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Gravitons to Photons – attenuation of gravitational waves ^{*}

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Abstract

In this essay we examine the response of an Unruh-DeWitt detector (a quantum two-level system) to a gravitational wave background. The spectrum of the Unruh-Dewitt detector is of the same form as some scattering processes or three body decays such as muon-electron scattering or muon decay. Based on this similarity we propose that the Unruh-DeWitt detector response implies a “decay” or attenuation of gravitons, G , into photons, γ , via $G + G \rightarrow \gamma + \gamma$ or $G \rightarrow \gamma + \gamma + G$. Over large distances such a decay/attenuation may have consequences in regard to the detection of gravitational waves.

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In the absence of an acceptable theory of quantum gravity the semi-classical theory of quantum fields in curved space-times [1–3] is the best available approach to the study of the interaction of quantum fields with gravitational phenomena. One of the key results of this semi-classical approach is the phenomenon of particle creation in a gravitational background. The usual example of this is Hawking radiation [4] - the creation of photons in the background of a black hole. One of the most direct ways to check if field quanta/photons are created by a given gravitational background is through the use of an Unruh-DeWitt (UD) detector [3, 5]. An UD detector is a quantum system with two energy levels. A simple, realistic model of an UD detector is an electron placed in a uniform magnetic fields [6]. By placing the UD detector in a given gravitational background, and observing how the upper energy level of the quantum system is populated, one can determine the emission/absorption spectrum for a given space-time. The UD detector method can be used to find the radiation associated with several well-known space-times: (i) the thermal radiation, with Hawking temperature $T = \frac{\hbar c^3}{8k_B \pi M G}$, of a black hole of mass M ; (ii) the thermal radiation, with temperature $T = \frac{\hbar a}{2\pi c k_B}$, observed by an eternally accelerating Rindler observer.

In this essay we want to examine the response of an UD detector to a gravitational plane wave background. The background of a gravitational plane wave is different from a black hole space-time in that the gravitational plane wave has no horizon. In this regard the response of an UD detector to a gravitational plane wave background is similar to the response of an UD detector undergoing circular motion as studied in [6–9]. Neither the rotating space-time nor the gravitational wave space-time has a horizon (although the circular rotating metric has a light surface). The spectrum of the UD detector in the background of a plane gravitational wave is obtained and found to have the same form as some Standard Model scattering or three body decays. Based on this similarity we propose that this represents, in Feynman diagram language, either a “decay” of the initial two gravitons into two photons (*i.e.* $G + G \rightarrow \gamma + \gamma$) or the attenuation of one initial graviton into two photons plus a lower frequency/energy graviton (*i.e.* $G \rightarrow \gamma + \gamma + G$). Such processes, as well as related processes like graviton-photon “Compton scattering” $G + \gamma \rightarrow G + \gamma$, have been investigated in [10–12] using Feynman diagrammatic methods or other techniques. The attenuation process, $G \rightarrow \gamma + \gamma + G$, is related to the previously studied graviton decay [13] process $G \rightarrow G + G + G$. Both $G + G \rightarrow \gamma + \gamma$ and $G \rightarrow \gamma + \gamma + G$ would lead to a weakening of the gravitational wave amplitude/graviton intensity with distance from the source.

In order to examine particle production from quantum vacuum fluctuations we consider the special case of a gravitational plane wave [14] traveling in the $x_3 = +z$ direction with diagonal metric. Taking the UD detector to be located as $z = 0$ the metric at this point is $g_{00} = -1$, $g_{11} = 1 - h(\omega t)$, $g_{22} = 1 + h(\omega t)$, $g_{33} = 1$ with $h(\omega t) = \sin^2(\theta) \sin(2\psi) h_0 \sin(\omega t)$. The angles θ and ψ represent the orientation of the direction of the gravitational wave with respect to the detector axis. These angles take values from $\theta = 0 \dots \pi$ and $\psi = 0 \dots 2\pi$ [15]. As an example, for an UD detector consisting of a particle with a magnetic moment in a uniform magnetic field the angles of orientation, θ and ψ , are between the direction of the magnetic field and the direction of travel of the incoming plane wave. The constant h_0 is the dimensionless amplitude of the gravitational wave, which can be written as $h_0 = G \frac{m_0}{c^2 r_0}$ where m_0 and r_0 are the relevant mass and distance of the system and $G \frac{m_0}{r_0}$ is a potential energy per mass. The amplitude squared of the wave is proportional to the energy density [16], $T_{00} = \frac{1}{32\pi} \frac{c^2}{G} \omega^2 h_0^2$. The graviton is taken as the quanta of the gravitational wave with energy $\hbar\omega$.

The spectrum, $S(E)$, observed by the UD detector can be viewed as a production rate of photons per unit volume. It is given by the product of the density of states, $\rho(E)$, and the response function $F(E)$ [1, 7, 8, 17, 18],

$$\begin{aligned} S(E) &= n_{general} - n_{inertial} = 2\pi\rho(E)F(E) \\ &= 2\pi\rho(E) \int_{-\infty}^{+\infty} e^{-i\Delta E\Delta\tau} (G^+(s) - G^+(\Delta\tau)) d(\Delta\tau). \end{aligned} \quad (1)$$

In this expression for $S(E)$ we have taken $\hbar = 1$ and $c = 1$. In (1) $\Delta E = E_{up} - E_{down}$ is the difference between the energies of the upper and lower energy level of the UD detector [8]. We will take $E_{down} = 0$ so that from now on we can write $\Delta E \rightarrow E_{up} \equiv E$. The number density of the inertial detector, $n_{inertial}$, has been subtracted out to eliminate the singularity in the integrand [7, 18] of equation (1). The trajectory for a particle in an inertial frame is $x(\Delta\tau) = (\Delta\tau, 0, 0, 0)$ which has an associated Wightman function of $G^+(\Delta\tau) = \frac{1}{4\pi^2\Delta\tau^2}$. The particle trajectory in the curved space of a passing gravitational wave is given by $x(\Delta\tau) = (\gamma\Delta\tau, \Delta x, 0, 0)$ where $\gamma^{-2} = 1 - \Delta\dot{x}^2$ and Δx is the deviation of a particle's trajectory due to the gravitational wave background. Substituting this into the general number spectra in equation (1) the integral becomes

$$\begin{aligned}
F(E, \theta, \psi) &= \int_{-\infty}^{+\infty} e^{-iE\Delta\tau} (G^+(s) - G^+(\tau)) d(\Delta\tau) \\
&= \frac{1}{2\pi^2} \int_0^\infty \cos(E\Delta\tau) \left(\frac{1}{\gamma^2\Delta\tau^2 - \Delta x^2} - \frac{1}{\Delta\tau^2} \right) d(\Delta\tau).
\end{aligned} \tag{2}$$

Equation (2) is correct in general for strong gravitational waves since up to now we have placed no restrictions on h_0 or γ . The dependence of $F(E, \theta, \psi)$ on the angles θ, ψ comes from Δx as we will see shortly. Without any approximation this $F(E, \theta, \psi)$ for the plane wave background, as given in equation (2), must be evaluated numerically.

In order to make an analytical evaluation of $F(E, \theta, \psi)$ we take the limit of a weak gravitational wave with $\gamma \approx 1$ and $h_0 \ll 1$. To first order the trajectory along a null geodesic [14] in the x direction is $\dot{x} = (1 + \frac{1}{2}h)$. The separation, Δx , can be calculated from the difference in the null geodesic between particles in the inertial and curved space-times, $\Delta x = (1 + \frac{1}{2}h) \Delta\tau - \Delta\tau = \frac{1}{2}h\Delta\tau$. Substituting this Δx into equation (2) and using $\gamma \approx 1$ and $h_0 \ll 1$ we obtain

$$F(E, \theta, \psi) = \frac{1}{8\pi^2} \int_0^\infty \cos(E\Delta\tau) \frac{h^2}{\Delta\tau^2(1 - \frac{1}{4}h^2)} d(\Delta\tau) \approx \frac{1}{8\pi^2} \int_0^\infty \frac{\cos(E\Delta\tau)h^2}{\Delta\tau^2} d(\Delta\tau). \tag{3}$$

With this approximation the RHS of equation (3) can be integrated. Recalling that $h(\Delta\tau, \theta, \psi) = \sin^2(\theta) \sin(2\psi) h_0 \sin(\omega\Delta\tau)$ (this dependence of h on θ, ψ is the reason F in (2) and (3) depends on θ, ψ) we find,

$$F(E, \theta, \psi) = \frac{1}{64\pi} \left(2\omega - E - \frac{(E-2\omega)}{\sqrt{(E-2\omega)^2}} (2\omega - E) \right) \sin^4(\theta) \sin^2(2\psi) h_0^2. \tag{4}$$

Finally we take the integral of (4) over all possible orientations of the direction of the gravitational wave with the orientation of the UD detector, (*i.e.* integrating over $\theta = 0 \dots \pi$ and $\psi = 0 \dots 2\pi$) to find

$$F(E) = \frac{3\pi}{256} (2\omega - E) h_0^2 \quad \text{for } E < 2\hbar\omega \quad ; \quad \text{and } F(E) = 0 \quad \text{for } E > 2\hbar\omega. \tag{5}$$

The response function $F(E)$ vanishes for energies above the cutoff $E > 2\hbar\omega$. Note that this cut-off is also the same as that seen in muon decay [19] [20]. The meaning of the cut-off in (5) can be seen to have a similar kinematic origin – for the processes $G + G \rightarrow \gamma + \gamma$ and $G \rightarrow G + \gamma + \gamma$ there is at most $2\hbar\omega$ of energy available (each graviton contributes $\hbar\omega$ of energy). Thus for $E > 2\hbar\omega$ neither process can proceed energetically.

The density of states in equation (1) is $\rho(E) = \frac{E^2}{2\pi^2}$ [7]. Using this along with the response function in equation (5), and restoring the factors of \hbar and c , gives the spectra detected by the UD detector

$$S(E) = \frac{3}{512\pi\hbar^3 c^3} E^2 (2\hbar\omega - E) h_0^2 \quad \text{for } E < 2\hbar\omega \quad ; \quad \text{and} \quad S(E) = 0 \quad \text{for } E > 2\hbar\omega. \quad (6)$$

The result in (6) gives the spectra of the UD detector $S(E)$. Equation (6) is the main result we want to emphasize in this essay. As previously mentioned $S(E)$ has the same form as certain Standard Model processes (see for example [19] [20]). The functional form of $S(E)$ in equation (6) is a *Beta*(3,2) distribution and such distributions are common for particle decay processes. In Feynman diagram language, the processes implied by $S(E)$ from (6) would be $G + G \rightarrow \gamma + \gamma$ or $G \rightarrow G + \gamma + \gamma$ where the gravitons take the place of the muon and electron and the two photons take the places of the neutrinos. For $G + G \rightarrow \gamma + \gamma$ the gravitons are completely transformed into photons while for $G \rightarrow G + \gamma + \gamma$ the initial graviton is transformed into a lower energy graviton. For either process energy is transferred completely or partly into photons and the gravitational wave will be attenuated.

Recalling that $S(E)$ from (6) can be viewed as a production rate of photons per unit volume for the UD detector we can use it to calculate the decay/attenuation of gravitational waves via the above processes. By energy conservation the photon production spectrum for the UD detector will come from the gravitons. The “decay” rate for gravitons per unit volume is then given by,

$$\hbar \frac{d\Gamma}{dV} = \int_0^{2\hbar\omega} S(E) dE. \quad (7)$$

The production rate is a function of the dimensionless gravitational wave amplitude, h_0 , but the physical amplitude falls off as r^{-1} for an expanding gravitational wave. The dimensionless gravitational wave amplitude can be written as $h_0 = G \frac{m_0}{c^2 r_0}$ where m_0 and r_0 are an effective mass and distance respectively. The physical amplitude for an expanding gravitational wave can then be written as $h_r = \frac{\xi_0}{r}$ where $\xi_0 = \frac{G m_0}{c^2}$, which has dimensions of length. Replacing h_0 with h_r in equation (7) and integrating over all energies in the spectra yields,

$$d\Gamma = \frac{1}{128\pi c^3} \frac{\xi_0^2}{r^2} \omega^4 dV = \frac{\xi_0^2}{32c^3} \omega^4 dR. \quad (8)$$

One can immediately integrate the RHS of equation (8) from $R = 0$ to some radius $R = r$ to obtain a decay rate of $\Gamma = \frac{\xi_0^2}{32c^3} \omega^4 r$. The four factors of \hbar coming from $\hbar\omega$ are canceled by the three factors of \hbar coming from $S(E)$ and the one factor coming from the LHS of equation (7).

The decay of gravitons can be expressed in terms of a mean path length, $\langle r \rangle = c \langle \tau \rangle = c\Gamma^{-1}$. Integrating equation (8) from 0 to $\langle r \rangle$ and substituting $\Gamma = \frac{\xi_0^2}{32c^3} \omega^4 r$ in the mean path length expression gives,

$$\langle r \rangle = \frac{c}{\Gamma} = \frac{32c^4}{\xi_0^2 \omega^4 \langle r \rangle} \rightarrow \langle r \rangle = \frac{2\sqrt{8}c^2}{\xi_0 \omega^2}. \quad (9)$$

Due to the inverse relation between the mean free path, $\langle r \rangle$, and ξ_0 and ω^2 , gravitational waves are effected less, and therefore attenuated less, by this conversion of gravitons into photons for low frequency ($\omega \ll 1$) and small amplitudes ($\xi_0 \ll 1$). This is similar to the conclusion reached by Mondanese [13] using “general kinematical arguments”. For a gravitational wave source on the order of a Solar mass, $m_0 \sim 2 \times 10^{30}$ kg, we find $\xi_0 \sim 10^3 m$. Next assuming a gravitational wave frequency of $\omega \sim 10^2 s^{-1}$, the approximate mean decay length would be $\langle r \rangle \sim 10^{11} m = 10^{-5} ly$. This is a short attenuation distance especially considering that the decay rate, Γ , is normally relatively low. However, this short attenuation length is not completely surprising since a small decay rate can be compensated for if it acts over a large distance traveled by the gravitational wave. Also for strong gravitational wave sources with large amplitudes, ξ_0 , and large frequencies, ω , the decay rate, $\Gamma = \frac{\xi_0^2}{32c^3} \omega^4 r$, becomes larger. Thus for strong gravitational waves one would need to numerically integrate (2) without the approximations and then repeat the above development.

The mean free path for graviton decay in equation (9) is based on a semi-classical calculation with the gravitational wave treated classically and the vacuum quantum mechanically. The quantum nature for the graviton appears here only via the particle energy, $\hbar\omega$, and through the cutoff in the energy distribution, $2\hbar\omega$. It would be interesting to compare this result to a more complete quantum field theory calculation of graviton decay following DeWitt [21] or Bjerrum-Bohr *et al.* [12]. While such calculations have the advantage of including spin and more carefully taking account of energy-momentum conservation, any

mean free path calculation for graviton transition rates would include a similar calculation over the volume of space as in equation (8). The UD detector calculation, followed by the mean free path estimate given above, indicates that detecting gravitational waves at large distances may be hard or impossible due to the attenuation of the gravitational wave coming from the conversion of gravitons to photons.

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