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Quentin G. Bailey
Indiana University - Bloomington, baileyq@erau.edu

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QUENTIN G. BAILEY

Physics Department,
Indiana University,
Bloomington, IN 47405, USA
E-mail: qbailey@indiana.edu

In this talk, the stationary limit of Lorentz-violating electrodynamics is discussed. As illustrated by some simple examples, the general solution includes unconventional mixing of electrostatic and magnetostatic effects. I discuss a high-sensitivity null-type measurement, exploiting Lorentz-violating electromagnetostatic effects, that could improve existing limits on parity-odd coefficients for Lorentz violation in the photon sector.

1. Introduction

Experiments to date have shown that Lorentz symmetry is an exact symmetry of all known forces in nature. However, many ongoing experiments are searching for small violations of Lorentz symmetry that could arise in the low-energy limit of a unified theory of nature at the Planck scale.[1] Much of the analysis of these experiments is performed within a theoretical framework called the Standard-Model Extension (SME).[2] The SME is an effective field theory that extends the Standard Model (SM) and general relativity to include small violations of particle Lorentz and CPT symmetry while preserving observer Lorentz symmetry and the coordinate invariance of physics. The CPT and Lorentz-violating terms in the SME lagrangian have coupling coefficients with Lorentz indices which control the Lorentz violation, and can be viewed as low-energy remnants of the underlying physics at the Planck scale.[3] Tests of this theory include ones with photons,[4,5,6,7,8] electrons,[9] protons and neutrons,[10] mesons,[11] muons,[12] neutrinos,[13] and the Higgs.[14]

In the photon sector of the minimal SME, recent Lorentz symmetry tests have focused on the properties of electromagnetic waves in resonant cavities and propagating *in vacuo*. I show in this talk, however, that there are unconventional effects associated with the stationary, non-propagating limit

of the photon sector. I also discuss experimental possibilities based on these effects in high-sensitivity null-type measurements. A detailed discussion of this topic is contained in Ref. 16.[a]

2. Framework

The lagrangian density for the photon sector of the minimal SME can be written as

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \tfrac{1}{4}(k_F)_{\kappa\lambda\mu\nu}F^{\kappa\lambda}F^{\mu\nu}$$
$$+ \tfrac{1}{2}(k_{AF})^{\kappa}\epsilon_{\kappa\lambda\mu\nu}A^{\lambda}F^{\mu\nu} - j^{\mu}A_{\mu}. \tag{1}$$

In this equation, $j^{\mu} = (\rho, \vec{J})$ is the 4-vector current source that couples to the electromagnetic 4-potential A_{μ}, and $F_{\mu\nu} \equiv \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$ is the electromagnetic field strength. From this definition the conventional homogeneous Maxwell equations are automatically satisfied. The coefficients $(k_F)_{\kappa\lambda\mu\nu}$ and $(k_{AF})^{\kappa}$ are assumed constant and control the CPT and Lorentz violation. The current j_{μ} is taken to be conventional, thus assuming Lorentz violation is only present in the photon sector. The CPT-odd coefficients $(k_{AF})^{\kappa}$ are stringently bounded by cosmological observations and are set to zero in this analysis.[7] The lagrangian (1) yields the inhomogeneous equations of motion

$$\partial_{\alpha}F_{\mu}{}^{\alpha} + (k_F)_{\mu\alpha\beta\gamma}\partial^{\alpha}F^{\beta\gamma} + j_{\mu} = 0. \tag{2}$$

These equations can be written as Maxwell equations in terms of $\vec{D}, \vec{H}, \vec{E}$ and $\vec{B}$ by defining appropriate vacuum constituency relations.[6,16] Equation (2) can be generalized to include regions of isotropic matter. The usual linear response of matter to applied fields is modified by Lorentz violation and additional matter coefficients appear in the constituency relations.[16]

3. Electromagnetostatics

The stationary solutions of the modifed Maxwell equations (2) *in vacuo* will satisfy the time-independent equation of motion

$$\tilde{k}^{j\mu k\nu}\partial_j\partial_k A_{\nu}(\vec{x}) = j^{\mu}(\vec{x}), \tag{3}$$

where the coefficients $\tilde{k}^{j\mu k\nu}$ are defined by

$$\tilde{k}^{j\mu k\nu} = \eta^{jk}\eta^{\mu\nu} - \eta^{\mu k}\eta^{\nu j} + 2(k_F)^{j\mu k\nu}. \tag{4}$$

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[a]See Ref. 15 for theoretical literature on the photon sector of the SME.

From the homogeneous Maxwell equations the electrostatic and magneto-static fields can be written in terms of the 4-potential $A^\mu = (\Phi, A^j)$ as $\vec{E} = -\vec{\nabla}\Phi$ and $\vec{B} = \vec{\nabla} \times \vec{A}$. The metric terms in Eq. (4) are the conventional terms that split (3) into separate equations for the scalar potential from charge density and the vector potential from current density. The presence of the $(k_F)^{j\mu k\nu}$ term implies that a static charge density generates a small vector potential and a modified scalar potential and similarly a steady-state current density generates a small scalar potential and a modified vector potential. Electrostatics and magnetostatics, while distinct in the conventional case, become convoluted in the presence of Lorentz violation. Discussing the static limit of Lorentz-violating electrodynamics therefore requires the simultaneous treatment of both electric and magnetic phenomena.[17]

To obtain a general solution for the potentials Φ and $\vec{A}$ I introduce Green functions $G_{\mu\alpha}(\vec{x}, \vec{x}')$ that solve Eq. (3) for a point source. Once a suitable Green theorem that incorporates the differential operator in Eq. (3) is found, the formal solution can be constructed for a spatial region V in terms of the Green functions, the 4-current density and the values of the potential on the boundary S. The general solution is

$$
\begin{aligned}
A_\lambda(\vec{x}) = &\int_V d^3x' G_{\mu\lambda}(\vec{x}', \vec{x}) j^\mu(\vec{x}') \\
&- \int_S d^2 S' \hat{n}'^j [G_{\mu\lambda}(\vec{x}', \vec{x}) \tilde{k}^{j\mu k\nu} \partial_k' A_\nu(\vec{x}') \\
&\qquad\qquad - A_\mu(\vec{x}') \tilde{k}^{j\mu k\nu} \partial_k' G_{\nu\lambda}(\vec{x}', \vec{x})].
\end{aligned}
\tag{5}
$$

Manipulation of Eq. (5) reveals four classes of boundary conditions that establish unique solutions for the electric and magnetic fields: $(\Phi, \hat{n} \times \vec{A})$, $(\Phi, \hat{n} \times \vec{H})$, $(\hat{n} \cdot \vec{D}, \hat{n} \times \vec{A})$, $(\hat{n} \cdot \vec{D}, \hat{n} \times \vec{H})$. With each of these sets of boundary conditions there are corresponding constraints on the Green functions.[16] The mixing of Φ and $\vec{A}$ in the boundary conditions comes from the unconventional definitions of the fields $\vec{D}$ and $\vec{H}$.[6,16] The solution (5) can be generalized to regions of isotropic matter using a modified version of Eq. (4).[16]

4. Applications

As a first application of Eq. (5) I consider the case of boundary conditions at infinity in which the surface terms are dropped. Imposing the Coulomb gauge, the explicit form for the Green functions can be extracted from

fourier decomposition in momentum space.[16] For the case of a point charge at rest at the origin the scalar potential[6] and vector potential are given by

$$\Phi(\vec{x}) = \frac{q}{4\pi|\vec{x}|}\Big(1 - (k_F)^{0j0k}\hat{x}^j\hat{x}^k\Big),$$

$$A^j(\vec{x}) = \frac{q}{4\pi|\vec{x}|}\Big((k_F)^{0kjk} - (k_F)^{jk0l}\hat{x}^k\hat{x}^l\Big). \tag{6}$$

Equation (6) shows explicitly that a point charge at rest produces a magnetic field in the presence of Lorentz violation, which is obtained from $B^j = \epsilon^{jkl}\partial^k A^l$.[16]

Consider now an example motivated by a possible experimental application. I seek the fields from a magnetic source surrounded by a conducting shell. In the idealized solution presented the magnetic source is a sphere of radius a and uniform magnetization $\vec{M}$ surrounded by a grounded conducting shell of radius $R > a$. The fields for this configuration can be obtained from (5) using the $(\Phi, \hat{n} \times \vec{A})$ set of boundary conditions and treating the magnetic source as a current density $\vec{J} = \vec{\nabla} \times \vec{M}$. The leading order solution for the scalar potential Φ in the region $a < r < R$, where r is the radial coordinate from the center of the sphere, is given by

$$\Phi(\vec{x}) = \frac{\hat{r}\cdot\tilde{\kappa}_{o+}\cdot\vec{m}}{4\pi}\left(\frac{1}{r^2} - \frac{r}{R^3}\right), \tag{7}$$

where $\vec{m} = 4\pi a^3 \vec{M}/3$. Here we have made use of the zero-birefringence approximation that $(\tilde{\kappa}_{o+})^{jk} = (k_F)^{0jpq}\epsilon^{kpq}$ is an anti-symmetric matrix.[6,16] The solution (7) becomes modified in the more realistic scenario with the magnet consisting of matter obeying Lorentz-violating matter constituency relations.[16]

5. Experiment

Recent experiments in the photon sector are least sensitive to $\tilde{\kappa}_{o+}$ and $\tilde{\kappa}_{\text{tr}} = -\frac{2}{3}(k_F)^{0j0j}$. This is due to the parity-odd nature of the corresponding Lorentz-violating effects from $\tilde{\kappa}_{o+}$ and the scalar nature of $\tilde{\kappa}_{\text{tr}}$ to which recent experiments are only indirectly sensitive. The setup of the second example in Sec. 4 is designed to be directly sensitive to parity-odd effects. It can be seen directly from (7) that the scalar potential, if taken to be the observable, is proportional to $\tilde{\kappa}_{o+}$.[18] A suitable experiment would measure the potential from Eq. (7) in the space between the magnet and outer shell $(a < r < R)$. The outer conducting shell then serves to sheild the apparatus from external electric fields. For an estimate of the sensitivity that might

be attainable I assume the source is a ferromagnet with strength 10^{-1} T near its surface and the voltage sensitivity is at the level of nV. A null measurement could then achieve a sensitivity $\tilde{\kappa}_{o+} \lesssim 10^{-15}$. This represents an improvement by 10^4 over the best existing sensitivities.[5]

Equation (7) is written in the laboratory frame. Since this frame is fixed to the earth it is not inertial on the time scale of the earth's rotation and revolution. The resultant time dependence of the signal can be obtained by transforming the laboratory-frame coefficients $(\tilde{\kappa}_{o+})^{jk}_{\text{lab}}$ to a Sun-centered inertial frame following Ref. 6. Thus, with upper-case letters denoting Sun-centered coordinates,

$$
\begin{aligned}
(\tilde{\kappa}_{o+})^{jk}_{\text{lab}} = {}& T_0^{jkJK}(\tilde{\kappa}_{o+})^{JK} + 2T_1^{kjJJ}\tilde{\kappa}_{\text{tr}} \\
& + (T_1^{kjJK} - T_1^{jkJK})(\tilde{\kappa}_{e-})^{JK},
\end{aligned} \tag{8}
$$

where $T_0^{jkJK} = R^{jJ}R^{kK}$ and $T_1^{jkJK} = R^{jP}R^{kJ}\epsilon^{KPQ}\beta^Q$ are tensors containing the time dependence from the rotations R^{jJ} and the boost β^J. One can also consider rotating the entire apparatus to produce a signal with a shorter time variation, which may increase sensitivity and reduce systematics. With these considerations one can attain time-dependent sensitivity to all three independent components of $\tilde{\kappa}_{o+}$ and time-dependent sensitivity to $\tilde{\kappa}_{\text{tr}}$ suppressed by a single power of $|\vec{\beta}| \simeq 10^{-4}$.

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