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Electromagnetic Radiation From Temporal Variations in Space-Time and Progenitors of Gamma Ray Burst and Millisecond Pulsars

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Abstract

A time varying space-time metric is shown to be a source of electromagnetic radiation even in the absence of charge sources. The post-Newtonian approximation is used as a realistic model of the connection between the space-time metric and a time varying gravitational potential. Rapid temporal variations in the metric from the coalescence of relativistic stars are shown to be likely progenitors of gamma ray burst and millisecond pulsars.

1 Introduction

With the observation of the bending of light around the sun by Eddington in 1919 it has been generally accepted that gravity and electromagnetism are interrelated. In the intervening years this relation between gravity and electromagnetism has been established beyond the association of gravity with curvature of space-time and the path of a beam of light. For example recent research [1] has established a theoretical connection between the gravity wave predictions of general relativity and stimulation of electromagnetic radiation in charged plasmas. A more fundamental relationship between gravity and electromagnetic radiation will be established here by solving the covariant Maxwell's equations in the presence of a rapidly time varying gravitational potential and in the absence of any charge sources. In particular rapid variations in space-time, as a consequence of the collision of astronomical objects, will be shown to be a source of observed electromagnetic emissions.

It is well established that the homogeneous and inhomogeneous 4-space equations of electrodynamics reduce to the familiar four Maxwell equations for the special case of a Lorentz inertial frame and flat space-time metric. Where the space-time is not flat these equations are complicated by space-time variations in the metric. This complication follows from the appearance of the metric in the relation between the electric and magnetic fields and the differential 4-space operator in the 4-space equations. Because of this explicit dependence on the metric in the 4-space equations of electrodynamics variations in space-time would be expected to contribute to the electrodynamic fields. It would also be expected that temporal variations in the metric will contribute to the electric and magnetic fields independent of the local charge distribution.

The 4-space equations of electrodynamics, in order to insure the invariance of the equations, must relate the 4-space invariant electric and magnetic fields and differential operator to the invariant charge sources. This is most commonly achieved by first relating a 2-rank tensor, the Electromagnetic Tensor, to the electric and magnetic fields [2]. The association between the Electromagnetic Tensor, the 4-space metric, and the electromagnetic fields in the equations of electrodynamics is then somewhat obscure since the fields only appear indirectly in the equations. It is much easier to identify the relation between the electric and magnetic fields and the 4-space metric in an alternative representation of the equations of electrodynamics that includes the fields directly in the invariant equations. The explicit appearance of the fields in the 4-space equations was established by Ellis [3][4] where the electromagnetic tensor is replaced with the direct products of the fields and the local 4-velocity.

In order to examine the relation between a time varying space-time metric and the electromagnetic fields the 4-space electrodynamic equations are expanded here under the conditions of a rapidly time varying metric in a space-time region that is free of charge sources. Solving for the divergence of the electromagnetic power the time variations in space-time are shown to be a source of electromagnetic radiation. These rapid time variations in the space-time metric can be related to realistic astronomical phenomena by expressing the metric in terms of the gravitational potential using the post-Newtonian approximation. From this approximation the rapid changes in the gravitational potential, associated with collisions of black-holes, are shown to be a probable source for the power spectra from gamma ray burst. As a second example of this phenomenon, temporal changes in the gravitational potential during ring-down following neutron star collisions are shown to be the possible progenitor of millisecond pulsars.

2 Covariant Equations of Electrodynamics

The equations of electrodynamics developed by Ellis [3] exhibit an implicit connection between the electric and magnetic fields and the 4-space metric through the covariant derivatives. Following Ellis [3] [5], the homogeneous Maxwell equations can be expressed in covariant form as the direct product of the fields and the local 4-velocity,

$$\left[B^{\gamma}u^{\delta}\right]_{;\gamma} - \left[u^{\gamma}B^{\delta}\right]_{;\gamma} + \left[e^{\gamma\delta\alpha\beta}u_{\alpha}E_{\beta}\right]_{;\gamma} = 0. \tag{1}$$

The 4-velocity u_{α} is the velocity of the differential volume element associated with the fields and B^γ and E_β are the components of the 4-vector magnetic and electric fields respectively. The Levi-Civita tensor $e^{\alpha\beta\gamma\delta}$ is defined in terms of the permutation symbol $E^{\alpha\beta\gamma\delta}$, which is zero if any indices are repeated, one for even permutations of 0, 1, 2, 3, and negative one for odd permutations,

$$e^{\alpha\beta\gamma\delta} = -\frac{1}{\sqrt{-g}} E^{\alpha\beta\gamma\delta}, \tag{2}$$

where $g = \det\left(\hat{\mathbf{e}}_\alpha \cdot \hat{\mathbf{e}}_\beta\right)$ is the determinant of the metric components. The inhomogeneous Maxwell equations are written in a similar fashion [5],

$$\left[E^\gamma u^\delta\right]_{;\gamma} - \left[u^\gamma E^\delta\right]_{;\gamma} - \left[e^{\gamma\delta\alpha\beta} u_\alpha B_\beta\right]_{;\gamma} = 4\pi J^\delta, \tag{3}$$

where J^δ is the 4-vector electric current density. Assuming a Lorentz inertial frame (LIF) the covariant derivatives can be replaced with partial derivatives. Adopting the convention of a negative time part for the metric the 4-velocity is $u_\alpha = u_0 = -1$. Expanding the homogeneous equation with $\delta = 0$ leads to

$$B^k_{,k} = 0 \tag{4}$$

and in the inhomogeneous equation to

$$E^k_{,k} = 4\pi J^0, \tag{5}$$

where the Roman indices range over 1, 2, 3. Taking $\delta = i$ in the homogeneous and inhomogeneous equations produces the remaining Maxwell equations,

$$B^i_{,0} + E^{ijk} E_{k,j} = 0 \tag{6}$$

and

$$E^{ijk} B_{k,j} - E^i_{,0} = 4\pi J^i. \tag{7}$$

This expansion demonstrates that the covariant equations 1 and 3 are the correct form invariant equations of electrodynamics.

3 Time Dependent Field in Post Newtonian Approximation

In order to examine the relation between the covariant equations of electrodynamics and variations in the gravitational potential the 4-space metric will be expanded in terms of the potential. The Post-Newtonian approximation assumes that only the lowest order terms in this expansion contribute appreciably to the metric. Making this assumption and requiring that the gravitational potential be time dependent, the time component of the metric is

$$g_{00} = -\left(1 + 2\phi\right), \quad g^{00} = -\left(1 - 2\phi\right). \tag{8}$$

In this post-Newtonian approximation the spacial components are,

$$g_{ii} = 1 - 2\phi, \quad g^{ii} = 1 + 2\phi, \tag{9}$$

with all other components equal to zero. The Christoffel symbols will be needed in the expansion of the covariant derivatives and the nonzero terms are

$$\Gamma^0_{00} = -\Gamma^0_{ii} = \dot{\phi} - 2\phi\dot{\phi}, \quad \Gamma^i_{i0} = -\dot{\phi} - 2\phi\dot{\phi}. \tag{10}$$

where the dots represent time derivatives and there is no sum on i. It will be helpful later to define a sum of two Christoffel symbols,

$$\Gamma^0_{00} + \Gamma^i_{i0} = -2\frac{\partial}{\partial t}\phi^2 = -\Sigma. \tag{11}$$

In a comoving frame, $u_i = 0$, the covariant derivatives in the equations of electrodynamics can be expanded in terms of this sum. The time derivatives in terms of some arbitrary vector A^k and the direct product with the 4-velocity are

$$\left[u^0 A^k\right]_{;0} = A^k_{,0} - \Sigma A^k. \tag{12}$$

The covariant space derivatives are simply the partial derivatives,

$$\left[u^0 A^k\right]_{;k} = A^k_{,k}. \tag{13}$$

The dual term in the equations requires a bit more attention. Since the metric is not position dependent the Christoffel symbols will vanish and the covariant derivatives are again partial derivatives,

$$\left[e^{ij0k} u_0 A_k\right]_{;i} = \left[\left(-\frac{1}{\sqrt{-g}} E^{ij0k}\right)(-1)\, A_k\right]_{,i}. \tag{14}$$

With our assumptions of a time dependent metric in the post-Newtonian approximation the spacial derivatives of the determinate make no contribution and to first order the covariant derivatives of the dual terms,

$$\left[e^{ij0k} u_0 A_k\right]_{;i} = -E^{ijk} A_{k,j}, \tag{15}$$

are simply the negative of the curl.

4 Electromagnetic Power Spectra

The post-Newtonian approximation for a time varying 4-space metric will be used to demonstrate the connection between the gravitational potential and the electromagnetic fields in the covariant equations of electrodynamics. The equations of electrodynamics will be expanded assuming a comoving reference frame and small second order terms in the metric, $\phi^2 \prec\prec 1$ and $\phi^2 \prec\prec \frac{\partial}{\partial t}\phi^2$. To demonstrate that a time varying gravitational potential generates electromagnetic radiation assume also that there are no charge sources, $J^\alpha = 0$. Taking the equations for $\delta = i$ and no charge sources,

$$\left[u^0 B^i\right]_{;0} + E^{ijk} E_{k,j} = 0 \tag{16}$$

and

$$\left[u^0 E^i\right]_{;0} - E^{ijk} B_{k,j} = 0. \tag{17}$$

Making use of the expansions of the covariant derivatives from the previous section and contracting each equation with the appropriate field,

$$B_i B^i_{,0} + B_i E^{ijk} E_{k,j} = \Sigma B^2 \tag{18}$$

and

$$E_i E^i_{,0} - E_i E^{ijk} B_{k,j} = \Sigma E^2. \tag{19}$$

Adding these two equations and rearranging the terms,

$$B_i E^{ijk} E_{k,j} - E_i E^{ijk} B_{k,j} + \frac{1}{2}\frac{\partial}{\partial t}\left(B^2 + E^2\right) = \Sigma\left(B^2 + E^2\right). \tag{20}$$

The terms on the left hand side can be simplified using the triple product identity, $\nabla \cdot (\mathbf{a} \times \mathbf{b}) = \mathbf{b} \cdot (\nabla \times \mathbf{a}) - \mathbf{a} \cdot (\nabla \times \mathbf{b})$, dividing by $\frac{1}{4\pi}$, and substituting the energy density $U = \frac{1}{8\pi}\left(E^2 + B^2\right)$,

$$\nabla \cdot \mathbf{S} + \frac{\partial U}{\partial t} = \frac{4}{c^4} U \frac{\partial}{\partial t}\phi^2, \tag{21}$$

where use has been made of the Poynting vector, $\mathbf{S} = \frac{c}{4\pi}\mathbf{E} \times \mathbf{B}$, and the factor of c has been reinstated to illustrate the order of magnitude of this effect. This demonstrates that in a region of nonzero internal energy density U and rapidly time varying gravitational potential, ϕ, electromagnetic radiation will be generated. Due to the square of the gravitational potential in the time derivative the period of the electromagnetic power spectra would be half the period of the variation in the gravitational potential. This relationship between electromagnetic radiation and a time varying metric is formally similar to the expression for the interaction between gravity waves and charged plasmas [1]. The current development is a more general consequence of the covariant derivatives in the Maxwell equations and in particular does not assume a charged plasma or any other charge sources.

5 Gamma Ray Burst and Millisecond Pulsars

Let us consider the electromagnetic radiation that will be generated from the collision of relativistic stars. Relativistically the internal energy density U would be associated with the total mass-energy density of the binary system. During coalescence a fraction of the rest mass energy will be radiated from the system as electromagnetic radiation. Since the gravitational potential increases most rapidly at the end of the collision process this is a likely progenitor of the radiation spike seen in gamma ray burst. Collisions of neutron star binaries have been previously identified as a potential source of gamma ray burst [1][6][7]. However, the magnitude of the radiated power calculated from the collisions of neutron stars is much smaller than the observed radiated power.

Since the typical energy radiated from gamma ray burst is much greater than the internal energy density of neutron star binaries the progenitor of the these burst are much more likely to be black hole collisions. Hawking [8] has estimated that as much as 50 percent of the original rest mass energy escapes during the coalescence of black hole binaries. Hawking's estimate would place a lower bound on the radiated energy of the order of one solar mass. However, the internal energy associated with a black hole binary does not have an upper bound and explains the magnitude of the energy radiated in even the largest gamma ray burst. The period of the coalescence of black hole binaries should be associated with the interval of the transition from two separate event horizons to a single event horizon which is consistent with the short duration of the peak energy of gamma ray burst. This association between black holes and gamma ray burst provides a method of determining the order of magnitude of the masses of black holes and could also provide a method for determining the distribution of black holes in the universe and the associated mass distribution.

Gamma ray bursts are one example of an electromagnetic power spectra that would be associated with an astronomical event involving a rapidly varying gravitational potential. However, the generation of electromagnetic radiation by time variations in a gravitational potential should be ubiquitous. In the example of the collision of a neutron star binary there is also the possibility of a ring-down of the coalesced object, provided that the combined mass does not produce a black hole. During this ring-down the gravitational potential would be expected to generate an electromagnetic power spectra due to the time variation in the square gravitational potential. The pulsation modes of the metric and associated gravitational potential, in the ring down of a coalesced neutron star, have been calculated by Allen et al [7][9]. The calculated modes predict millisecond time variations in the gravitational potential which would explain the mechanism for electromagnetic radiation from millisecond pulsars.

6 Conclusion

Expanding the covariant equations of electrodynamics, in terms of the direct products of the electromagnetic fields and the local 4-velocity and assuming the post-Newtonian approximation, a time varying metric was shown to be a source of electromagnetic radiation even in the absence of charge sources. Electromagnetic radiation produced by rapid time variations in the gravitational potential of coalescing black holes is the likely progenitor of gamma ray burst. This gravity induced electromagnetic radiation is also the likely mechanism for producing the radiation from millisecond pulsars. The phenomena of gravity induced electromagnetic radiation would not be expected to be limited to these two examples. However, due to the factor of c^4 in the relation between the radiated power and the time rate of change of the potential this phenomenon will be observable only in astronomical events where space-time variation in the potential and the internal energy are great.

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