the Colombian Aeronautical Authority (Aerocivil, 2022) and socioeconomic data were obtained from related official sources existing in the country (BRC, 2022; DANE, 2022).

Although the data –both air traffic and socioeconomic— for 2020 are very low compared to previous years due to the reasons explained previously, they are included in the calculation to impose the model to consider the COVID-19 effect.

Finally, as far as data is concerned, an important part of preparing the dataset has consisted in extracting the temporal sequences of input to the model and split up them into selected parametric subsequences relative to the number of previous steps (*lags* or past) of each independent variable or feature, necessary to make the predictions (*outs* or future steps). Although the code trains the complete historical time series of 42 years, the data are prepared sequentially into temporal subdivisions of the steps of the previous years (called *n_lags*) before considering future prediction steps (called *n_outs*). These *n_lags* grouping can be seen as time context to extract time cycle features. In addition, these values have been hyper-parametrized in the algorithm to study their cost sensitivity to them. Likewise, output data corresponding to a national or international passenger is extracted from the dataset, with the historical subsequences corresponding to the number of steps in future years to be predicted. For simplicity, the same number of steps from previous years has been considered as future steps to be predicted (*n_lags* = *n_outs*).

The model that is developed in the present research, which is one derived from Artificial Neural Networks, is called ConvLSTM2D (<Conv> of Convolutional and <LSTM2D> of long-short-term memory) (Hermans & Schrauwen, 2013; Malhotra et al., 2015; Millstein, 2018; Sewak et al., 2018; Yang et al., 2015). What the Conv2LSTM2D model aims, as expressed in equation (1), is that once the model has been trained or adjusted in the connections (*weights*) within the neurons of the different layers designed, symbolized by the function $f_{\text{model forecast}}$, this predicts (output variables denoted $\hat{\boldsymbol{Y}}$) the *n_outs* temporary outputs after time t corresponding to the j feature selected as output ($\hat{y}_{it+n\_outs}$), from the input of the *n_lags* observations of previous times of the whole i input features considered ($x_{it-n_{lags}}$).

$$\hat{y}_{jt+n\_outs}, \hat{y}_{jt+n\_outs-1}, \dots, \hat{y}_{jt} = f_{model\, forecast}\left(x_{it-1}, x_{it-2}, \dots, x_{it-n_{lags}} \right) \quad (1)$$

To describe the equations of the ConvLSTM2D model, we begin by outlining the basic architecture of ANN, on which the ConvLSTM2D model is based, called Multilayer Perceptron Networks (MLP) (Vang-Mata, 2020). These are made up of arbitrary several layers of neurons, with a first layer corresponding to the input (associated with the **X** tensor) and a last output layer (associated with the **Y** tensor), the hidden layers are interspersed between them, to modelized the output variables from the input ones. Each layer contains an arbitrary variable number of neurons (or nodes) that can be activated with different activation functions (Hastie et al., 2013).

The ConvLSTM2D model adopted here is of the Deep Learning (DL) type and has an encoder-decoder structure in which the **X** inputs are first encoded, with hybrid layers of convolutional (CNN) and recurrent (RNN) characteristics (Aggarwal, 2018; Bianchi et al., 2017; Blokdyk, 2017; Mandic & Chambers, 2001). On the other hand, the decoding is performed only with the recurring layers of long-short-term memory (LSTM). The coding provided by the convolutional layers facilitates the representation of the invariances of the sequences of the features of the inputs, while the contribution of the LSTM cells in the coding allows memorizing the context of the temporality of the inputs. The decoding part of the model is performed only by the LSTM layers, which allows extracting and identifying the patterns of the features and the present temporality, such as trend and seasonality (Pedrycz & Chen, 2020). Finally, the output of the LSTM networks is delivered to other dense layers of neurons, MLP type (where all neurons are connected between them) which are the ones that conduct the learning of the feature patterns extracted by the previous encoder-decoder block, until reaching the last layer, where the output tensor **Y** is obtained. In summary, the application of encoding-decoding with convolutional CNN and recurring LSTM layers in the ConvLSTM2D model is equivalent to performing double deep learning (Brownlee, 2018; Trifa et al., 2017; Yang et al., 2015).

The group of equations (2) that define the layers of the ConvLSTM2D model, was proposed as an improvement of the traditional RNN (Donahue, 2015; Hu et al., 2019), which managed the hidden states of the cell to map the output from the input. LSTM networks were proposed to solve the problems that appeared by gradient vanishing and exploding (Hochreiter & Schmidhuber, 1997). The superiority of the ConvLSTM2D layered architecture lies in its ability to handle short and long-term memory provided by the LSTM architecture by managing the three types of gates: input, forget and output, plus the new gate coming from the contribution of the convolutional layers with which they are combined to produce the new ConvLSTM2D layer (Shi et al., 2015). Then:

$$i_t = \sigma(W_{xi} * x_t + W_{hi} * h_{t-1} + W_{ci} \circ c_{t-1} + b_i) \quad (2a)$$

$$f_t = \sigma(W_{xf} * x_t + W_{hf} * h_{t-1} + W_{cf} \circ c_{t-1} + b_f) \quad (2b)$$

$$o_t = \sigma(W_{xo} * x_t + W_{ho} * h_{t-1} + W_{co} \circ c_t b_o) \quad (2c)$$

$$g_t = tanh(W_{xc} * x_t + W_{hc} * h_{t-1} + b_c) \quad (2d)$$

$$c_t = f_t \circ c_{t-1} + i_t \circ g_t \quad (2e)$$

$$h_t = O_t \circ \tanh(c_t) \quad (2f);$$

in which: i_t, f_t, o_t represent respectively the input gate, the forget gate and the output gate of the ConvLSTM2D cell; x_t, the temporary t inputs of the cell; h_{t-1} and c_{t-1}, the output and the state respectively of the previous cell; * is the convolution operator and *W* is the convolution filter; with $W_{\bullet i}$ and b_i the equivalent of the weights and biases of the input gate; $W_{\bullet f}$ and b_f, the equivalents to the weights and biases of the forget gate; $W_{\bullet o}$ and b_o, those at the output gate, and $W_{\bullet c}$ and b_c the weights and biases of the cell state. The dimensions and processing of the ConvLSTM2D layers, while being analogous to those of the LSTM cell, are different, since the hybrid ConvLSTM2D cell also performs the convolution operation (Hu et al., 2019). Finally, the operator ∘ denotes the Hadamard product and σ the activation function.

It is worth mentioning that, in the algorithm developed for the present research, the activation function σ chosen has been one of the Rectified Linear Unit Function (ReLU) (Nair & Hinton, 2010), because it offers better results for the selected case study than the function of the hyperbolic tangent (*tanh*) present in the equations described in (2).

Before training the model, the compilation method is implemented. In the compilation method, the model's arguments are defined, such as: gradient optimizers, cost or error function, and metric evaluation. The next step in the methodology involves training the ConvLSTM2D model. Training the ConvLSTM2D model is under supervised learning, that involves defining the error metric, which measures the 'distance' or norm between the value of the real outputs of the time series used from the **Y** tensor, and the predicted or estimated $\widehat{\mathbf{Y}}$ value by the ConvLSTM2D model, after applying it to the tensor input **X** (Goodfellow et al., 2016). In this research, the mean square error (MSE) has been used for the cost function. Equation (3) defines this error with the norm 2 || ||, where ***E*** is the error and ***n*** is the number of samples from the mean.

$$E = \frac{1}{n} \sum_{x} \|(y(x) - \hat{y}(x))\|^2 \qquad (3)$$

In ML, Backpropagation (BP) is the fundamental process for the automatic adjustment of the model weights going backward from the cost function (Goodfellow et al., 2016). BP calculates the gradient of said total error (in this case the MSE) compared to each of the weights or connections of the neurons of each layer of the model. For this, the chain rule in the derivation applies, to propagate said error backward, to distribute the error among all the weights of the model, neuron by neuron, layer by layer. With this, the new adjustment of weights of the model is sought to produce a smaller error when it is iterated with the next batch of input samples **X**. If the solution converges, said error becomes smaller or asymptotic, as is the case analyzed here. The group of equations (4) shows the derivation chain rule in ANN networks (Aggarwal, 2018; Hassoun, 1995):

$$\frac{\partial E}{\partial w_{ij}} = \frac{\partial E}{\partial o_j} \frac{\partial o_j}{\partial w_{ij}} = \frac{\partial E}{\partial o_j} \frac{\partial o_j}{\partial net_j} \frac{\partial net_j}{\partial w_{ij}} \qquad (4a)$$

$$\frac{\partial net_j}{\partial w_{ij}} = \frac{\partial}{\partial w_{ij}} \left(\sum_{k=1}^{n} w_{kj} o_k \right) = \frac{\partial}{\partial w_{ij}} \, w_{ij} o_i = \, o_i \qquad (4b)$$

The problem of minimum searching of the cost function is implemented through different optimizers, generically known as Stochastic Gradient Descent (SGD) (Kingma & Ba, 2014). These optimizers try to avoid that the solution is trapped in some of the local minima that the cost function presents when it is not a convex function. To this end, each of them has implemented different strategies to seek the global optimum.

For the present research, one of the most efficient optimizers has been used, given its high convergence speed and robust stability, known as Adaptive Moment Estimation (ADAM) (Kingma & Ba, 2014). This is an improved version of the RMSProp that implements a variable/adaptive speed (or learning step) and a moment or inertia that tries to avoid getting stuck in the local minima of the cost function. The group of equations (5) describes the update of the weights by the ADAM optimizer (Kingma & Ba, 2014):

$$m_w{}^{(t+1)} \leftarrow \beta_1 m_w + (1+\beta_1)\, \nabla_w\, l^{(t)} \tag{5a}$$

$$v_w{}^{(t+1)} \leftarrow \beta_2 v_w{}^{(t)} + (1-\beta_2)\, (\nabla_w\, l^{(t)})^2 \tag{5b}$$

$$\widehat{m}_w = \frac{m_w{}^{(t+1)}}{1+\beta_1{}^{(t+1)}} \tag{5c}$$

$$\hat{v}_w = \frac{v_w{}^{(t+1)}}{1+\beta_2{}^{(t+1)}} \tag{5d}$$

$$w^{(t+1)} \leftarrow w^t - \eta \frac{\widehat{m}_w}{\sqrt{\hat{v}_w}+\epsilon} \qquad \text{(5e);}$$

in which $w^{(t+1)}$ represents the weights in iteration t+1; η, the learning speed or step; $\nabla_w\, l^{(t)}$ represents the gradient of the cost function l with respect to the weights; ϵ, a minimum value to avoid dividing by 0 (in Keras 10^{-7} is used); β_1 and β_2 are the forget factors for the gradient or the first and second moments, respectively; $m_w{}^{(t+1)}$ is the moving mean of the weights or first moments in the iteration *t+1*, and $v_w{}^{(t+1)}$ is the moving variance (or second moment) of the weights in the iteration *t+1*. It should be noted that in the notation of equations (5) the superscripts with the variable *t* correspond to the iterations (or learning cycles), while the notation of the subscripts *t* of the rest of the equations corresponds to the different instants of time dataset features from the time series.

In this work, both the number of iterations and the learning batch size have been hyper-parametrized. A batch is a unit or subset of the dataset from which the error of the cost function is estimated and, therefore, the weights are adjusted. Also, the gradient has the stochastic attribute or name, since it cannot be calculated for the full dataset, due to time, process, and memory computational constraint, then SGD choose a stochastic batch instead. All of this makes the learning process sensitive to the batch size (and the number of epochs or iterations), so these values were hyper-parameterized in the algorithm (Pedrycz and Chen, 2020; Calin, 2020; Yi and Tan, 2004) to optimize learning process via cost-sensitivity analysis.

The next step in the methodology is the evaluation of the model, in which the goodness of the training or adjustment of the model weights is compared and evaluated. The sensitivity of training variations is taken into account through performing different hyperparameters configurations defining both the model and the training process itself. In the field of ML/DL, the search for the best solution is a heuristic problem, in which different solutions of architectures and model training processes are proposed and tested, extracted from the infinite space of possible solutions to, then, measure their performances and be able to select among them the best models and configurations (Aggarwal, 2018; Patterson & Gibson, 2017).

For this reason, in the evaluation process it is required, on the one hand, to define the space of the hyperparameters used to configure the model and its training, and, on the other hand, to define the metrics used to evaluate and compare the accuracy of the results. As evaluation metrics for regression problems the Mean Absolute Percentage Error (MAPE), which measures the prediction error as a percentage, has been implemented here. This is an advantage since it provides an intuitive way to evaluate the error of the model; the smaller the MAPE, the better the prognosis (Kim & Kim, 2016; Ren & Glasure, 2009; Rodríguez et al., 2020). Additionally, the Root Mean Square Error (RMSE) was also implemented. To implement this, once the model has been trained with all the data, so that the weights of all the connections of the neurons/layers of the model are finally fixed, the full output values $\mathbf{Y}$ of the dataset are again compared to the predicted or estimated $\widehat{\mathbf{Y}}$ values, after applying the model to the full input values $\mathbf{X}$.

Regarding the ConvLSTM2D model, the hyperparameters initially managed to define the possible space of solutions have been the number of filters per convolutional-recurrent layer, the size of the sliding windows of the convolution, the number of times the encoder of ConvLSTM2D layers is repeated, the presence or absence of specific layers such as dropouts and batch normalization (which help to avoid overfitting and be trained robustly) and, finally, varying the number of neurons in the LSTM layer (Donahue et al., 2015). With all this, a (relative to space research) robust model has finally been obtained regarding its performance.

In relationship to the process of training and splitting up the time series dataset into sequences of *n_lags* length to predict future sequences *n_outs*, after performing several internal tests the framework analysis has focused on two forecast scenarios: the first one, 6 years (short-term) and the second, 12 years (medium term), with the two hyperparameters *n-lags* and *n_outs* equal. Given that implementing Keras callbacks techniques, such as EarlyStopping and ModelCheckpoint, related to stopping learning when the threshold error is reached and saving the optimal learning model found in training respectively, the algorithm is, to some extent, independent of the number of iterations used. Finally, the hyperparameters that have been taken into account have been the number of subsequences, and the number of features used as inputs to predict national passengers or international passengers, together with previous steps *n_lags* and future steps to predict *n_outs*.

Finally, and as the last stage of the methodology, the prediction of the model is developed, which consists of making the predictions of the future series (*n_outs*) with the trained model, from the series of the previous years (*n_lags*) chosen and from the features taken into account as independent input variables.

For this evaluation, applying said prediction to the historical (real) series $\mathbf{Y}$ and, thus, be able to observe the deviation with the estimated outputs $\widehat{\mathbf{Y}}$ or predicted by the ConvLSTM2D model was considered good enough and promising (since the final optimization was not performed due to time and resources constraint). This prediction process is ultimately the final objective of this methodology, since the development of the algorithm to predict the demand for air passengers for future years is what is sought. Although the objective has been focused on two of the features, national (or domestic) passenger and international passenger (annually), the model could predict for the coming years any of the other five additional features included in the historical data set (as are socioeconomic data).

It should be noted that the model weights for each of the possible combinations between any number of the seven possible output features, depending on the seven input features, are different, since the training $\mathbf{Y} = f(\mathbf{X})$, or adjustment of weights of the ConvLSTM2D model, is done only with the appropriate features of the $\mathbf{X}$ and $\mathbf{Y}$ tensors extracted from the historical data set.

Results

The results of the prediction of passenger demand are presented, interpreted, and discussed, for the short and medium term, corresponding to the domestic and international cases gathered from the Colombian airport's system dataset, and getting with the defined, developed, trained, and evaluated own specific algorithm.

The two hyperparameters to which the possible prediction scenarios have been reduced are those relative to the number of previous input and prediction years, respectively *n_lags* and *n_outs*. On the other hand, said pre-post times are subdivided into subsequences, because results are sensitive to those number of subsequences, among other hyperparameters, since they define the temporality context from which the patterns of the short-term cycles may exist and have to be extracted, within those longer historical periods.

This sensitivity study is common in the analysis of ConvLSTM2D networks and it is derived from the fact that the input data set $\mathbf{X}$ has many (five in this case) dimensions: the number of samples, the number of subsequences, the time series, the number of years per subsequence associated to *n_lags*, and finally the number of features selected as $\mathbf{X}$ input tensor variables (e.g. GDP, Population, IPI, IPC, TRM). On the contrary, output tensor $\mathbf{Y}$ has 3 dimensions: the number of samples, the number of forecast years (*n_outs*) and the number of dependent variables to be forecasted (domestic or international passenger). In the scenarios considered, the time series itself of the dependent variable for the next years to be predicted is always, taken also as an input variable (of previous years), to provide greater robustness and predictability to the model.

Before describing the hyperparameters and results achieved in each of the most relevant scenarios, Figure 1 shows the learning curve of the model, relative to the iterations of learning cycles (where the robustness of convergence towards asymptotic values of MAPE is observed, with small fluctuations). While Figure 2 shows the prediction of the historical series with the model already trained where we can realize how the decrease in passenger traffic in 2020 (full pandemic situation) was not predicted, as a result of pre-pandemic values. Both curves have been taken as examples of scenario 1, which is described later.

Figure 1

Evolution of MAPE According to the Learning Curve of the Trained Model Described in Scenario 1. Source: authors

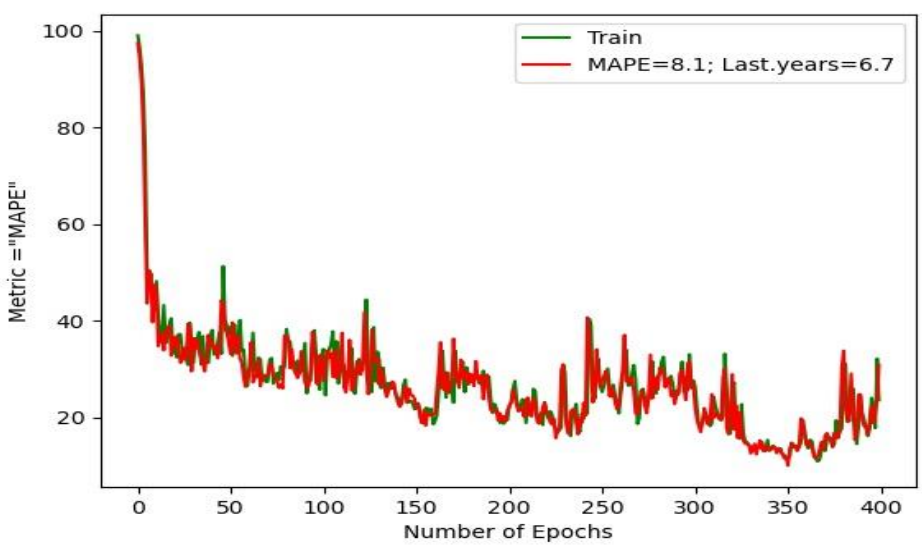
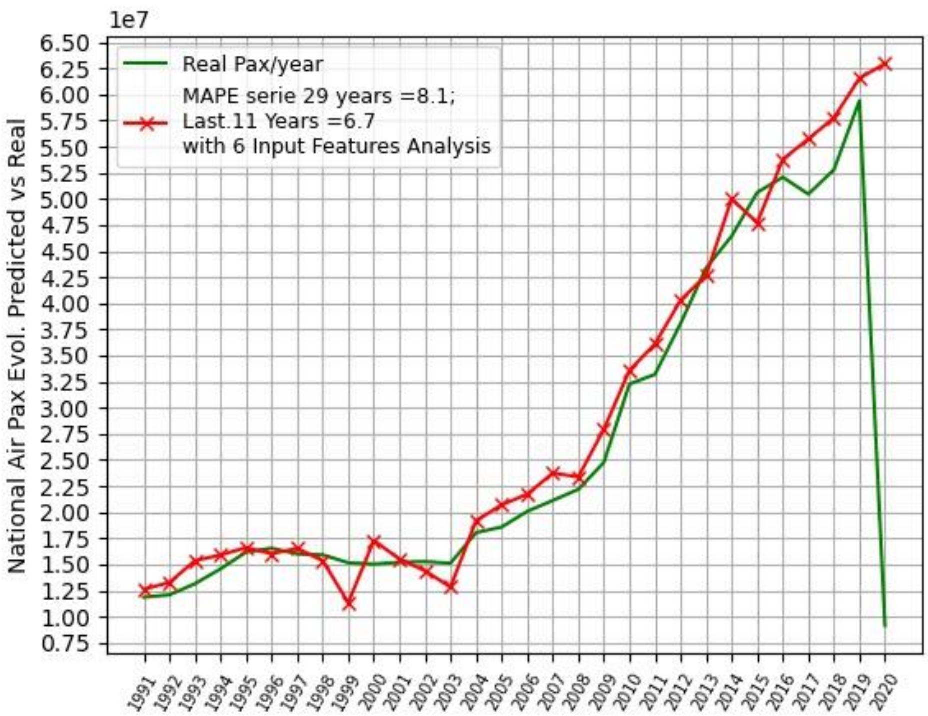


Figure 2

Evolution of Passenger Prediction vs. the Real Historical Series, Started n_lags Years After 1979 of the Model Already Trained and Described in Scenario 1. Source: authors



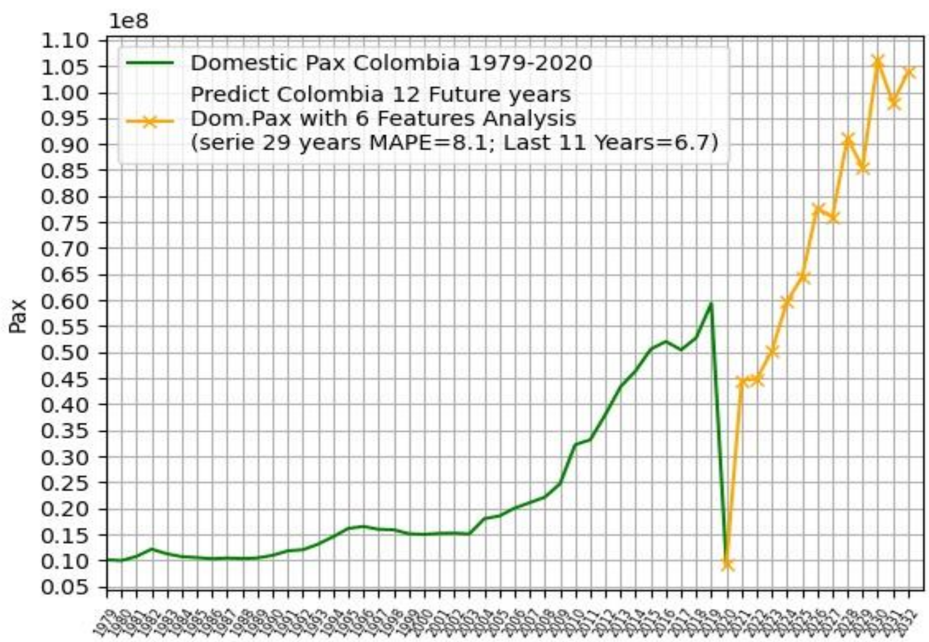
Results are generated and presented in various scenarios to deal with the fact that ML/DL models are stochastics in nature. It means, results are sensitive to many external factors, such as dataset quality (veracity on historical values collected without noise and biases), or the number of inputs (features) and output selected, but also to the internal factors associated with the own ConvLSTM2D model such as the hyperparameters deeply described here as, the number of subsequences, length of years considered as previous inputs (*n_lags*) and predicted outputs (*n_outs*), number of model training epochs (or iterations), training batch size, training, and test dataset size, optimizers and cost function chosen, number and type of layers implemented, number of neurons per layer, to mention the import ones.

Scenario 1

A first prediction is presented, based on sequences of the previous 12 years and the following 12 years, from the total historical series, with the time series of six features chosen as input variables (national passenger, GDP, population, IPI, IPC, and TRM) and with national passengers as the only output variable to be predicted. The previous 12 years' *n_lags* are grouped into three subsequences (or possible sub-cycles) of 4 years each. Figure 3 shows this medium-term prediction with a MAPE value of 6.7% for the last 11 years and 8.1% for the whole historical series of 29 years, where 12 years after 1979 are counted as previous *n_lags* steps to predict the following 12 years *n_outs*. In Figure 3 it can be observed that by the beginning of 2024 the traffic (or demand) of domestic passengers at the country level, would recover the existing level of the pre-pandemic period (2019), with a recovery of the trend in demand slightly higher than the one corresponding to 2019 year, but showing oscillations that could indicate potential echoes of the impact of COVID-19.

Figure 3

Forecast of Domestic Passenger Demand at the Country Level (Colombia), Period 2021-2032; Scenario 1. Source: authors

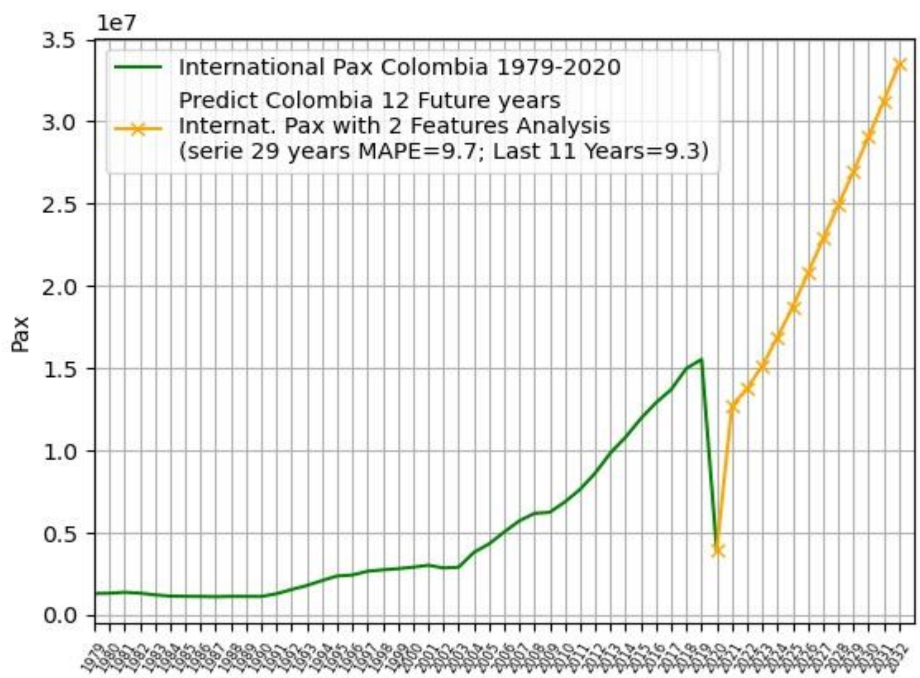


Scenario 2

A second prediction is presented considering sequences of the previous 12 years and the following 12 years, from the total historical series, with the time series but with only two features chosen as input variables (international passenger and GDP) and with the international passenger as the only output variable to be predicted. The previous 12 years *n_lags* are grouped into three subsequences (or possible sub-cycles) of 4 years each. Figure 4 shows this medium-term prediction with a MAPE value of 9.3% for the last 11 years and 9.7% for the total historical series of 29 years (12 years after 1979 are counted as previous *n_lags* steps to predict the following 12 years *n_outs*). In Figure 4 it can be observed that by the beginning of the year 2024 the traffic (or demand) of an international passenger in Colombia would recover to the level existing in the pre-pandemic period (2019) and with a recovery of the trend in demand very similar to that of the year 2019 but without fluctuations.

Figure 4

Forecast of the Demand for International Passengers in Colombia, Period 2021-2032; Scenario 2. Source: authors

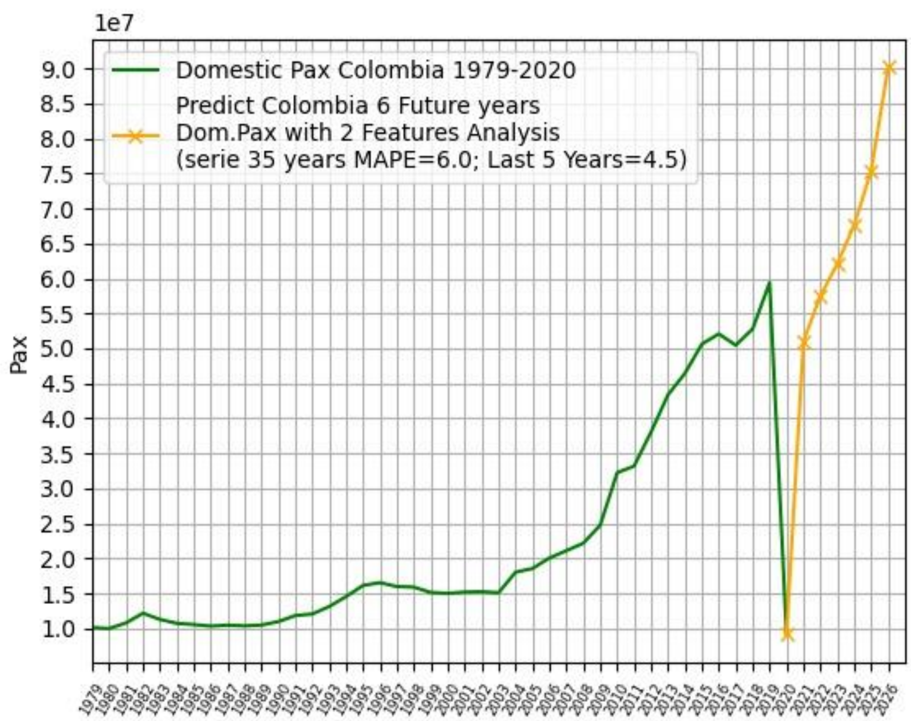


Scenario 3

A third prediction is presented based on sequences of six previous years and the six following years, out of the total historical series, with the time series for only two features chosen input variables (domestic passenger and GDP) and domestic passenger as the only output variable to be predicted. The previous six years *n_lags* are grouped into two subsequences (or possible sub-cycles), therefore, of three years each. Figure 5 represents this short-term prediction with a MAPE value of 4.5% for the last five years (2020 is excluded) and 6.0% for the total 35-year historical series (starting six years after 1979, the initial year of the available history, to predict the following *n_outs* years). The results show that by the end of 2022 or the beginning of 2023, the traffic (or volume of demand) of national passengers at the country level, would have recovered to the previous pre-pandemic level (2019).

Figure 5

Forecast of the Demand of Domestic Passenger at the Country Level (Colombia), Period 2021-2026; Scenario 3. Source: authors

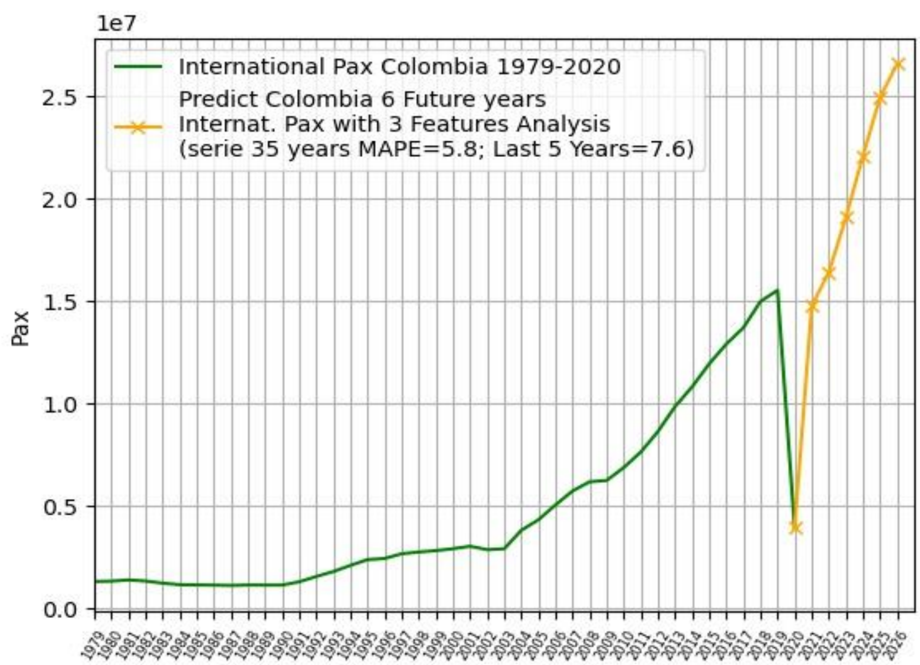


Scenario 4

A fourth prediction scenario is presented, calculated with the trends of the time series of domestic and international passengers used as input features. To analyze its impact the number of features used as inputs is left at three. Sequences of the previous six years and the following six years have been considered from the total historical series, with the time series for only 3 features chosen as input variables (international passenger, domestic passenger, GDP) and with the international passenger as the unique output variable to be predicted. The previous six years *n_lags* are also grouped into two subsequences (or possible sub-cycles), therefore, of three years one. The results of this calculation are shown in Figure 6. The resulting MAPE values are 7.6% for the last five years (2020 is excluded) and 5.8% for the total 35-year historical series (starting six years after 1979, the initial year of the available data history, to predict the following *n_outs* years). The results indicate by mid-2022 the demand for international passengers, at the country level, would recover the volume of pre-pandemic demand (the year 2019) and even with a higher trend (in demand growth).

Figure 6

Forecast of the Demand for the International Passenger in Colombia, Period 2021-2026; Scenario 4. Source: authors

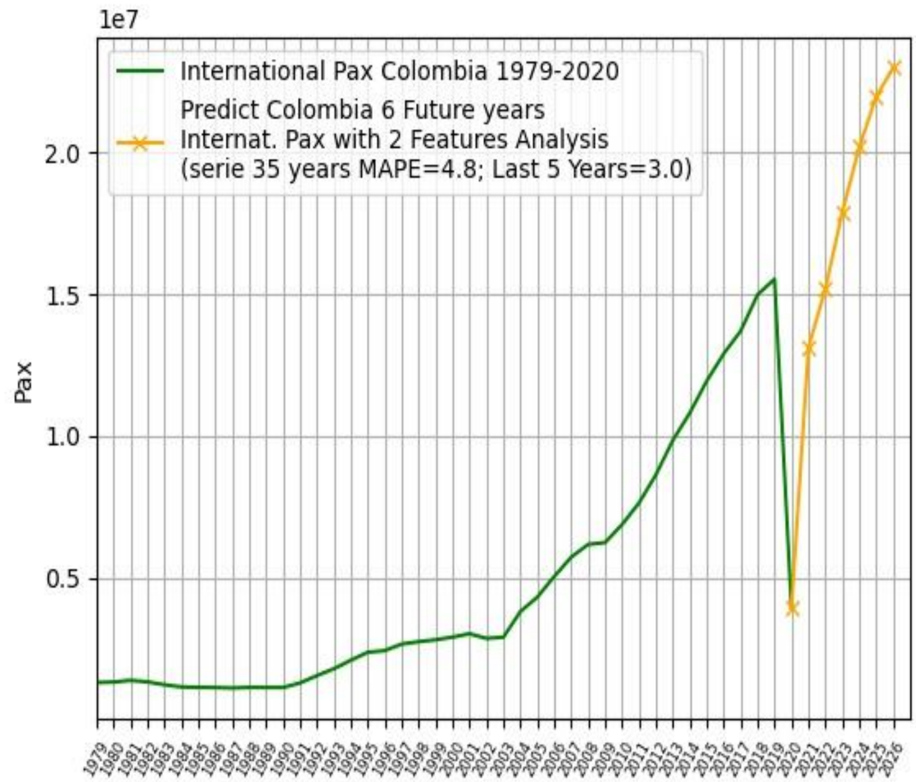


Scenario 5

Finally, a fifth prediction is presented with sequences of six previous years and the six following years, out of the total historical series, with the time series of only two features chosen as input variables (international passenger, GDP) and with the international passenger as the only output variable to predict. The previous six years *n_lags* were also grouped into two subsequences (or possible sub-cycles), therefore, of three years each. The results are shown in Figure 7, the MAPE values are 3% for the last five years (2020 is excluded) and 4.8% for the total 35-year historical series (starting six years after 1979, the initial year of the available data history, and predicting the following *n_outs* years). Under this fifth scenario, the results indicate that by mid-2022, Colombia's international passenger demand would have recovered to the level of pre-pandemic demand (2019).

Figure 7

Forecast of the Demand of International Passengers in Colombia, Period 2021-2026; Scenario 5. Source: authors



As a conclusion to the analysis of the results, three observations are made. In the first place, the algorithm developed here has a basic-medium complexity, characterized by the presence of 1,920,009 neural connections, which must be adjusted as unknowns in each iteration during the learning process. This value is specified for scenario 5 (with two features and six years of forecast) but grows to 3,149,833 neural connections when six features and 12 years of forecast are considered for scenario 1. Second, as has been exhaustively shown in the description of the ML/DL model of ConvLSTM2D, these are stochastic processes, that implicitly originated in the same process of calculating the Stochastic Gradient Descent (SGD), which has been improved here by choosing ADAM as loss (or cost function) optimizer (Kingma & Ba, 2014) and, therefore, subject to uncertainties or errors. Third, to constraint the uncertainties associated with the stochastic processes the algorithm has been executed several times, with the same hyperparameters – that process is known as cross-validation techniques (K-fold cross-validation) (Chang & Lin, 2011; Pedrycz & Chen, 2020).

Finally, the results obtained here coincide with the forecasts made in a recent study (Gudmundsson et al., 2021) on the recovery of air transport worldwide, which predicts that the demand for air passengers will recover to pre-pandemic levels at the end of 2022 (optimistic scenario). And the results also coincide with the IATA study (2020) which foresees the recovery of worldwide air passenger traffic at the 2019 level by the end of 2022 for domestic passenger and the beginning of 2024 for the international passenger.

Discussion on Methodology

Due to the non-linear characteristics of air traffic demand, classical time series such as econometric-statistical approaches are currently not considered the most convenient methodology, as these approaches are severely criticized due to their poor and limited forecasting capacity (Ensafi et al., 2022; Li et al., 2020; Liu et al., 2020; Rodríguez et al., 2020; Suryani et al., 2012; Tascón & Díaz Olariaga, 2021; Tsui et al. al., 2014). For this reason, a methodology based on a type of Artificial Neural Network architecture called ConvLSTM2D is proposed, which, although they have been very successful in areas of Machine Learning (such as computer vision and natural language processing (Alayba & Palade, 2022; Chaiani et al., 2022; Elboushaki et al., 2020; Fang et al., 2021; Kumar et al., 2022; Xingjian et al., 2015), have not yet been tested in time series forecasting of air passenger demand until now. The reason for choosing this methodology lies in the fact that these Artificial Neural Network architectures have shown to be very robust and successful in the fields of Deep Learning (DL) mentioned, which they come, by automatically extracting the intrinsic patterns of the non-linear relationships between the variables considered, without a priori knowledge about the existing relationships between the input variables among themselves and between these and the output variables considered, so, in short, it is estimated that this model can be a viable and promising tool to be explored for new flexible modeling of the forecast (Dingari et al., 2019; Gupta et al., 2019). Therefore, this research pursues (with a certain specific scope) to demonstrate the feasibility of applying these Artificial Neural Network models defined by the ConvLSTM2D architecture, which are of the Deep Learning (DL) type, and show it can be obtained acceptable and hopefully results when applied to multivariate time series forecasting such as our air traffic demand forecasting (Agga et al., 2022; Ensafi et al., 2022; Huang et al., 2022; Prince, 2022; Shastri et al., 2020).

As mentioned before, this promise ConvLSTM2D-ML/DL model needs to be optimized in terms of tuning the hyperparameters and architecture design of the model (number and type of layers, number, and types of neurons in each layer, batch size, number of training epochs, different input features, number of subsequences, etc.). Therefore, this study has been limited to performing certain variability analyses considering different scenarios (number of subsequences, number of years to be predicted, number and types of input features), due to time and resource constraints to work within the unlimited search ML/DL hyperspace. But on the contrary, the different and similar MAPE obtained for the variability analysis performed on the model vs. different scenarios seems to work sufficiently robust and stable, concluding that this model is appropriate and convenient to keep on developing for air passenger demand.

On the other hand, it is considered appropriate to make certain observations and constraints on the development of the proposed model and/or its operativity. In the first place, 42 years could be not considered the ideal amount of data to gather all possible air passenger demand patterns. Even to have only one year (2020) of COVID-19 time series included, is not enough information about COVID-19 patterns, but due to currently available dataset constraints on Colombia air passenger demand at the moment to perform the study, we must accept this (not as extensive as we would have liked) dataset and study limitation.

Conclusions

Firstly, estimating the demand for air passengers at the national level in the short and medium term provides valuable information so that the aviation/air transport planners of a country can well in advance design, plan, and implement: (a) development strategies (infrastructures, facilities, equipment, technological modernization, training of technical personnel, etc.); (b) a capital investment calendar (to address the proposed development programs); and (c) related public policies (to consolidate and reinforce the development of the local air transport industry).

Secondly, academics, analysts, planners, and decision-makers in the international civil aviation industry around the world are currently developing forecast studies that allow them to know when and how air traffic will recover (mainly the volume of the air passenger demand) to pre-pandemic levels (the year 2019), due to the importance of the aviation industry, not only in terms of local/regional/global connectivity but also because of its contribution to the global economy. The present work had the objective of contributing from the academic point of view to these two mentioned points. On the other hand, this work provides a novel and original geographic character, as one of the first studies (of an academic nature) in the Latin American subcontinent on air traffic forecasting for the post-COVID-19 era.

Regarding the approach, it was decided to develop a model based on an Artificial Neural Network architecture compared to other possible classical statistical techniques, considering that ML/DL prediction techniques can not only incorporate more elements of analysis but also extract more complex patterns from historical time series dataset, without any previous feature engineering and, therefore, be more potentially effective and accurate. In particular, for the analysis of the time series, techniques of encoder-decoder networks of the ConvLSTM2D type have been applied. The ConvLSTM2D model developed here admits a multivariate treatment (the same does not happen with the classic methods based on autoregression). On the other hand, the ConvLSTM2D model developed is of the supervised learning type, which means that the model is trained with tensor inputs and tensor outputs extracted from the historical series of the dataset. The possibility of hybridization of the convolutional CNN networks with the recurrent networks of long-short-term memory LSTM has allowed the implementation of the ConvLSTM2D model in the present research (novel in terms of its application for air traffic prediction time series). The application of encoder-decoder blocks with hybrid convolutional-recurrent networks represents a novelty for the treatment of time series. Finally, the results of the application of the model based on ML/DL present very acceptable MAPE values (in the order of 3% to 9%, depending on the scenario), which makes the model developed here a feasible alternative to develop reliable air traffic forecasts, at least in the short and medium term.

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