

8-2013

Field Localization and Mass Generation in an Alternative 5-Dimensional Brane Model

Preston Jones

California Polytechnic State University, San Luis Obispo, jonesp13@erau.edu

Doug Singleton

California State University, Fresno

Gerardo Muñoz

California State University, Fresno

Triyanta

Institut Teknologi Bandung

Follow this and additional works at: <https://commons.erau.edu/publication>



Part of the [Astrophysics and Astronomy Commons](#)

Scholarly Commons Citation

Jones, P., Singleton, D., Muñoz, G., & Triyanta (2013). Field Localization and Mass Generation in an Alternative 5-Dimensional Brane Model. , (). Retrieved from <https://commons.erau.edu/publication/1076>

This Conference Proceeding is brought to you for free and open access by Scholarly Commons. It has been accepted for inclusion in Publications by an authorized administrator of Scholarly Commons. For more information, please contact commons@erau.edu.

DPF2013-84
January 13, 2018

Field localization and mass generation in an alternative 5-dimensional brane model

PRESTON JONES¹

Department of Physics, Cal Poly, San Luis Obispo, CA 93407

DOUG SINGLETON² AND GERARDO MUÑOZ³

*Physics Department, California State University Fresno, 2345 East San Ramon
Avenue M/S 37, Fresno, California 93740-8031*

TRİYANTA⁴

*Department of Physics, Faculty of Mathematics and Natural Sciences, Institut
Teknologi Bandung, Jalan Ganesha 10 Bandung 40132, Indonesia*

This proceedings is from a talk given at the APS DPF 2013 on a 5-dimensional brane world model. This alternative brane world model is formally related but physically distinct from the Randall-Sundrum brane world model. The spin dependent localization of 5D fields for the alternative model are different and in some ways superior to the Randall-Sundrum. The alternative model also exhibits a cutoff in the localization of massive scalar fields not seen in the Randall-Sundrum model. This revision includes a correction to the integrand for the scalar field action appearing in the principle reference [1].

PRESENTED AT

DPF 2013

The Meeting of the American Physical Society
Division of Particles and Fields
Santa Cruz, California, August 13–17, 2013

¹pjones02@calpoly.edu

²dougs@csufresno.edu

³gerardom@csufresno.edu

⁴triyanta@fi.itb.ac.id

1 Introduction

Recently, the authors [1] proposed an alternative to the Randall-Sundrum brane world model [2, 3] (RS model) that is formally similar to the original but has different physical properties. This alternative model does not require fine tuning of the cosmological constant and has a constant energy-momentum in the bulk instead of a vacuum. The alternative model will be identified as the r-metric model following the coordinate of the bulk dimension. This r-metric model shares all the important features of the RS model and in particular the model includes a 4D brane as a topological defect in a uniform 5D bulk. The r-metric model also shares with the RS model spin dependent (spin $= 0, 1/2, 1$) gravitational confinement of particles, a large extra dimension, 5D and 4D scale proportionality independent of the size of the extra dimension, and constant energy-momentum tensor.

2 Formal similarity to the Randall-Sundrum model

The formal similarity between the RS model and r-metric model is apparent by considering the two metrics together. The RS metric with the commonly used y and z coordinate systems is

$$ds^2 = e^{-2k|y|} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2, \quad (1)$$
$$ds^2 = e^{-2A(z)} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2), \quad e^{-A(z)} = \frac{1}{1-2k|z|},$$

and the r-metric is

$$ds^2 = e^{-2A(r)} (\eta_{\mu\nu} dX^\mu dX^\nu - dr^2), \quad (2)$$
$$A(r) = k|r|.$$

Both metrics are conformally flat and transformations can be readily identified between all three sets of coordinates, with the RS metric most obviously conformally flat in the z coordinates. The RS metric in the z coordinates is sufficiently similar to the r-metric that some care must be exercised to distinguish between the two. The transformation between the RS metric and r-metric $dX^\mu = \frac{e^{-k|y|}}{1-k|y|} dx^\mu$ and $e^{-k|r|} = 1 - k|y|$ is not an exact differential, $dX^\mu = A(x^\mu, y) dx^\mu + B(x^\mu, y) dy$, since $\partial_y A \neq 0$ and the y and r coordinates do not represent the same geometry. However, on any foliation $y = \text{constant}$ the two geometries are connected by a simple coordinate transformation.

We will now show how both models can be considered topological defects in a 5D bulk. Solving the Einstein-Hilbert equation, $G_{AB} + g_{AB} = \kappa^2 T_{AB}$, the energy-momentum tensor for the RS model is

$$\begin{aligned} \eta_{\mu\nu} e^{-2k|y|} (6k^2 + \lambda_{[y]}) - 6k\eta_{\mu\nu}\delta(y) &= \kappa^2 T_{\mu\nu} \\ - (6k^2 + \lambda_{[y]}) &= \kappa^2 T_{55} \end{aligned} \quad (3)$$

and for the r-metric

$$\begin{aligned} \eta_{\mu\nu} (3k^2 + e^{-2k|r|}\lambda_{[r]}) - 6k\eta_{\mu\nu}\delta(r) &= \kappa^2 T_{\mu\nu} \\ -6k^2 - e^{-2k|r|}\lambda_{[r]} &= \kappa^2 T_{55}. \end{aligned} \quad (4)$$

Requiring that the RS model be consistent with a 4D brane that is a topological defect in a uniform 5D bulk the cosmological constant must be fine tuned, $\lambda_{[y]} = -6k^2$. For the r-metric model to be consistent with a 4D brane as a topological defect in a uniform 5D bulk the cosmological constant must vanish $\lambda_{[r]} = 0$.

3 Spin dependent localization of fields to the brane

The condition for the localization of fields to the brane is that the Wick rotated propagator does not vanish, $\int Dx e^{-S} \rightarrow S = \text{finite}$. Considering the action for spinor fields $S_\psi = \int d^5x \sqrt{g} \bar{\Psi} i \Gamma^M D_M \Psi$ and expanding the integral produces three terms. The two kinetic terms are

$$\begin{aligned} c^2 \int_0^\infty dr e^{-2mr} \int d^4x \bar{\psi}_R i \gamma^\mu \partial_\mu \psi_R &\rightarrow \text{finite} \\ d^2 \int_0^\infty dr e^{2mr} \int d^4x \bar{\psi}_L i \gamma^\mu \partial_\mu \psi_L &\rightarrow \infty \end{aligned} \quad (5)$$

and a gamma 5 term is

$$c d \int_0^\infty dr \int d^4x (\psi_L^\dagger \psi_R + \psi_R^\dagger \psi_L) \rightarrow \infty \quad (6)$$

where c and d are constants. Since the action for the spinor field is not finite there can be no localized spinor fields due to gravitational confinement. Localization of spinor fields to the brane would require a mechanism other than gravitational confinement which is also the case for the RS model with decreasing warping as shown in Figure 1.

The action for the massless gauge fields is finite for decreasing warping as can be seen by expanding the action with the r-metric (2),

$$\begin{aligned} S_A &= -\frac{1}{4} \int d^5x \sqrt{g} g^{MN} g^{RS} (\partial_M A_N - \partial_N A_M) (\partial_R A_S - \partial_S A_R) \\ &= -\frac{c^2}{4} \int_0^\infty dr e^{-kr} \int d^4x (\partial^\mu a_\mu - \partial^\nu a_\nu) (\partial^\sigma a_\sigma - \partial^\rho a_\rho), \end{aligned} \quad (7)$$

where $A_r = \text{constant}$ and $A_\mu(x^M) = a_\mu(x^\nu) c(r)$. The finite action for gauge fields localizes the massless spin 1 particles to the brane. This gravitational confinement of massless gauge fields is an improvement over the RS model as shown in Figure 1.

<i>Spin</i>	<i>RS model</i>	<i>r – metric</i>
0	$-2k$	$-2k \rightarrow 2m < 3k$ $+2k \rightarrow 2m < 3k$
1	<i>none</i>	$-2k$
$\frac{1}{2}$	$+2k$	<i>none</i>

Figure 1: Comparison of spin dependent localization for the Randall-Sundrum and r-metric models. The negative values $-2k$ are for decreasing warping and positive values $+2k$ are for increasing warping.

The action for scalar fields in the r-metric model using equation (2) is

$$\begin{aligned}
S_{\Phi} &= \int d^5x \sqrt{g} g^{MN} \partial_M \Phi^* \partial_N \Phi = N \int d^4x \partial_{\mu} \phi \partial_{\nu} \phi + M_0^2 \int d^4x \phi^2 \\
N &= \int_0^{\infty} dr \sqrt{g} \chi^2 g^{\mu\nu} = \begin{cases} 1 & \text{if } 3|k| > 2m \\ \infty & \text{if } 3|k| < 2m \end{cases} \\
M_0^2 &= \int_0^{\infty} dr \sqrt{g} g^{rr} \partial_r \chi^* \partial_r \chi = \begin{cases} M_0^2(m, k) & \text{if } 3|k| > 2m \\ \infty & \text{if } 3|k| < 2m \end{cases}
\end{aligned} \tag{8}$$

where $\Phi = \phi(x^{\mu}) \chi(r)$ and

$$M_0(m, k) = \frac{3}{2}k - \frac{1}{2}\sqrt{9k^2 - 4m^2}. \tag{9}$$

The action is finite and the field localized to the brane for masses below $m < \frac{3}{2}k$ with decreasing warping and $m < \frac{3}{2}k$ for increasing warping as shown in reference [1]. The comparison of the localization of scalar fields in the r-metric and Randall-Sundrum models is provided in Figure 1 which includes both decreasing and increasing warping. Unlike the RS model with decreasing warping the r-metric model exhibits a cutoff in the mass spectra localized to the brane.

Setting the mass in the action for the scalar boson in equation (9) equal to the mass in the 4D equation of motion $M_0(m, k) = m$ has the solution, $m_{(\pm)}^2 = 0$. This is in contrast to the conclusion reached earlier in reference [1] due to an error in the integrand for the scalar field action equation (30). This leaves the parameters m and k undetermined. One way to establish the magnitude of k is in the reduction of the 5D action to an effective 4D action [3, 5],

$$\begin{aligned}
S_{gravity} &= -2M^3 \int dx^4 \int_{-\infty}^{\infty} dr \sqrt{g} R = -2M^3 \left(\frac{2}{3k}\right) \int dx^4 \sqrt{{}^{(4)}g} {}^{(4)}R \\
&\rightarrow M_{Pl}^2 = M^3 \left(\frac{2}{3k}\right)
\end{aligned} \tag{10}$$

where $\sqrt{g} = e^{-5k|r|}$ and $R = e^{2k|r|} \eta^{\mu\nu} R_{\mu\nu}$ and integrating over an infinite extra dimension. The bulk warping is $k = \frac{2M^3}{3M_{Pl}^2}$ which is proportional to the ratio between

the 5D and 4D scales.

References

- [1] P. Jones, G. Munoz, D. Singleton, Triyanta, Phys. Rev. D **88**, 025048 (2013).
- [2] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999).
- [3] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999).
- [4] ATLAS collaboration, Phys. Lett. B **716** 1-29 (2012), arXiv:hep-th/0407276v1.
- [5] K. Benson, arXiv:hep-th/0407276v1.