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Main Sequence Masses and Radii from Gravitational Redshifts

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ABSTRACT

Modern instrumentation makes it possible to measure the mass to radius ratio for main sequence stars in open clusters from gravitational redshifts. For stars where independent information is available for either the mass or the radius, this application of general relativity directly determines the other quantity. Applicable examples are: 1) measuring the radii of solar metallicity main sequence stars for which the mass - luminosity relation is well known, 2) measuring the radii for stars where model atmospheres can be used to determine the surface gravity (the mass to radius squared ratio), 3) refining the mass - radius relation for main sequence stars, and 4) measuring the change in radius as stars evolve off the main sequence and up the giant branch.

Subject headings: gravitation - open clusters and associations: general - relativity - stars: fundamental parameters (masses, radii) - techniques: radial velocities

1. Introduction

Einstein (1911) predicted a gravitational redshift for light escaping the Sun equivalent to 0.636 km s^{-1} based on his General Theory of Relativity (Einstein 1916). Early pioneering work (Evershed 1931) demonstrated that absorption lines near the solar limb were systematically redshifted, although the measurements did not entirely agree with Einstein's theory as the experiment was too difficult for the instrumentation of the time. Subsequent work (see Vessot et al. 1980; LoPresto et al. 1991; Krisher, Morabito, & Anderson 1993) yielded results which, within the errors (now 1%), were indistinguishable from Einstein's predictions. While measuring the solar gravitational redshift has been a better means of testing General Relativity than of measuring the mass of the Sun, subsequent gravitational

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redshift work has been aimed primarily at measuring the masses of white dwarf stars in binaries (e.g. Adams 1925; Wegner 1980; Wegner & Reid 1991) and open clusters (for the early work see Greenstein & Trimble 1967 and Trimble & Greenstein 1972; for a modern work see Reid 1995). The high surface gravities of white dwarfs produce redshifts of $\approx 30 \text{ km s}^{-1}$ (Wegner & Reid 1991). The procedure is to measure the radial velocity of a white dwarf relative to a known systemic velocity, such as that of an open cluster or binary system.

Modern instrumentation (see Walker 1992) now makes it possible to extend gravitational redshift measurements to a new regime and measure the gravitational redshifts of main sequence stars. Such measurements would directly determine the mass - radius ratio (M/R) for these stars. While the mass of main sequence stars as a function of luminosity, color or spectral type is well-determined for solar metallicity stars from binary star studies (Popper 1980), it is not well-determined for metal-poor or metal-rich stars. Furthermore, stellar radii have not been measured for most types of stars. Generally, the techniques used to determine stellar radii measure the angular size of a star (e.g. lunar occultation and interferometric techniques, see McAlister (1985)), and thus also require a distance determination. These techniques also generally favor nearby giant stars. Detailed studies of double-line spectroscopic eclipsing binaries (e.g. Nordström & Johansen 1994) can yield stellar radii without the need for distance measurements, but few such eclipsing binaries exist. Besides being a fundamental parameter, the radii of stars with convective outer layers cannot be determined adequately from currently understood physics (see below).

Following, I suggest two methods for measuring M/R for main sequence stars in open clusters. Both methods are based on the very low velocity dispersions in many open clusters, for instance $\sigma = 0.44 \pm 0.04 \text{ km s}^{-1}$ in the Hyades (Zhao & Chen 1994) and $\sigma = 0.48 \pm 0.09 \text{ km s}^{-1}$ in M67 (Mathieu 1985).

2. Method 1: Relative measurement along the Main Sequence

This method uses the intrinsic, but slow, variation of the mass - radius relation along the main sequence to measure the relative increase in M/R as a function of increasing stellar mass. The gravitational redshift is given by Greenstein & Trimble (1967) as:

$$K = 0.635(M/R) \tag{1}$$

where K = gravitational redshift in km s^{-1} , and M and R are in solar units. (Note that the coefficient here is slightly different from the 0.636 km s^{-1} value predicted by Einstein. This

is simply a commonly-used round-off of the Einstein value.)

The mass - radius relation (Mihalas & Binney 1981) can be approximated as:

$$R \sim M^{0.7} \tag{2}$$

again with M and R in solar units, yielding:

$$K = 0.635M^{0.3} \tag{3}$$

Thus a G2 V star ($1.0 M_{\odot}$) should have $K = 0.635 \text{ km s}^{-1}$ (and the Sun does, see above) and a B3 V star ($\approx 10 M_{\odot}$) should have $K = 1.267 \text{ km s}^{-1}$. In practice one would determine the radial velocities of all available cluster members, remove binary stars, and correlate the measured stellar velocities with color, effective temperature, or luminosity, which themselves correlate with mass. In the example cited here, one is attempting to measure a difference of $\approx 0.6 \text{ km s}^{-1}$ (G2 V to B3 V) within a system with a velocity dispersion of $\approx 0.5 \text{ km s}^{-1}$. Modern velocity measurements can be made with a precision significantly better than 0.5 km s^{-1} for F-type and later stars (Duquennoy, Mayor, & Halbwachs 1991), while for A-type and earlier stars the paucity of absorption lines and the generally high rotation velocities make it difficult to achieve a precision better than 1.0 km s^{-1} (Morse, Mathieu, & Levine 1991). Thus for most cluster stars the cluster dispersion dominates the random errors. Systematic errors which are a function of stellar surface temperature present a larger observational challenge, however. Such systematic errors arise primarily from line blends which change in relative strength as a function of atmospheric temperature, inducing small, but real, line centroid shifts. There appears to be, for example, a systematic dependence on temperature in the radial velocities measured with the CfA speedometers (Latham 1995) at the few tenths of a km s^{-1} level. This systemic error is most likely due to the small wavelength range used, so that mismatches in blends do not average out adequately. Increased wavelength coverage should greatly reduce this source of systematic error. Increased resolution may also help to de-blend these lines.

I assume that the errors in color, effective temperature, or luminosity can be made small enough so that the measurements are limited by the velocity measurements. This is reasonable since internal photometric or spectroscopic temperature errors can be reduced to $\leq 1\%$ per star, and with $\leq 2\%$ external accuracy (Young 1993). A suitably chosen color index, such as V-I, will span a magnitude between a B3 V and a G2 V star, for example, and the effective temperature difference is a factor of three.

Assume for this discussion that insignificant systematic errors can be achieved². Then high-quality velocity measurements will yield errors $\approx 0.3 \text{ km s}^{-1}$ for F-type and later stars and $\approx 1.0 \text{ km s}^{-1}$ for A-type and earlier stars. Combining these with the $\approx 0.5 \text{ km s}^{-1}$ velocity dispersion yields $\sigma_{\text{late-type}} \approx 0.6 \text{ km s}^{-1}$ and $\sigma_{\text{early-type}} \approx 1.1 \text{ km s}^{-1}$. The uncertainty for the late-type stars is the same size as the gravitational redshift difference across this mass range, $\approx 0.6 \text{ km s}^{-1}$. In this example the goal is the slope in a velocity - mass (or color, etc.) diagram. The early-type and late-type stars each cover approximately a factor of 3.3 in mass range in this example, and so an approximation of the number of stars needed for a reliable measurement comes from acquiring a ratio (=3.4) of early-type to late-type stars to give each mass bin the same total error, then increasing the number of stars to the point where \sqrt{N} statistics yield a meaningful result. Thus ≈ 12 stars along the main sequence mass range from G2 V to A0 V and 40 from A0 V to B3 V would produce a 5σ ($=0.12 \text{ km s}^{-1}$) measurement, statistically determining the ratio of M/R at $10 M_{\odot}$ to M/R at $1 M_{\odot}$ to within 20%.

This method would yield M/R for main sequence stars spanning a range in mass. If the cluster distance, reddening and metallicity are well known, and if the metallicity is near solar, then the mass - luminosity relation can be used to give stellar masses, from which stellar radii can be determined. Alternatively, model atmosphere fits to the spectra of individual stars can yield the surface gravity, which is proportional to M/R^2 , and again radii can be determined³.

As a final note to this method, the mass - radius relation for main sequence stars is not entirely a power law, but has some subtle structure, especially for the lowest mass stars with spectral types later than M3. Careful work along a number of cluster main sequences to very faint magnitudes may eventually be possible, allowing a refined mass - radius relation.

3. Method 2: Relative measurement from the main sequence turn-off through the giant branch stars

This method uses the rapid increase in radius at a nearly constant mass as stars evolve off the main sequence and up the giant branch. Typically stellar radii double from the

²If systematic errors cannot be greatly reduced by increased wavelength coverage, then this problem can be inverted and the gravitational redshift effects discussed here should be *assumed* to determine the level of systematic error still resident in the data.

³See, however, Bergeron, Liebert & Fulbright 1995 for a discussion of the small, but significant, inconsistencies between gravitational redshift and atmospheric mass determinations.

main sequence turn-off to the blue hook, then remain nearly constant across the subgiant branch to the base of the red giant branch (see Bressan et al. 1993). Once on the giant branch, stellar radii rapidly increase. Since stellar surface temperatures differ very little as stars increase their radii and move up the giant branch, this technique is not as sensitive to the systematic velocity errors as Method 1. This method employs velocities in luminosity intervals to provide stellar radii at those points in the HR diagram.

From the arguments presented above, it is easy to see that a cluster with a turn-off at $\approx 1.6 M_{\odot}$ (i.e. at F0 and with age ≈ 2 Gyrs) would exhibit a gravitational redshift velocity falling from $\approx 0.73 \text{ km s}^{-1}$ on the main sequence to essentially 0 km s^{-1} somewhere along the giant branch. With $\sigma_{late-type} \approx 0.6 \text{ km}$ and ≈ 18 stars per luminosity interval (i.e. in each of the turn-off region, the subgiant branch, and the various luminosity bins up the giant branch), for example, velocities would be determined to $\approx 0.15 \text{ km s}^{-1}$, providing a 5σ measurement across the applicable luminosity range, and measuring stellar radii to within 20% on the main sequence and to within 40% at the point where the radius has doubled from its main sequence value. A determination of the run of radii as a star evolves would provide a valuable constraint on stellar models, since current theory cannot precisely determine stellar radii for most stars. This weakness arises because the physics of convection is poorly understood, and thus one cannot calculate from first principals the radius of any star with surface convection. The convection theory used in most stellar models, known as mixing length theory, is a simple parameterization of convective cells rising some ratio of the pressure scale height. The single parameter of this model is fit to the standard solar model, but detailed studies (e.g. Taylor 1986) of its applicability to other stars provide significant evidence that this theory is too simplistic.

This technique potentially could be inverted if the radii of some subset of the cluster stars were known, e.g. by lunar occultation measurements, in which case the mass of the turn-off stars, and thus their age, could be measured. This would be easiest for very young clusters with higher mass turn-offs, since stellar lifetimes are a steep function of mass, though the number of high mass stars available would be small.

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