

Bifurcation Analysis for Complex Systems Resilience



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Abstract

As technology advances, systems are growing in complexity and interactivity. This complexity in systems can become a problem when operating and understanding them. Resilience Engineering studies system behavior in the face of impacts. However, existing methods to measure resilience are limited by the non-linear and dynamic characteristics of contemporary systems. Bifurcation analysis can present a different perspective on Resilience Engineering by examining mathematically how systems behave under changing situations. The hypothesis of this work is: *if Bifurcation analysis is performed on a nonlinear system, then it is possible to get information on the resilient properties of the system.* The poster aims to bridge the gap between bifurcation analysis and Resilience Engineering, offering a framework for integrating both approaches. Future efforts will focus on using the framework practically, to improve the framework to make it more accessible, versatile, and reliable.

Framework

- Navigates users to analyze a system's resilience starting from the preprocessing of its data after a disruption (Figure 1)
- Presents a novel approach to integrate Bifurcation Analysis into Resilience Engineering discussions with a guide (Figure 2)
- Allows flexibility. Steps can be omitted if enough information is gathered

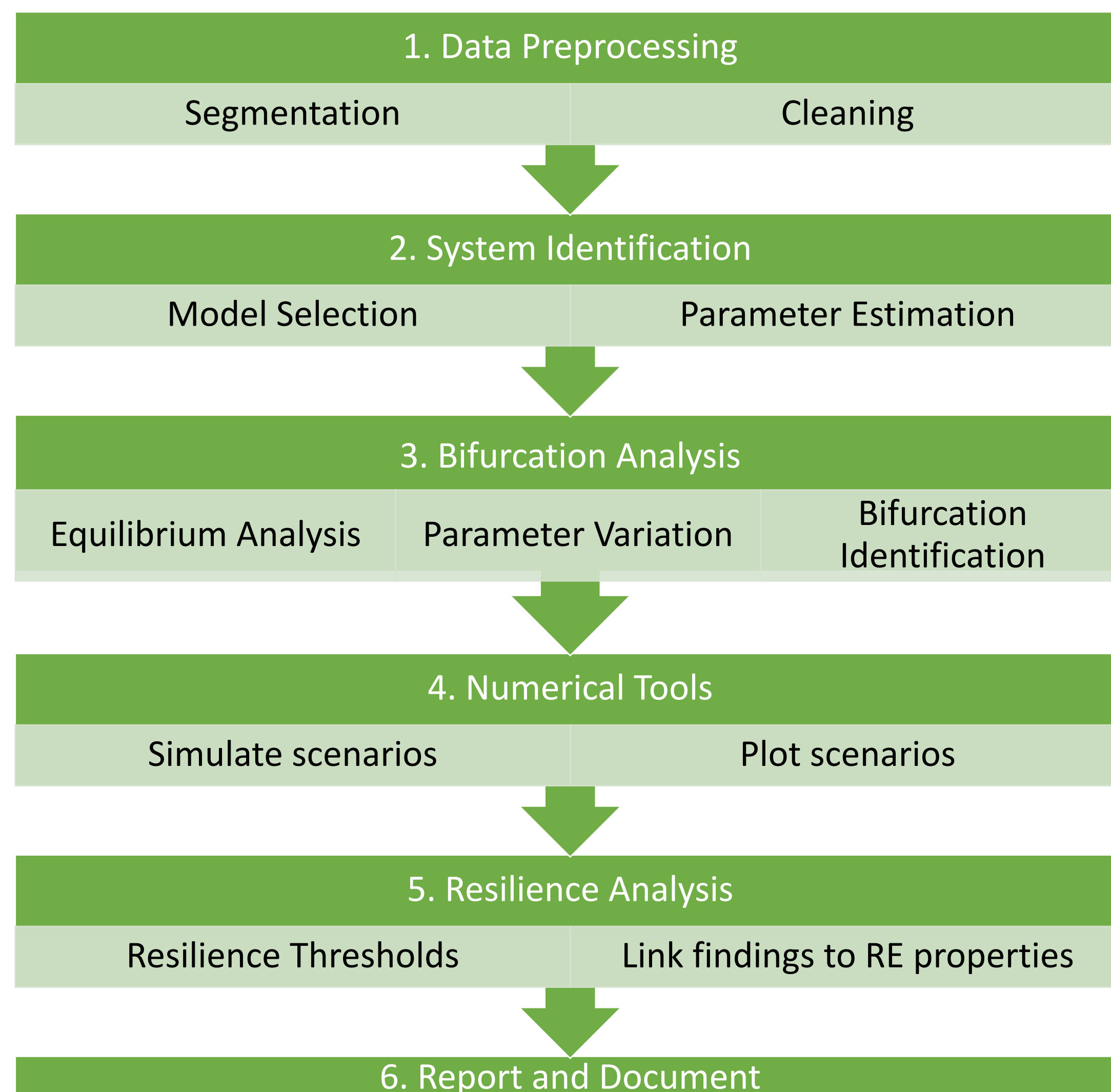


Figure 1: Diagram of the proposed framework

Discussion Guide

BIFURCATION ANALYSIS

		Time Series Analysis	Phase Diagram	Bifurcation Diagram
RESILIENCE CAPABILITIES	Absorptive Capabilities	High absorptive capacity is indicated if the system experiences a disturbance (visible as a deviation from a steady-state) but quickly returns to its baseline or changes minimally.	A system with high absorptive capacity might show trajectories in the phase diagram that quickly return to a stable orbit or fixed point after a disturbance.	A system with high absorptive capacity would demonstrate similar qualitative behavior over a wide range of parameter values.
	Adaptive Capabilities	A system that presents different states and can smoothly transition from one to another shows high adaptability.	A system transitioning smoothly between different orbits or fixed points as conditions change suggests high adaptability, as it can function across a range of dynamic states.	A system is adaptable if it can transition smoothly between different types of behavior (e.g., from stable to periodical states) as parameters change, indicating flexibility.
	Recovery Capabilities	If the system settles into a new patterns or fails to stabilize after a disturbance, it suggests limited recovery capability.	If paths leading back to a stable state or fixed point following a perturbation, it indicates good recovery properties. Conversely, if the system moves to a different attractor or becomes chaotic, it suggests limited recovery.	Good recovery will be shown if after passing through a bifurcation point and then reversing the parameter change, the system returns to its original behavior. If it is not the case, there could be hysteresis in the system, limiting recovery.

Figure 2: Guide to discussing Resilience Engineering properties after the Bifurcation Analysis of a system.

Conceptual Demonstration: Logistic Map

The Logistic Map Function (to the right) for population systems is used as an example to demonstrate how to approach complex system resilience discussions from bifurcation analysis results. To fully utilize the guide in Figure 2, all 3 bifurcation analysis diagrams are presented (Figures 3 – 5).

$$x_{n+1} = r * x_n * (1 - x_n)$$

Logistic Map Formula

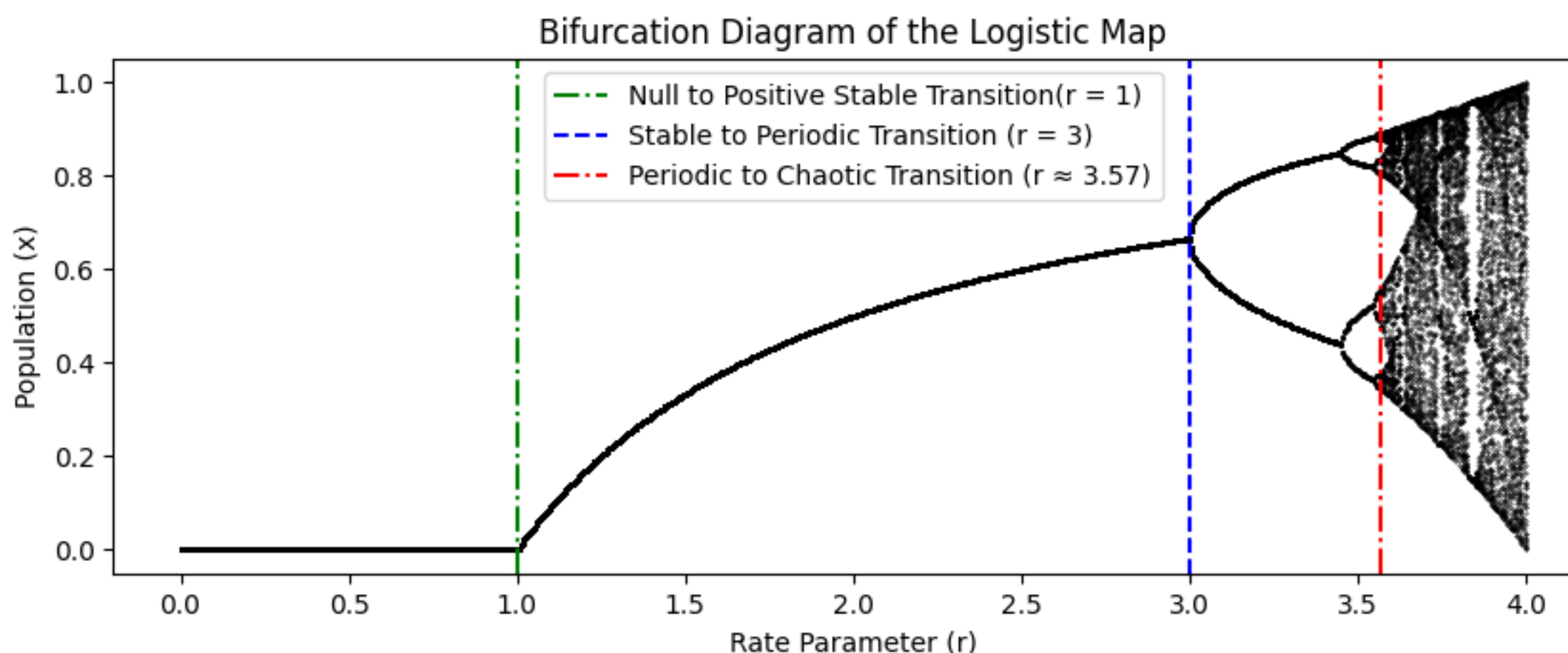


Figure 3: Bifurcation Diagram of the Logistic Map. It shows 4 different regions: no population, stable population, periodic populations, and chaotic populations based on parameter r

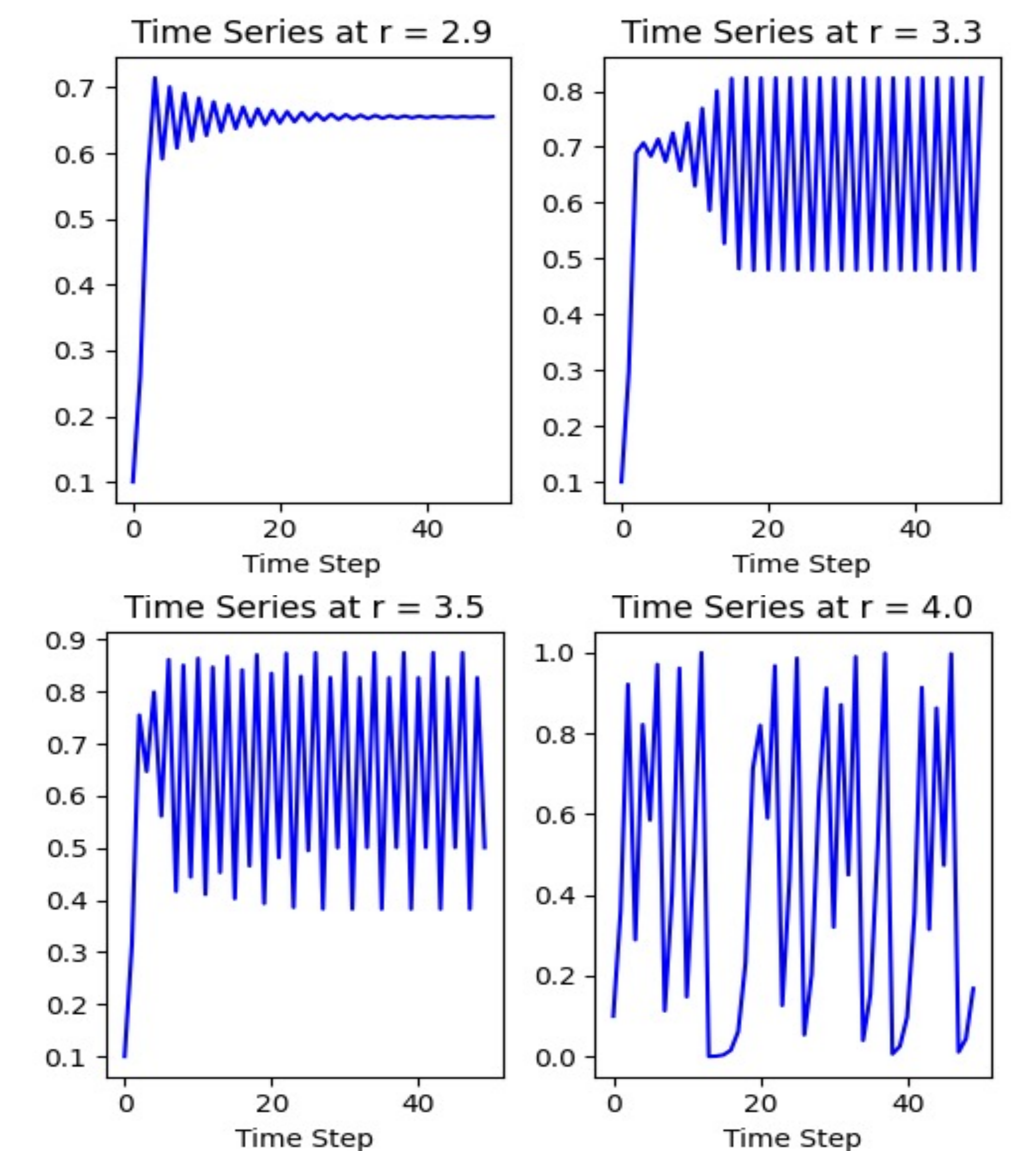


Figure 4: Time series for 4 r values. They show how the system evolves from stable to unstable

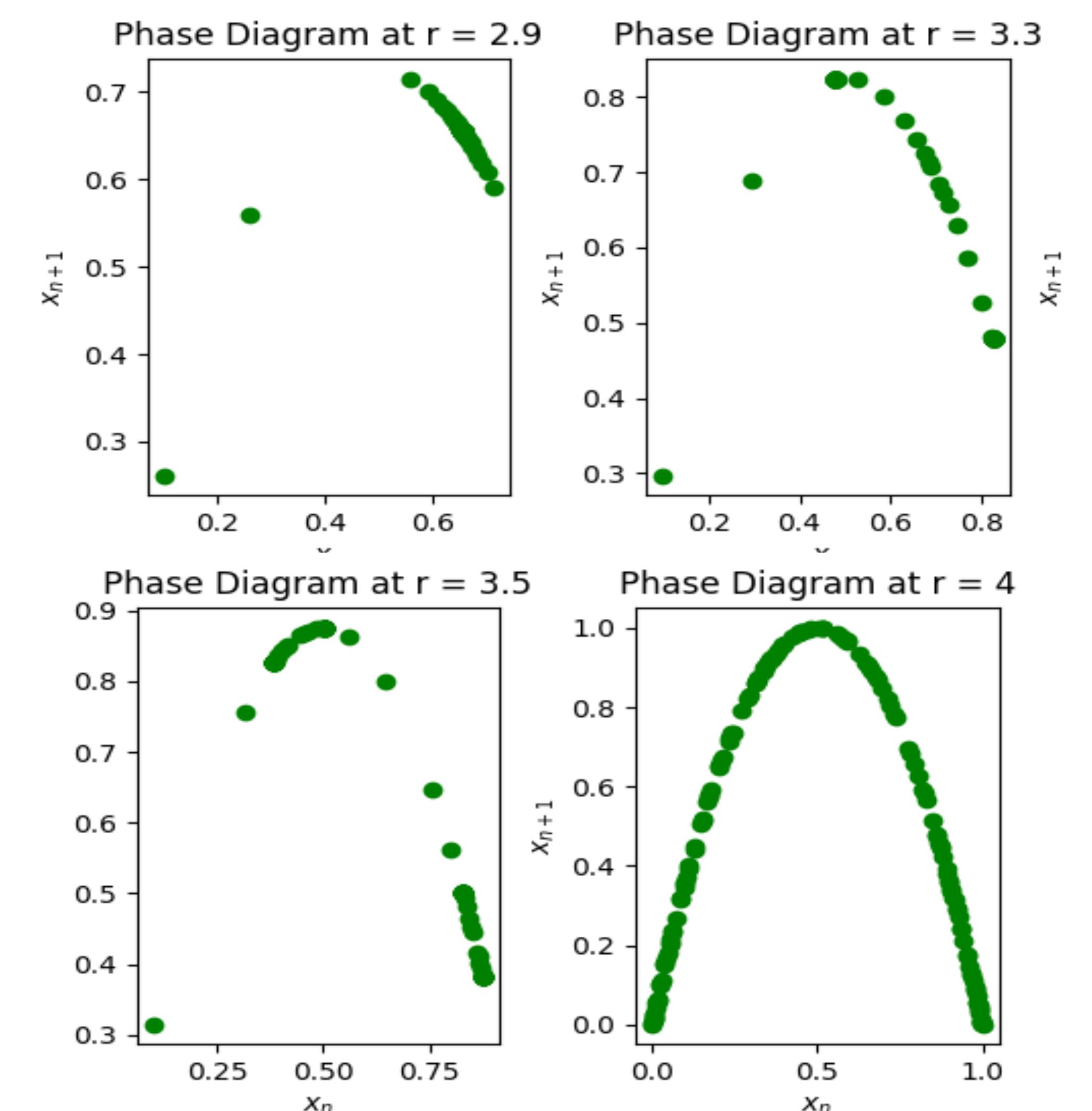


Figure 5: Phase Diagrams for 4 r values. They show how the system transitions from one step to the next one for the different values

Conclusion

- The proposed framework for system resilience discussion helps analysts understand complex systems when in the face of disruptions and changes
- Future work can test the framework with other systems, adding or modifying the suggested steps to accommodate larger, more complex systems



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