Nonadiabatic Heating in Magnetic Reconnection

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Nonadiabatic heating in magnetic reconnection
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Abstract Plasma transport process as a fundamental problem in magnetospheric physics is often associated with strong nonadiabatic heating. At the magnetopause, observations show an increase of specific entropy (i.e., $S = p/\rho\gamma$) by 2 orders of magnitude from the magnetosheath into the magnetosphere. In the near-Earth magnetotail, particle injection requires strongly entropy depleted plasma bubbles, and their evolution can be strongly modified in the presence of nonadiabatic heating. In this study, one of the critical plasma transport mechanisms, magnetic reconnection, is investigated as a nonadiabatic process in the framework of MHD. It is important to examine whether magnetic reconnection can provide sufficient nonadiabatic heating to explain the observed plasma properties and to identify plasma conditions that allow such strong nonadiabatic heating. We demonstrate that the entropy can indeed strongly increase associated with magnetic reconnection provided that the plasmabeta (i.e., the ratio of thermal to magnetic energy density) is low in the inflow region of reconnection.

1. Introduction
Magnetic reconnection is a fundamentally important process in space physics, because it converts the stored magnetic energy into the kinetic energy and changes the magnetic topology such that plasma can be transported across magnetic boundaries [Dungey, 1961; Parker, 1957; Sweet, 1958]. It occurs in the presence of sufficiently large antiparallel magnetic field components. At the dayside magnetopause, magnetic reconnection is the prime mechanism for a transfer of magnetic flux and energy into the magnetosphere during periods of southward interplanetary magnetic field (IMF). During times of northward IMF, magnetic reconnection occurs at higher latitudes [Kessel et al., 1996]. The concept of magnetic reconnection is based on the breakdown of the so-called “frozen-in” condition. In Petschek’s reconnection model [Petschek, 1964], the frozen-in condition breaks in a tiny diffusion region, which is bound by inflow and outflow regions. The inflow and outflow regions are separated by two pairs of slow shocks (see Figure 1), where the plasma is mostly accelerated by the $j \times B$ force and magnetic energy is converted into bulk and thermal energy.

The energy conversion is one of the most important aspects of magnetic reconnection. This process can also be considered as a nonadiabatic process, which can be characterized by a variety of computable plasma entropy measures [Balasis et al., 2009; Kaufmann and Paterson, 2009]. In this study, we focus on the so-called “specific entropy,” $S = p/\rho\gamma$, where $p$ is the plasma thermal pressure, $\rho$ is the plasma density, and $\gamma = 5/3$ is the ratio of specific heats [Birn et al., 2006, 2009]. Hereafter, we simply refer to this quantity as entropy. Note that a value of $\gamma = 5/3$ corresponds to 3 degrees of freedom for the motion of charged particles. In the MHD description, entropy is an invariant in the absence of Ohmic or viscous heating and shocks. Nevertheless, in the context of magnetic reconnection, the breakdown of ideal MHD by local dissipation also implies a breakdown of the entropy conservation. A significant entropy increase in Petschek’s model may be provided by the two pairs of slow shocks bounding the outflow regions. These represent the major entropy sources, as will be demonstrated in this study for different types of reconnection geometries.

It is believed that magnetic reconnection is the dominant process for southward IMF conditions. For northward IMF conditions, reconnection driven by nonlinear Kelvin-Helmholtz modes at the low-latitude boundary layer and high-latitude reconnection become more important [Scholer and Treumann, 1997]. On the other side, satellite observations show that ions in the Earth’s plasma sheet become cold and dense during prolonged periods of northward IMF, which has been attributed to the massive transport of the solar wind or magnetosheath ions into the plasma sheet [Fujimoto et al., 1997]. There are two components of plasma in this cold dense plasma sheet, a hot distribution, which is considered a remnant of the original plasma sheet plasma, and a cold distribution, which is believed to be of magnetosheath origin [Wang et al., 2007]. The typical flank magnetosheath ion density and temperature for fast solar wind conditions

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are about 5 cm$^{-3}$ and 50 eV. For slow solar wind conditions, the ion density and temperature are about 8 cm$^{-3}$ and 10 eV [Borovsky and Cayton, 2011]. Density and temperature for the cold plasma component are about 0.5 cm$^{-3}$ and 500 eV [Wang et al., 2007], which yield an entropy increase of about 1 to 2 orders of magnitude compared to the magnetosheath. Therefore, the plasma entry process is accompanied with strong nonadiabatic heating. One may expect that this nonadiabatic heating is mainly caused by magnetic reconnection, because reconnection is believed to be the prime mechanism for this entry process. However, studies by Birn et al. [2006, 2009] and Wing and Johnson [2009] demonstrated that entropy appears more or less unchanged after reconnection. This raises the important question, can magnetic reconnection provide any significant nonadiabatic heating? Directly related is the issue of any specific conditions, for which significant nonadiabatic heating may occur.

Here we present a systematic study of the entropy changes in magnetic reconnection based on analytical theory and numerical MHD simulation. The numerical methods are introduced in section 2. In section 3, we discuss entropy sources in different subregions in a Petschek-type reconnection geometry. For applications to the actual magnetospheric boundary, the influence of a guide field, of an asymmetry in the inflow regions, and of the shear flow are discussed in section 4. Section 5 presents a summary and discussion.

2. Numerical Model

The full set of the resistive MHD equations and its numerical solver have been discussed by Otto [1990]. In the computations all quantities are normalized to the typical values, that is, the length scales $L$ to a typical length $L_0$, the density $\rho$ to $\rho_0 = n_0 m_0$ with the number density $n_0$ and the ion mass $m_0$, the magnetic field $B$ to $B_0$, the velocity $V$ to the typical Alfvén velocity $V_A = B_0/\sqrt{\mu_0 \rho_0}$, the thermal pressure $p$ to $P_0 = B_0^2/(2\mu_0)$, and the time $t$ to a typical Alfvén transit time $T_A = L_0/V_A$. To enclose MHD equations, one can use either the entropy equation (1)

$$\frac{\partial h}{\partial t} = -\nabla \cdot (hV) + \frac{\gamma - 1}{\gamma} h^{1-\gamma} \eta^2,$$

where $h = (p/2)^{1/\gamma}$ or the energy equation (2):

$$\frac{\partial w}{\partial t} = -\nabla \cdot \left( (w + \frac{1}{2}\rho_i) V - (V \cdot B) B + \eta j \times B \right),$$

where $w = [\rho_v^2 + B^2 + p]/(\gamma - 1)/2$ is the total energy density of the plasma, and $\rho_i = (p + B^2)$ is the total pressure. Note that in this normalization, the unitless entropy is $S = p/(2\rho')$. The difference between the energy equation and the entropy equation will be discussed in section 3.1.1.

We present the results from four selected two-dimensional simulations to study the entropy enhancement for typical magnetic reconnection configurations using a symmetric reference case and including characteristic variations in terms of the magnetic field, plasma flow, and symmetry of the configuration. The simulation domain is a rectangular box with $|x| \leq 20$, and $0 \leq y \leq 80$, and is resolved by using 323 $\times$ 353 grid points with a nonuniform grid in both directions. To sufficiently resolve the diffusion region, the best resolution is set to 0.025 and 0.1 in the $x$ and $y$ direction in the diffusion region. Free boundary conditions ($\phi_x = 0$, where $\phi_x$ is the partial derivative in the direction normal to the boundary) are applied to the $x$ maximum and minimum boundary and $y$ maximum boundary. The $y$ minimum boundary is determined by symmetry properties of the MHD equations [Otto et al., 2007]. The initial equilibrium is a one-dimensional
modified Harries sheet [Harris, 1962], which is given by 

\[ B = \tanh(x)\hat{e}_y + B_g\hat{e}_x, \quad V = V_{A}\tanh(x)\hat{e}_x, \quad \rho = \rho_{\infty} + 1 - B_g^2, \]

\[ \rho = \rho_0 + \delta \rho \tanh(x), \]

where the parameters \( B_g, V_{A} \), and \( \delta \rho \) are used to introduce a guide field, flow shear, and density asymmetry. The values of all parameters are listed in Table 1. To trigger Petschek-type magnetic reconnection (except case B), we use a localized resistivity given by

\[ \eta = \eta_0 \left[ 1 - \exp\left( -t/\tau \right) \right] \cosh^{-1}(x) \cosh^{-1}(y) + \eta_b, \]

where \( \eta_0 = 0.05, \tau = 5 \), and the small background resistivity \( \eta_b = 0.002 \).

Case A is the reference case. At the real magnetopause, the magnetosheath density is about 10 times larger than the magnetospheric density. Therefore, it is important to investigate this asymmetry for the entropy production. In the simulation (case B), we set \( \rho_{\infty} = 0.025, \rho_0 = 1.5 \), and \( \delta \rho = 0.5 \), which yields to \( \rho_M : \rho_{\infty} = 1 : 2 \), where \( \rho_M \) refers to the magnetosphere \((x < 0)\) density, and \( \rho_{\infty} \) is the magnetosheath \((x > 0)\) density. We only localized the resistivity along the \( y \) direction, because the diffusion region may move along the \( x \) direction:

\[ \eta = \eta_0 \left[ 1 - \exp\left( -t/\tau \right) \right] \cosh^{-1}(y) + \eta_b. \]

At dayside magnetopause, a guide field or a large perpendicular shear flow is always present. The influence of a guide field and of shear flow on the entropy production is investigated in the cases C and D, respectively.

### 3. Symmetric Petschek Reconnection

Figure 2 shows the thermal pressure (left) at \( t = 150 \) for Case A. In the inflow region, the pressure is about 0.1 (the reddish region in Figure 2), which yields an initial inflow region plasma beta of 0.1. In the outflow region, the pressure increases to about unity (the greenish region in Figure 2). Black lines are magnetic field lines (contours for the \( z \) component of the vector potential). The strongly bend magnetic field lines at the transition between inflow and outflow regions illustrates the presence of a strong surface current, and the inspection of the plasma properties (section 3.1) of the inflow and outflow regions demonstrates that this boundary represents a slow switch-off shock. Black arrows represent the flow velocity. The inflow velocity only has a normal component (\( x \) direction) with a relatively small magnitude, and it carries flux into the outflow region. The fast jetting plasma in the outflow region reaches the inflow Alfvén speed, which indicates that reconnection is well developed. There are three subregions [La Belle-Hamer et al., 1995] that can be identified in the outflow region. The diffusion region is located at the origin \((x = y = 0)\), and it is barely visible in Figure 2, which is consistent with the assumption of a small diffusion region in Petschek’s model [Petschek, 1964]. The steady outflow region \((y < 50 \text{ in Figure 2})\) is bounded by a pair of slow shocks being characterized by the thin current layers. And the nonsteady bulge region \((y > 50 \text{ in Figure 2})\) is the transition region between the fast reconnection jet and the unperturbed plasma.

### 3.1. The Dominant Entropy Source: Slow Shocks at the Steady Outflow Region

The transition between the inflow and outflow regions is almost one-dimensional, because the angle of the shocks with the \( y \) axis is small, and the transition represents the matching of the two asymptotic plasma conditions of the inflow regions by MHD waves and discontinuities (the so-called “Riemann problem” [Lin and Lee, 1993]). In Figure 2 (right), we present a profile of this transition layer \((x \geq 0 \text{ part of the cut, due to the symmetry})\), where the cut is taken from the blue line in Figure 2 (left). The five panels in Figure 2 (right) show the profiles of the plasma density, thermal pressure, normalized entropy, \( S/S_0 \), the magnetic field \( B_y \) component, and current density \( z \) component. Here \( S_0 = 0.05 \) is the inflow region entropy. The depletion of the density in the vicinity of the \( y \) axis, (shaded in gray and labeled as D in Figure 2) is a contact discontinuity and its cause will be discussed in section 3.2. The large current density is consistent with the switch off of the magnetic field, which indicates that these current layers represent the slow switch-off shock layers. These shock layers are shaded in gray and labeled S in Figure 2. A slow shock in the reconnection layer of the dayside magnetopause has for instance been observed by ISEE 2 spacecraft [Walthour et al., 1995].

### Table 1. Values of the Simulation

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
<th>Case D</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_0 )</td>
<td>1.0</td>
<td>0.5</td>
<td>0.5</td>
<td>0.1</td>
</tr>
<tr>
<td>( \delta \rho )</td>
<td>0.0</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>( B_g )</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.1</td>
</tr>
<tr>
<td>( V_{A} )</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.1</td>
</tr>
<tr>
<td>( \rho_{\infty} )</td>
<td>1.0</td>
<td>1.5</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>( \delta \rho_{\infty} )</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>

In the simulation (case B), we set \( \rho_{\infty} = 0.025, \rho_0 = 1.5 \), and \( \delta \rho = 0.5 \), which yields to \( \rho_M : \rho_{\infty} = 1 : 2 \), where \( \rho_M \) refers to the magnetosphere \((x < 0)\) density, and \( \rho_{\infty} \) is the magnetosheath \((x > 0)\) density.
Figure 2. (left) The thermal pressure $p$ at $t = 150$ for Case A. Black lines are the magnetic field lines (contours for the $z$ component of the vector potential), and black arrows indicate the flow velocity $V$. (right) The five panels show the profiles of the plasma density, thermal pressure, normalized entropy, $S/S_0$, magnetic field $B_y$ component, and current density $j_z$ component, which are taken from the blue line in Figure 2 (left). The red dashed lines are the results from the Rankine-Hugoniot (RH) relations. The black dashed line is the total pressure. The slow shock and density depletion layer are shaded in gray and labeled S and D, respectively.

The plasma density, thermal pressure, and entropy strongly increase through the shock layers, while the total pressure (black dashed line in the pressure plot) remains constant across the shock layers. The properties of the slow switch-off shock can be described by the so-called “Rankine-Hugoniot (RH) relations,” which are inferred from MHD equations (enclosed by energy equation) with homogeneous assumption for both upstream and downstream. Here we list the compression ratio, the ratio of downstream and upstream pressure and the corresponding ratio of the entropy for slow switch-off shocks:

\[
\frac{\rho_d}{\rho_u} = 1 + \frac{1}{\gamma \beta + \gamma - 1}, \tag{5}
\]

\[
\frac{p_d}{p_u} = \frac{\rho_d}{\rho_u} \left( 1 + \frac{\gamma - 1}{\gamma \beta} \right), \tag{6}
\]

\[
\frac{S_d}{S_u} = \left( \frac{\rho_d}{\rho_u} \right)^{1-\gamma} \left( 1 + \frac{\gamma - 1}{\gamma \beta} \right), \tag{7}
\]

where the index $d$ and $u$ indicate the downstream and upstream, respectively, and $\beta = \rho_u/B^2_u$ is the upstream plasma beta. Figure 3 presents the compression ratio, the ratio of downstream and upstream pressure, and the corresponding ratio of the entropy as a function of upstream plasma beta by using dash-dotted, dashed, and solid lines, respectively. Note that all the ratios increase with decreasing upstream
plasma beta, and the maximum plasma compression is 2.5, which means that the plasma entropy can increase significantly if the upstream plasma beta is sufficiently small. For $\beta < 10^{-2}$, the entropy ratio between outflow and inflow regions $S_d/S_u$ is greater than 20.

To compare the theoretical results with the two-dimensional simulation results, we labeled the downstream results from the RH relations by the red lines in Figure 2. It shows that right after slow shock (downstream region), all quantities are consistent with the theoretical results, although density increases further deeper into the outflow region. This is possibly because fluid elements deep in the outflow region have entered much closer to the diffusion region at a location or time when the slow shock had not been fully developed. We carry out a one-dimensional simulation of the Riemann problem. This has the advantage of much higher resolution and short execution times. In this study, one-dimensional simulations are also used to compare the influence of different forms of the equation of state which closes the MHD equations.

### 3.1.1. Difference Between Energy Equation and Entropy Equation

The energy equation (1) and the energy equation (2) are commonly used to model magnetic reconnection and other plasma problems. However, the RH conditions are based on the set of ideal MHD equations using energy conservation. Therefore, it is important to understand the implications of using the entropy equation (1) in resistive MHD particularly for applications involving nonadiabatic heating in magnetic reconnection.

The slow shock transition can be simulated in a one-dimensional configuration by adding a small constant $B_n$ component in the Harries sheet [Lin and Lee, 1993, 1999; Wu and Lee, 2000; Ma and Otto, 2013]. The initial conditions are given by $B = B_n \hat{e}_x + \tanh(x) \hat{e}_y$, $V = 0$, $\rho = 1$, and initial plasma thermal pressure is determined by total pressure balance. Figure 4 presents the results from the one-dimensional simulation with the upstream plasma beta of 0.1. The three panels show the density (top), the thermal pressure (middle), and the entropy (bottom). The dashed and dash-dotted lines are the simulation results by using the entropy equation and energy equation, respectively. As a reference, the theoretical results from the RH relations (equations (5)–(7)) are labeled by the solid lines. For convenience, we have transformed the system to the shock frame. Thus, $x > 0$ is the upstream (inflow region) and $x < 0$ is the downstream (outflow region).

Figure 4 shows that energy conservation yields correct jump relations; however, using the entropy equation generates an artificial density increase and an insufficient entropy change. Note that this density increase associated with the shock usingentropy conservation satisfies mass conservation.

Clearly energy and entropy conservations generate different shock properties in ideal MHD ($\eta = 0$). The entropy increase through the shock is due to the energy conservation and does not require explicit Ohmic heating. However, the use of entropy conservation in ideal MHD strictly maintains entropy conservation and the resulting "slow shock equivalent" discontinuity is violating energy conservation. For resistive MHD ($\eta \neq 0$), the entropy equation (including the resistive term in equation (1)) allows entropy to increase through Ohmic heating. This can be shown clearly by combining the entropy equation and the continuity equation, which yields to

$$\frac{dS}{dt} = \frac{d}{dt} \left( \frac{\rho}{2\rho'} \right) = (\gamma - 1) \eta \frac{j^2}{\rho'}. \quad (8)$$

where $d/dt = \partial_t + V \cdot \nabla$. Equation (8) indicates that a correct approximation to the slow shock solution is possible in the presence of sufficient Ohmic heating, which requires a very high resolution of the boundary...
The three panels show (top) the density, (middle) the thermal pressure, and (bottom) the entropy. The dashed and dash-dotted lines are the simulation results by using the entropy equation and energy equation, respectively. As the reference, the theoretical results from the RH relations are indicated by the solid lines.

For the width of the shock current sheet if the dissipation is determined by a constant resistivity. This estimate uses normalized upstream values of $B = 1$, $\rho = 1$, and $\beta = B^2 = 0.1$. In the cases of $\eta = 2 \times 10^{-3}$ and $1 \times 10^{-3}$, and $B_n = 0.025$, $\delta \approx 0.1$ and 0.05, respectively. The following result tests equation (10) based on these parameters.

Figure 4. MHD slow shocks with upstream plasma beta of 0.1, and $\Delta x = 0.04$. The three panels show (top) the density, (middle) the thermal pressure, and (bottom) the entropy. The dashed and dash-dotted lines are the simulation results by using the entropy equation and energy equation, respectively. As the reference, the theoretical results from the RH relations are indicated by the solid lines.

3.1.2. Estimate of the Entropy Increase in the Magnetic Reconnection

It is demonstrated that slow switch-off shocks generate strong nonadiabatic heating for low upstream (inflow) plasma beta using analytical theory and simulation. However, this conclusion is based on one-dimensional shock configurations. To extend this argument to two-dimensional magnetic reconnection and more general inflow conditions including asymmetries requires a more general estimate. In magnetic reconnection, pressure balance implies

$$p_i + B_i^2 = p_o + B_o^2,$$

where $i$ and $o$ indicate the inflow and outflow region, respectively. Therefore, the entropy increase can be expressed as

$$\frac{S_o}{S_i} = \frac{p_o \rho_i^{\gamma}}{p_i \rho_o^{\gamma}} = \left( \frac{p_i + B_i^2 - B_o^2}{p_i} \right) \left( \frac{\rho_i}{\rho_o} \right)^{\gamma},$$

Since $B_o^2 \geq 0$, and $\rho_i < \rho_o$ due to compression, it follows that

$$S_o/S_i < 1 + 1/\beta_i.$$
Figure 5. The (top) compression ratio, \( \rho_d/\rho_u \), and (bottom) entropy ratio, \( S_d/S_u \), as a function of the grid separation, \( \Delta x \), from the one-dimensional slow shock simulation by using the entropy equation. The solid lines indicate the theoretical results from the RH relations. A result for the energy equation by using resistivity \( \eta = 2 \times 10^{-3} \) is labeled by a star. The circle and square markers are the results by using resistivity \( \eta = 2 \times 10^{-3} \) and \( 1 \times 10^{-3} \), respectively.

3.2. Other Entropy Sources: The Diffusion Region and the Weak Fast Shock

The diffusion region is critical for the magnetic reconnection. In this region, the resistive term in Ohm’s law is dominant, and nonadiabatic heating is prominent. Since the pressure is determined by total pressure balance with the inflow region, the density must decrease to balance the increasing plasma temperature. The low-density plasma is convected along streamlines. Due to the symmetry, the central streamline is the \( y \) axis, which is consistent with the result in Figure 2. This density depletion layer has also been reported in other reconnection simulations [Ma and Bhattacharjee, 2001; Hesse et al., 2001; Shay et al., 2001; Pritchett, 2001; Yang et al., 2006]. However, the typical width of the diffusion region is on the order of an ion inertia scale for Hall physics and even smaller (electron inertia scale or gyroscale) in a kinetic model, which is negligible compared with macroscales of the whole system. Thus, we argue that the nonadiabatic heating in the diffusion region occupies a very small volume in a real space plasma and only a tiny fraction of the total plasma transported into the outflow region is actually going through the diffusion region. Therefore, only a small fraction of the plasma transported into the magnetosphere would be exposed to this nonadiabatic heating source.

The plasma in the steady outflow region is accelerated to the inflow region Alfvén speed by the slow switch-off shock. However, the plasma in the nonsteady bulge region moves at a velocity significantly lower than Alfvén speed. Due to the compression and heating, the acoustic speed in the outflow region can be lower than the velocity of the outflow jet. Therefore, it is possible for the fast shock to exist in the transition from the steady outflow to the bulge [Zenitani and Miyoshi, 2011] (in Figure 2, \( y \in (50, 55) \)). To characterize these fast shocks, we estimate the outflow Mach number \( M_o = V_o/c_{so} \), where \( V_o \) is outflow speed, which equates to the inflow Alfvén speed \( V_{Ai} = B_i/\sqrt{\rho_i} \), and \( c_{so} = \sqrt{\gamma p_o/(2\rho_o)} \) is the outflow acoustic speed. The pressure in the outflow region can be represented by the magnetic field in the inflow region from pressure balance \( p_o = \rho_i + B_i^2 \). By using the jump condition for density in a switch-off shock (equation (5)), we find

\[
M_o = \sqrt{2/(\gamma \beta_i + \gamma - 1)} \leq \sqrt{3}.
\]
Figure 6. (left) The normalized entropy at \( t = 100 \) for Case B. Black lines are the magnetic field lines. (right) The five panels show the profiles of the thermal pressure, normalized entropy, magnetic field \( B_y \) component, current density \( j_z \) component, and velocity \( V_y \) component, which are taken from the magenta line in Figure 6 (left). The total pressure is presented by a dashed line. The estimated value of \( B_0 \) from equation (18) is indicated by a red dashed line.

This Mach number does not take into account the shock speed. Therefore, the actual Mach number is smaller, and a weak fast shock is not expected to be a significant source for the entropy increase. We note that in the high-resolution simulation by Zenitani and Miyoshi [2011], the maximum velocity of outflow jet in this fast shock region is higher than the inflow region Alfvén speed. However, the entropy remains almost constant across this fast shock.

4. More General Configurations

Although we have mostly discussed Petschek reconnection, the conclusions drawn from the estimate for the entropy increase (equation (13)) apply to more general and asymmetric configurations. In this section, we will discuss three different reconnection configurations, which more closely reflect to the real magnetopause geometry.

4.1. Asymmetric Density

An asymmetric configuration, with different densities and an inflow plasma beta of 0.025, is examined in case B. The normalized entropy \( S/S_0 \) at \( t = 100 \) is presented in Figure 6 (left), where \( S_0 \) is the initial magnetosheath side entropy (\( x > 0 \)), which is \( 2^{\gamma} \approx 3.17 \) times lower than magnetospheric entropy, due to the different densities. However, on both sides of the outflow region, entropy increases about 4 times compared to its original value. The magnetic field lines (the black contours in Figure 6 (left)) indicate that the slow shock on the magnetosheath side is replaced by an intermediate shock [Yong and Lee, 1990]. For a better illustration of the outflow, Figure 6 (right) shows profiles of the thermal pressure, normalized entropy, magnetic field \( B_y \) component, current density \( j_z \) component, and velocity \( V_y \) component, which are taken...
from the magenta line in Figure 6 (left). The current density is higher on the magnetosheath side than on the magnetospheric side, which is consistent with the larger rotation of magnetic field on the magnetosheath side. This is caused by the larger inertia of the higher density magnetosheath material which requires a larger $\mathbf{j} \times \mathbf{B}$ force to change the momentum and accelerate the plasma into the outflow region [La Belle-Hamer et al., 1995]. This also generates a nonzero outflow magnetic field $B_y = B_y \neq 0$, which contributes to the total pressure in the outflow region. The outflow total pressure (labeled by the dashed line in the first panel of Figure 6 (right)) is larger than the thermal pressure. Compared with the result from the RH relations, the entropy increase in the asymmetric case is less than the corresponding symmetric case.

The rigorous manner to determine $B_y$ in the outflow region is to solve the Riemann problem [Lin and Lee, 1993, 1999]. However, a reasonable estimate of $B_y$ in the outflow region can be obtained by assuming an approximately constant outflow velocity [La Belle-Hamer et al., 1995], which is consistent with the fifth panel in Figure 6 (right). From the momentum equation and ignoring the pressure gradient along the $y$ direction, we have

$$\frac{dv_y}{d\tau} = j_z B_x. \quad (15)$$

By integrating both sides from the inflow region to outflow region

$$\Delta v_y = \int_{\text{inflow}}^{\text{outflow}} j_z \frac{B_x}{\rho} \, dx, \quad (16)$$

where $d\tau = dx/|v_x|$ and $v_x = V_x = B_y/\sqrt{\rho}$, which yields

$$\Delta v_y = \pm \int_{\text{inflow}}^{\text{outflow}} \frac{j_z}{\rho^{\frac{1}{2}}} \, dx = \pm \frac{B_o - B_i}{\sqrt{\rho}}. \quad (17)$$

The “±” is determined by whether $B_y$ is parallel or antiparallel to the shock normal direction. Here $B_i$ and $B_o$ are the tangential magnetic field components on the inflow and outflow sides at the outflow boundary. This equation ignores compressibility if the density is interpreted as the density in the respective inflow region. Alternatively, the density can be interpreted as suitable average density at the respective boundary, which, however, scales with the inflow density, inasmuch as the maximum compression is 2.5. We apply this equation to both sides of the boundary, which yields to

$$\frac{B_i - B_o}{\sqrt{\rho_1}} = \frac{B_o + B_i}{\sqrt{\rho_2}}$$

$$B_o = \frac{1 - r}{1 + r} B_i, \quad (18)$$

where $\rho_1$ and $\rho_2$ refers to the density of the two inflow regions (or a suitably averaged density if plasma compression is considered), and $r = \sqrt{\rho_1/\rho_2}$. Equation (18) shows that the magnitude of $B_o$ is always smaller than $B_i$, and $B_o$ vanishes, if the density is symmetric ($\rho_1 = \rho_2$). Interchanging $\rho_1$ and $\rho_2$ changes the sign of $B_o$. The simulation result in Figure 6 uses a density ratio of $r = \sqrt{2}$ for the inflow regions, and $B_o$ is indicated by red dashed line in the third panel in Figure 6 (right), which shows a good agreement with the simulation result. Combining equations (18) and (12), we obtain

$$\frac{S_o}{S_i} < 1 + \frac{1}{\rho^{\frac{1}{2}}} \frac{4r}{(1 + r)^2}, \quad (19)$$

where $S_i$ can be the entropy on either side. This relation (19) implies a reduction of the entropy increase in the presence of density asymmetry for the same inflow plasma beta.

4.2. Magnetic Shear

Magnetic reconnection without guide field is a singular situation in space plasma systems. A guide field component is present almost everywhere at the dayside magnetopause. In a two-dimensional configuration, the guide field component ($B_z$ component in this study) does not change the reconnection dynamics significantly, because it can be considered as an additional pressure and is convected by the plasma. This $B_z$ component often increases in the outflow region, which may cause the reduction of the entropy increase...
in the outflow region. Figure 7 shows the results from Case C, where the initial configuration is the same as in Case A, except for the addition of a uniform guide field $B_z = 1$. Figure 7 (left) is the normalized entropy $S/S_0$ at $t = 150$, where $S_0$ is the inflow entropy. It shows that the outflow entropy is about 1.5 times higher than the inflow entropy, which is lower than in Case A. To better understand this transition region, a rigorous treatment is to solve the Riemann problem [Lin and Lee, 1993, 1999]. Lin and Lee [1993] demonstrated that the switch-off shock is replaced by a rotational discontinuity and a slow shock for an asymmetric guide field, which is consistent with the observational results [Biernat et al., 1998] and is also seen in our two-dimensional simulation. The five panels in Figure 7 (right) show the profiles of the normalized entropy, plasma beta, two components of the tangential magnetic field $B_{t1}$ and $B_{t2}$, and the total tangential magnetic field, which are taken from the magenta line in Figure 7 (left). Due to the symmetry, we only show the $x \geq 0$ part of the cut. For a better representation of the rotational discontinuity, we rotate the frame by an angle $\theta = \arctan (1/B_y) / 2 = \pi/8$, thus

$$B_{t1} = B_z \cos \theta + B_y \sin \theta$$

$$B_{t2} = -B_z \sin \theta + B_y \cos \theta.$$ 

The rotational discontinuity layer is shaded in dark gray and labeled R, where $B_{t1}$ is almost constant and $B_{t2}$ changes from $+0.5$ to $-0.5$. Theoretically, the rotational discontinuity does not involve an entropy increase. However, the presence of the resistivity replaces the rotational discontinuity by an intermediate shock, thus leading to an increase in entropy and decrease in the total magnetic field, which is consistent with the results in Figure 7. The slow shock layer is shaded in light gray and labeled S, where the total tangential magnetic field $B_t = \sqrt{B_y^2 + B_z^2}$.
magnetic field strongly decreases. This remnant magnetic field, however, is still larger than the $y$ component of the magnetic field $B_y$ in the inflow region. The larger $B_y$ field implies a smaller increase of the thermal pressure in the outflow region, and for this reason, a reduction in the entropy increases. Therefore, Case C demonstrates that the presence of a guide field reduces the entropy increase.

4.3. Magnetic Reconnection With Shear Flow

Large shear flow always exists close to the magnetopause, due to the solar wind velocity. This sheared flow general has components along the $y$ and $z$ directions, and here we consider shear along the $z$ (invariant in the two-dimensional configuration) direction. Flow shear along the $y$ direction has an effect similar to density asymmetry in that it generates a configuration similar to density asymmetry with a nonzero $B_y$ in the outflow region [La Belle-Hamer et al., 1995]. For flow shear along the $z$ direction, the frozen-in condition implies a drag of reconnected magnetic field lines into opposite directions on the two sides of the outflow region, which generates a $B_z$ component. This $B_z$ component contributes an additional magnetic pressure to the total pressure in the steady outflow region. This mechanism is demonstrated in Case D, where the initial configuration is the same as in Case A, except for the perpendicular flow shear. In Case D, the total perpendicular velocity jump is equal to the Alfvén speed. Figure 8 (left) shows the normalized entropy $S/S_0$ at $t = 110$, where $S_0$ is the inflow entropy. The figure shows that the outflow entropy is about twice the inflow entropy, which is lower than in Case A. Similar to the guide field case, the presence of the shear flow replaces the switch-off shock by an intermediate shock and a slow shock [Sun et al., 2005]. The five panels in Figure 8 (right) show the profiles of the normalized entropy, plasma beta, two components of the tangential magnetic field $B_{t1}$ and $B_{t2}$, and the total tangential magnetic field, which are taken from the magenta line in Figure 8 (left).
intermediate shock layer and the slow shock layer. Again, the magnetic pressure in the outflow region reduces the thermal pressure and the entropy increase.

5. Summary and Discussion

The plasma entropy measure $\rho' / \rho$ is a useful physical quantity to identify adiabatic and nonadiabatic processes in a physical system, and satellite observations demonstrate that the access of solar wind plasma to the Earth's magnetosphere involves strong nonadiabatic heating. Magnetic reconnection is often suggested as the dominant process for this transport of plasma into the magnetosphere. In this study, we focus mainly on Petschek-type magnetic reconnection, and we demonstrate that entropy can strongly increase only when the plasma $\beta \ll 1$ by means of theoretical analysis and numerical simulation using one-dimensional and two-dimensional configurations. Key arguments for this $\beta$ limitation are (1) that the compression rate is usually around unity ($\rho_i / \rho_o = \mathcal{O}(1)$) and (2) that the thermal pressure is limited by the total pressure in an approximately pressure balanced system. A large entropy increase in such a total pressure balanced system requires a very small thermal pressure (compared to magnetic pressure) in the inflow region. It can be expected that the beta limitation for the entropy increase is applicable also to kinetic models of reconnection as long as such models satisfy approximate total pressure balance and plasma compression of order unity.

The more specific quantitative estimates of entropy changes are mostly based on applications of steady state reconnection. Frequently, reconnection is considered to be a time-dependent process (such as the formation of magnetic flux transfer events (FTEs) [Russell and Elphic, 1978, 1979] or plasmoids in the terrestrial magnetosphere [Hones et al., 1984; Moldwin and Hughes, 1992]). It is trivial that a steady state assumption is always violated on sufficiently large spatial or temporal scales. A steady state is defined by the condition that system changes are slow (time scale $\tau_L$) compared to typical wave travel times $\tau_V$ across the system. For a reconnection geometry, this implies $\tau_L \ll \tau_V$, with $\tau_V = d / V$ (d is the width of the outflow region and $V$ is the speed of group velocity of typical wave), and $\tau_r = r (dr/dt)^{-1}$ is characterized by the change of the reconnection rate $r$. For instance, the scale size of typical FTEs [Kawano and Russell, 1996] or typical plasmoids in the magnetotail [Ieda et al., 1998; Imber et al., 2011] requires several minutes of sustained fast reconnection. The Alfvén speed in these region is typically 100 to several 100 km s$^{-1}$, such that typical wave travel times are of the order of 10 s, i.e., much shorter than the required duration of fast reconnection. It is also noted that the asymmetric current layers for magnetopause reconnection and for large-scale solar wind reconnection events [Eriksson et al., 2009] appear reasonably consistent with basic predictions by MHD reconnection.

In two-dimensional MHD magnetic reconnection, the entropy increase occurs mostly through the shock layers. Such shocks do not exist or are modified in the presence of kinetic physics [Lin and Lee, 1993; Lin, 2001; Hesse et al., 2008; Ma and Otto, 2013; Zenitani et al., 2013], which may involve additional entropy sources/sinks. However, kinetic models are still governed by the fundamental conservation laws (i.e., mass, energy, and momentum conservation). Therefore, the asymptotic behavior on sufficiently large scales of the transition from the inflow to the outflow region should be somewhat consistent with the MHD shock prediction provided that the plasma remains sufficiently isotropic. This is an ultimately unresolved problem worthy of a more detailed large-scale kinetic study.

Other entropy sources include the small reconnection diffusion region which can heat plasma locally. However, it has no volume filling effect because of its microscopic (ion or electron inertia scale) size. A fast shock may exist in the transition region between steady outflow region and nonsteady bulge region, when the inflow plasma beta is low. However, the Mach number is too small (marginally above 1), such that the fast shock is not expected to be a major entropy source.

For more realistic configuration, density asymmetry leads to $B_y \neq 0$ in the outflow region which reduces the entropy increase. The presence of a guide field and a perpendicular shear flow also reduce the entropy increase in the outflow region. This reduction of nonadiabatic heating is caused the replacement of the switch-off shock in the steady outflow region by a rotational discontinuity (or intermediate shock) and a slow shock, which does not switch off the tangential component of the magnetic field for both configurations. Therefore, any asymmetry, guide field, or shear flow increases the magnetic field magnitude in the outflow region and reduces the thermal pressure increase from the inflow to the outflow regions, which results in a lower entropy increase than in the symmetric (Petschek) situation.
The presented results demonstrate that magnetic reconnection can indeed generate a strong nonadiabatic heating. However, the condition for this is a sufficiently small plasma \( \beta \ll 1 \). Typical magnetosheath conditions imply a plasma beta of order 0.1 to 1. This is insufficient to explain the observed entropy increase of at least 1 to 2 orders of magnitude between the magnetosheath and the magnetosphere. It is also noted that reconnection causes plasma compression where the original magnetosheath densities are already too high in comparison with plasma sheet. A subsequent adiabatic expansion would lower the temperature of the plasma far below values observed in the magnetosphere. Therefore, it is concluded that additional physics is required to explain density and temperature of the plasma sheet. Such physics can include the following: (1) plasma conditions that are not typical magnetosheath. For instance, density and plasma beta are typically significantly lower in the plasma depletion layer just outside of the magnetopause, (2) plasma conditions that are modified by additional processes such of Kelvin-Helmholtz instability, and (3) other processes that contribute to the plasma entry, such as diffusion or other microphysical processes [Johnson and Wing, 2009].

There are two additional highly important applications for low-beta reconnection. In the magnetotail lobes (similar in other magnetospheres such as the giant planets), the plasma beta is extraordinarily low. Here reconnection of lobe magnetic field can be expected to increase the entropy very significantly by orders of magnitude. Observations indicate that bursty earthward directed flows are likely entropy-depleted flux tubes, which are generated by lobe reconnection because of the low lobe plasma beta [Angelopoulos et al., 1992; Birn et al., 2011]. However, observations indicate that only few of these flows have a sufficiently low flux tube entropy to reach geosynchronous distances where particle injection is observed during substorms [Sergeev et al., 2012; Kim et al., 2012; Dubyagin et al., 2011]. A significant nonadiabatic heating in the outflow region is a critical element to better understand the penetration depth of such reconnection flows [Lyons et al., 2009]. The second application is reconnection in the solar corona where again the plasma beta is very low, and magnetic reconnection can potentially increase the plasma entropy by orders of magnitude.

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References